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## Communication and Agreement Abstractions in the Presence of Byzantine Processes

Achour Mostéfaouil\* Michel Raynal\*\* \*\*\*\*

**Abstract:** A Byzantine process is a process that –intentionally or not– behaves arbitrarily (Byzantine failures include crash and omission failures). Considering message-passing systems, this paper presents communication and agreement abstractions that allow non-faulty processes to correctly cooperate, despite the uncertainty created by the net effect of asynchrony and Byzantine failures. The world is distributed. Consequently more and more applications are distributed, and the “no Byzantine failure” assumption is no longer reasonable. Hence, due to both the development of clouds and security requirements, such abstractions are becoming more and more important.

The aim of this paper is to be a simple and homogeneous introduction to (a) communication and agreement abstractions, and (b) algorithms that implement these abstractions, in the context of asynchronous distributed message-passing systems where an a priori unknown subset of processes may exhibit Byzantine failures. To that end the paper presents existing abstractions and algorithms, and new ones. In this sense the paper has a mixed “pedagogical/survey/research” flavor.

**Key-words:** Abstraction level, Agreement, Asynchronous message-passing system, Broadcast abstraction, Byzantine process, Consensus, Fault-tolerance, Intrusion-tolerance, Message validation, Reliable broadcast, Signature-free algorithm.

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### *Communication et accord en présence de processus byzantins*

**Résumé :** *Cet article présente des abstractions de communication et d'accord en présence de processus byzantins.*

**Mots clés :** *Accord, Diffusion fiable, Niveau d'abstraction, Processus byzantin, Système asynchrone, Tolérance aux intrusions.*

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# 1 Introduction

**Distributed computing** Distributed computing occurs when one has to solve a problem in terms of physically distinct entities (usually called nodes, processors, processes, agents, sensors, etc.) such that each entity has only a partial knowledge of the many parameters involved in the problem. In the following, we use the term *process* to denote any computing entity. From an operational point of view this means that the processes of a distributed system need to exchange information, and agree in some way or another, in order to cooperate to a common goal. If processes do not cooperate, the system is no longer a distributed system. Hence, a distributed system has to provide the processes with communication and agreement abstractions.

Understanding and designing distributed applications is not an easy task [3, 18, 25, 26, 27]. This is due to the fact that, due its very nature, no process can capture instantaneously the global state of the application it is part of. This is due to the fact that, as processes are geographically localized at distinct places, distributed applications have to cope with the uncertainty created by asynchrony and failures. As a simple example, it is impossible to distinguish a crashed process from a very slow process in an asynchronous system prone to process crashes.

As in sequential computing, a simple approach to facilitate the design of distributed applications consists in designing appropriate abstractions. With such abstractions, the application designer can think about solutions to solve problems at a higher conceptual level than the basic send/receive communication level.

**Communication and agreement abstractions** One of the most important communication abstractions encountered in fault-tolerant distributed computing is *Reliable Broadcast* [3, 9, 10, 18, 25]. Roughly speaking, reliable broadcast allows processes to broadcast messages, in such a way that all the non-faulty processes eventually deliver the same set of messages, and this set includes *all* the messages they have broadcast plus a subset of messages broadcast by faulty processes.

*Consensus* is the most important agreement abstraction of fault-tolerant distributed computing [13]. Assuming each process proposes a value, it allows the non-faulty processes to agree on the same value, which has to satisfy some validity condition depending on both the proposed values and the failure model [18, 26].

**Byzantine failure** This failure type has first been introduced in the context of synchronous distributed systems [17, 23, 26], and then investigated in the context of asynchronous distributed systems [3, 18, 25]. A process has a *Byzantine* behavior when it arbitrarily deviates from its intended behavior; it then commits a Byzantine failure. Otherwise it is non-faulty (or non-Byzantine). This bad behavior can be intentional (malicious) or simply the result of a transient fault that altered the local state of a process, thereby modifying its behavior in an unpredictable way. Let us notice that process crashes (unexpected halting) define a strict subset of Byzantine failures.

**Content of the paper: Byzantine-tolerant broadcast abstractions** This paper presents communication and agreement abstractions suited to distributed systems made up of  $n$  processes, and where up to  $t$  processes may exhibit Byzantine failures.

As far as communication is concerned, three abstractions are presented. The first two abstractions, which have been proposed in [5, 30], ensure that a message broadcast by a non-faulty process is delivered by all the non-faulty processes. They differ in their requirement on the messages broadcast by faulty processes. More precisely, we have the following.

- No-duplicity broadcast (ND-broadcast). As far a message broadcast by a Byzantine process  $p$  is concerned, ND-broadcast ensures that no two non-faulty processes deliver different messages from process  $p$  [30]. Let us observe that this delivery rule allows a subset of non-faulty processes to deliver the same message  $m$  from the faulty process  $p$ , while other non-faulty processes do not deliver a message from  $p$ .
- Reliable broadcast (RB-broadcast). This abstraction is stronger than the previous one. As far a message broadcast by a Byzantine process  $p$  is concerned, the RB-broadcast abstraction ensures that all the non-faulty processes deliver the same message from  $p$ , or none of them delivers a message from  $p$  [5]. This is an “all-or-none” delivery rule.

Let us notice that, if a message is delivered from a faulty process  $p$ , there is no requirement on its content. It is only required that the same message content be delivered to all the non-faulty processes or none of them. This is due to the fact that, while a process is supposed to broadcast the same message to all, a Byzantine process can send distinct messages (i.e., with different contents) to different processes.

While the previous communication abstractions are one-to-all abstractions, the third abstraction (called VB-broadcast) is an all-to-all communication abstraction.

- Validated broadcast (VB-broadcast). Each process is assumed to broadcast a message. For a message to be delivered by a non-faulty process, its content needs to be *validated* [21]. A message content is validated by its sender as soon as it knows that at least one non-faulty process broadcast a message with the same content. As a process does not know if it is faulty or not (e.g., while it executes correctly its algorithm, a process may crash unexpectedly), a message content is validated by its sender as soon as it has received messages with the very same content from  $(t + 1)$  distinct processes (such a set contains at least one non-faulty process). If the message broadcast by a process cannot be validated, the default value  $\perp$  is delivered instead of it.

Each of these three broadcast abstractions requires  $n > 3t$ , and are consequently resilience-optimal (with respect to the maximal number of processes that can be faulty) [5, 25, 30]. Moreover, (as we will see) an algorithm implementing RB-broadcast can be obtained by using a relatively simple “echo” mechanism [5, 29, 30], and an algorithm implementing each of the  $n$  broadcasts of a VB-broadcast instance can be obtained from two instances the RB-broadcast abstraction.

**Content of the paper: Byzantine-tolerant agreement abstraction** The most important agreement abstraction encountered in distributed systems prone to process failures is consensus ([3, 8, 18, 27], to cite only books). It is well-known that consensus is impossible to solve in the basic asynchronous message-passing system model prone to even a single process crash failure [13]. This means that solving consensus despite both asynchrony and Byzantine processes requires to enrich the system with additional computational power. We consider here that this additional power is given by an underlying algorithm solving the binary consensus problem (consensus instance where the only values that can be proposed are values 0 and 1). Such an algorithm (in short BBC, for Binary Byzantine Consensus) is described in the paper (this algorithm assumes that the processes can access an oracle called *common coin*, which outputs random numbers [2, 24]).

The paper presents three multivalued Byzantine consensus algorithms (i.e., consensus algorithms where the set of values is not restricted to the values 0 and 1, and processes can be Byzantine). These algorithms, whose constructions are highly modular, are based on the previous broadcast abstractions and (as announced) a BBC algorithm. More precisely, The first two multivalued Byzantine consensus algorithms are obtained from a generic algorithm whose genericity parameter is a broadcast abstraction (namely, UB-broadcast –which captures unreliable broadcast, and RB-broadcast). The third algorithm is based on the VB-broadcast abstraction. Interestingly, all these algorithms are signature-free (no underlying cryptography mechanism is used).

These Byzantine consensus algorithms differ in their cost (number and size of messages they use), and their requirement on  $t$ , from a “lower bound on  $t$ ” (or resilience) point of view. The instance of the generic Byzantine consensus algorithm based on UB-broadcast requires  $n > 5t$ , while its instance based on RB-broadcast requires  $n > 4t$ . Finally, the Byzantine consensus algorithm based on VB-broadcast requires  $n > 3t$ , and is consequently resilience-optimal.

These Byzantine consensus algorithms have a noteworthy property, namely, if the Byzantine processes collude and propose the very same value  $v$ , while  $v$  is proposed by none of the non-faulty processes, then  $v$  cannot be decided. This property is called *intrusion-tolerance* in [21]. More generally, if the most proposed value is proposed by too few non-faulty processes, the default value  $\perp$  may be decided. To our knowledge, the only algorithm known so far that considers the intrusion-tolerance property is the one described in [11] (which requires messages to carry a vector of proposed values, which –as shown here– is not necessary). The proposed binary Byzantine consensus algorithm BBC is also signature-free, requires  $t < n/3$  (and is consequently optimal with respect to resilience), and its expected number rounds is four.

**Road map** The paper is made up of 9 sections. Section 2 presents the computation model. Then Section 3 presents the broadcast abstractions which have been previously sketched, and algorithms implementing them. Section 4 presents the intrusion-tolerant Byzantine consensus problem. Then, Sections 5 and 6 present a suite of intrusion-tolerant multivalued Byzantine consensus algorithms that differ mainly in the underlying broadcast abstraction they use. Section 7 discusses the previous algorithms, and Section 8 presents a new binary consensus algorithm based on random numbers and the VB-broadcast abstraction. Finally, Section 9 concludes the paper. Last but not least, presenting existing and new (a) abstractions and (b) algorithms implementing them, the paper is self-contained, which gives it an additional “introductory survey” flavor.

## 2 Computation Model

**Asynchronous processes** The system is made up of a finite set  $\Pi$  of  $n > 1$  asynchronous sequential processes, namely  $\Pi = \{p_1, \dots, p_n\}$ . “Asynchronous” means that each process proceeds at its own speed, which can vary arbitrarily with time, and remains always unknown to the other processes.

**Communication network** The processes communicate by exchanging messages through an asynchronous reliable point-to-point network. “Asynchronous” means that a message that has been sent is eventually received by its destination process, i.e., there is no bound on message transfer delays. “Reliable” means that the network does not lose, duplicate, modify, or create messages. “Point-to-point” means that there is a bi-directional communication channel between each pair of processes. Hence, when a process receives a message, it can identify its sender.

A process  $p_i$  sends a message to a process  $p_j$  by invoking the primitive “send TAG( $m$ ) to  $p_j$ ”, where TAG is the type of the message and  $m$  its content. To simplify the presentation, it is assumed that a process can send messages to itself. A process receives a message by executing the primitive “receive()”.

**Failure model** Up to  $t$  processes can exhibit a *Byzantine* behavior. A Byzantine process is a process that behaves arbitrarily: it can crash, fail to send or receive messages, send arbitrary messages, start in an arbitrary state, perform arbitrary state transitions, etc. Hence, a Byzantine process, which is assumed to send a message  $m$  to all the processes, can send a message  $m_1$  to some processes, a different message  $m_2$  to another subset of processes, and no message at all to the other processes. Moreover, Byzantine processes can collude to “pollute” the computation.

Let us notice that, as each pair of processes is connected by a channel, no Byzantine process can impersonate another process. Moreover, it is assumed that the Byzantine processes do not control the network.

**Terminology** A process that exhibits a Byzantine behavior is called *faulty*. Otherwise, it is *non-faulty*. Given an execution,  $\mathcal{C}$  denotes the set of processes that are non-faulty in that execution, and  $\mathcal{F}$  denotes the set of processes that are faulty.

**Multiset** Distributed algorithms presented in the paper use multisets. A *multiset* (also called *bag*) differs from a set in that it can contain several copies of the same value. Given a multiset  $rec_i$ , the operation  $\#equal(v, rec_i)$  denotes the number of occurrences of  $v$  in  $rec_i$ , while  $\#differ(v, rec_i)$  denotes the number of occurrences of values different from  $v$  in  $rec_i$ , namely,  $\#differ(v, rec_i) = |rec_i| - \#equal(v, rec_i)$ .

**Notation** This process model is denoted  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$ . In the following, this model is enriched with a constraint on  $t$  and a specific broadcast abstraction. As an example,  $\mathcal{BZ\_AS}_{n,t}[n > 5t, \text{RB}]$  is  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$  in which less than  $n/5$  processes are assumed to be faulty and processes communicate using the operations of the RB-broadcast abstraction.

**Lemma 1.** *Let  $n > 3t$ . We have*

- (a)  $n - t > \frac{n+t}{2}$ ,
- (b) *any set containing more than  $\frac{n+t}{2}$  distinct processes, contains at least  $(t + 1)$  non-faulty processes, and*
- (c) *the intersection of any two sets, each containing more than  $\frac{n+t}{2}$  distinct processes, contains at least one non-faulty process.*

**Proof** Proof of (a).  $n > 3t \Leftrightarrow 2n > n + 3t \Leftrightarrow 2n - 2t > n + t \Leftrightarrow n - t > \frac{n+t}{2}$ .

Proof of (b). We have  $\frac{n+t}{2} \geq \frac{4t+1}{2} = 2t + \frac{1}{2}$ , from which it follows that any set of more than  $\frac{n+t}{2}$  distinct processes contains at least  $2t + 1$  processes. The proof then follows from the fact that any set of  $2t + 1$  distinct processes contains at least  $t + 1$  non-faulty processes.

Proof of (c). Let  $\Pi_1$  and  $\Pi_2$  be two sets, each consisting of more than  $\lceil \frac{n+t}{2} \rceil$  distinct processes. It follows that  $|\Pi_1| + |\Pi_2| > n + t$ , thus  $|\Pi_1 \cup \Pi_2| + |\Pi_1 \cap \Pi_2| > n + t$ . Moreover,  $|\Pi_1 \cup \Pi_2| = n - |\Pi \setminus (\Pi_1 \cup \Pi_2)|$  and  $|\Pi_1 \cap \Pi_2| = |\mathcal{C} \cap \Pi_1 \cap \Pi_2| + |\mathcal{F} \cap \Pi_1 \cap \Pi_2|$ . Consequently:

$$n - |\Pi \setminus (\Pi_1 \cup \Pi_2)| + |\mathcal{C} \cap \Pi_1 \cap \Pi_2| + |\mathcal{F} \cap \Pi_1 \cap \Pi_2| > n + t,$$

$$\text{hence, } |\mathcal{C} \cap \Pi_1 \cap \Pi_2| > t + |\Pi \setminus (\Pi_1 \cup \Pi_2)| - |\mathcal{F} \cap \Pi_1 \cap \Pi_2|.$$

According to the definition of  $t$  we have:

$$|\mathcal{F}| = |\mathcal{F} \setminus (\Pi_1 \cap \Pi_2)| + |\mathcal{F} \cap \Pi_1 \cap \Pi_2| \leq t, \quad \text{i.e., } |\mathcal{F} \cap \Pi_1 \cap \Pi_2| \leq t - |\mathcal{F} \setminus (\Pi_1 \cap \Pi_2)|.$$

Hence,  $|\mathcal{C} \cap \Pi_1 \cap \Pi_2| > t + |\Pi \setminus (\Pi_1 \cup \Pi_2)| - (t - |\mathcal{F} \setminus (\Pi_1 \cap \Pi_2)|) \geq 0$ , i.e.,  $|\mathcal{C} \cap \Pi_1 \cap \Pi_2| > 0$ .  $\square_{\text{Lemma 1}}$

### 3 Broadcast Abstractions

This section defines the broadcast abstractions sketched in the Introduction, and presents algorithms implementing each of them. The first two, ND-broadcast and RB-broadcast, are from [30] and [5], respectively. The third one (VB-broadcast) has been introduced in [21].

All broadcast abstractions are implemented from the basic send/receive network primitives, which means that, while they provide us with distinct abstraction levels, they do not provide processes with additional computing power.

**Notation** When considering the broadcast abstraction XX (where XX stands for UB, ND, RB, or VB, see below), we say that a process “XX-broadcasts” or “XX-delivers” a message.

**Unreliable broadcast** The simple broadcast (UB-broadcast) is defined by a pair of operations denoted `UB_broadcast()` and `UB_deliver()`. `UB_broadcast TAG(m)` is used as a shortcut for

**for each**  $j \in \{1, \dots, n\}$  **send** `TAG(m)` **to**  $p_j$  **end for**,

and `UB_deliver()` is synonym with `receive()`. This means that a message UB-broadcast by a non-faulty process is UB-delivered at least by all the non-faulty processes. Differently, while it is assumed to send the same message to all the processes, a faulty process can actually send different messages to distinct processes and no message to others. Hence the name “unreliable broadcast” (sometimes also called “best effort broadcast”).

Trivially, an invocation of `UB_broadcast TAG(m)` costs one communication step and  $O(n)$  messages (more precisely,  $n - 1$  messages). The corresponding system model is denoted  $\mathcal{BZ}_{n,t}[\text{UB}]$ .

**Remark** When measuring the cost of a broadcast abstraction we do not take into account the size of the “data message” that is broadcast. This is because this size is independent of the way the broadcast is implemented. We only consider the size of the additional control information required by the corresponding broadcast implementation.

**The no-duplication property** The definition of each XX-broadcast abstraction includes the following no-duplication property: a non-faulty process  $p_i$  XX-delivers at most one message from any process  $p_j$ . This property states that the corresponding XX-broadcast abstraction is not allowed to create message duplicates. As this property follows trivially from the implementation of each broadcast abstraction, it is no longer mentioned in the following.

### 3.1 The no-duplicity broadcast abstraction

**No-duplicity broadcast** The ND-broadcast communication abstraction has been introduced by S. Toueg [30]. It is defined by the operations `ND_broadcast()` and `ND_deliver()`, which provide the processes with a higher abstraction level than UB-broadcast but do not add computational power ( $\mathcal{BZ\_AS}_{n,t}[\text{UB}]$  and  $\mathcal{BZ\_AS}_{n,t}[\text{ND}]$  have the same computational power, namely the same power as  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$ ).

Considering an instance of ND-broadcast where `ND_broadcast()` is invoked by a process  $p_i$ , this communication abstraction is defined by the following properties.

- ND-Validity. If a non-faulty process ND-delivers a message from  $p_i$ , then  $p_i$  invoked ND-broadcast.
- ND-no-duplicity. No two non-faulty processes ND-deliver distinct messages from  $p_i$ .
- ND-Termination. If the sender  $p_i$  is non-faulty, all the non-faulty processes eventually ND-deliver its message.

Let us observe that, if the sender  $p_i$  is faulty, it is possible that some non-faulty processes ND-deliver a message from  $p_i$  while others do not. The no-duplicity property prevents the non-faulty processes from ND-delivering different messages from a faulty sender.

**An algorithm implementing ND-broadcast** Assuming  $t < n/3$ , the algorithm presented in Figure 1 (from [30]) implements the ND-broadcast abstraction. It is shown in [30] that  $t < n/3$  is an upper bound on the model parameter  $t$  when one has to implement ND-broadcast in an asynchronous message-passing system prone to process Byzantine failures.

```

operation ND_broadcast MSG( $v_i$ ) is
(01) UB_broadcast INIT( $i, v_i$ ).

when INIT( $j, v$ ) is UB_delivered do
(02) if (first UB_delivery of INIT( $j, -$ )) then UB_broadcast ECHO( $i, v$ ) end if.

when ECHO( $j, v$ ) is UB_delivered do
(03) if (ECHO( $j, v$ ) UB_delivered from more than  $\frac{n+t}{2}$  different processes and MSG( $j, v$ ) not yet ND_delivered)
(04)   then ND_deliver MSG( $j, v$ )
(05) end if.

```

Figure 1: An algorithm implementing ND-broadcast ( $t < n/3$ ) [30]

The algorithm considers that a process is allowed to ND-broadcast only one message. Adding sequence numbers allows processes to ND-broadcast several messages. In that case, the process identity associated with each message has to be replaced by a pair made up of a sequence number and a process identity.

When a process  $p_i$  wants to ND-broadcast a message whose content is  $v_i$ , it UB-broadcasts the message `INIT( $i, v_i$ )` (line 01). When a process  $p_i$  receives (UB-delivers) a message `INIT( $j, -$ )` for the first time, it UB-broadcasts a message `ECHO( $j, v$ )` where  $v$  is the data content of the `INIT()` message (line 02). If the message `INIT( $j, v$ )` received is not the first message `INIT( $j, -$ )`,  $p_j$  is Byzantine and the message is discarded. Finally, when  $p_i$  has received the same message `ECHO( $j, v$ )` from more than  $(n + t)/2$  processes, it locally ND-delivers `MSG( $j, v$ )` (lines 03-04).

**Theorem 1.** *The algorithm described in Figure 1 implements the ND-broadcast abstraction in the system model  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \text{UB}]$ .*

**Proof** (See also [30].)

To prove the ND-termination property, let us consider a non-faulty process  $p_i$  that ND-broadcasts the message `MSG( $v_i$ )`. As  $p_i$  is non-faulty, the message `INIT( $i, v_i$ )` is received by all the non-faulty processes, which are at least  $n - t$ , and every non-faulty process UB-broadcasts `ECHO( $i, v_i$ )` (line 02). Hence, each non-faulty process UB-delivers  $n - t$  copies of `ECHO( $i, v_i$ )`. As  $n - t > \frac{n+t}{2}$  (Item (a) of Lemma 1), it follows that every non-faulty process eventually ND-delivers the message `MSG( $i, v_i$ )` (lines 03-04).

To prove the ND-no-duplicity property, let us assume by contradiction that two non-faulty processes  $p_i$  and  $p_j$  ND-deliver different messages  $m_1$  and  $m_2$  from some process  $p_k$  (i.e.,  $m_1 = \text{MSG}(k, v)$  and  $m_2 = \text{MSG}(k, w)$ , with

$v \neq w$ ). It follows from the predicate of line 03, that  $p_i$  received  $\text{ECHO}(k, v)$  from a set of more than  $\frac{n+t}{2}$  distinct processes, and  $p_j$  received  $\text{ECHO}(k, w)$  from a set of more than  $\frac{n+t}{2}$  distinct processes. Moreover, it follows from Item (c) of Lemma 1 that the intersection of these two sets contains a non-faulty process. But, as it is non-faulty, this process has sent the same message  $\text{ECHO}()$  to  $p_i$  and  $p_j$  (line 02). Hence,  $m_1 = m_2$ , which contradicts the initial assumption.

The ND-validity follows from the fact that, to be ND-delivered, a message from  $p_j$  has first to be UB-delivered, which –as the network does not create messages– implies that it has been sent.  $\square_{\text{Theorem 1}}$

It is easy to see that this implementation uses two consecutive communication steps and  $O(n^2)$  underlying messages ( $n - 1$  in the first communication step, and  $n(n - 1)$  in the second one). Moreover, the size of the control information added to a message is  $\log_2 n$  (sender identity).

**Remark** Let us notice that replacing at line 04 “more than  $\frac{n+t}{2}$  different processes” by “ $(n - t)$  different processes” leaves the algorithm correct. As  $n - t > \frac{n+t}{2}$  (Item (a) of Lemma 1), it follows that using “more than  $\frac{n+t}{2}$  different processes” provides a weaker ND-delivery condition, and consequently a more efficient algorithm from message ND-delivery point of view. As a simple numerical example, considering  $n = 21$  and  $t = 2$ , we have  $n - t = 19$ , which is much greater than the required value  $12 (> \frac{n+t}{2} = 11.5)$ .

### 3.2 The reliable broadcast abstraction

**Reliable broadcast** The RB-broadcast abstraction has been proposed by G. Bracha [5]. It is proved in [6] that  $t < n/3$  is an upper bound on  $t$  when one has to implement such an abstraction. RB-broadcast provides the processes with the operations  $\text{RB\_broadcast}()$  and  $\text{RB\_deliver}()$  defined by the following properties.

- RB-Validity. If a non-faulty process RB-delivers a message from  $p_x$ , then  $p_x$  invoked the operation  $\text{RB\_broadcast}()$ .
- RB-Uniformity. If a non-faulty process RB-delivers a message from  $p_i$  (possibly faulty) then all the non-faulty processes eventually RB-deliver the same message from  $p_i$ .
- RB-Termination. If the sender is non-faulty, all the non-faulty processes eventually RB-deliver its message.

Let us observe that, from an abstraction level point of view, the RB-uniformity property is strictly stronger than the ND-no-duplicity property: not only two non-faulty processes cannot RB-deliver different messages from a given process, but it is no longer possible that one of them RB-delivers a message while the other does not. From a computational point of view,  $\mathcal{BZ\_AS}_{n,t}[\text{RB}]$  and  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$  have the same power.

**An algorithm implementing RB-broadcast** The algorithm presented in Figure 2, which assumes  $t < n/3$ , implements RB-broadcast. It is a simple variant of an algorithm proposed in [5]. It is presented here in an incremental way from the previous ND-broadcast algorithm.

While the ND-broadcast algorithm of Figure 1 requires two sequential communications steps (message  $\text{INIT}()$  followed by messages  $\text{ECHO}()$ ), the implementation of RB-broadcast requires three consecutive communications steps: message  $\text{INIT}()$ , followed by messages  $\text{ECHO}()$ , followed by messages  $\text{READY}()$ .

The first five lines of the algorithm are similar to the corresponding lines of the ND-broadcast algorithm. The only difference lies in the lines 03-04, where the ND-delivery is replaced by the UR-broadcast of the message  $\text{READY}(j, v)$ .

The aim of the last step of the algorithm (lines 06-11) is to ensure that all or none of the non-faulty processes RB-deliver the message  $\text{MSG}(j, v)$  from  $p_j$ . To that end, the RB-delivery predicate requires that  $p_i$  UB-delivers  $(2t + 1)$  copies of  $\text{READY}(j, v)$ , which means at least  $(t + 1)$  copies from non-faulty processes (line 09).

**Theorem 2.** *The algorithm described in Figure 2 implements the RB-broadcast abstraction in the system model  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \text{UB}]$ .*

**Proof** (See also [5].)

Claim 1. If two non-faulty processes UB-broadcast the messages  $\text{READY}(j, v)$  and  $\text{READY}(j, w)$ , respectively, we have  $v = w$ .



```

operation RB_broadcast MSG( $v_i$ ) is
(01) UB_broadcast INIT( $i, v_i$ ).

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when ECHO( $j, v$ ) is UB_delivered do
(03) if (ECHO( $j, v$ ) UB_delivered from more than  $\frac{n+t}{2}$  different processes and READY( $j, v$ ) not yet UB_broadcast)
(04) then UB_broadcast READY( $j, v$ )
(05) end if.

when READY( $j, v$ ) is UB_delivered do
(06) if (READY( $j, v$ ) UB_delivered from ( $t + 1$ ) different processes and READY( $j, v$ ) not yet UB_broadcast)
(07) then UB_broadcast READY( $j, v$ )
(08) end if;
(09) if (READY( $j, v$ ) UB_delivered from ( $2t + 1$ ) different processes and MSG( $j, v$ ) not yet UB_delivered)
(10) then RB_deliver MSG( $j, v$ )
(11) end if.

```

Figure 2: An algorithm implementing RB-broadcast ( $t < n/3$ ) [5]

Proof of the claim. Let  $p_i$  and  $p_k$  be two non-faulty processes that UB-broadcast at line 04 the messages READY( $j, v$ ) and READY( $j, w$ ), respectively. The claim follows then from the observation that  $p_i$  and  $p_j$  execute lines 01-05, which implement the ND-broadcast where the UB-broadcast of READY( $j, -$ ) replaces the NB-delivery of MSG( $j, -$ ). Consequently, as no two different messages MSG( $j, -$ ) can be ND-delivered in the ND-broadcast algorithm, it follows that no two different messages READY( $j, v$ ) and READY( $j, w$ ) can be UB-broadcast by the non-faulty processes  $p_i$  and  $p_k$ . Let us finally observe that a non-faulty process that UB-broadcasts READY( $j, u$ ) at line 07 has necessarily UB-delivered a message READY( $j, -$ ) whose UB-broadcast originated at line 04, i.e., we necessarily have  $u = v$  such that READY( $j, v$ ) was UB-broadcast at line 04. End of the proof of the claim.

Claim 2. If two non-faulty processes RB-deliver MSG( $j, v$ ) and MSG( $j, w$ ), respectively, then  $v = w$ .

Proof of the claim. If a process RB-delivers MSG( $j, v$ ), it has RB-delivered READY( $j, v$ ) from ( $2t + 1$ ) processes, hence from at least one non-faulty process. Similarly, if a process RB-delivers MSG( $j, w$ ), it has RB-delivered READY( $j, w$ ) from at least one non-faulty process. It follows from Claim 1 that the non-faulty processes UB-broadcast the same message READY( $j, -$ ), from which we conclude that  $v = w$ . End of the proof of the claim.

Claim 3. If a non-faulty process RB-delivers MSG( $j, v$ ), then any non-faulty process RB-delivers MSG( $j, v$ ).

Proof of the claim. If a non-faulty process RB-delivers MSG( $j, v$ ), it has received the message READY( $j, v$ ) from ( $t + 1$ ) non-faulty processes. It follows that every non-faulty process receives at least ( $t + 1$ ) copies of READY( $j, v$ ) and consequently every non-faulty process UB-broadcasts READY( $j, v$ ) at the latest at line 07 (if not previously done at line 04). As there are at least  $n - t \geq 2t + 1$  non-faulty processes, each non-faulty process eventually receives at least  $2t + 1$  copies of READY( $j, v$ ) and RB-delivers MSG( $j, v$ ) (lines 09-11). End of the proof of the claim.

Claim 4. If a non-faulty process  $p_i$  RB-broadcasts MSG( $v$ ), then all the non-faulty process RB-deliver MSG( $i, v$ ).

Proof of the claim. If a non-faulty process  $p_i$  RB-broadcasts MSG( $v$ ), every non-faulty process receives INIT( $i, v$ ), UB-broadcasts ECHO( $i, v$ ), and, as  $n - t > \frac{n+t}{2}$ , UB-broadcasts READY( $i, v$ ) (let us notice that, as  $t < \frac{n+t}{2}$ , even if they collude and UB-broadcast the same message READY( $i, w$ ) where  $w \neq v$ , the faulty processes cannot prevent non-faulty processes from UB-broadcasting READY( $i, v$ )). Finally, as  $n - t \geq 2t + 1$ , all the non-faulty processes RB-deliver MSG( $i, v$ ). End of the proof of the claim.

RB-termination follows from claim 4, while RB-uniformity follows from claim 2. RB-validity is as in Theorem 1.

□*Theorem 2*

As we have seen, this algorithm uses three consecutive communication steps and  $O(n^2)$  underlying messages ( $n - 1$  in the first communication step, and  $n(n - 1)$  in the second and third steps). Moreover, the size of the control information added to a message is  $\log_2 n$  (sender identity).

**Improvement** This algorithm can be improved to be more efficient with respect to asynchrony and message UB-delivery order (i.e., to favor early RB-delivery). More precisely, we have the following.

- When a message  $\text{ECHO}(j, v)$  is UB-delivered, the following statement is added before line 03:
 

```

if ( $\text{ECHO}(j, v)$  UB_delivered from more than  $\frac{n+t}{2}$  different processes and  $\text{ECHO}(j, v)$  not yet UB_broadcast)
  then UB_broadcast  $\text{ECHO}(i, v)$ 
end if.

```
- When  $\text{READY}(j, v)$  is UB-delivered, the following statement is added before line 06:
 

```

if ( $\text{READY}(j, v)$  UB_delivered from  $(t + 1)$  different processes and  $\text{ECHO}(j, v)$  not yet UB_broadcast)
  then UB_broadcast  $\text{ECHO}(i, v)$ 
end if.

```

The fact that these two “**if ... end if**” statements leave the algorithm correct follows from the following observations. If the predicate of the first additional statement is true, it follows from Item (b) of Lemma 1 that at least  $(t + 1)$  copies of the message  $\text{ECHO}(j, v)$  come from non-faulty processes (directly or forwarded through a path of non-faulty-processes). Moreover, each of these processes necessarily UB-broadcast  $\text{ECHO}(j, v)$  at line 02, or in the additional statement (after having UB-delivered the message  $\text{ECHO}(j, v)$  from more than  $\frac{n+t}{2}$  different processes).

Similarly, if the predicate of the second additional statement is true, it follows that at least one copy of the message  $\text{READY}(j, v)$  comes from a non-faulty process (directly or forwarded through a path of non-faulty-processes). Moreover, each of these messages was necessarily UB-broadcast at line 04. Hence, there is a process that UB-delivered  $\text{ECHO}(j, v)$  from more than  $\frac{n+t}{2}$  processes, i.e., due to Item (b) of Lemma 1, from at least  $(t + 1)$  non-faulty processes.

### 3.3 The validated broadcast abstraction

**Validated broadcast** The VB-broadcast communication abstraction has been introduced in [21]. It is an *all-to-all* communication abstraction designed to be used in the implementation of distributed agreement abstractions. VB-broadcast integrates a notion of message *validation*, namely, assuming that each non-faulty process VB-broadcasts a message, it requires that, for a message to be VB-delivered, its content  $v$  be validated; otherwise the default value  $\perp$  is VB-delivered instead of it. For a message with content  $v$  to be valid, a message with the same content  $v$  has to be VB-broadcast by at least one non-faulty process. As no process knows if it is itself faulty or non-faulty (e.g., if a process executes correctly its algorithm and then unexpectedly crashes, it is faulty), for a message  $m$  to be valid in the presence of up to  $t$  faulty processes, messages with the same content need to be VB-broadcast by “enough” processes, where “enough” means “at least  $(t + 1)$ ”. As already indicated, if a message is not validated, the default value  $\perp$  is delivered instead of it.

VB-broadcast provides the processes with two operations denoted  $\text{VB\_broadcast}()$  and  $\text{VB\_deliver}()$ . In a VB-broadcast instance each non-faulty process invokes  $\text{VB\_broadcast}()$  once, and VB-delivers at least  $(n - t)$  messages, one from each non-faulty process and at most one from each faulty process. The content of a message that is VB-delivered can be a value that has been VB-broadcast or the default value  $\perp$ . VB-broadcast is defined by the following properties.

- VB-Validity. As previously, this property relates the outputs (VB-delivered messages) to the inputs (VB-broadcast messages). It is made up of two sub-properties.
  - VB-Justification. Let  $p_i$  be a non-faulty process that VB-delivers a message  $m$  as the value VB-broadcast by some (faulty or non-faulty) process. If  $m \neq \perp$ , there is at least one non-faulty process that invoked  $\text{VB\_broadcast MSG}(m)$ .
  - VB-Obligation. If all the non-faulty processes VB\_broadcast the same value  $v$ , each non-faulty process VB-delivers  $m = v$  from each non-faulty process.

- VB-Uniformity. If a non-faulty process VB-delivers a message from  $p_i$  (possibly faulty), all the non-faulty processes eventually VB-deliver the same message from  $p_i$  (which can be a validated non- $\perp$  value or the default value  $\perp$ ).
- VB-Termination. If  $p_i$  is non-faulty and VB-broadcast  $m$ , all the non-faulty processes eventually VB-deliver the same message  $m'$ , where  $m'$  is  $m$  or  $\perp$ .

### 3.4 An algorithm implementing VB-broadcast

Assuming  $t < n/3$ , the algorithm presented in Figure 3 implements the all-to-all VB-broadcast abstraction. Let us recall that all-to-all means here that all the non-faulty processes are assumed to invoke `VB_broadcast()`. This means that a process VB-delivers at least  $n - t$  messages. This implementation uses consecutively two RB-broadcast abstractions. It is made up of two parts.

```

operation VB_broadcast( $v_i$ )
(01)  RB_broadcast INIT( $i, v_i$ );
(02)  let  $rec_i$  = multiset of values RB_delivered to  $p_i$ ;
(03)  wait until ( $|rec_i| \geq n - t$ );
(04)  if ( $\#equal(v_i, rec_i) \geq n - 2t$ ) then  $aux_i \leftarrow$  "yes" else  $aux_i \leftarrow$  "no" end if;
(05)  RB_broadcast VALID( $i, aux_i$ ).

for  $1 \leq j \leq n$  VB-delivery background task  $T_i[j]$ :
(06)  wait until (VALID( $j, x$ ) and INIT( $j, v$ ) are RB_delivered from  $p_j$ );
(07)  if ( $x =$  "yes") then wait ( $\#equal(v, rec_i) \geq n - 2t$ );  $d \leftarrow v$ 
(08)           else wait ( $\#differ(v, rec_i) \geq t + 1$ );  $d \leftarrow \perp$ 
(09)  end if;
(10)  VB_deliver( $d$ ) at  $p_i$  as the value VB-broadcast by  $p_j$ .

```

Figure 3: VB-broadcast on top of reliable broadcast ( $t < n/3$ , code of  $p_i$ )

- In the first part, a process  $p_i$  first invokes `RB_broadcast INIT( $i, v_i$ )` and waits until it has RB-delivered messages from at least  $n - t$  processes (lines 01-03). The values RB-delivered are deposited in a multiset denoted  $rec_i$ . Then, if value  $v_i$  has been RB-delivered from at least  $n - 2t \geq t + 1$  processes (which means that it was RB-broadcast by at least one non-faulty process),  $p_i$  validates it by assigning "yes" to  $aux_i$ . Otherwise  $p_i$  sets  $aux_i$  to "no" at line 04 (in this case,  $v_i$  is not validated). Then,  $p_i$  issues a second RB-broadcast (line 05) to disseminate  $aux_i$  to all processes.
- The second part is made up of  $n$  tasks, which execute in the background. The task  $T_i[j]$  is associated with the VB-delivery of the message from  $p_j$ . It starts by the wait statement for both the value  $v$  RB-broadcast by  $p_j$  and the value  $x$  RB-broadcast also by  $p_j$  ( $x$  indicates the validation status attached to  $v$  by its sender  $p_j$ ). Let us remember that each time a message `INIT( $-, w$ )` is RB-delivered to  $p_i$ , the corresponding value  $w$  is added to  $rec_i$ , which means that, after the predicate  $|rec_i| \geq n - t$  has become true at line 03, the set  $rec_i$  still keeps on being updated when new messages `INIT()` are RB-delivered to  $p_i$ .
  - If  $x =$  "yes", as  $p_j$  can be Byzantine,  $v$  has not necessarily been validated by a non-faulty process. Hence,  $p_i$  has to check it. To that end,  $p_i$  waits until the predicate  $\#equal(v, rec_i) \geq n - 2t$  becomes true (line 07). When this predicate becomes true (if ever it does), it follows from  $n - 2t \geq t + 1$  that  $\#equal(v, rec_i) \geq t + 1$ . If this occurs,  $v$  is VB-delivered to  $p_i$  as being the value VB-broadcast by  $p_j$ .
  - Similarly, if  $x =$  "no",  $p_i$  waits until  $rec_i$  contains more than  $t$  occurrences of values different from  $v$  (the value RB-delivered from  $p_j$ ), which means that at least one non-faulty process did not validate  $v$ . When this occurs (if ever it does, line 08),  $p_i$  VB-delivers  $\perp$  as the value VB-broadcast by  $p_j$ .

It is possible that the waiting predicate used at line 07 or line 08 never becomes satisfied. When this occurs, the corresponding sender process  $p_j$  is necessarily a faulty process. The waiting condition becomes always satisfied when  $p_j$  is a non-faulty process, and can become satisfied for some faulty senders  $p_j$ .

As two instances of RB-broadcast are used, the algorithm requires  $2 \times 3 = 6$  communication steps, and as VB-broadcast is an all-to-all abstraction, the algorithm uses  $n \times O(n^2)$  messages.

**Theorem 3.** *The algorithm described in Figure 3 implements the validated broadcast abstraction in the system model  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \text{RB}]$ .*

**Proof** Proof of the VB-Termination property. This property states that, if a process  $p_i$  is non-faulty and VB-broadcast  $m$ , then all the non-faulty processes eventually VB-deliver the same message  $m'$  from  $p_i$ , where  $m'$  is  $m$  or  $\perp$ .

As there are at least  $n - t$  non-faulty processes, and each non-faulty process VB-broadcasts a value, we eventually have  $|rec_j| \geq n - t$  at every non-faulty process  $p_j$ . Hence, no non-faulty process blocks forever at line 03. It consequently RB-broadcasts a message  $\text{VALID}()$  at line 05. We now consider two cases.

- The non-faulty process  $p_i$  RB-broadcasts  $\text{VALID}(i, \text{"yes"})$ . It follows from line 07 that (a)  $d = v_i$  (the value VB-broadcast by  $p_i$ ), and (b)  $rec_i$  contains at least  $n - 2t$  copies of  $v = v_i$ , i.e.,  $p_i$  has RB-delivered messages  $\text{INIT}(-, v)$  from  $n - 2t$  different processes. Due to the RB-Uniformity of RB-broadcast, each non-faulty process  $p_j$  eventually RB-delivers both these  $n - 2t$  messages  $\text{INIT}(-, v)$ , and the message  $\text{VALID}(i, \text{"yes"})$  from  $p_i$ . It follows that  $p_j$  eventually VB-delivers  $v = v_i$  at line 07.
- The non-faulty process  $p_i$  RB-broadcasts  $\text{VALID}(i, \text{"no"})$ . It follows from the termination property of RB-broadcast that each non-faulty process  $p_j$  RB-delivers  $\text{VALID}(i, \text{"no"})$  from  $p_i$ . Moreover, it follows from the test line 04 that, if  $p_i$  RB-broadcast  $\text{VALID}(i, \text{"no"})$ , that, among the  $n - t$  values in  $rec_i$ , less than  $n - 2t$  values are equal to  $v_i$ , i.e. more than  $t$  values are different from  $v_i$ . Hence due to the RB-Uniformity property of RB-broadcast, every non-faulty process  $p_j$  eventually RB-delivers at least  $t + 1$  values different from  $v_i$ , and consequently VB-delivers  $\perp$  at line 08.

Proof of the VB-Uniformity property. This property states that, if a non-faulty process  $p_i$  VB-delivers a message from  $p_j$  –possibly faulty–, then all the non-faulty processes eventually VB-deliver the same message from  $p_j$ .

Let  $p_i$  be a non-faulty process that VB-delivers a value  $d$  from  $p_j$ . This means that  $p_i$  has previously RB-delivered a message  $\text{INIT}(j, v)$  and a message  $\text{VALID}(j, x)$  from  $p_j$  at the latest in its delivery task  $T_i[j]$ . The proof of this property is very similar to the previous one.

It follows that  $p_i$  has RB-delivered (1) a message  $\text{VALID}(j, x)$  and a message  $\text{INIT}(j, v)$  from  $p_j$ , and (2) a multiset  $rec_i$  of values that satisfies some property (depending on the value of  $x$ ). As  $p_i$  is non-faulty, it follows from the RB-Uniformity property of RB-broadcast, that every non-faulty process  $p_k$  eventually RB-delivers (1) both  $\text{VALID}(j, x)$  and  $\text{INIT}(j, v)$ , and (2) a multiset  $rec_k$  of values such that eventually  $rec_k = rec_i$ . As the value  $x$  RB-delivered to  $p_i$  and  $p_k$  is the same, it follows from the waiting condition (used at line 07 or line 08, according to the value of  $x$ ) that  $p_k$  eventually VB-delivers at line 10 the same value  $d$  as  $p_i$ .

Proof of the VB-Obligation property. This property states that if all the non-faulty process VB-broadcast the same value  $v$ , each of them VB-delivers  $v$  as the value VB-broadcast by each of them.

As each non-faulty process  $p_j$  VB-broadcasts the value  $v$ , it follows that it RB-broadcasts  $\text{INIT}(j, v)$  (line 05). Consequently this value  $v$  eventually appears at least  $(n - 2t)$  times in the multiset  $rec_i$  of every non-faulty process  $p_i$ . Hence, each non-faulty process  $p_i$  VB-broadcasts the message  $\text{VALID}(i, \text{"yes"})$  and (from the RB-termination property) each non-faulty process  $p_k$  RB-delivers the message  $\text{VALID}(i, \text{"yes"})$ . Consequently, each non-faulty process  $p_k$  executes the task  $T_k[i]$  with respect to each non-faulty process  $p_i$  (and possibly also with respect to faulty processes). The waiting predicate of line 07 is then eventually satisfied at  $p_k$ , and this is true for value  $v$  only. When this occurs, each non-faulty process  $p_k$  VB-delivers  $v$  as the value VB-broadcast by the non-faulty process  $p_i$ .

Proof of the VB-Justification property. This property states that, if the VB-delivered value  $m$  is such that  $m \neq \perp$ , there is at least one non-faulty process that invoked  $\text{VB\_broadcast MSG}(m)$ .

If  $m \neq \perp$  is VB-delivered by a non-faulty process  $p_i$  as the value VB-broadcast by  $p_j$ , this value appears at least  $(n - 2t)$  times in  $rec_i$  (waiting predicate of line 07). As  $n - 2t \geq t + 1$ , it follows that at least one non-faulty process has VB-broadcast  $m$ . □<sub>Theorem 3</sub>

### 3.5 Comparing the broadcast abstractions

Table 1 compares the costs of the three previous broadcast abstractions. Considering one broadcast instance, the second column indicates the broadcast type (1-to- $n$  or  $n$ -to- $n$ ). The third column indicates the number of (sequential) communication steps required by the corresponding algorithms. The fourth column presents the size of the additional control information that each message has to carry (the  $\log_2 n$  comes from the fact that the identity of the process that broadcasts a message has to be sent together with it when forwarded by another process). The fifth column indicates the number of implementation messages used by the corresponding algorithms. Finally, the last column states the constraint on  $t$  required to implement the corresponding abstraction in  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$ .

broadcast	$x$ -to- $y$ type	# comm. steps	message size	# msgs	constraint on $t$
UB	1-to- $n$	1	constant	$n - 1$	$n > t$
ND	1-to- $n$	2	$\log_2 n$	$O(n^2)$	$n > 3t$
RB	1-to- $n$	3	$\log_2 n$	$O(n^2)$	$n > 3t$
VB	$n$ -to- $n$	6	$\log_2 n$	$n \times O(n^2)$	$n > 3t$

Table 1: Cost and constraint on the different broadcast abstractions

## 4 Intrusion-Tolerant Byzantine Consensus and Underlying Enriched Model

### 4.1 Byzantine consensus

**Byzantine consensus** The consensus problem has been informally stated in the Introduction. Assuming that at least each non-faulty process proposes a value, each of them has to decide on a value in such a way that the following properties are satisfied.

- C-Termination. Every non-faulty process eventually decides on a value.
- C-Agreement. No two non-faulty processes decide on different values.
- C-Obligation (validity). If all the non-faulty processes propose the same value  $v$ , then  $v$  is decided.

**Intrusion-tolerant Byzantine (ITB) consensus** In Byzantine consensus, if not all the non-faulty processes propose the same value, any value can be decided. As indicated in the Introduction, we are interested here in a more constrained version of the consensus problem in which a value proposed only by faulty processes cannot be decided. This consensus problem instance is defined by the C-Termination, C-Agreement and C-Obligation properties stated above plus the following C-Non-intrusion property (which is a validity property).

- C-Non-intrusion (validity). A decided value is a value proposed by a non-faulty process or  $\perp$ .

The fact that no value proposed only by faulty processes can be decided gives its name (namely *intrusion-tolerant*) to that consensus problem instance.

**Binary consensus** The consensus is *binary* when only two values (e.g., 0 and 1) can be proposed. When more than two values can be proposed, consensus is *multivalued*.

Interestingly, the fact that only *two* values can be proposed to a binary Byzantine consensus, combined with the C-obligation property, provides the binary consensus problem with the following interesting property (which is no longer true for multivalued consensus).

**Property 1.** *The binary Byzantine consensus problem is such that: C-obligation  $\Rightarrow$  C-non-intrusion  $\wedge$  ( $\perp$  is never decided).*

## 4.2 Enriched model for multivalued ITB consensus

**Additional power is required** It is well-known that Byzantine consensus cannot be solved when  $t \leq n/3$  in synchronous systems [17, 23]. Moreover, consensus cannot be solved in asynchronous systems as soon as even only one process may crash [13], which means that Byzantine consensus cannot be solved either as soon as one process can be faulty. Said another way, additional computational power is needed if one wants to solve Byzantine consensus in an asynchronous system.

Such an additional power can be obtained by randomization (e.g., [4, 11, 15, 24, 30]), failure detectors (e.g., [14, 16, 20]), additional synchrony assumptions (e.g., [12, 19]), or even the assumption that there is a binary consensus algorithm that is given for free by the underlying system (e.g., [7, 11, 22, 28, 31]).

**Enriched model for multivalued ITB consensus** In the following, BBC denotes any algorithm that solves the Byzantine binary consensus problem. (Such algorithms are described in [5, 11, 15, 30]. A novel BBC algorithm based on the VB-broadcast abstraction is presented in Section 8). Let  $\mathcal{BZ\_AS}_{n,t}[\text{XX}, \text{BBC}]$  denote the system model  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$  enriched with BBC (which adds computational power) and the broadcast abstraction XX (which provides a higher abstraction level than send/receive).

As announced in the Introduction, the aim is to design a generic multivalued ITB consensus algorithm on top of  $\mathcal{BZ\_AS}_{n,t}[\text{XX}, \text{BBC}]$ .

## 5 Generic Consensus Based on the UB or ND-Broadcast Abstractions

This section presents a generic multivalued ITB consensus algorithm that can be instantiated with UB-broadcast or ND-broadcast. It uses two rounds for each process to compute a value it proposes to the underlying binary consensus. The instantiation based on UB-broadcast requires  $n > 5t$ , while the one based on ND-broadcast requires  $n > 4t$ .

### 5.1 Principles and description of the algorithm

The generic algorithm is presented in Figure 4. A process invokes `propose( $v_i$ )` where  $v_i$  is the value it proposes to the consensus. It terminates when it executes the statement `return()` (line 14), which supplies it with the decided value. (In order to prevent confusion, the operation of the underlying binary consensus is denoted `bin_propose()`.)

```

operation propose( $v_i$ )
(01)  XX_broadcast EST1( $v_i$ );
(02)  wait until (EST1(-) messages XX_delivered from ( $n - t$ ) processes);
(03)  let  $rec1_i$  = multiset of values XX_delivered and carried by EST1 messages;
(04)  if ( $\exists v : \#equal(v, rec1_i) \geq n - 2t$ ) then  $aux_i \leftarrow v$  else  $aux_i \leftarrow \perp$  end if;
(05)  XX_broadcast EST2( $aux_i$ );
(06)  wait until (EST2(-) messages XX_delivered from ( $n - t$ ) processes);
(07)  let  $rec2_i$  = multiset of values XX_delivered and carried by EST2 messages;
(08)  if ( $\exists v \neq \perp : \#equal(v, rec2_i) \geq n - 2t$ ) then  $bp_i \leftarrow 1$  else  $bp_i \leftarrow 0$  end if;
(09)  if ( $\exists v \neq \perp : v \in rec2_i$ ) then let  $v$  = most frequent non- $\perp$  value in  $rec2_i$ ;
(10)   $res_i \leftarrow v$ 
(11)  else  $res_i \leftarrow \perp$ 
(12)  end if;
(13)   $b\_dec_i \leftarrow \text{bin\_propose}(bp_i)$ ;    % underlying BBC algorithm %
(14)  if ( $b\_dec_i = 1$ ) then return( $res_i$ ) else return( $\perp$ ) end if.

```

Figure 4: Generic algorithm for intrusion-tolerant Byzantine multivalued consensus algorithm

In order to reduce the Byzantine consensus problem to its binary counterpart to benefit from BBC, the processes first exchange the values they propose. If a process sees that a value  $v$  has been proposed “enough” times, it proposes 1 to BBC, otherwise it proposes 0. Then, if 1 is decided from BBC, the non-faulty processes decide the value  $v$  that has been proposed “enough” times, otherwise they decide  $\perp$  (lines 09-14). For this to work, If a process  $p_i$  proposes 1 to

the underlying BBC algorithm because it has seen enough copies of a value  $v$ , it must be sure that any other non-faulty process  $p_j$  will be able to decide  $v$  even if it has proposed 0 to BBC (because it has not seen enough copies of  $v$ ).

This issue is solved by two asynchronous rounds executed before invoking the underlying BBC algorithm (lines 01-12). The messages of the first round and the second round are tagged EST1 and EST2, respectively. Interestingly, we will state below two properties  $PR1$  and  $PR2$  that are the same as the properties used in [20, 25] to solve consensus on top of an asynchronous system enriched with any of Chandra and Toueg’s failure detectors [10].

It is important to remark that, at the abstraction level of the consensus algorithm, a message carries only a tag (EST1 or EST2) and a proposed value or  $\perp$ . Hence, considering that proposed values have constant size, the size of the messages used by the algorithm is  $O(1)$  (no message is required to carry array-like data structures whose size would depend on  $n$ ).

## 5.2 First additional round

The aim of this round (lines 01-04) is to direct each process  $p_i$  to define a “new” proposed value  $aux_i$  in such a way that the values  $aux_i$  of the non-faulty processes satisfy the following property (Lemma 2):

$$PR1 \equiv [\forall i, j \in \mathcal{C} : ((aux_i \neq \perp) \wedge (aux_j \neq \perp)) \Rightarrow (aux_i = aux_j = v) \wedge (v \text{ has been proposed by a non-faulty process})].$$

Hence this round replaces (for the non-faulty processes) the set of values they propose by a non-empty set including at most two values (namely, a value  $v$  proposed by a non-faulty process and  $\perp$ ).

From an operational point of view, this is obtained as follows. The processes first exchange (with the help of the underlying broadcast facility) the values they propose (lines 01-02). The values delivered at  $p_i$  are kept in the multiset  $rec1_i$ . Then, if there is a value  $v$  in  $rec1_i$  such that  $\#equal(v, rec1_i) \geq n - 2t$ ,  $v$  is assigned to  $aux_i$ . Otherwise  $aux_i = \perp$ .

## 5.3 Second additional round

The aim of the second round (lines 05-12) is to establish the following property denoted  $PR2$  (Lemma 3) in order the result of the underlying BBC algorithm can be safely exploited as described previously (lines 13-14). The local variable  $bp_i$  contains the value proposed by  $p_i$  to the underlying BBC algorithm, and  $res_j$  contains the non- $\perp$  value that any non-faulty process  $p_j$  will decide if the default value  $\perp$  is not decided.

$$PR2 \equiv [(\exists i \in \mathcal{C} : bp_i = 1) \Rightarrow (\forall j \in \mathcal{C} : res_j = res_i = v \neq \perp)].$$

Operationally, this is obtained as follows. With the help of the underlying broadcast abstraction the non-faulty processes exchange the values of their  $aux_i$  variables. The values delivered at  $p_i$  are saved in the multiset  $rec2_i$ . (This multiset contains  $n - t$  values, and, due to  $PR1$ , those can be  $\perp$ , a non- $\perp$  value  $v$  proposed by a non-faulty process, and at most  $t$  arbitrary values sent by faulty processes.)

If there is a non- $\perp$  value  $v$  such that  $\#equal(v, rec2_i) \geq n - 2t$ ,  $p_i$  proposes  $bp_i = 1$  to the binary consensus BBC. Otherwise,  $p_i$  has not seen enough copies of a value  $v \neq \perp$  and consequently proposes  $bp_i = 0$ . In all cases,  $p_i$  defines  $res_i$  as the most frequent non- $\perp$  value it has received. As the proof of Lemma 3 will show, if a non-faulty process  $p_i$  invokes `bin_propose(1)`, each non-faulty process will have the same non- $\perp$  value in its local variable  $res_j$ .

## 5.4 Proof of the algorithm

Let us recall that  $\mathcal{C}$  denotes the set of processes that are non-faulty in the considered execution.

**Lemma 2.** *PR1 holds in both system models  $\mathcal{BZ\_AS}_{n,t}[t < n/5, \text{UB}]$  and  $\mathcal{BZ\_AS}_{n,t}[t < n/4, \text{ND}]$ .*

**Proof** Let  $p_i$  and  $p_j$  be two non-faulty processes such that  $aux_i = v \neq \perp$ . We consider separately each case stated in the lemma assumption.

- Case 1:  $t < n/5$  and the non-faulty processes use the UB-broadcast abstraction.

As  $aux_i = v \neq \perp$ , it follows that  $\#equal(v, rec1_i) \geq n - 2t$  (line 04). Hence, due to the UB-broadcast, among the  $n - t$  messages it has UB-delivered (from different processes), at least  $n - 2t$  are  $EST1(v)$ . As at most  $t$  processes are faulty, it follows that at least  $n - 3t$  non-faulty processes have UB-broadcast a message  $EST1(v)$ . Consequently, at most  $n - (n - 3t) = 3t$  processes may send  $w \neq v$  to  $p_j$ . As  $3t < n - 2t$ ,  $p_j$  never assigns  $w$  to  $aux_j$ .

Finally, the proof that  $v$  has been proposed by a non-faulty process follows from the observation that  $v$  has been sent by at least  $n - 2t > t$  non-faulty processes.

- Case 2:  $t < n/4$  and the non-faulty processes use the ND-broadcast.

In that case,  $p_i$  has ND-delivered at least  $(n - 2t)$  messages  $EST1(v)$ , from different processes, and  $p_j$  has ND-delivered at least  $n - 2t$  messages  $EST1(w)$  from different processes. As  $n > 4t$ , it follows that there is a process  $p_x$  such that  $p_i$  has ND-delivered  $EST1(v)$  from  $p_x$  and  $p_j$  has ND-delivered  $EST1(w)$  from  $p_x$ . But, be  $p_x$  faulty or non-faulty, this is impossible due to the ND-duplicity property (if a non-faulty process ND-delivers a value from a process  $p_x$ , any other non-faulty process either ND-delivers the same value from  $p_x$  or does not ND-deliver a message from  $p_x$ ). It follows that we have  $v = w$ .

Finally, similarly to the previous case, the proof that  $v$  has been proposed by a non-faulty process follows from the observation that  $v$  has been ND-broadcast by at least  $n - 2t > t$  non-faulty processes.

□*Lemma 2*

**Lemma 3.** *PR2 holds in both system models  $\mathcal{BZ\_AS}_{n,t}[t < n/5, \text{UB}]$  and  $\mathcal{BZ\_AS}_{n,t}[t < n/4, \text{ND}]$ .*

**Proof** Let  $p_i$  be a process such that  $bp_i = 1$ . It follows from lines 06-08 that the multiset  $rec2_i$  contains  $n - t$  values (including  $\perp$ ). From line 08 we also have  $(bp_i = 1) \Rightarrow (\exists v \neq \perp : \#equal(v, rec2_i) \geq n - 2t)$ , from which we conclude that  $p_i$  has delivered at least  $n - 2t$  messages  $EST2(v)$ . Moreover, due to Lemma 2, the values sent by non-faulty processes are only  $v$  or  $\perp$ . Let us consider separately each case stated in the lemma assumption.

- Case 1:  $t < n/5$  and the non-faulty processes use UB-broadcast.

As there are at most  $t$  faulty processes, at most  $t$  messages  $EST2(v)$  UB-delivered by  $p_i$  are from faulty processes. Consequently, at least  $n - 3t$  non-faulty processes have UB-broadcast  $EST2(v)$  to  $p_j$ . As  $p_j$  waits for  $n - t$  messages, it can miss at most  $t$  messages  $EST2(v)$  from non-faulty processes (this is because, in the worst case, the  $t$  messages missed by  $p_j$  are from non-faulty processes that UB-broadcast  $EST2(v)$ ). Consequently,  $p_j$  UB-delivers at least  $n - 4t$  messages  $EST2(v)$  from non-faulty processes. As  $n > 5t$ , we have  $\#equal(v, rec2_j) > n - 4t \geq t + 1$ . Let us finally notice that, as at most  $t$  processes are faulty,  $p_j$  UB-delivers at most  $t$  messages  $EST2(-)$  carrying values different from  $v$  and  $\perp$ . Hence,  $\forall w \neq \perp$  we have  $\#equal(v, rec2_j) > t \geq \#equal(w, rec2_j)$ , which proves the lemma.

- Case 2:  $t < n/4$  and the non-faulty processes use ND-broadcast.

In that case, due to ND-broadcast, no two non-faulty processes can ND-deliver different values from the same faulty process. The worst case is then when (a)  $t$  processes are faulty and ND-broadcast the same value  $w \notin \{v, \perp\}$ , and (b)  $p_j$  ND-delivers these  $t$  messages  $EST2(w)$ . We trivially have  $t \geq \#equal(w, rec2_j)$ . On another side, as  $\#equal(v, rec2_i) \geq n - 2t \geq 2t + 1$ , and  $p_j$  misses at most  $t$  messages  $EST2(v)$ , we have  $\#equal(v, rec2_j) \geq t + 1$ . Hence, we have  $\#equal(v, rec2_j) > t \geq \#equal(w, rec2_j)$ , which concludes the proof of the lemma.

□*Lemma 3*

**Theorem 4.** *The algorithm described in Figure 4 solves the ITB multivalued consensus problem in both  $\mathcal{BZ\_AS}_{n,t}[t < n/5, \text{UB}, \text{BBC}]$  and  $\mathcal{BZ\_AS}_{n,t}[t < n/4, \text{ND}, \text{BBC}]$ .*

**Proof** Proof of the C-Termination property (every non-faulty process decides). As at most  $t$  processes are faulty, no non-faulty process blocks forever at line 02 or line 06. Finally, due to the C-termination property of the underlying



binary consensus algorithm BBC, every non-faulty process decides.

Proof of the C-Agreement property (no two non-faulty processes decide differently). If BBC returns 0, all the non-faulty processes decide  $\perp$ , and C-Agreement trivially follows. Hence, let us consider that BBC returns 1. It then follows from Property 1 of BBC that there is a non-faulty process  $p_i$  such that  $bp_i = 1$ . Hence, due to Lemma 3, any non-faulty process  $p_j$  is such that  $res_j = v$ , and all the non-faulty processes decide  $v$ .

Proof of the C-Obligation property (if all the non-faulty processes propose the same value, that value is decided). Let us assume that all the non-faulty processes propose value  $v$ . Let  $p_i$  be any non-faulty process. We then have  $\#equal(v, rec1_i) \geq n - 2t$  at each non-faulty process  $p_i$ , and consequently each of the (at least)  $n - t$  non-faulty process sends  $EST2(v)$  (line 05). So, each non-faulty process delivers at least  $n - 2t$  of these messages and we have  $\#equal(v, rec2_i) \geq n - 2t$ . Hence, any non-faulty  $p_i$  is such that  $bp_i = 1$  and sets  $res_i$  to  $v$ . Due to the C-obligation property of the underlying BBC algorithm, value 1 is decided, and consequently all the non-faulty processes decide  $v$ .

Proof of the C-Non-intrusion property (a non- $\perp$  value proposed only by faulty processes cannot be decided). If a non- $\perp$  value is decided, it follows from Property 1 of the underlying BBC that a non-faulty process  $p_i$  has proposed 1. Hence, we have  $bp_i = 1$ , and consequently  $\#equal(v, rec2_i) \geq n - 2t$ . As there are at most  $t$  faulty processes, it follows that non-faulty processes have broadcast  $EST2(v)$ , which in turn implies that  $n - 2t$  processes have broadcast  $EST1(v)$ , i.e., at least  $n - 3t \geq t + 1$  processes have broadcast  $EST1(v)$ , from which we finally conclude that  $v$  has been proposed by non-faulty processes.  $\square_{Theorem 4}$

## 6 A Consensus Algorithm Based on the VB-Broadcast Abstraction

This section presents an intrusion-tolerant Byzantine consensus algorithm based on the VB-broadcast abstraction. This algorithm requires  $t < n/3$  and has consequently an optimal resilience. It requires a single round (instead of two as in Figure 4). As it is based on VB-broadcast, this round requires six communication steps.

**Principles and description of the algorithm** The algorithm is presented in Figure 5. After it has VB-broadcast its value, a process  $p_i$  waits for  $EST()$  messages from  $n - t$  processes and deposits the corresponding values in the multiset  $rec_i$ . Then,  $p_i$  checks if (1) (in addition to  $\perp$ ) it has VB-delivered exactly one non- $\perp$  value  $v$ , and (2) that value has been VB-broadcast by at least  $n - 2t$  processes (line 04). If there is such a value,  $p_i$  proposes 1 to the underlying binary consensus, otherwise it proposes 0 (line 05).

Finally,  $p_i$  decides  $\perp$  if the underlying binary consensus BBC returns 0 (lines 11). Differently, if 1 is returned,  $p_i$  waits until it has VB-delivered  $(n - 2t)$  messages  $EST()$  carrying the very same value  $v$  (line 09) and then decides that value (line 10). Let us notice that, among these  $(n - 2t)$  messages, some have been already VB-delivered at line 02. The important point is (as shown in the proof) that the net effect of (a) the VB-broadcast, (b) the predicate used at line 04, and (c) the predicate used in the wait statement at line 09, ensures that if a non-faulty process invokes  $bin\_propose(1)$ , then all the non-faulty processes eventually VB-deliver  $(n - 2t)$  times the very same value  $v$  and decide it.

**On the predicate “ $rec_i$  contains a single non- $\perp$  value” used at line 04** The aim of this predicate is to ensure that, if  $bp_i = bp_j = 1$  (where  $p_i$  and  $p_j$  are two non-faulty processes), then the multisets  $rec_i$  and  $rec_j$  contain only instances of the very same value  $v$  (plus possibly instances of  $\perp$ ).

To motivate this predicate, let us consider that the predicate of line 04 is restricted to its first part, namely, “ $\exists v : \#equal(v, rec_i) \geq n - 2t$ ”. Assuming  $n = 10$  and  $t = 3$ , let us consider the case where, at line 01, four processes VB-broadcast the message  $EST(v)$ , while six processes VB-broadcast the message  $EST(w)$ . Moreover, let us consider the following execution:

- On the one side,  $p_i$  VB-delivers  $n - t = 7$  messages  $EST()$ , four that carry  $v$  and three that carry  $w$ . As  $\#equal(v, rec_i) = 4 \geq n - 2t = 4$ , the restricted predicate is satisfied for  $v$ , and consequently  $p_i$  assigns 1 to  $bp_i$ .

- On the other side,  $p_j$  VB-delivers  $n - t = 7$  messages  $\text{EST}()$ , four that carry  $w$  and three that carry  $v$ . As  $\# \text{equal}(w, \text{rec}_i) = 4 \geq n - 2t = 4$ , the restricted predicate is satisfied for  $w$ , and consequently  $p_j$  assigns 1 to  $bp_i$ .

It follows that we have  $bp_i = bp_j = 1$  ( $p_i$  and  $p_j$  being non-faulty processes), while  $v$  is the value that will be decided by  $p_i$  if the underlying BBC algorithm returns 1, and the value decided by  $p_j$  will be  $w \neq v$ . It is easy to see that the second part of predicate of line 04 prevents this bad scenario from occurring.

```

operation propose( $v_i$ )
(01)  VB_broadcast EST( $v_i$ );
(02)  wait until (EST(-) messages VB_delivered from ( $n - t$ ) processes);
(03)  let  $\text{rec}_i = \text{multiset of the values } v \text{ such that EST}(v) \text{ is VB\_delivered to } p_i$ ;
(04)  if ( $\exists v \neq \perp : \# \text{equal}(v, \text{rec}_i) \geq n - 2t$ )  $\wedge$  ( $\text{rec}_i$  contains a single non- $\perp$  value)
(05)    then  $bp_i \leftarrow 1$  else  $bp_i \leftarrow 0$ 
(06)  end if;
(07)   $b\_dec_i \leftarrow \text{bin\_propose}(bp_i)$ ;    % underlying BBC consensus %
(08)  if ( $b\_dec_i = 1$ )
(09)    then wait until ( $\exists v \neq \perp$  such that EST( $v$ ) VB_delivered from ( $n - 2t$ ) processes);
(10)      return( $v$ )
(11)    else return( $\perp$ )
(12)  end if.

```

Figure 5: Intrusion-tolerant Byzantine consensus algorithm based on VB-broadcast ( $t < n/3$ )

**Theorem 5.** *The algorithm described in Figure 5 solves the ITB multivalued consensus problem in the system model  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \text{VB}, \text{BBC}]$ .*

**Proof** Proof of the C-Termination property (every non-faulty process decides). If the underlying BBC algorithm returns 0, termination is trivial. Hence, let us consider that 1 is returned. Due to Property 1 of BBC, there is a non-faulty process  $p_i$  such that  $bp_i = 1$ , which in turn implies that, at line 02,  $p_i$  has received at least  $(n - 2t)$  messages  $\text{EST}(v)$ . Due to the VB-Uniformity property of VB-broadcast, any non-faulty process eventually VB-delivers these  $(n - 2t)$  messages  $\text{EST}(v)$ . Hence, no non-faulty process  $p_j$  blocks forever at line 09, which concludes the proof of the termination property.

Proof the C-Agreement property (no two non-faulty processes decide differently). The proof is similar to the previous one. If BBC returns 0, agreement is trivial. If 1 is returned, it follows from  $n - 2t > t$  and the fact that –at any non-faulty process  $p_i$ – there is no  $w \neq v$  such that  $w \in \text{rec}_i$  (second predicate of line 04), that the value  $v$  the processes are waiting for at line 09 is unique, which completes the proof of the agreement property.

Proof of the C-Obligation property (if all the non-faulty processes propose the same value, that value is decided). If all the non-faulty processes propose the same value  $v$ , it follows from the VB-Obligation property that  $v$  is necessarily validated, and from the VB-Termination property that all the non-faulty processes VB-deliver at least  $(n - 2t)$  messages  $\text{EST}(v)$ . Moreover, as  $n - 2t > t$ , there is a single such value  $v$ . Due to VB-Justification property, a value VB-broadcast only by faulty processes cannot be validated and consequently no non-faulty process can VB-deliver it. This means that only  $v$ ,  $\perp$  or nothing at all can be VB-delivered from a faulty process. It follows that, at each non-faulty process  $p_i$ , the predicate of line 04 is satisfied and  $p_i$  proposes  $bp_i = 1$ . Due to the C-Obligation property of BBC, they all decide 1 and consequently decide the same proposed value  $v$ .

Proof of the C-Non-intrusion property (a non- $\perp$  value proposed only by faulty processes cannot be decided). If a value  $w$  is proposed only by faulty processes, it follows from the VB-Justification property that no non-faulty process  $p_i$  VB-delivers it. If the underlying BBC algorithm returns 0,  $w$  is not decided. If BBC returns 1, we have seen in the proof of the C-Agreement property that the processes decide a value  $v$  such that at least  $(n - 2t)$  messages  $\text{EST}(v)$  have been VB-delivered. As  $n - 2t > t$ , it follows that  $w$  cannot be decided.  $\square_{\text{Theorem 5}}$

## 7 Discussion

### 7.1 An interesting property of the previous ITB consensus algorithms

Let  $v$  be the value most proposed by the non-faulty processes (if several values are equally most proposed,  $v$  is any of them), and let  $\#(v)$  be the number of non-faulty processes that propose it. The previous algorithms have the following noteworthy property. (This follows from Lemma 2, Lemma 3 and Theorem 4 for the ITB instances obtained from the generic algorithm described in Figure 4, and from Theorem 5 for the VB-based algorithm described in Figure 5.)

- If  $\#(v) \geq n - t$ , then  $v$  is always decided by the non-faulty processes (let us observe that, in that case, there is a single most proposed value).
- If  $\#(v) < n - 2t$ , then  $\perp$  is always decided by the non-faulty processes.
- If  $n - 2t \leq \#(v) < n - t$ , then which value ( $v$  or  $\perp$ ) is decided by the non-faulty processes depends on both the behavior of Byzantine processes and the asynchrony pattern.

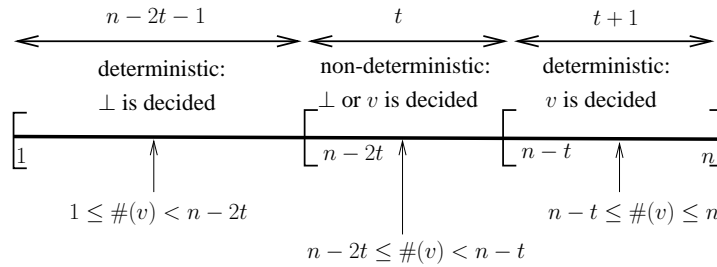


Figure 6: Deterministic vs non-deterministic scenarios

Let us consider an omniscient observer that would know which are the proposed values proposed by the non-faulty processes. In the first and the second cases, this omniscient observer can compute the result in a deterministic way. Differently, it cannot in the last case. The value that is decided depends then on the behavior of Byzantine processes (that can favor the most proposed value, or entail a  $\perp$  decision). These different possibilities are depicted on Figure 6. As we have seen, a value proposed only by Byzantine processes is necessarily proposed by less than  $(n - 2t)$  processes and, consequently, cannot be decided.

### 7.2 Comparing the previous signature-free multivalued ITB algorithms

Table 2 presents a summary of the cost and the constraint on  $t$  associated with the previous signature-free ITB multivalued consensus algorithms. As they all use the same underlying BCC algorithm, the comparison does not take this algorithm into account.

It is easy to see that, due the weaker constraint on  $t$ , the algorithm of Figure 5 instantiated with VB-broadcast outperforms the generic algorithm of Figure 4 instantiated with ND-broadcast. On another side, in a system where the number of Byzantine processes remains small (i.e.,  $t < n/5$ ), the generic algorithm instantiated UB-broadcast is the most efficient.

Consensus algorithm instantiated with	# communication steps	message size at send/receive level	# msgs at send/receive level	constraint on $t$
Generic algorithm with UB	$1 \times 2$	constant	$O(n^2)$	$n > 5t$
Generic algorithm with ND	$2 \times 2$	$\log_2 n$	$O(n^3)$	$n > 4t$
Specific algorithm based on VB	$1 \times 6$	$\log_2 n$	$O(n^3)$	$n > 3t$

Table 2: Cost of the ITB consensus algorithms

## 8 A Randomized VB-Based Byzantine Binary Consensus Algorithm

This section presents a particularly simple randomized Byzantine binary consensus algorithm (that can be used as the underlying BBC algorithm on which rely the multivalued Byzantine consensus algorithms previously described). The additional power needed to solve consensus is given here by random coins. This algorithm, which is optimal from a resilience point of view ( $t < n/3$ ), is based on the validated broadcast abstraction. More precisely, each round requires one VB-broadcast instance.

When looking at Byzantine consensus algorithms that are optimal from a resilience point of view (i.e., algorithms able to cope with up to  $\lfloor (n-1)/3 \rfloor$  faulty processes), the best consensus algorithm we are aware of has rounds made up of three communication steps [8]. Moreover, this algorithm is based on signatures (public key cryptography). As far as signature-free algorithms are concerned, the best resilience-optimal algorithm we are aware of, that uses control information whose size is only  $O(\log_2 n)$ , is the one described in [30], which requires five communication steps per round. The algorithm presented in Figure 7 is signature-free and requires only six communication steps per round.

### 8.1 Randomized model

**Common coin** The asynchronous system is equipped with a *common coin* as defined by Rabin [24], and improved in [8] in order to get rid of the trusted dealer. Such an oracle is denoted CC. The corresponding enriched –from a computational power point of view– system model is consequently denoted  $\mathcal{BZ}_{\mathcal{AS}_{n,t}}[t < n/3, \text{CC}]$ . A common coin can be seen as a global entity that delivers a sequence of random bits  $b_1, b_2, \dots, b_r, \dots$  to processes (each bit  $b_r$  has the value 0 or 1, with probability 1/2).

More precisely, this oracle provides the processes with a primitive denoted  $\text{random}()$  that returns a bit each time it is called by a process. In addition to being random, this bit has the following global property: the  $r$ th invocation of  $\text{random}()$  by any non-faulty process  $p_i$  returns it the bit  $b_r$ . This means that the  $r$ th invocations of  $\text{random}()$  by any pair of non-faulty processes  $p_i$  and  $p_j$  return them  $b_r$ , whatever the times at which each of these invocations occur. It is important to notice that the network has no access to the common coin, which corresponds to the *oblivious scheduler* model [2]. (The reader interested in the implementation of a common coin can consult [2, 8].)

**On randomized consensus** When using additional computing power provided by random coins, the consensus termination property can no longer be deterministic. *Randomized consensus* is defined by C-Validity (Obligation), C-Agreement, plus the following termination property [4, 24]: Every non-faulty process decides with probability 1. For round-based algorithms, this termination property can be re-stated as follows:

$$\text{For any non-faulty process } p_i: \lim_{r \rightarrow +\infty} (\text{Probability } [p_i \text{ decides by round } r]) = 1.$$

### 8.2 The algorithm

**Underlying principles and description of the algorithm** In the algorithm described in Figure 7, a process  $p_i$  invokes the function  $\text{bin\_propose}(v_i)$  where  $v_i$  is the value it proposes. It decides when it executes the statement  $\text{decide}(v)$  (line 08). The design of this algorithm is close to an algorithm proposed in [15]. Its fundamental difference is that it is resilience-optimal ( $t < n/3$ ), while the one described in [15] requires  $t < n/5$ .

The local variable  $est_i$  of a process  $p_i$  keeps its current estimate of the decision value (initially,  $est_i = v_i$ ). The processes proceed by consecutive asynchronous rounds. Thus, the pair  $(r_i, est_i)$  of a non-faulty process  $p_i$  describes its current state ( $r_i$  is  $p_i$ 's current round number). The first part of the algorithm (lines 01-04) is devoted to communication occurring during a round. The second part (lines 05-10) defines the management of the local estimate  $est_i$  and the decision rule. There is one VB-broadcast instance per round. To distinguish the messages  $\text{EST}()$  associated with different VB-broadcast instances, these messages are tagged by their round number, namely  $\text{EST}[r](v)$  denotes a round  $r$  message carrying the value  $v$ . More precisely, we have the following.

- At every round  $r_i$ , each non-faulty process  $p_i$  VB-broadcasts  $\text{EST}[r_i](est_i)$ , and waits until it has VB-delivered  $\text{EST}[r_i](-)$  from at least  $n - t$  processes (lines 02-04).
- In the second part,  $p_i$  first computes the random number  $s$  associated with the current round  $r_i$  (line 05). Then,  $p_i$  checks if (a) it has received a non- $\perp$  value  $v$  from at least  $n - 2t$  different processes, and (b)  $v$  is the only

non- $\perp$  value in  $rec_i$  (predicate at line 06). If this predicate holds,  $p_i$  adopts  $v$  as new estimate (line 07) and decides the random value  $s$  if  $v = s$  (line 08). If the predicate is false,  $p_i$  updates its estimate  $est_i$  to the random value  $s$ . In all cases,  $p_i$  starts a new asynchronous round.

The statement `decide()` allows the invoking process to decide but does not stop its execution. Hence, a process executes rounds forever. This facilitates the description of the algorithm. Using techniques such as the one developed in [15] allows a process to both decide and stop.

**Remark** It is possible to add the following test after line 04:

**if**  $(\exists v : \#equal(v, rec_i) \geq n - t)$  **then** `decide( $v$ )` **end if**.

This allows the algorithm to always terminate in a single round whatever the value of the common coin when no process commits Byzantine failure and all processes propose the same value. (This scenario is likely to happen in actual executions.)

```

operation bin_propose( $v_i$ )
 $est_i \leftarrow v_i; r_i \leftarrow 0;$ 
repeat forever
(01)  $r_i \leftarrow r_i + 1;$ 
(02) VB_broadcast EST[ $r_i$ ]( $est_i$ );
(03) let  $rec_i =$  multiset of values  $est$  such that EST[ $r_i$ ]( $est$ ) has been VB_delivered to  $p_i$ ;
(04) wait until  $(|rec_i| \geq n - t)$ ;
(05)  $s_i \leftarrow \text{random}();$ 
(06) if  $(\exists v \neq \perp : \#equal(v, rec_i) \geq n - 2t) \wedge (rec_i \text{ contains a single non-}\perp \text{ value})$ 
(07)   then  $est_i \leftarrow v;$ 
(08)       if  $(v = s) \wedge (p_i \text{ has not yet decided})$  then decide( $v$ ) end if
(09)   else  $est_i \leftarrow s$ 
(10)   end if
end repeat.

```

Figure 7: A binary Byzantine consensus algorithm based on VB-broadcast ( $t < n/3$ )

### 8.3 Proof

**Lemma 4.** *Let  $n > 3t$ . Consider the situation where, at the beginning of a round  $r$ , all the non-faulty processes have the same estimate value  $v$ . These processes will never change their estimates, thereafter.*

**Proof** As all the non-faulty processes VB-broadcast the same value  $v$  at the beginning of round  $r$  (line 02), it follows from the VB-obligation property of VB-broadcast, that the only values that can be VB-delivered are  $v$  (VB-broadcast by each of them and possibly from Byzantine processes) and  $\perp$  (from Byzantine processes). Moreover, as each non-faulty process  $p_i$  waits for  $n - t$  messages (line 04), it will VB-deliver at least  $n - 2t$  values  $v$ ; as  $n > 3t$ , at most  $t$  of them can be VB-broadcast by Byzantine processes (due to the VB-validity property, a value  $w \neq v$  VB-broadcast by a Byzantine process  $p_j$  cannot be validated, and consequently  $\perp$  or no value at all is VB-delivered from such a process  $p_j$ ). Hence, the predicate of line 06 is satisfied, and  $p_i$  sets  $est_i$  to  $v$  (line 07), which concludes the proof of the lemma.

□*Lemma 4*

Let  $COND(v, i)$  be the predicate that process  $p_i$  tests at line 06.

**Lemma 5.** *Let  $n > 3t$ . If two non-faulty processes  $p_i$  and  $p_j$  are such that both  $COND(v, i)$  and  $COND(w, j)$  hold at round  $r$ , then  $v = w$ .*

**Proof** By the VB-Uniformity property of VB-broadcast, no two non-faulty processes VB-deliver different values from the same process. Hence, if  $COND(v, i)$  holds for some non-faulty process  $p_i$ , no other non-faulty process  $p_j$  can VB-deliver a value  $w \neq v$  from the set of  $(n - t)$  processes whose VB-broadcasts built the set  $rec_i$ . Consequently, if

$p_j$  VB-delivers a value  $w \neq v$ , the number of occurrences of  $w$  is necessarily at most  $t < n - 2t$ , and consequently  $COND(w, j)$  cannot be satisfied.  $\square_{\text{Lemma 5}}$

**Lemma 6.** *Let  $n > 3t$ . If all the non-faulty processes propose the same value  $v$ , then no value  $v' \neq v$  can be decided.*

**Proof** This lemma is an immediate consequence of Lemma 4. As all estimates of the non-faulty processes remain equal to  $v$ , it follows from line 08 that no value  $v' \neq v$  can be returned by a non-faulty process.  $\square_{\text{Lemma 6}}$

**Lemma 7.** *No two non-faulty processes decide different values.*

**Proof** Let  $r$  be the first round during which non-faulty processes decide. If two processes  $p_i$  and  $p_j$  decide at round  $r$ , they decide at line 08 the value  $s$  computed by the common coin for that round. Moreover, before deciding during round  $r$ , a process updated its estimate to the decided value  $s$ . Hence, all processes that decide during round  $r$  decide the same value  $s$ , and have their estimates equal to the decided value.

Let us now consider the case of a processes  $p_x$  for which, during round  $r$ , (a) the predicate of line 06 is satisfied (i.e.,  $COND(w, x)$  is true), (b) while the decision predicate at line 08 is not. As the predicate of line 06 is satisfied for both  $p_x$  and any process  $p_i$  that decides at line 08 (i.e., both  $COND(w, x)$  and  $COND(i, v)$  are true), it follows from Lemma 5 that  $w = v$ , which means that it is not possible that the decision predicate of  $p_x$  be false. Hence,  $p_x$  decides during round  $r$ , exactly as  $p_i$ .

Let us finally consider the case of a non-faulty process  $p_k$  such that  $COND(-, k)$  does not hold at line 06 during round  $r$ . It follows from line 09 that  $p_k$  updates its estimate to the random value  $s$  associated with round  $r$ . Hence, all such processes  $p_k$  start round  $r + 1$  with their estimates equal to the decided value  $s$ .

It then follows from Lemma 4 that, from round  $r + 1$ , the estimates of all the non-faulty processes keep forever the same value (namely, the decided value). Hence, no value different from this estimate value can be decided.  $\square_{\text{Lemma 7}}$

**Lemma 8.** *Each non-faulty process decides with probability 1.*

**Proof** No non-faulty process remains blocked forever during a round  $r$ . This follows from the fact that, at every round, a non-faulty process  $p_i$  waits for the VB-delivery of a message  $EST(r, -)$  from  $n - t$  distinct processes, and at every round each non-faulty process VB-broadcasts such a message that (due to the VB-Termination property) entails a corresponding VB-delivery at each non-faulty process.

**Claim.** With probability 1, there is a round  $r$  at the end of which all the non-faulty processes have the same estimate value. (End of the claim.)

Assuming the claim holds, it follows from Lemma 4 that all the non-faulty processes  $p_i$  keep their estimate value  $est_i = v$  and consequently the predicate  $COND(v, i)$  (line 06) is true at every round. Due to common coin CC, it follows that, with probability 1, there is eventually a round in which  $random()$  outputs  $v$ . Then, the condition of line 08 evaluates to true, and all the non-faulty processes decide.

**Proof of the claim.** We need to prove that, with probability 1, there is a round at the end of which all the non-faulty processes have the same estimate value. Let us consider a round  $r$ .

- Observe that if all the non-faulty processes execute line 09 then, at the end of  $r$ , they all adopt the same value (defined by the common coin) by the end of  $r$ . The claim directly follows.
- If all the non-faulty processes execute line 07, due to Lemma 5 they adopt the same value  $v$  as their estimate, and the claim follows.
- The third case is when some non-faulty processes execute line 07 and (by Lemma 5) adopt the same value  $v$ , while others execute line 09 and adopt the same value  $s$ .

Due to the properties of the common coin, the value it computes at a given round is independent from the values it computes at the other rounds (and also from the Byzantine behavior and the network scheduler). Thus,  $s$  is equal to  $v$  with probability  $p = 1/2$ . Let  $P(r)$  be the following probability (where  $var^r$  is the value of  $var$  at round  $r$ ):  $P(r) = \text{Probability}[\exists r' : r' \leq r : v^{r'} = s^{r'}]$ . We have  $P(r) = p + (1 - p)p + \dots + (1 - p)^{r-1}p$ . So,  $P(r) = 1 - (1 - p)^r$ . As  $\lim_{r \rightarrow +\infty} P(r) = 1$ , the claim follows. (End of the proof of the claim.)

□ Lemma 8

**Theorem 6.** *The algorithm described in Figure 7 solves the randomized binary consensus problem in the system model  $\mathcal{BZ}\text{-}\mathcal{AS}_{n,t}[t < n/3, \text{VB}, \text{CC}]$ .*

**Proof** Follows from lemmas 6, 7 and 8.

□ Theorem 6

**Theorem 7.** *Let  $n > 3t$ . The expected decision time is constant.*

**Proof** As indicated in the proof of Lemma 8, termination is obtained in two phases. First, all the non-faulty processes must adopt the same value  $v$ . Second, the outcome of the common coin has to be the same as the commonly adopted value  $v$ .

It follows from the proof of Lemma 8 that there is only one situation in which the non-faulty processes do not adopt the same value. This is when the predicate of line 06 is satisfied for a subset of non-faulty processes and not for the other non-faulty processes. Thus, the expected number of rounds for this to happen is 2. As for the second phase, here again, the probability that the value output by the common coin is the same as the value held by all the non-faulty processes is  $1/2$ . Thus, the expected time for this to occur is also 2. Combining the two phases, the expected termination time is 4 rounds (i.e., a small constant).

□ Theorem 7

## 9 Conclusion

Considering distributed message-passing systems made up of  $n$  processes, and where up to  $t$  processes may commit Byzantine failures, the aim of the paper was to present in a simple and homogeneous way (a) existing and new broadcast and agreement abstractions, and (b) algorithms implementing them. These broadcast abstractions are UB-broadcast (unreliable broadcast), ND-broadcast (no-duplicity broadcast), RB-broadcast (reliable broadcast), and VB-broadcast (validated broadcast). They have been used to design three multivalued intrusion-tolerant Byzantine consensus algorithms. Moreover, all these algorithms are signature-free. As we have seen, the intrusion-tolerance property means that no value proposed only by Byzantine processes can ever be decided. As a consequence, a default value can be decided when the same value is not proposed by enough processes<sup>1</sup>.

The intrusion-tolerant consensus algorithm based on VB-broadcast has several noteworthy features: it is optimal from a resilience point of view ( $t < n/3$ ), each round requires only a single VB-broadcast instance, which costs six communication steps, and the size of control information attached with each message is  $O(\log_2 n)$ . The paper has also presented a novel randomized binary Byzantine consensus algorithm that is resilient-optimal and, in a very interesting way, is also based on the VB-broadcast abstraction. Let us finally notice that an important feature of the paper lies in its “partial survey” flavor.

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<sup>1</sup>Let us remark that this is similar to the notion of an *abortable* object as defined in [27] (which is different from the abortable object notion defined in [1]). Such an abortable object  $X$  allows any operation  $X.op()$  to return the default value  $\perp$  in “bad circumstances”; moreover, when this occurs, the object  $X$  is not modified by the operation  $op()$ . Given an object  $X$ , “bad circumstances” have to be defined and belong to the specification of  $X$ . A “bad circumstance” can be for example “in the presence of concurrent operations”. Here, “bad circumstance” means “when not enough non-faulty processes have proposed the same value”.

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