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INFLUENCE OF PERIODIC DIFFUSIVE INCLUSIONS ON THE BIDOMAIN MODEL

Andjela Davidović 1a,2 with advisers: Yves Coudiere 1a,2,3 and Clair Poignard 1b,2 1.Inria Bordeaux Sud-Ouest: 1a.Carmen team, 1b.MC2 team; 2.Institut de Mathematiques de Bordeaux (IBM); 3.IHU-Liryc

Introduction

We present a new mathematical model of the electric activity of the heart. The main **drawback of** the standard bidomain model is that it assumes the existence of excitable cells (myocytes) everywhere in the heart, while it is known that there exist non small regions where non-excitable cells (fibroblasts and collagen) take place. The problems that we are trying to address with our model are:

• The **laminar structure** of the myocardium that shows the presence of collagen, especially between muscle layers [2].



Figure 1: The laminar structure of myocardium. [3]

• The **infarct border zone**. After infraction in the heart some number of myocytes die and they are replaced by collagen and a few cells of fibroblasts. In our current model we consider as if there are no cells of fibroblasts in the extracellular space.



Figure 2: Fibroblast organization in sheep normal ventricular myocardium (A), infarct border zone (B) and centre (C), 1 week after infarction. [4]







I - Mesoscopic model

Modelling assumptions:

- Periodic distribution of added extracellular space
- Extracellular space is a passive conductor

Work on the domain $\Omega = B_{\varepsilon} \cup D_{\varepsilon}$

- B_{ε} represent the bidomain layer.
- given intra- and extracellular conductivities σ_{ε}^{i} and σ_{ε}^{e}
- unknown potentials u_{ε}^{i} and u_{ε}^{e}
- define $v_{\varepsilon} := u_{\varepsilon}^i u_{\varepsilon}^e$
- D_{ε} represent the diffusive inclusions.
- conductivity σ^d_{ε} • unknown potential u^d_{ε}
- $\Sigma_{\varepsilon} = \partial B_{\varepsilon} \cap \partial D_{\varepsilon}$ is the interface.

The bidomain model

$$\begin{array}{ll} \partial_t v_{\varepsilon} + c v_{\varepsilon} = \nabla \cdot (\sigma_{\varepsilon}^i \nabla u_{\varepsilon}^i), & B_{\varepsilon}, \\ \partial_t v_{\varepsilon} + c v_{\varepsilon} = -\nabla \cdot (\sigma_{\varepsilon}^e \nabla u_{\varepsilon}^e), & B_{\varepsilon}, \end{array} \qquad \begin{array}{ll} P_{\varepsilon} \\ \end{array}$$

The diffusive inclusion

$$0 = -\nabla \cdot (\sigma_{\varepsilon}^{d} \nabla u_{\varepsilon}^{d}), \qquad D_{\varepsilon}, \qquad \text{of}$$

The transmission conditions

$$\begin{aligned} \sigma_{\varepsilon}^{i} \nabla u_{\varepsilon}^{i} \cdot n_{\Sigma_{\varepsilon}} &= 0, \\ \sigma_{\varepsilon}^{e} \nabla u_{\varepsilon}^{e} \cdot n_{\Sigma_{\varepsilon}} &= \sigma_{\varepsilon}^{d} \nabla u_{\varepsilon}^{d} \cdot n_{\Sigma_{\varepsilon}}, \\ u_{\varepsilon}^{e} &= u_{\varepsilon}^{d}, \end{aligned} \right\} \qquad \Sigma_{\varepsilon}, \qquad \qquad \sigma_{\varepsilon}^{i}(x) = \sigma_{i} \left(\frac{x}{\varepsilon}\right), \quad \sigma_{\varepsilon}^{e}(x) = \sigma_{e} \left(\frac{x}{\varepsilon}\right), \quad \sigma_{\varepsilon}^{d}(x) = \sigma_{d} \left(\frac{x}{\varepsilon}\right). \\ \text{Having a priori estimates on the norms of the un-$$

The initial and the boundary conditions given.

• We have a well posed problem up to a constant.

• Numerical simulations expensive.

III - Macroscopic equations

Define	$\sigma_i^* := \sigma_i / Y_B ,$	$\sigma_e^* := \sigma_e / Y_B ,$	$\sigma_d^* := \sigma_d^*$
In the limit		$u_0^e(t,x) = u_0^d(t,x)$	(x) =: u
And the limit r	broblem is again t	he bidomain r	nodel v

 $\nabla_x \cdot \left(\left(\sigma_e^* + \rho \sigma_d^* + \left(\sigma_e - \sigma_d \right) \left(A_e + \rho A_d \right) \right) \nabla_x u_0 \right) = -(\partial_t v_0 + c v_0), \text{ in } \Omega,$

Where A_e and A_d are constant matrices that depend only on the geometry of the unit cell. **Remark:**

- The modified conductivities depend on the volume fraction of the diffusive part and on the geometry of the unit cell.
- If $\sigma_e = \sigma_d$ the problem simplifies and the modified conductivities depend only on the volume fraction.



II - Homogenisation

Principle: Redefine the problem in two scales and obtain the limit problem that depends only on a large scale. It gives a global, macroscopic behaviour the unknown functions.

Assumption

known functions and their derivatives, and following the work in [1], there is a limit for $\varepsilon \to 0$

$$u^i_{\varepsilon} \to u^i_0, \quad u^e_{\varepsilon} \to u^e_0, \quad u^d_{\varepsilon} \to u^d_0.$$

- $\sigma_d/|Y_D|, \quad \rho = |Y_D|/|Y_B|.$ $u_0(t,x).$
- with updated conductivities
- $\nabla_x \cdot \left((\sigma_i^* + Id) \nabla_x u_0^i \right) = \partial_t v_0 + c v_0, \text{ in } \Omega,$

We expect to observe the convergence of the mesoscopic model to the macroscopic model.

- model.

IV - Numerical verifications

We work on the numerical simulations

• On simple geometries. • Using FEM method. • Values of parameters obtained from the literature. • Using Gmsh mesh generator ([6]) and FreeFem + + PDE solver ([5]).

Perspectives

We can develop our work in several directions.

• Perform numerical experiments on different geometries, simulating only the macroscopic

• Use real data obtained by clinicians. For

example, the late enhancement MRI provides the volume fraction of the extracellular space, ρ .

• Apply the ionic current model.

• Develop a model for the infarct scar transiting from the classical bidomain, via our new model, to the purely diffusive centre of the scar.

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Contact: andjela.davidovic@inria.fr, yves.coudiere@inria.fr, clair.poignard@inria.fr