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# On the Scalability of Snapshot Isolation<sup>\*</sup>

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**Abstract.** Many distributed applications require transactions. However, transactional protocols that require strong synchronization are costly in large scale environments. Two properties help with scalability of a transactional system: genuine partial replication (GPR), which leverages the intrinsic parallelism of a workload, and snapshot isolation (SI), which decreases the need for synchronization. We show that under standard assumptions (data store accesses are not known in advance, and transactions may access arbitrary objects in the data store), it is impossible to have both SI and GPR. Our impossibility result is based on a novel decomposition of SI which proves that, like serializability, SI is expressible on plain histories.

## 1 Introduction

Large scale transactional systems have conflicting requirements. On the one hand, strong transactional guarantees are fundamental to many applications. On the other, remote communication and synchronization are costly and should be avoided.<sup>5</sup>

To maintain strong consistency guarantees while alleviating the cost of synchronization, Snapshot Isolation (SI) is a popular approach in both distributed database replications [1–3], and software transactional memories [4, 5]. Under SI, a transaction accesses its own *consistent snapshot* of the data, which is unaffected by concurrent updates. A read-only transaction always commits unilaterally and without synchronization. An update transaction synchronizes on commit to ensure that no concurrent conflicting transaction has committed before it.

Our first contribution is to prove that SI is equivalent to the conjunction of the following properties: *(i)* no cascading aborts, *(ii)* strictly consistent snapshots, i.e., a transaction observes a snapshot that coincides with some point in (linear) time, *(iii)* two concurrent write-conflicting update transactions never

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<sup>5</sup> We address general-purpose transactions, i.e., we assume that a transaction may access any object in the system, and that its read- and write-sets are not known in advance.

both commit, and *(iv)* snapshots observed by transactions are monotonically ordered. Previous definitions of SI [6, 7] extend histories with abstract snapshot points. Our decomposition shows that in fact, like serializability, SI can be defined on plain histories [8].

Modern data stores replicate data for both performance and availability. Full replication does not scale, as every process must perform all updates. *Partial replication* (PR) aims to address this problem, by replicating only a subset of the data at each process. Thus, if transactions would communicate only over the minimal number of replicas, synchronization and computation overhead would be reduced. However, in the general case, the overlap of transactions cannot be predicted; therefore, many PR protocols perform system-wide global consensus [2, 3] or communication [9]. This negates the potential advantages of PR; hence, we require *genuine* partial replication [10] (GPR), in which a transaction communicates only with processes that replicate some object accessed in the transaction. With GPR, independent transactions do not interfere with each other, and the intrinsic parallelism of a workload can be thus exploited.

Our second contribution is to show that SI and GPR are incompatible. More precisely, we prove that an asynchronous message-passing system supporting GPR, even if it is failure-free, cannot compute monotonically ordered snapshots, nor strictly consistent ones.

This paper proceeds as follows. We introduce our system model in Section 2. Section 3 presents our decomposition of SI. Section 4 shows that GPR and SI are mutually incompatible. We discuss implications of this result and related work in Section 5. Section 6 closes this paper. Due to space constraints, some proofs are deferred to our companion technical report [11].

## 2 Model

This section defines the elements in our model and formalizes SI and GPR .

### 2.1 Objects & transactions

Let *Objects* be a set of objects, and  $\mathcal{T}$  be a set of transaction identifiers. Given an object  $x$  and an identifier  $i$ ,  $x_i$  denotes *version*  $i$  of  $x$ . A *transaction*  $T_{i \in \mathcal{T}}$  is a finite permutation of read and write operations followed by a *terminating* operation, commit ( $c_i$ ) or abort ( $a_i$ ). We use  $w_i(x_i)$  to denote transaction  $T_i$  writing version  $i$  of object  $x$ , and  $r_i(x_j)$  to mean that  $T_i$  reads version  $j$  of object  $x$ . In a transaction, every write is preceded by a read on the same object, and every object is read or written at most once.<sup>6</sup> We note  $ws(T_i)$  the write set of  $T_i$ , i.e., the set of objects written by transaction  $T_i$ . Similarly,  $rs(T_i)$  denotes the read set of transaction  $T_i$ . The *snapshot* of  $T_i$  is the set of versions read by  $T_i$ . Two transactions *conflict* when they access the same object and one of them modifies it; they *write-conflict* when they both write to the same object.

<sup>6</sup> These restrictions ease the exposition of our results but do not change their validity.

## 2.2 Histories

A *complete history*  $h$  is a partially ordered set of operations such that (1) for every operation  $o_i$  appearing in  $h$ , transaction  $T_i$  terminates in  $h$ , (2) for every two operations  $o_i$  and  $o'_i$  appearing in  $h$ , if  $o_i$  precedes  $o'_i$  in  $T_i$ , then  $o_i <_h o'_i$ , (3) for every read  $r_i(x_j)$  in  $h$ , there exists a write operation  $w_j(x_j)$  such that  $w_j(x_j) <_h r_i(x_j)$ , and (4) any two write operations over the same objects are ordered by  $<_h$ . A *history* is a prefix of a complete history. For some history  $h$ , order  $<_h$  is the *real-time order* induced by  $h$ . Transaction  $T_i$  is *pending* in history  $h$  if  $T_i$  does not commit, nor abort in  $h$ . We note  $\ll_h$  the version order induced by  $h$  between different versions of an object, i.e., for every object  $x$ , and any two transactions  $T_i$  and  $T_j$ ,  $x_i \ll_h x_j = w_i(x_i) <_h w_j(x_j)$ . Following Bernstein et al. [12], we depict a history as a graph. We illustrate this with history  $h_1$  below in which transaction  $T_a$  reads the initial versions of objects  $x$  and  $y$ , while transaction  $T_1$  (respectively  $T_2$ ) updates  $x$  (resp.  $y$ ).<sup>7</sup>

$$\begin{array}{l}
 h_1 = r_a(x_0) \longrightarrow r_1(x_0).w_1(x_1).c_1 \\
 \searrow \hspace{10em} \longrightarrow r_a(y_0).c_a \longrightarrow r_2(y_0).w_2(y_2).c_2
 \end{array}$$

When order  $<_h$  is total, we shall write a history as a permutation of operations, e.g.,  $h_2 = r_1(x_0).r_2(y_0).w_2(y_2).c_1.c_2$ .

## 2.3 Snapshot Isolation

Snapshot isolation (SI) was introduced by Berenson et al. [8], then later generalized under the name GSI by Elnikety et al. [7]. In this paper, we make no distinction between SI and GSI.

Let us consider a function  $\mathcal{S}$  which takes as input a history  $h$ , and returns an extended history  $h_s$  by adding a *snapshot point* to  $h$  for each transaction in  $h$ . Given a transaction  $T_i$ , the snapshot point of  $T_i$  in  $h_s$ , denoted  $s_i$ , precedes every operation of transaction  $T_i$  in  $h_s$ . A history  $h$  is in SI if, and only if, there exists a function  $\mathcal{S}$  such that  $h_s = \mathcal{S}(h)$  and  $h_s$  satisfies the following rules:

### D1 (Read Rule)

$$\forall r_i(x_{j \neq i}), w_{k \neq j}(x_k), c_k \in h_s :$$

$$c_j \in h_s \quad (D1.1)$$

$$\wedge c_j <_{h_s} s_i \quad (D1.2)$$

$$\wedge (c_k <_{h_s} c_j \vee s_i <_{h_s} c_k) \quad (D1.3)$$

### D2 (Write Rule)

$$\forall c_i, c_j \in h_s :$$

$$ws(T_i) \cap ws(T_j) \neq \emptyset$$

$$\Rightarrow (c_i <_{h_s} s_j \vee c_j <_{h_s} s_i)$$

## 2.4 System

We consider a message-passing distributed system of  $n$  processes  $\Pi = \{p_1, \dots, p_n\}$ . We shall define our synchrony assumptions later. Following Fischer et al. [13], an execution is a sequence of steps made by one or more processes. During an execution, processes may fail by crashing. A process that does not crash is said

<sup>7</sup> Throughout the paper, read-only transactions are specified with an alphabet subscript, and update transactions are shown with numeric subscript.

*correct*; otherwise it is *faulty*. We note  $\mathfrak{F}$  the refinement mapping [14] from executions to histories, i.e., if  $\rho$  is an execution of the system, then  $\mathfrak{F}(\rho)$  is the history produced by  $\rho$ . A history  $h$  is *acceptable* if there exists an execution  $\rho$  such that  $h = \mathfrak{F}(\rho)$ . We consider that given two sequences of steps  $U$  and  $V$ , if  $U$  precedes  $V$  in some execution  $\rho$ , then the operations implemented by  $U$  precedes (in the sense of  $<_h$ ) the operations implemented by  $V$  in the history  $\mathfrak{F}(\rho)$ .<sup>8</sup>

## 2.5 Partial Replication

A data store  $\mathcal{D}$  is a finite set of tuples  $(x, v, i)$  where  $x$  is an object (data item),  $v$  a value, and  $i \in \mathcal{T}$  a version. Each process in  $\Pi$  holds a data store such that initially every object  $x$  has version  $x_0$ . For an object  $x$ ,  $Replicas(x)$  denotes the set of processes, or *replicas*, that hold a copy of  $x$ . By extension for some set of objects  $X$ ,  $Replicas(X)$  denotes the replicas of  $X$ ; given a transaction  $T_i$ ,  $Replicas(T_i)$  equals  $Replicas(rs(T_i) \cup ws(T_i))$ .

We make no assumption about how objects are replicated. The coordinator of  $T_i$ , denoted  $coord(T_i)$ , is in charge of executing  $T_i$  on behalf of some client (not modeled). The coordinator does not know in advance the read set or the write set of  $T_i$ . To model this, we consider that every prefix of a transaction (followed by a terminating operation) is a transaction with the same id.

Genuine Partial Replication (GPR) aims to ensure that, when the workload is parallel, throughput scales linearly with the number of nodes [10]:

- **GPR.** For any transaction  $T_i$ , only processes that replicate objects accessed by  $T_i$  make steps to execute  $T_i$ .

## 2.6 Progress

The read rule of SI does not define what is the snapshot to be read. According to Adya [6], “transaction  $T_i$ ’s snapshot point needs not be chosen after the most recent commit when  $T_i$  started, but can be selected to be some (convenient) earlier point.” To avoid that read-only transactions always observe outdated data, we add the following rule:

- **Non-trivial SI.** Consider an acceptable history  $h$  and a transaction  $T_i$  pending in  $h$  such that the next operation invoked by  $T_i$  is a read on some object  $x$ . Note  $x_j$  the latest committed version of  $x$  prior to the first operation of  $T_i$  in  $h$ . Let  $\rho$  be an execution satisfying  $\mathfrak{F}(\rho) = h$ . If  $h.r_i(x_j)$  belongs to SI then *there exists* an execution  $\rho'$  extending  $\rho$  such that in history  $\mathfrak{F}(\rho')$ , transaction  $T_i$  reads at least (in the sense of  $\ll_h$ ) version  $x_j$  of  $x$ .

In addition, we consider that the system provides the following progress guarantees on transactions:

<sup>8</sup> Notice that since steps to implement operations may interleave,  $<_h$  is not necessarily a total order.

- **Obstruction-free Updates.** For every update transaction  $T_i$ , if  $coord(T_i)$  is correct then  $T_i$  eventually terminates. Moreover, if  $T_i$  does not write-conflict with some concurrent transaction, then  $T_i$  eventually commits.
- **Wait-free Queries.** If  $coord(T_i)$  is correct and  $T_i$  is a read-only transaction, then transaction  $T_i$  eventually commits.

### 3 Decomposing SI

This section defines four properties, whose conjunction is necessary and sufficient to attain SI. We later use these properties in Section 4 to derive our impossibility result.

#### 3.1 Cascading Aborts

Intuitively, a read-only transaction must abort if it observes the effects of an uncommitted transaction that later aborts. By guaranteeing that every version read by a transaction is committed, rules D1.1 and D1.2 of SI prevent such a situation to occur. In other words, these rules *avoid cascading aborts*. We formalize this property below:

**Definition 1 (Avoiding Cascading aborts).** *History  $h$  avoids cascading aborts, if for every read  $r_i(x_j)$  in  $h$ ,  $c_j$  precedes  $r_i(x_j)$  in  $h$ . ACA denotes the set of histories that avoid cascading aborts.*

#### 3.2 Consistent and Strictly Consistent Snapshots

Consistent and strictly consistent snapshots are defined by refining causality into a dependency relation as follows:

**Definition 2 (Dependency).** *Consider a history  $h$  and two transactions  $T_i$  and  $T_j$ . We note  $T_i \triangleright T_j$  when  $r_i(x_j)$  is in  $h$ . Transaction  $T_i$  depends on transaction  $T_j$  when  $T_i \triangleright^* T_j$  holds.<sup>9</sup> Transaction  $T_i$  and  $T_j$  are independent if neither  $T_i \triangleright^* T_j$ , nor  $T_j \triangleright^* T_i$  hold.*

This means that a transaction  $T_i$  depends on a transaction  $T_j$  if  $T_i$  reads an object modified by  $T_j$ , or such a relation holds by transitive closure. To illustrate this definition, consider history  $h_3 = r_1(x_0).w_1(x_1).c_1.r_a(x_1).c_a.r_b(y_0).c_b$ . In  $h_3$ , transaction  $T_a$  depends on  $T_1$ . Notice that, even if  $T_1$  causally precedes  $T_b$ ,  $T_b$  does not depend on  $T_1$  in  $h_3$ .

We now define consistent snapshots with the above dependency relation. A transaction sees a consistent snapshot iff it observes the effects of all transactions it depends on [15]. For example, consider the history  $h_4 = r_1(x_0).w_1(x_1).c_1.r_2(x_1).r_2(y_0).w_2(y_2).c_2.r_a(y_2).r_a(x_0).c_a$ . In this history, transaction  $T_a$  does not see a

<sup>9</sup> We note  $\mathcal{R}^*$  the transitive closure of some binary relation  $\mathcal{R}$ .

consistent snapshot:  $T_a$  depends on  $T_2$ , and  $T_2$  also depends on  $T_1$ , but  $T_a$  does not observe the effect of  $T_1$  (i.e.,  $x_1$ ). Formally, consistent snapshots are defined as follows:

**Definition 3 (Consistent snapshot).** *A transaction  $T_i$  in a history  $h$  observes a consistent snapshot iff, for every object  $x$ , if (i)  $T_i$  reads version  $x_j$ , (ii)  $T_k$  writes version  $x_k$ , and (iii)  $T_i$  depends on  $T_k$ , then version  $x_k$  is followed by version  $x_j$  in the version order induced by  $h$  ( $x_k \ll_h x_j$ ). We write  $h \in \text{CONS}$  when all transactions in  $h$  observe a consistent snapshot.*

SI requires that a transaction observes the committed state of the data at some *point* in the past. This requirement is stronger than consistent snapshot. For some transaction  $T_i$ , it implies that (SCONSa) there exists a snapshot point for  $T_i$ , and (SCONSb) if transaction  $T_i$  observes the effects of transaction  $T_j$ , it must also observe the effects of all transactions that precede  $T_j$  in time. A history is called strictly consistent if both SCONSa and SCONSb hold.

To illustrate this, consider the following history:  $h_5 = r_1(x_0).w_1(x_1).c_1.r_a(x_1).r_2(y_0).w_2(y_2).c_2.r_a(y_2).c_a$ . Because  $r_a(x_1)$  precedes  $c_2$  in  $h_5$ ,  $y_2$  cannot be observed when  $T_a$  takes its snapshot. As a consequence, the snapshot of transaction  $T_a$  is not strictly consistent. This issue is disallowed by SCONSa. Now, consider history  $h_6 = r_1(x_0).w_1(x_1).c_1.r_2(y_0).w_2(y_2).c_2.r_a(x_0).r_a(y_2).c_a$ . Since  $c_1$  precedes  $c_2$  in  $h_6$  and transaction  $T_a$  observes the effect of  $T_2$  (i.e.,  $y_2$ ), it should also observe the effect of  $T_1$  (i.e.,  $x_1$ ). SCONSb prevents history  $h_6$  to occur.

**Definition 4 (Strictly consistent snapshot).** *Snapshots in history  $h$  are strictly consistent, when for any committed transactions  $T_i, T_j, T_{k \neq j}$  and  $T_l$ , the following two properties hold:*

- $\forall r_i(x_j), r_i(y_l) \in h : r_i(x_j) \not\prec_h c_l$  (SCONSa)
- $\forall r_i(x_j), r_i(y_l), w_k(x_k) \in h :$   
 $c_k <_h c_l \Rightarrow c_k <_h c_j$  (SCONSb)

We note *SCONS* the set of strictly consistent histories.

### 3.3 Snapshot Monotonicity

In addition, SI requires what we call monotonic snapshots. For instance, although history  $h_7$  below satisfies SCONS, this history does not belong to SI. Indeed, since  $T_a$  reads  $\{x_0, y_2\}$ , and  $T_b$  reads  $\{x_1, y_0\}$ , there is no extended history that would guarantee the read rule of SI.

$$\begin{array}{ccccc}
 h_7 = r_a(x_0) & \longrightarrow & r_1(x_0).w_1(x_1).c_1 & \longrightarrow & r_b(x_1).c_b \\
 & & \searrow & & \nearrow \\
 r_b(y_0) & \longrightarrow & r_2(y_0).w_2(y_2).c_2 & \longrightarrow & r_a(y_2).c_a
 \end{array}$$

SI requires monotonic snapshots. However, the underlying reason is intricate enough that some previous works [4, for instance] do not ensure this property, while claiming to be SI. Below, we introduce an ordering relation between snapshots to formalize snapshot monotonicity.

**Definition 5 (Snapshot precedence).** Consider a history  $h$  and two distinct transactions  $T_i$  and  $T_j$ . The snapshot read by  $T_i$  precedes the snapshot read by  $T_j$  in history  $h$ , written  $T_i \rightarrow T_j$ , when  $r_i(x_k)$  and  $r_j(y_l)$  belong to  $h$  and either (i)  $r_i(x_k) <_h c_l$  holds, or (ii) transaction  $T_i$  writes  $x$  and  $c_k <_h c_l$  holds.

For more illustration, consider histories  $h_8 = r_1(x_0).w_1(x_1).c_1.r_2(y_0).w_2(y_2).r_a(x_1).c_2.r_b(y_2).c_a.c_b$  and  $h_9 = r_1(x_0).w_1(x_1).c_1.r_a(x_1).c_a.r_2(x_1).r_2(y_0).w_2(x_2).w_2(y_2).c_2.r_b(y_2).c_b$ . In history  $h_8$ ,  $T_a \rightarrow T_b$  holds because  $r_a(x_1)$  precedes  $c_2$  and  $T_b$  reads  $y_2$ . In  $h_9$ ,  $c_1$  precedes  $c_2$  and both  $T_1$  and  $T_2$  modify object  $x$ . Thus,  $T_a \rightarrow T_b$  also holds. We define snapshot monotonicity using snapshot precedence as follows:

**Definition 6 (Snapshot monotonicity).** Given some history  $h$ , if the relation  $\rightarrow^*$  induced by  $h$  is a partial order, the snapshots in  $h$  are monotonic. We note *MON* the set of histories that satisfy this property.

According to this definition, both  $T_a \rightarrow T_b$  and  $T_b \rightarrow T_a$  hold in history  $h_7$ . Thus, history  $h_7$  does not belong to *MON*.

Non-monotonic snapshots are observed under update serializability [16], that is when queries observe consistent state, but only updates are serializable.

### 3.4 Write-Conflict Freedom

Rule D2 of SI forbids two concurrent write-conflicting transactions from both committing. Since in our model we assume that every write is preceded by a corresponding read on the same object, every update transaction depends on a previous update transaction (or on the initial transaction  $T_0$ ). Therefore, under SI, concurrent conflicting transactions must be independent:

**Definition 7 (Write-Conflict Freedom).** A history  $h$  is write-conflict free if two independent transactions never write to the same object. We denote by *WCF* the histories that satisfy this property.

### 3.5 The decomposition

Theorem 1 below establishes that a history  $h$  is in SI iff (1) every transaction in  $h$  sees a committed state, (2) every transaction in  $h$  observes a strictly consistent snapshot, (3) snapshots are monotonic, and (4)  $h$  is write-conflict free. A detailed proof appears in our companion technical report [11].

**Theorem 1.**  $SI = ACA \cap SCONS \cap MON \cap WCF$

To the best of our knowledge, this result is the first to prove that SI can be split into simpler properties. Theorem 1 also establishes that SI is definable on plain histories. This has two interesting consequences: (i) a transactional system does not have to explicitly implement snapshots to support SI, and (ii) one can compare SI to other consistency criterion without relying on a phenomena based characterization.<sup>10</sup>

<sup>10</sup> Contrary to, e.g., the work of Adya [6].



## 4 The impossibility of SI with GPR

This section leverages our previous decomposition result to show that SI is inherently non-scalable. In more details, we prove that none of MON, SCONSa or SCONsb is attainable in some asynchronous failure-free GPR system  $\Pi$  when updates are obstruction-free and queries are wait-free. To prove these results, we first characterize below histories acceptable by  $\Pi$ .

**Lemma 1.** *Let  $h = \mathfrak{F}(\rho)$  be an acceptable history by  $\Pi$  such that a transaction  $T_i$  is pending in  $h$ . Note  $X$  the set of objects accessed by  $T_i$  in  $h$ . Only processes in  $\text{Replicas}(X)$  make steps to execute  $T_i$  in  $\rho$ .*

*Proof.* (By contradiction.) Consider that a process  $p \notin \text{Replicas}(X)$  makes steps to execute  $T_i$  in  $\rho$ . Since the prefix of a transaction is a transaction with the same id, we can consider an extension  $\rho'$  of  $\rho$  such that  $T_i$  does not execute any additional operation in  $\rho'$  and  $\text{coord}(T_i)$  is correct in  $\rho'$ . The progress requirements satisfied by  $\Pi$  imply that  $T_i$  terminates in  $\rho'$ . However, process  $p \notin \text{Replicas}(X)$  makes steps to execute  $T_i$  in  $\rho'$ . A contradiction to the fact that  $\Pi$  is GPR.

We now state that monotonic snapshots are not constructable in  $\Pi$ . Our proof holds because objects accessed by a transaction are not known in advance.

**Theorem 2.** *No asynchronous failure-free GPR system implements MON*

*Proof.* (By contradiction.) Let us consider (i) four objects  $x, y, z$  and  $u$  such that for any two objects in  $\{x, y, z, u\}$ , their replica sets do not intersect; (ii) four queries  $T_a, T_b, T_c$  and  $T_d$  accessing respectively  $\{x, y\}$ ,  $\{y, z\}$ ,  $\{z, u\}$  and  $\{u, x\}$ ; and (iii) four updates  $T_1, T_2, T_3$  and  $T_4$  modifying respectively  $x, y, z$  and  $u$ .

Obviously, history  $r_b(y_0)$  is acceptable, and since updates are obstruction-free,  $r_b(y_0).r_2(y_0).w_2(y_2).c_2$  is also acceptable. Applying that  $\Pi$  satisfies non-trivial SI, we obtain that history  $r_b(y_0).r_2(y_0).w_2(y_2).c_2.r_a(x_0).r_a(y_2)$  is acceptable. Since  $T_a$  must be wait-free,  $h = r_b(y_0).r_2(y_0).w_2(y_2).c_2.r_a(x_0).r_a(y_2).c_a$  is acceptable as well. Using a similar reasoning, history  $h' = r_d(u_0).r_4(u_0).w_4(u_4).c_4.r_c(z_0).r_c(u_4).c_c$  is also acceptable. We note  $\rho$  and  $\rho'$  respectively two sequences of steps such that  $\mathfrak{F}(\rho) = h$  and  $\mathfrak{F}(\rho') = h'$ .

System  $\Pi$  is GPR. As a consequence, Lemma 1 tells us that only processes in  $\text{Replicas}(x, y)$  make steps in  $\rho$ . Similarly, only processes in  $\text{Replicas}(u, z)$  make steps in  $\rho'$ . By hypothesis,  $\text{Replicas}(x, y)$  and  $\text{Replicas}(u, z)$  are disjoint. Applying a classical indistinguishability argument [13, Lemma 1], both  $\rho'.\rho$  and  $\rho.\rho'$  are admissible by  $\Pi$ . Thus, histories  $h'.h = \mathfrak{F}(\rho'.\rho)$  and  $h.h' = \mathfrak{F}(\rho.\rho')$  are acceptable.

Since updates are obstruction-free, history  $h'.h.r_3(z_0).w_3(z_3).c_3$  is acceptable. Note  $U$  the sequence of steps following  $\rho'.\rho$  with  $\mathfrak{F}(U) = r_3(z_0).w_3(z_3).c_3$ . Observe that by Lemma 1  $\rho'.\rho.U$  is indistinguishable from  $\rho'.U.\rho$ . Then consider history  $\mathfrak{F}(\rho'.U.\rho)$ . In this history,  $T_b$  is pending and the latest version of object  $z$  is  $z_3$ . As a consequence, because  $\Pi$  satisfies non-trivial SI, there exists an extension of  $\rho'.U.\rho$  in which transaction  $T_b$  reads  $z_3$ . From the fact that queries are

wait-free and since  $\rho'.\rho.U$  is indistinguishable from  $\rho'.U.\rho$ , we obtain that history  $h_1 = h'.h.r_3(z_0).w_3(z_3).c_3.r_b(z_3).c_b$  is acceptable. We note  $U_1$  the sequence of steps following  $\rho'.\rho$  such that  $\mathfrak{F}(U_1)$  equals  $r_3(z_0).w_3(z_3).c_3.r_b(z_3).c_b$ .

With a similar reasoning, history  $h_2 = h'.h.r_1(x_0).w_1(x_1).c_1.r_d(x_1).c_d$  is acceptable. Note  $U_2$  the sequence satisfying  $\mathfrak{F}(U_2) = r_1(x_0).w_1(x_1).c_1.r_d(x_1).c_d$ .

Executions  $\rho'.\rho.U_1$  and  $\rho'.\rho.U_2$  are both admissible. Because  $\Pi$  is GPR, only processes in  $\text{Replicas}(y, z)$  (resp.  $\text{Replicas}(x, u)$ ) make steps in  $U_1$  (resp.  $U_2$ ). By hypothesis, these two replica sets are disjoint. Applying again an indistinguishably argument,  $\rho'.\rho.U_1.U_2$  is an execution of  $\Pi$ . Therefore, the history  $\hat{h} = \mathfrak{F}(\rho'.\rho.U_1.U_2)$  is acceptable. In this history, relation  $T_a \rightarrow T_b \rightarrow T_c \rightarrow T_d \rightarrow T_a$  holds. Thus,  $\hat{h}$  does not belong to MON. Contradiction.

Our next theorem states that SCONSb is not attainable. Similarly to Atiya et al. [17], our proof builds an infinite execution in which a query  $T_a$  on two objects never terminates. We first define a finite execution during which we interleave between any two consecutive steps to execute  $T_a$ , a transaction updating one of the objects read by  $T_a$ . We show that during such an execution, transaction  $T_a$  does not terminate successfully. Then, we prove that asynchrony allows us to continuously extend such an execution, contradicting the fact that queries are wait-free.

**Definition 8 (Flippable execution).** Consider two distinct objects  $x$  and  $y$ , a query  $T_a$  over both objects, and a set of updates  $T_{j \in [1, m]}$  accessing  $x$  if  $j$  is odd, and  $y$  otherwise. An execution  $\rho = U_1 V_2 U_2 \dots V_m U_m$  where,

- transaction  $T_a$  reads in history  $h = \mathfrak{F}(\rho)$  at least version  $x_1$  of  $x$ ,
- for any  $j$  in  $\llbracket 1, m \rrbracket$ ,  $U_j$  is the execution of transaction  $T_j$  by processes  $Q_j$ ,
- for any  $j$  in  $\llbracket 2, m \rrbracket$ ,  $V_j$  are steps to execute  $T_a$  by processes  $P_j$ , and
- both  $(Q_j \cap P_j = \emptyset) \oplus (P_j \cap Q_{j+1} = \emptyset)$  and  $Q_j \cap Q_{j+1} = \emptyset$  hold,

is called *flippable*.

**Lemma 2.** Let  $\rho$  be an execution admissible by  $\Pi$ . If  $\rho$  is flippable and histories accepted by  $\Pi$  satisfy SCONSb, query  $T_a$  does not terminate.

*Proof.* Let  $h$  be the history  $\mathfrak{F}(\rho)$ . In history  $h$  transaction  $T_j$  precedes transaction  $T_{j+1}$ , it follows that  $h$  is of the form  $h = w_1(x_1).c_1.*.w_2(y_2).c_2.*\dots$ , where each symbol  $*$  corresponds to either no operation, or to some read operation by  $T_a$  on object  $x$  or  $y$ .

Because  $\rho$  is flippable, transaction  $T_a$  reads at least version  $x_1$  of object  $x$  in  $h$ . For some odd natural  $j \geq 1$ , let  $x_j$  denote the version of object  $x$  read by  $T_a$ . Similarly, for some even natural  $l$ , let  $y_l$  be the version of  $y$  read by  $T_a$ . Assume that  $j < l$  holds. Therefore,  $h$  is of the form  $h = \dots w_j(x_j) \dots w_l(y_l) \dots$ .

Note  $k$  the value  $l + 1$ , and consider the sequence of steps  $V_k$  made by  $P_k$  right after  $U_l$  to execute  $T_a$ . Applying the definition of a flippable execution, we know that (F1)  $(Q_l \cap P_k = \emptyset) \oplus (P_k \cap Q_k = \emptyset)$ , and (F2)  $Q_l \cap Q_k = \emptyset$ . Consider now the following cases:

(Case  $Q_l \cap P_k = \emptyset$ .) It follows that  $\rho$  is indistinguishable from the execution  $\rho'' = \dots U_j \dots V_k U_l U_k \dots$ . Then from fact F2,  $\rho$  is indistinguishable from execution  $\rho' = \dots U_j \dots V_k U_k U_l \dots$

(Case  $P_k \cap Q_k = \emptyset$ ) With a similar reasoning, we obtain that  $\rho$  is indistinguishable from  $\rho' = \dots U_j \dots U_k U_l V_k \dots$

(Case  $P_k \cap (Q_l \cup Q_k) = \emptyset$ .) This case reduces to any of the two above cases. Note  $h'$  the history  $\mathfrak{F}(\rho')$ . Observe that since  $\rho'$  is indistinguishable from  $\rho$ , history  $h'$  is acceptable. In history  $h'$ ,  $c_k <_{h'} c_l$  holds. Moreover,  $c_j <_{h'} c_k$  holds by the assumption  $j < l$  and the fact that  $k$  equals  $l + 1$ . Besides, operations  $r_i(x_j)$ ,  $r_i(y_l)$  and  $w_k(x_k)$  all belong to  $h'$ . According to the definition of SCONSb, transaction  $T_a$  does not commit in  $h'$ . (The case  $j > l$  follows a symmetrical reasoning to the case  $l > j$  we considered previously.)

**Theorem 3.** *No asynchronous failure-free GPR system implements SCONSb.*

*Proof.* (By contradiction.) Consider two objects  $x$  and  $y$  such that  $\text{Replicas}(x)$  and  $\text{Replicas}(y)$  are disjoint. Assume a read-only transaction  $T_a$  that reads successively  $x$  then  $y$ . Below, we exhibit an execution admissible by  $\Pi$  during which transaction  $T_a$  never terminates. We build this execution as follows:

[Construction.] Consider some empty execution  $\rho$ . Repeat for all  $i \geq 1$ : Let  $T_i$  be an update of  $x$ , if  $i$  is odd, and  $y$  otherwise. Start the execution of transaction  $T_i$ . Since no concurrent transaction is write-conflicting with  $T_i$  in  $\rho$  and updates are obstruction-free, there must exist an extension  $\rho.U_i$  of  $\rho$  during which  $T_i$  commits. Assign to  $\rho$  the value of  $\rho.U_i$ . Execution  $\rho$  is flippable. Hence, Lemma 2 tells us that transaction  $T_a$  does not terminate in this execution. Consider the two following cases: (Case  $i = 1$ ) Because  $\Pi$  satisfies non-trivial SI, there exists an extension  $\rho'$  of  $\rho$  in which transaction  $T_a$  reads at least version  $x_1$  of object  $x$ . Notice that execution  $\rho'$  is of the form  $U_1.V_2.s\dots$  where (i) all steps in  $V_2$  are made by processes in  $\text{Replicas}(x)$ , and (ii)  $s$  is the first step such that  $\mathfrak{F}(U_1.V_2.s) = r_1(x_0).w_1(x_1).c_1.r_a(x_1)$ . Assign  $U_1.V_2$  to  $\rho$ . (Case  $i > 2$ ) Consider any step  $V_{i+1}$  to terminate  $T_a$  and append it to  $\rho$ .

Execution  $\rho$  is admissible by  $\Pi$ . Hence  $\mathfrak{F}(\rho)$  is acceptable. However, in this history transaction  $T_a$  does not terminate. This contradicts the fact that queries are wait-free.

SCONSa disallows some real time orderings between operations accessing different objects. Our last theorem shows that this property cannot be maintained under GPR.

**Theorem 4.** *No asynchronous failure-free GPR system implements SCONSa.*

*Proof.* (By contradiction.) Consider two distinct objects  $x$  and  $y$  such that  $\text{Replicas}(x)$  and  $\text{Replicas}(y)$  are disjoint. Let  $T_1$  be an update accessing  $y$ , and  $T_a$  be a query reading both objects.

Obviously, history  $h = r_a(x_0)$  is acceptable. Note  $U_a$  a sequence of steps satisfying  $U = \mathfrak{F}(r_a(x_0))$ . Because  $\Pi$  supports obstruction-free updates, we know the existence of an extension  $U_a.U_1$  of  $U_a$  such that  $\mathfrak{F}(U_1) = r_1(y_0).w_1(y_1).c_1$ . By Lemma 1, we observe that  $U_a.U_1$  is indistinguishable from  $U_1.U_a$ . Then, since  $\Pi$  satisfies non-trivial SI and read-only transactions are wait-free, there must exist an extension  $U_1.U_a.V_a$  of  $U_1.U_a$  admissible by  $\Pi$  and such that  $\mathfrak{F}(V_a) = r_a(y_1).c_a$ . Finally, since  $U_a.U_1$  is indistinguishable from  $U_1.U_a$  and  $U_1.U_a.V_a$  is admissible,  $U_a.U_1.V_a$  is admissible too. The history  $\mathfrak{F}(U_a.U_1.V_a)$  is not in SCONSa. Contradiction.

As a consequence of the above, no asynchronous system, even if it is failure-free, can support both GPR and SI. In particular, even if the system is augmented with failure detectors [18], a common approach to model partial synchrony, SI cannot be implemented under GPR. This fact strongly hinders the usage of SI at large scale. In the following section, we further discuss implications of this result.

## 5 Discussion

A straightforward corollary of any of the theorems we proved in Section 4 is that neither strict serializability [19], nor opacity [20] is attainable under GPR. In the case of opacity, this answers negatively to a problem posed by Peluso et al. [21].

The classical (non-genuine) solution for building strictly consistent monotonic snapshots is to use total order broadcast (e.g., [2, 3]).

When a transaction declares objects it accesses in advance, a GPR system can install a snapshot just after the start of the transaction. As a consequence, such an assumption sidesteps our impossibility result.

A transactional system  $\Pi$  is *permissive* with respect to a consistency criterion  $\mathcal{C}$  when every history  $h \in \mathcal{C}$  is acceptable by  $\Pi$ . Permissiveness [22] measures the optimal amount of concurrency a system allows. If we consider again histories  $h_1$  and  $h_2$  in the proof of Theorem 2, we observe that both histories are serializable. Hence, every system permissive with respect to SER accepts both histories. By relying on the very same argument as the one we exhibit to close the proof of Theorem 2, we conclude that no transactional system is both GPR and permissive with respect to SER. For instance, none of the systems presented in [10, 23] accept history  $h_{10} = r_1(x_0).w_1(x_1).c_1.r_2(x_0).r_2(y_0).w_2(y_2).c_2$ .

Recent distributed transactional systems (e.g., [9, 24]) support weaker consistency criteria than SI or SER. In particular, Walter [9] supports Parallel Snapshot Isolation (PSI). PSI is weaker than SI, and allows snapshots to be non-monotonic. But, it still requires SCONS<sub>a</sub> to be ensured. Sovran et al. justify the use of PSI in Walter by the fact that SI is too expensive in a geographically distributed environment [9, page 4]. Our impossibility result establishes that, in order to scale, a transactional system needs supporting both non-monotonic *and* non-strictly consistent snapshots.

## 6 Conclusion

Partial replication and genuineness are two key factors of scalability in replicated systems. This paper shows that ensuring snapshot isolation (SI) in a genuine partial replication system is impossible. To state this impossibility result, we prove that SI is decomposable into a set of simpler properties. We show that two of these properties, namely snapshot monotonicity and strictly consistent snapshots cannot be ensured. As a corollary of our results, a GPR system cannot support neither strict serializability, nor opacity.

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