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Optimal control of interbank contagion under complete information

Andreea Minca*, Agnès Sulem†

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Abstract

We study the optimal control of interbank contagion, when the government has complete information on interbank exposures. Financial institutions are prone to insolvency risk channeled through the network of exposures and to liquidity risk through bank runs. The government seeks to maximize, under budget constraints the total value of the financial system or, equivalently, to minimize the dead-weight loss induced by bank runs. The problem can be expressed as a convex optimization problem with a combinatorial aspect, tractable when the set of banks eligible for intervention is sufficiently, yet realistically, small.

Keywords: Systemic risk, Liquidity risk, Bank runs, Financial contagion, Financial Networks, Optimal Intervention.

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1 Introduction

During the recent financial crisis, systemic risk has emerged as a major concern for governments, financial regulators and risk managers. By contrast with the traditional approach in risk management, the focus is no longer on modeling and managing the risks faced by a single financial institution, but on taking into account the interrelations between financial institutions and the complex mechanisms of distress propagation.

Limiting systemic risk requires building new analytical and computational tools for macroprudential regulation. Most research in this area focuses on the important topic of systemic risk measurement and attribution. Giesecke and Kim (2011) propose such measures within the framework of reduced form models (in reduced form models, the ensemble of firms in the economy is represented as a portfolio in which failures arrive according to some exogenous process). Some works propose systemic risk measures within the framework of network models for insolvency risk in banking systems, see e.g. Lehar (2005), Amini et al. (2011), Cont et al. (2012). Blanchet and Shi (2012) extend the application field of the network framework to insurance-reinsurance markets. Chen et al. (2012) propose an axiomatic approach to systemic risk measures.

Network models for systemic risk have the advantage of being both structural (they integrate explicitly the details of banks balance sheets and the interbank exposures) and tractable. Such features are crucial for determining best regulatory responses to systemic crisis. A recent example in this sense is Rogers and Veraart (2012), who use networks as decision tools for the establishment of rescue consortia.

Our contribution is within the area of systemic risk management. We use network models to investigate a government's problem of optimal intervention in the form of equity infusions. This problem is motivated by the government interventions during the recent crisis, which took forms of recapitalizations, see e.g. Swagel (2009), Veronesi and Zingales (2010). The rationale and the gain from these equity infusion programs is an important topic of notable research in the finance literature, both empirical and theoretical, see Philippon and Skreta (2010), Veronesi and Zingales (2010), Bayazitova and Shivdasani (2012), Philippon and Schnabl (2009).

Several possible reasons for equity infusion programs have been advanced in this literature (see Veronesi and Zingales (2010)). One theory is that the government recapitalized a banking sector that restricted lending because of debt overhang. The resulting optimization program is investigated in Philippon and Schnabl (2009). The second possible reason is that government intervened in order to avoid runs by short term creditors, since runs destroy value and are inefficient. Runs by short term creditors played a central role during the crisis, see Gorton and Metrick (2012). In this paper we investigate this rationale.

Some of the questions we ask are: Is an equity infusion program such as the British equity infusion program or the US CPP (Capital Purchase Program) of 2008 justified when we take into account the runs of short term creditors? How to optimally inject equity in each bank? Is the intervention budget constraint saturated at the optimal solution?¹ To answer these questions, we

¹In the US, the intervention budget is imposed to the government by Congress vote, and therefore it is important to determine in a quantitative model whether this constraint is saturated.

build a model in which runs by short term creditors can be triggered by banks' exposure to insolvency risk channeled through the network. We set up an optimal equity infusion program by a government with constrained budget who aims at maximizing the total value of the financial system. The crucial ingredient is the fact that this value accounts for dead-weight losses induced by bank runs, on top of the writedowns of exposures to insolvent banks and losses to outside creditors.

The feature that dead-weight losses may drive firms into insolvency is a new feature that this model adds to network-based contagion literature. This also discussed in Diamond and Rajan (2011). The underlying network of exposures transmits losses among financial institutions, which then see their capital depleted and consequently may face withdrawals in the funding provided to them by market participants (e.g., money market funds). A run by short term creditors is a severe shortage in funding, causing a bank to fail. The failure due to bank runs of some participants prompts more writedowns to their counterparties, thus leading to large feedback effects.

The government problem is set under full information by which we mean that the controller observes the entire interbank exposure network. We also consider that only an exogenously chosen set of banks are eligible to receive government equity infusions. Indeed, the network of financial institutions contains banks, mutual funds, hedge funds, etc. and it is reasonable to assume that not all financial institutions are eligible for equity infusions. We will call the subset of institutions that are eligible to receive such equity infusions *core banks* which can be defined according to an exogenous rule, e.g., the top US banks in terms of assets. Finally, it is a model assumption that the decision of the government cannot be refused by the banks².

Our analysis (Proposition 9) shows that under the considered optimization program, all banks achieve a higher capital with intervention than without intervention. The increase in capitalization is due to the elimination of dead-weight losses in part and to a transfer of capital among the government and banks and among banks themselves. The transfer of capital from the banks to the government is done through a fixed dividend rate on the preferred equity received from the government. It follows that a pooling of losses occurs in the financial network in which these losses are first absorbed by the dividends paid by the banks, and then by the government.

We show that under the proposed optimization program, the government does not make equity infusions if the system is not prone to dead-weight losses induced by banks runs (Proposition 4). Our results offer the methodology to test the theory that the US equity infusion was set up to avoid bank runs. The fact that a government finds it optimal to make equity infusions to avoid banks runs does not necessarily create moral hazard and incentives for banks to expose themselves to bank runs. Indeed, if the dividend is sufficiently large, banks that are fundamentally healthy in absence of bank runs and that receive equity infusions to avoid bank runs effectively participate with the government in the pooling of losses. Since the equity infusion is larger when the bank has both a large amount of illiquid asset and a large fraction of unstable short term debt to total debt, it is in this case precisely that the bank will have to pay a larger dividend.

Our main result (Proposition 10) shows that the optimal equity infusion problem becomes a

²During the 2008 crisis, amid the Capital Purchase Program, the US banks that received equity infusions did not have the option to refuse the government decision, see e.g. Veronesi and Zingales (2010), Landler and Dash (2008). The Capital Assistance Program (CAP) replaced the Capital Purchase Program in 2009, offering participating banks redemption options. CAP securities valuation is investigated in Glasserman and Wang (2011).

convex optimization problem with a combinatorial aspect. In our numerical examples, the solution to the convex optimization problem shows that there exists a maximum budget beyond which the marginal cost from intervention surpasses its marginal gain. As a consequence, beyond a certain point, the budget constraint will not be saturated. This result indicates that if the government would make equity infusions beyond a certain limit, it would, in effect, transfer the losses from the existing creditors and shareholders of failed banks to the taxpayer.

The combinatorial aspect of the optimization problem becomes relevant when the continuum of losses of the paper by Eisenberg and Noe (2001a) is lost. This continuum is lost as soon as there are fixed and important bankruptcy costs, see e.g. Rogers and Veraart (2012), Amini et al. (2011). This aspect is highly relevant to systemic crisis which have an illiquidity component (such as the crisis of 2007 – 2009).

Our paper is organized as follows. In the next section we present a model for distress propagation in a network of financial institutions that are prone both to insolvency risk and to bank runs. In Section 2.3 we introduce the fixed point equation that determines the equilibrium asset-to-debt ratio for all banks and the equilibrium dead-weight losses. In Section 3 we state and solve the government optimization problem. In Section 4 we perform numerical experiments. The paper finishes with a technical appendix.

2 The model

We consider a one period network model for the cross-section of a financial system, $t = 0, 1$. The financial system is constituted of a set $[n] := \{1, \dots, n\}$ of interrelated financial institutions (“Banks”) and a continuum of outside creditors. There exists a government that makes instantaneous equity infusions in the form of cash at time 0.

At time 0, banks come with capital, they have debt to each other which matures at time 1 (long-term debt) and they have short-term debt to the continuum of outside creditors. The debt to the outside creditors is of two types, stable, i.e., it is not withdrawn at time 0, or *unstable*, i.e., the outside creditors may decide to withdraw funding, depending on the state of the financial system and the financial health of the bank in question.

On the asset side, bank hold investments in cash and an illiquid asset. The illiquid asset matures at time 1 and has a *fundamental price normalized at 1 at time 0*. However, if the asset needs to be prematurely liquidated at time 0, it is then valued at a *fire-sale* price $0 < p \leq 1$. The price of the asset at time 1 is unknown at time 0 and market participants may have different beliefs about a fundamental shock that may change the fundamental value of the illiquid asset from time 0 to time 1. The government expects that the price of the illiquid asset will be 1 at time 1, i.e., the government does not expect a shock in the fundamental price of the illiquid asset between times 0 and 1. We do not model directly the beliefs of the outside creditors, but we assume an exogenous short term creditor outflow schedule that incorporates these beliefs.

We are interested in the equilibrium reached at time 0.

2.1 Market participants and their interrelations

We now describe the financial system at time 0.

Debt. For a bank $i \in [n]$, ℓ_i denotes the sum of its long term debt and stable short term debt and s_i its *unstable short term debt*. A proportion π_{ij} of the debt ℓ_i is toward bank j , with $\sum_{j \in [n]} \pi_{ij} \leq 1$. The remaining fraction $1 - \sum_{j \in [n]} \pi_{ij}$ of ℓ_i gives the proportion of stable short term debt.

Assets. We let γ_i and y_i denote the holdings in cash and respectively the holdings in illiquid asset of bank i . One bank's interbank liability is another bank's interbank asset. The total value of the interbank assets of i is given by $\sum_{j \in [n]} \ell_j \pi_{ji}$.

The capital of a bank is given by the value of its assets minus the value of its liabilities. A bank is said to be **solvent** if the value of its assets is larger than the value of its liabilities, i.e., if its capital is positive. If no debtor bank is insolvent, the capital of bank i at time 0 is given by

$$c_i := y_i + \gamma_i + \sum_{j \in [n]} \ell_j \pi_{ji} - \ell_i - s_i.$$

An important quantity in the sequel will be **the asset-to-debt ratio**. If no debtor bank is insolvent, the asset-to-debt ratio of bank i at time 0 is given by

$$\begin{aligned} r_i &:= \frac{y_i + \gamma_i + \sum_{j \in [n]} \ell_j \pi_{ji}}{\ell_i + s_i} \\ &= 1 + \frac{c_i}{\ell_i + s_i}. \end{aligned}$$

The capital and asset-to-debt ratio determined under the assumption that no debtor banks are insolvent are called *fundamental* capital and asset-to-debt ratio.

The fundamental solvency condition for a bank i can be expressed in term of the fundamental asset-to-debt ratio as $r_i \geq 1$.

Clearly, if there exists a bank that is insolvent, then its creditors cannot account the full value of their interbank assets and their asset-to debt ratio is no longer determined as above. The *equilibrium asset-to-debt ratios* for all banks will be determined from a fixed point equation, as we show in the next section.

2.2 Financial contagion

If $r_i < 1$ bank i is fundamentally insolvent and all holdings of the illiquid asset are sold at the fire-sale price $p \leq 1$. The proceeds are distributed to creditors *proportionally* to the outstanding debt as in Eisenberg and Noe (2001a). The creditors of an insolvent bank will suffer a loss and, in turn, may become insolvent themselves.

Importantly, even if a bank is fundamentally solvent, i.e. $r_i \geq 1$, it may be forced to liquidate some of its illiquid asset holdings due to the outflow of short term creditors. We model the *short term creditor outflow schedule* by an exogenous function $g(\cdot) : [1, \infty) \rightarrow [0, 1]$ of the asset-to-debt ratio, on which we make the following assumption.

Assumption 1. *The short term creditor outflow schedule g has the following properties*

- (i) There exists $\bar{r} \geq 1$ such that $g(x) = 0$, for $x \geq \bar{r}$;
- (ii) $x \mapsto g(x)$ is non-increasing, twice differentiable and convex;
- (iii) $g' \geq -1$.

The first condition ensures that for sufficiently large asset-to-debt ratios there are no short term creditor outflows. The second condition states that the creditor outflow is decreasing with the asset-to-debt ratio. This assumption is reasonable, since banks with higher asset-to-debt ratios have lower future insolvency risk. The convexity assumption can be interpreted as an increase in the outflow rate as we approach the insolvency level 1. The last assumption is a tractability assumption that puts a lower bound on the withdrawal rate.

Importantly, the short term creditor outflow schedule is assumed to incorporate all the beliefs of the short-term creditors about the fundamental value of the illiquid asset at its maturity date 1.³

As a consequence of the short-term creditors behavior, captured by the short term creditor outflow schedule, a bank whose asset-to-debt ratio falls below \bar{r} may be forced to liquidate part of its risky asset holdings at the fire-sale price p .

Consider now a bank with asset-to-debt ratio x , total short-term debt s , cash holdings γ and illiquid asset holdings y . The short term creditors will claim back an amount $sg(x)$ of the short term debt. If the cash available to the bank cannot cover the amount $sg(x)$, then the bank will have to liquidate a quantity $\frac{(\gamma - sg(x))^-}{p} \wedge y$ of the illiquid asset. In doing so, the bank will effectively have a loss in its' asset value, since the liquidated value will achieve a price $p \leq 1$ instead of its fundamental price 1. The total loss on the asset side in this case is $\left(\frac{(\gamma - sg(x))^-}{p} \wedge y\right) (1 - p)$.

In our model, an insolvent bank is completely liquidated and its entire illiquid asset holdings are valued at the fire-sale price p and not at the fundamental value 1. In this case there will be a loss on the asset side equal to $y(1 - p)$.

The losses arising on the asset side as a result of liquidations are called dead-weight losses and are defined as follows.

Definition 1. We define the dead-weight loss of a bank with asset-to-debt ratio x , total short-term debt s , cash holdings γ and illiquid asset holdings y as follows

$$\delta(x, s, \gamma, y) := \begin{cases} \left(\frac{(\gamma - sg(x))^-}{p} \wedge y\right) (1 - p) & \text{if } x \geq 1, \\ y(1 - p) & \text{if } x < 1. \end{cases} \quad (1)$$

If $g \equiv 0$, dead-weight losses occur only when firms become insolvent. This is like in Rogers and Veraart (2012), where there are fixed liquidation costs when firms become insolvent.

If $g > 0$, dead-weight losses can occur due to withdrawal of short term creditors (part of the risky asset is *de facto* valued at the fire-sale price p). Unlike Rogers and Veraart (2012), these dead-weight losses may occur even if all banks are solvent.

³Modeling the roll-over coordination game of the short term creditors is beyond the scope of this paper, and thus the short term creditor outflow schedule is exogenous. For models of insolvency and rollover risk we refer the reader to Morris and Shin (2009), Lütkebohmert et al. (2012), He and Xiong (2012), Krishenik et al. (2012).

Note that the dead-weight losses will *decrease* the asset-to-debt ratio of the bank, which will entail a larger short-term creditor outflow and consequently more liquidations. It is possible that the asset-to-debt ratio reaches 1 as a result of this process, at which point the bank is insolvent. If $p < 1$ then the dead-weight loss has a jump when the asset-to-debt ratio reaches 1, since all the illiquid asset is then sold.

A bank i with asset-to-debt ratio $x_i < 1$, i.e. a bank that is insolvent, will generate a loss on the asset side of its creditors, irrespective how it has reached insolvency. The creditors will then receive a percentage $x_i < 1$ of their debt, and thus contagion ensues. Note in particular that for an insolvent bank, under the proportionality assumption, the asset-to-debt ratio represent *the recovery rate* of the creditors.

2.3 Equilibrium with government intervention

We consider now that **there exists a government that makes instantaneous equity infusions in the form of cash** at time 0. We assume that only a subset $\mathcal{C} \subseteq [n]$ are eligible for intervention. This set represents the set of core banks in the financial system.

The intervention budget of the government is M . The government injects *preferred equity* $\xi_i \in [0, M]$ in bank $i \in [n]$, with $\xi_i = 0$ for $i \in [n] \setminus \mathcal{C}$. After the (instantaneous) intervention, a bank $i \in [n]$ holds $\gamma_i + \xi_i$ of cash. The investment pays to the government a *preferred dividend* rate d at time 1. The equilibrium values of the asset-to-debt ratios of all banks in the system are jointly given by the fixed point of the following function, similar as in Eisenberg and Noe (2001b), Cifuentes et al. (2005), Amini et al. (2013), $\Phi(R(\xi), \xi) = R(\xi)$, of the non-linear map Φ

$$\Phi(x, \xi)_i = \frac{y_i - \delta(x_i, s_i, \gamma_i + \xi_i, y_i) + \gamma_i + \xi_i + \sum_{j \in [n]} \ell_j \pi_{ji} (x_j \wedge 1)}{\ell_i + s_i}. \quad (2)$$

The difference between the above map and the baseline specification in Eisenberg and Noe (2001b) is that only a fraction of the risky asset is valued at its fundamental price. The liquidated quantity is evaluated at the fire-sale price. This means that, *in equilibrium, there is a dependence between the value of the assets and the value and the stability of the liabilities*. This dependence is induced by the behavior of the short term creditors captured by the short term creditor outflow schedule g and the network of interbank liabilities. Of course, if $p = 1$, then the equilibrium dead-weight losses are equal to zero and the fixed point of the map Φ is the same as in Eisenberg and Noe (2001b). In this case, the short term creditor behavior does not matter.

We have the following lemma.

Lemma 2. *The mapping Φ is monotone and bounded.*

Proof. Fix ξ . We have that $x \rightarrow \Phi(x, \xi)_i$ is a non-increasing function of x . The fact that equilibrium asset-to-debt ratios are bounded by fundamental asset-to-debt ratios, i.e., $\Phi(0, \xi) \leq r$, implies that the map Φ is bounded, which concludes the proof. \square

Lemma 2 and Tarski's fixed-point theorem Tarski (1955) implies the existence of an equilibrium asset-to-debt ratio and consequently an equilibrium amount of liquidations. In the sequel we consider

the largest fixed point of the map Φ .

The *equilibrium dead-weight losses* of bank i are given by

$$\Delta_i(\xi) := \delta(R_i(\xi), s_i, \gamma_i + \xi_i, y_i).$$

The equilibrium capital of bank i , including the government share, is $(R_i(\xi) - 1)(\ell_i + s_i)$. The face value of the government share is $\xi_i(1 + d)$. Since the government equity share is junior to the outside debt, the *equilibrium value of the government investment* in bank i is

$$\begin{aligned} & (R_i(\xi) - 1)^+(\ell_i + s_i) \wedge \xi_i(1 + d) \\ &= \xi_i(1 + d) - ((R_i(\xi) - 1)^+(\ell_i + s_i) - \xi_i(1 + d))^-. \end{aligned} \quad (3)$$

The *equilibrium capital* of bank i , net of the face value of the government share, is then given by

$$\begin{aligned} C_i(\xi) &:= (R_i(\xi) - 1)(\ell_i + s_i) - \xi_i(1 + d) \\ &= y_i + \gamma_i + \sum_{j \in [n]} \ell_j \pi_{ji} (R_j(\xi) \wedge 1) - \Delta_i(\xi) - (s_i + \ell_i) - \xi_i d. \end{aligned} \quad (4)$$

The equilibrium capital in absence of government intervention is given by

$$C_i(0) = y_i P + \gamma_i + \sum_{j \in [n]} \ell_j \pi_{ji} (R_j(0) \wedge 1) - \Delta_i(0) - (s_i + \ell_i). \quad (5)$$

We obtain thus the difference in the capital with and without intervention

$$C_i(\xi) - C_i(0) = (\Delta_i(0) - \Delta_i(\xi)) + \sum_{j \in [n]} \ell_j \pi_{ji} (R_j(\xi) \wedge 1 - R_j(0) \wedge 1) - \xi_i d. \quad (6)$$

The first two terms are positive.

We now compute the total value of the government investment. We assume that the part of the budget non-invested in the banks bears a return rate to which we refer as *outside return rate*. One can think of this not as a risk free rate but rather relate it to the value that the government can create by directly funding projects using the budget M .

Assumption 2. *We assume that the outside return rate is equal to the dividend rate d paid by the banks.*

Since the problem setup gives the banks no option to refuse the government equity infusion, if the outside return rate were smaller than d , then the government would have an incentive to invest in a healthy bank in order to earn the preferred dividend. Also if the outside return rate were larger than d , then the government would provide capital “too cheaply” for the banks.

Using Eq. (3) and by the outside return rate d , we obtain the total value of the government investment

$$M(1+d) - \sum_{i \in [n]} ((R_i(\xi) - 1)^+(s_i + \ell_i) - \xi_i(1+d))^- . \quad (7)$$

The total value of the investment of the outside creditors is given by

$$\sum_{i \in [n]} (R_i(\xi) \wedge 1)(s_i + \ell_i(1 - \sum_{j \in [n]} \pi_{ij})) . \quad (8)$$

3 Government optimization problem

3.1 Problem statement

The government has a constrained budget M and their objective is to maximize the overall value of the financial system. Recall that the government assigns probability one to the illiquid asset having the same fundamental value 1 at time 1. The value of the financial system (to the government) is thus deterministic and defined as the total equilibrium capital of the banks given in Eq. (4) plus the total value of the government investment as given in Eq. (7) plus the value of the outside creditors's investment given in Eq. (8).

The total value of the financial system is given by

$$\begin{aligned} V(\xi) &:= \sum_{i \in [n]} \left(y_i + \gamma_i + \sum_{j \in [n]} \ell_j \pi_{ji} (R_j(\xi) \wedge 1) - \Delta_i(\xi) - (\ell_i + s_i) - \xi_i d \right) \\ &\quad + \sum_{i \in [n]} (R_i(\xi) \wedge 1)(s_i + \ell_i(1 - \sum_{j \in [n]} \pi_{ij})) \\ &\quad + M(1+d) - \sum_{i \in [n]} ((R_i(\xi) - 1)^+(\ell_i + s_i) + \xi_i(1+d))^- \\ &= \sum_{i \in [n]} (y_i + \gamma_i) + M(1+d) \\ &\quad - \sum_{i \in [n]} \Delta_i(\xi) - \sum_{i \in [n]} (R_i(\xi) - 1)^-(s_i + \ell_i) \\ &\quad - \sum_{i \in [n]} \left(\xi_i d + ((R_i(\xi) - 1)^+(\ell_i + s_i) - \xi_i(1+d))^- \right) . \end{aligned} \quad (9)$$

Note that $\sum_{i \in [n]} (y_i + \gamma_i) + M(1+d)$ represents the fundamental value of the financial system (including the government intervention funds valued at the outside return rate). We let

$$\begin{aligned} L(\xi) &:= \sum_{i \in [n]} \Delta_i(\xi) + \sum_{i \in [n]} (R_i(\xi) - 1)^-(s_i + \ell_i) \\ &\quad + \sum_{i \in [n]} \left(\xi_i d + ((R_i(\xi) - 1)^+(\ell_i + s_i) + \xi_i(1+d))^- \right) . \end{aligned} \quad (10)$$

Since the total value in the system can be written as

$$V(\xi) = \sum_{i \in [n]} (y_i + \gamma_i) + M(1 + d) - L(\xi), \quad (11)$$

it follows that $L(\xi)$ can be interpreted as the loss from the fundamental value of the financial system.

We define the set of **admissible intervention strategies** as

$$\mathcal{A}_M := \left\{ \xi \mid \forall i \in [n] \setminus \mathcal{C}, \xi_i = 0; \sum_{i \in \mathcal{C}} \xi_i \leq M \right\}. \quad (12)$$

We consider the following problem of optimal equity infusions with maximum budget M

$$\underset{\xi \in \mathcal{A}_M}{\text{Maximize}} V(\xi)$$

or equivalently, using Eq. (11),

Problem 3.

$$\mathcal{P}_M := \underset{\xi \in \mathcal{A}_M}{\text{Minimize}} L(\xi).$$

We also make the following assumption.

Assumption 3. *If there are multiple solutions to the above problem, then the government prefers the one that maximizes the value of its investment in the banks.*

3.2 Problem solution

Consider a solution ξ^* to the government optimization problem. For any i such that $R_i(\xi^*) < 1$ we have that $\Delta_i(\xi^*)$ does not depend on ξ_i^* . If $\xi_i^* > 0$ we would have a direct transfer from the government to the outside creditors which would not increase the value of the system. Thus we immediately check that $R_i(\xi^*) < 1$ implies that $\xi_i^* = 0$, i.e., the government never invests in an insolvent bank.

The following proposition is without surprise and can be immediately checked. There are no interventions if the fire-sale price of the illiquid asset is the same as the fundamental price since in this case there are no dead-weight losses to reduce. Also, there are no intervention if all banks are initially solvent and liquid.

Proposition 4. *If $p = 1$ or for all $i \in [n]$ we have $r_i \geq 1$ and $\gamma_i \geq s_i g(r_i)$, then any solution ξ^* to \mathcal{P}_M is no intervention, i.e., $\xi^* \equiv 0$.*

In the main result of the paper Proposition 10 we quantify the optimal intervention in the case with a fire-sale price below the fundamental value and when there is a bank that is fundamentally insolvent or illiquid.

In order to be able to give the solution, we break the problem and each sub-problem represents the optimal equity infusion problem when a given set is guaranteed to be collectively solvent. As we will see, each sub-problem will be decoupled from the network fixed point.

Optimal equity infusion problem when a given set is guaranteed to be collectively solvent. We define, for a set of banks $S \subseteq \mathcal{C}$, the subset of admissible intervention strategies in which we allow only the banks in S to receive government intervention

$$\forall S \subseteq \mathcal{C}, \mathcal{A}_S = \{\xi \in \mathcal{A}_M, \forall i \in S, \xi_i > 0, R_i(\xi) \geq 1, \forall i \in [n] \setminus S, \xi_i = 0\}.$$

The constraint $R_i(\xi) \geq 1$ is set since the government does not invest in a bank that is insolvent in equilibrium.

Fix now an intervention strategy $\xi \in \mathcal{A}_S$. We make the crucial point that, for $i \in [n] \setminus S$, the recovery rate $R_i(\xi)$ can be computed as a fixed point of the following function

$$\phi(x, S)_{i, i \in [n] \setminus S} = \frac{y_i - \delta(x_i, s_i, \gamma_i, y_i) + \sum_{j \in S} \ell_j \pi_{ji} + \sum_{j \notin S} \ell_j \pi_{ji} (x_j \wedge 1)}{\ell_i + s_i}. \quad (13)$$

The above function is a modified version of the function Φ in which the set of banks S is guaranteed to be collectively solvent, i.e., the asset-to-debt ratio is larger than 1. We denote its largest fixed point $(R_i^S)_{i \in [n] \setminus S}$. In other words, we have established the recovery rates for the banks in $[n] \setminus S$, when the set of banks S is guaranteed to be collectively solvent and these asset-to-debt ratios are constant on the set of intervention strategies \mathcal{A}_S , i.e., for all $i \in [n] \setminus S$ we have

$$R_i(\xi) = R_i^S, \forall \xi \in \mathcal{A}_S.$$

We now note that for all $i \in S$ and $\forall \xi \in \mathcal{A}_S$ the asset-to-debt ratio of bank i is given as the fixed point of the real-valued function $f^S(\cdot, \xi_i) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$

$$f^S(x, \xi_i) = \frac{y_i - \delta(x, s_i, \gamma_i + \xi_i, y_i) + \gamma_i + \xi_i + \sum_{j \in S} \ell_j \pi_{ji} + \sum_{j \notin S} \ell_j \pi_{ji} (R_j^S \wedge 1)}{\ell_i + s_i}. \quad (14)$$

We have the following immediate result.

Proposition 5. Fix $i \in S$. We let $\underline{\xi}_i^S := \inf\{\xi_i \mid f^S(1, \xi_i) \geq 1\}$.

$$\mathcal{A}_S = \begin{cases} \emptyset & \text{if } \sum_{i \in S} \underline{\xi}_i^S \geq M, \\ \{\xi \in \mathbb{R}^n \mid \forall i \in [n] \setminus S, \xi_i = 0, \forall i \in S, \xi_i \geq \underline{\xi}_i^S, \sum_{i \in S} \xi_i \leq M\} & \text{else.} \end{cases} \quad (15)$$

If $\mathcal{A}_S \neq \emptyset$, then there is a unique fixed point larger than 1 for the map $f^S(x)$. We denote this fixed point by $R_i^S(\xi_i)$ and we have $R_i(\xi) = R_i^S(\xi_i)$.

Proof. From the fact that g is decreasing in ξ_i , we have that $\forall x, \xi_i \rightarrow f^S(x, \xi_i)$ is increasing with $\lim_{\xi_i \rightarrow \infty} f^S(x, \xi_i) = \infty$. For ξ_i sufficiently large, $f^S(1, \xi_i) \geq 1$.

We now fix $\xi \in \mathcal{A}_S \neq \emptyset$. We let

$$I_i^S(\xi_i) := \left(\frac{1}{p} - 1\right) \mathbf{1}_{0 < s_i g(R_i^S(\xi_i)) - \gamma_i - \xi_i < p y_i}.$$

We have

$$\frac{\partial f^S(x, \xi_i)}{\partial x} = -\frac{\partial \delta}{\partial x} \delta(x, s_i, \gamma_i + \xi_i, y_i) / (\ell_i + s_i) = -s_i g'(x) I_i^S(\xi_i) / (\ell_i + s_i) \geq 0.$$

From the Assumption 1 (iii) it also follows that $f^S(x) \leq s_i / (\ell_i + s_i) < 1$. We also have

$$\frac{\partial^2 f^S(x, \xi_i)}{\partial x^2} = -\frac{\partial \delta}{\partial x} \delta(x, s_i, \gamma_i + \xi_i, y_i) \leq 0,$$

where the inequality holds thanks to the convexity of the function g (Assumption 1 (ii)).

It follows thanks to the continuity, concavity, the bound of the slope and the asymptote at ∞ that the function f has either no fixed point or one fixed point, depending on whether $f^S(1, \xi_i)$ is smaller or greater than 1. Since $f^S(1, \xi_i) > 1$ by the assumption that $\xi \in \mathcal{A}_S \neq \emptyset$, there is a unique fixed point. \square

The previous proposition shows in particular that for $i \in S$ and $\xi \in \mathcal{A}_S$, $R_i(\xi)$ depends on ξ only through ξ_i .

For $i \in S$, the dead-weight losses are given by

$$\Delta_i(\xi) = \Delta_i^S(\xi_i) := \delta(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i). \quad (16)$$

From Eq. (10) and noting that $R_i^S(\xi_i) > 1$ for $i \in S$, we have

$$\begin{aligned} L^S(\xi) &= \sum_{i \in S} \left(\Delta_i^S(\xi_i) + \xi_i d + ((R_i^S(\xi_i) - 1)(\ell_i + s_i) - \xi_i(1 + d))^- \right) \\ &\quad + \sum_{i \in [n] \setminus S} \Delta_i^S + \sum_{i \in [n] \setminus S} (R_i^S - 1)^- (s_i + \ell_i). \end{aligned} \quad (17)$$

For a bank $i \in S$. We define the contribution of bank i to the loss in system fundamental value as

$$L_i^S(\xi) := \Delta_i^S(\xi_i) + \xi_i d + ((R_i^S(\xi_i) - 1)(\ell_i + s_i) - \xi_i(1 + d))^- . \quad (18)$$

Here we have divided the loss on the sets of banks in S (first two lines above) and respectively $[n] \setminus S$ (last line above). Remark in particular that the loss on banks in $[n] \setminus S$ is constant over \mathcal{A}_S .

We have the following lemma.

Lemma 6. *The map $\xi_i \rightarrow \Delta_i^S(\xi_i) + \xi_i d$ is convex over \mathcal{A}_S . The map $\xi_i \rightarrow L_i^S(\xi_i)$ is convex over \mathcal{A}_S .*

The proof of this lemma is given in Appendix A.

Remark 7. *If $\frac{d\Delta_i^S(\xi_i)}{d\xi_i} + d \geq 0$ then $\xi_i = \underline{\xi}_i$, i.e., the equity infusion is equal to to the minimum equity*

infusion for which the bank is solvent. The gradient $\frac{dL^S(\xi)}{d\xi_i} \leq 0$ at the minimum equity infusion is decreasing with the ratio of unstable short term debt to total debt $\frac{s_i}{s_i+l_i}$ whenever $g'(1) < 0$. This shows that banks with larger fraction of unstable short term debt receive equity infusions in priority, in order to minimize dead-weight losses.

We consider now the problem

$$\begin{aligned} \mathcal{P}_S &:= \underset{\xi \in \mathcal{A}_S}{\text{Minimize}} && L^S(\xi) \\ &= \underset{\xi \in \mathcal{A}_S}{\text{Minimize}} && \sum_{i \in S} L_i^S(\xi) + \sum_{i \in [n] \setminus S} \Delta_i^S + \sum_{i \in [n] \setminus S} (R_i^S - 1)^-(s_i + \ell_i), \end{aligned}$$

where $L^S(\xi_i)$ is defined in Eq. (17) and $L_i^S(\xi)$ is defined in Eq. (18).

Thanks to Lemma 6, this problem is a convex optimization problem.

This problem is also decoupled from the network fixed point. It has a unique solution whose value we denote $L^*(S)$.

The case when the set of core banks is guaranteed to be collectively solvent. Of particular interest is the solution of \mathcal{P}_C , i.e., all core banks are solvent. We assume that the budget is sufficiently large such that a nontrivial solution to \mathcal{P}_C exists, i.e., $\sum_{i \in C} \xi_i^C \leq M$.

We denote by ξ_i^* the minimum of the convex function $L_i^C(\xi_i)$ on the interval $\underline{\xi}_i^C$. Thanks to Eq. (20) and (21) it is defined by

$$\begin{cases} \frac{1+I_i^C(\xi_i)}{1+\frac{s_i}{\ell_i+s_i}g'(R_i^C(\xi_i))I_i^C(\xi_i)} = 1+d & \text{if } \frac{1+I_i^C(\xi_i^C)}{1+\frac{s_i}{\ell_i+s_i}g'(1)I_i^C(\xi_i^C)} \leq 1+d, \\ \underline{\xi}_i & \text{else.} \end{cases}$$

Remark 8. If $\sum_{i \in C} \xi_i^* \leq M$ then it immediately follows that the budget constraint is not saturated.

Otherwise it is given by the dual problem

$$\underset{\lambda \in \mathbb{R}, \xi_i \geq \underline{\xi}_i^C}{\text{Minimize}} \sum_i L_i^C(\xi_i) + \lambda(\sum_i \xi_i - M).$$

and the solution is given by

$$\xi_i^*(\lambda) := \begin{cases} \frac{1+I_i^C(\xi_i)}{1+\frac{s_i}{\ell_i+s_i}g'(R_i^C(\xi_i))I_i^C(\xi_i)} = 1+d+\lambda & \text{if } \frac{1+I_i^C(\xi_i^C)}{1+\frac{s_i}{\ell_i+s_i}g'(1)I_i^C(\xi_i^C)} \leq 1+d, \\ \underline{\xi}_i & \text{else,} \end{cases}$$

with the Lagrange multiplier λ obtained from the equation

$$\sum_{i \in C} \xi_i^*(\lambda) = M. \tag{19}$$

Main results. We now state the two main results of the paper. The first proposition states that, whatever the set of banks that are guaranteed not to fail, the government equity infusion is such that the capital of *all banks* increases after intervention.

Proposition 9. *Let ξ^* the solution to the problem \mathcal{P}_S . Then, for all $i \in [n]$ we have $C_i(\xi^*) \geq C_i(0)$.*

The proof is given in Appendix B.

The next proposition gives the solution to the government optimization problem.

Proposition 10. *Assume an illiquid asset with fire-sale price $p < 1$ and assume that there exists a bank i fundamentally insolvent or illiquid, i.e., $r_i \geq 1$ or $\gamma_i < s_i g(r_i)$. Let S^* the minimizer of $L^*(S)$ over $\mathcal{2}$ and ξ^* the solution to \mathcal{P}_{S^*} . Then ξ^* is the solution to the optimal equity infusion problem \mathcal{P} .*

The solution has a combinatorial aspect, requiring to assess for each set of eligible banks the cost of a collective guarantee. The solution is tractable when the number of banks that are eligible for infusions is sufficiently small. In the next section we study numerically the solution. In particular, it may be that it is not optimal to guarantee the whole set of core banks to be collectively solvent.

4 Numerical experiments

In the numerical examples we illustrate our theoretical findings on a simulated network generated similarly as in Amini et al. (2011).

Figure 1 illustrates the idea behind our solution to the optimal equity infusion problem. Each point represents a set of banks that are guaranteed to be collectively solvent (in total $2^{|\mathcal{C}|}$ such sets) and the value of the solution for the corresponding sub-problem. For each set we plot the total loss in the system versus the total equity infusion (when the respective set is guaranteed to be solvent).

We observe that the budget constraint is not saturated at the optimal solution, see Remark 8. This result suggests that injecting equity beyond a certain limit would not serve to mitigate systemic risk.

We next plot in Figure 2 the optimal value of the problem \mathcal{P}_M for varying M . We observe that the loss in the system is significantly mitigated as we increase M and that this effect decreases with the maximum budget M .

We now fix M and vary the liquidation price p (we could vary alternatively the parameters for the creditor outflow schedule so as to increase the outflow). We plot, for each value of the liquidation price p , the optimal equity infusion and the value of the solution. The results are shown in Figure 3.

The numerical results corroborate the theoretical results, namely that in absence of dead-weight losses, i.e., $p = 1$, there is no intervention, see Proposition 4. As the liquidation price decreases, the amount of equity infusions increases.

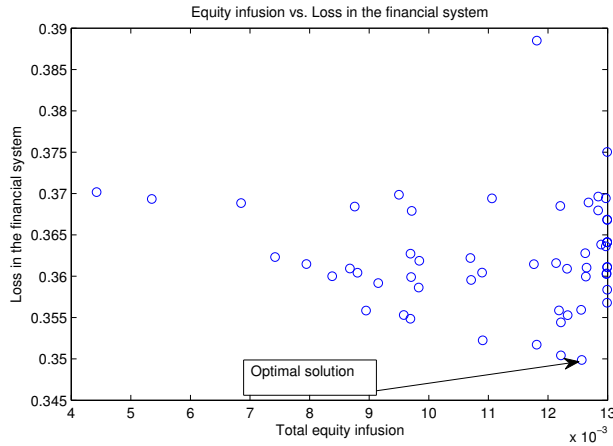


Figure 1: Total loss in the system vs. total equity infusion. Each point represents a set of banks that are guaranteed to be collectively solvent. All quantities are expressed as a percentage of the fundamental value of the financial system.

5 Conclusions

We analyzed the optimal equity infusions of a government with constrained budget. Our numerical results show that beyond a certain threshold for the intervention budget, the budget constraint is not saturated. We found that contagion is significantly mitigated by intervention.

Our results offer a methodology to test the theory that the US government equity infusion program in 2008 had as rationale avoiding bank runs, since runs induce dead-weight losses in the system.

There exist, of course, other kinds of interventions for supporting liquidity which do not take the form of equity infusions, but that of loans or credit guarantees. These are important topics for future research, which can be studied with methods similar to the current paper. Equity infusions in 2008, on the other hand, constituted the largest ever U.S. government intervention in the financial sector, and our results are intended to be used as tools for the decision process in this type of interventions.

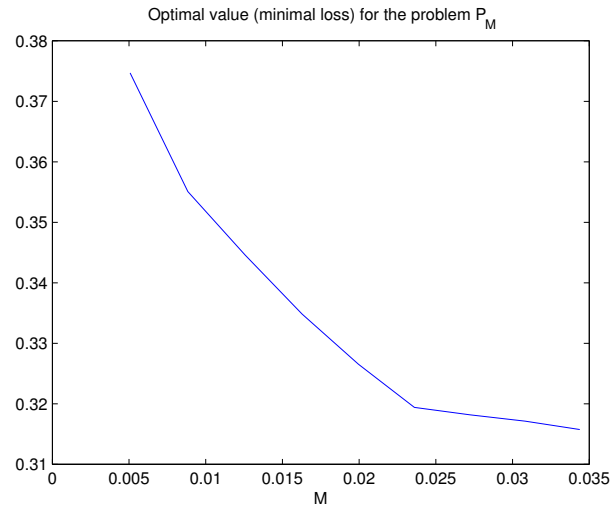


Figure 2: Optimal value for the problem \mathcal{P}_M for varying M .

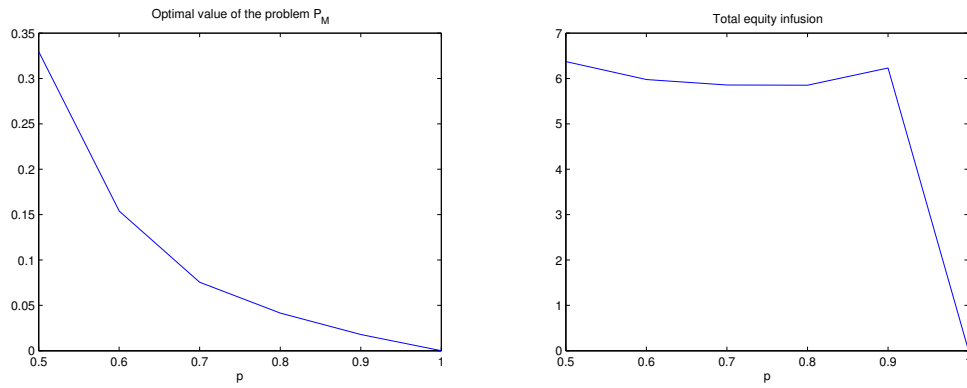


Figure 3: Optimal value and optimal total equity infusion for the problem \mathcal{P}_M for varying p .

A Proof of Lemma 6

Proof. From the definition of $R_i^S(\xi_i)$ as the unique fixed point in the interval $[1, \infty)$ in Eq. (14) we have

$$\begin{aligned}
(\ell_i + s_i) \frac{dR_i^S(\xi_i)}{d\xi_i} &= -\frac{\partial\delta}{\partial x}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i) \frac{dR_i^S(\xi_i)}{d\xi_i} - \frac{\partial\delta}{\partial\gamma}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i) + 1 \\
(\ell_i + s_i) \frac{d^2R_i^S(\xi_i)}{(d\xi_i)^2} &= -\frac{\partial\delta}{\partial x}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i) \frac{d^2R_i^S(\xi_i)}{(d\xi_i)^2} \\
&\quad - \frac{\partial^2\delta}{\partial x^2}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i) \left(\frac{dR_i^S(\xi_i)}{d\xi_i}\right)^2 \\
&\quad - 2\frac{\partial^2\delta}{\partial x\partial\gamma}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i) \frac{dR_i^S(\xi_i)}{d\xi_i} \\
&\quad - \frac{\partial^2\delta}{\partial\gamma^2}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i).
\end{aligned}$$

We let

$$I_i^S(\xi_i) := \left(\frac{1}{p} - 1\right) \mathbb{1}_{0 < s_i g(R_i^S(\xi_i)) - \gamma_i - \xi_i < p y_i}.$$

We obtain, using Assumption 1 (iii), that

$$\begin{aligned}
\frac{dR_i^S(\xi_i)}{d\xi_i} &= \frac{1 - \frac{\partial\delta}{\partial\gamma}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i)}{\ell_i + s_i + \frac{\partial\delta}{\partial x}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i)} \\
&= \frac{1 + I_i^S(\xi_i)}{\ell_i + s_i + s_i g'(R_i^S(\xi_i)) I_i^S(\xi_i)} \\
&\geq 0, \\
\frac{d^2R_i^S(\xi_i)}{(d\xi_i)^2} &= \frac{-\frac{\partial^2\delta}{\partial x^2}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i) \left(\frac{dR_i^S(\xi_i)}{d\xi_i}\right)^2}{\ell_i + s_i + \frac{\partial\delta}{\partial x}(R_i^S(\xi_i), s_i, \gamma_i + \xi_i, y_i)} \\
&\quad - \frac{-s_i g''(R_i^S(\xi_i)) I_i^S(\xi_i) \left(\frac{dR_i^S(\xi_i)}{d\xi_i}\right)^2}{\ell_i + s_i + s_i g'(R_i^S(\xi_i)) I_i^S(\xi_i)} \\
&\leq 0.
\end{aligned}$$

It then immediately follows

$$\begin{aligned}
\frac{d\Delta_i^S(\xi_i)}{d\xi_i} &= 1 - (\ell_i + s_i) \frac{dR_i^S(\xi_i)}{d\xi_i} \\
&= 1 - \frac{1 + I_i^S(\xi_i)}{1 + \frac{s_i}{\ell_i + s_i} g'(R_i^S(\xi_i)) I_i^S(\xi_i)} \\
&= \frac{I_i^S(\xi_i) \left(\frac{s_i}{\ell_i + s_i} g'(R_i^S(\xi_i)) - 1 \right)}{1 + \frac{s_i}{\ell_i + s_i} g'(R_i^S(\xi_i)) I_i^S(\xi_i)} \\
&\leq 0; \\
\frac{d^2\Delta_i^S(\xi_i)}{(d\xi_i)^2} &= -(\ell_i + s_i) \frac{d^2R_i^S(\xi_i)}{(d\xi_i)^2} \geq 0.
\end{aligned} \tag{20}$$

We have obtained the first part of the lemma, namely that the map

$$\xi_i \rightarrow \Delta_i^S(\xi_i) + \xi_i d$$

is convex. We now show the second part of the lemma. We have

$$\begin{aligned}
\frac{dL_i^S(\xi)}{d\xi_i} &= d + \frac{d\Delta_i^S(\xi_i)}{d\xi_i} + \mathbf{1}_{(R_i^S(\xi_i)-1)(\ell_i+s_i) < \xi_i(1+d)} \left(-(\ell_i + s_i) \frac{dR_i^S(\xi_i)}{d\xi_i} + 1 + d \right) \\
&= d + \frac{d\Delta_i^S(\xi_i)}{d\xi_i} + \mathbf{1}_{(R_i^S(\xi_i)-1)(\ell_i+s_i) < \xi_i(1+d)} \left(\frac{d\Delta_i^S(\xi_i)}{d\xi_i} + d \right) \\
&= \left(\frac{d\Delta_i^S(\xi_i)}{d\xi_i} + d \right) (1 + \mathbf{1}_{(R_i^S(\xi_i)-1)(\ell_i+s_i) < \xi_i(1+d)}). \\
\frac{d^2L_i^S(\xi)}{(d\xi_i)^2} &= \frac{d^2\Delta_i^S(\xi_i)}{(d\xi_i)^2} (1 + \mathbf{1}_{(R_i^S(\xi_i)-1)(\ell_i+s_i) < \xi_i(1+d)}) \\
&\geq 0.
\end{aligned} \tag{21}$$

In the second line above we have used Eq. (20). In the last inequality we have used the first part of the lemma. This completes the proof of the lemma. \square

B Proof of Proposition 9

Proof. In light of Eq. (6), we can write the condition $C_i(\xi) \geq C_i(0)$ as

$$\Delta_i(\xi^*) + \xi_i^* d \leq \Delta_i(0) + \sum_{j \in [n]} \ell_j \pi_{ji} (R_j(\xi^*) \wedge 1 - R_j(0) \wedge 1). \tag{22}$$

which becomes a simple constraint on ξ_i^* as follows

$$\begin{aligned}
\forall i \in S, \Delta_i^S(\xi_i^*) + \xi_i^* d &\leq \\
\Delta_i(0) + \sum_{j \in S} \ell_j \pi_{ji} (1 - R_j(0) \wedge 1) &+ \sum_{j \in [n] \setminus S} \ell_j \pi_{ji} (R_j^S \wedge 1 - R_j(0) \wedge 1).
\end{aligned}$$

Suppose now by way of contradiction that there exists i such that $C_i(\xi) < C_i(0)$, i.e.,

$$\Delta_i(\xi^*) + \xi_i^* d > \Delta_i(0) + \sum_{j \in [n]} \ell_j \pi_{ji} (R_j(\xi^*) \wedge 1 - R_j(0) \wedge 1).$$

This inequality cannot hold for $i \in [n] \setminus S$, who receive no infusion but have indirect gains from equity infusions in their debtors.

Suppose that above inequality holds for $i \in S$. Thanks to Lemma 6, the map $\xi_i \rightarrow \Delta_i(\xi_i) + \xi_i d$ is convex on \mathcal{A}_S . Since $\Delta_i(\xi^*) + \xi_i^* d \geq \Delta_i(0)$, it follows that the map attains its minimum for a $\xi_i' < \xi_i^*$. But since ξ_i^* is a solution to \mathcal{P}_S , it must be that

$$((R_i^S(\xi_i') - 1)(\ell_i + s_i) - \xi_i'(1 + d))^- \geq ((R_i^S(\xi_i^*) - 1)(\ell_i + s_i) - \xi_i^*(1 + d))^-.$$

This is equivalent to $(R_i^S(\xi_i') - 1)(\ell_i + s_i) - \xi_i'(1 + d) \leq 0$ and

$$(R_i^S(\xi_i') - 1)(\ell_i + s_i) - \xi_i'(1 + d) \leq (R_i^S(\xi_i^*) - 1)(\ell_i + s_i) + \xi_i^*(1 + d),$$

i.e. $\Delta_i^S(\xi_i') + \xi_i' d \geq \Delta_i^S(\xi_i^*) - \xi_i^* d$. If ξ_i' is a strict minimum, then we directly obtain a contradiction. If with ξ_i' is not a strict minimum then we have a contradiction with Assumption 3. \square

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