Signaling Game-based Approach to Power Control Management in Wireless Networks

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ABSTRACT

In this paper, we revisit the power control problem in wireless networks by introducing a signaling game approach. This game is known in the literature as "Cheap Talk". Under the considered scenario, we consider two players named player I and player II. We assume that player I only knows his channel state without any information about the channel state of player II and vice-versa. Player I moves first and sends a signal to player II which can be accurate or distorted. Player II picks up his power control strategy based on this information and his belief about the nature of the informed player's information. In order to analyze such a model, the proposed scheme game is transformed into 4×4 matrix game. We establish the existence of Nash equilibria and show by numerical results the equilibria and the performance of the proposed signaling game.

Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Miscellaneous; D.2.8 [**Software Engineering**]: Metrics—*complexity measures, performance measures*

General Terms

Theory, Performance

Keywords

Wireless networks; Power control; Partial information; Signaling game; Belief.

1. INTRODUCTION

A signal is a special sort of physical interaction between two agents which represents the product of a strategic dynamic between sender and receiver, each of whom is pursuing distinct but interrelated objectives. Moreover, a signal is a specific type of strategic interaction in which the content of the interaction is determined by the sender, and it changes the receiver's behavior by altering the way the receiver evaluates alternative actions. This situation type is known in the literature as a signaling game theory [2, 3]. A recent work is proposed in [5] to study two competition problems between service providers with asymmetric information by applying the signaling game approach.

Power control management is an important problem in wireless networks [14, 6]. This problem is well studied in the literature with different approaches [9, 4]. The power control game is one of these approaches in wireless networks and is a typical non-cooperative game where each mobile decides about his transmit power in order to optimize his performance [13, 10]. The authors in [7] study the power control problem by applying the evolutionary game theory for pairwise interaction networks. In [1], the authors study a competition between wireless devices with incomplete information about their opponents. They model such interactions as Bayesian interference games. Each wireless device selects a power profile over the entire available bandwidth to maximize his data rate, which requires mitigating the effect of interference caused by other devices. Such competitive models represent situations in which several wireless devices share spectrum without any central authority or coordinated protocol.

The main difference in this work is to use of signaling game theory for studying the power control game in the wireless networks context. To the best of our knowledge, this direction has not been done in this context. Specifically, we consider a situation where players compete to maximize their individual throughput by optimizing their transmit power. We assume that the power takes two values: high or low. We will restrict our study to the case of two players namely, player I and player II. We further assume that player I only knows his channel gain without any information about the channel gain of player II and vice-versa. Finally, the signal sent by player I to player II may be accurate or distorted which allows us to extend the original signalling model to situations where player II could receive a misleading information from player I. Although its simplicity, this scheme allows us to study the problem of distributively allocating transmit power in wireless systems using a signalling game and address some interesting features that allows us to gain insight on problems with partial and asymmetric information among players.

This is a natural setting for hierarchical wireless networks, where users have access to the medium in a hierarchical manner. For example, in cognitive radio networks [11] where primary (licensed) users have priority to access the medium and

then secondary (unlicensed) users access the medium after sensing the environment. At the core lies the idea that in signaling games, the informed player (player I) moves first and sends a signal about his decision with which the uninformed player (player II) may update his beliefs [12] about the nature of the informed player's information [12, 8]. We thus have a situation with *partial* and *asymmetric* information among players. Under this setting, the following questions may naturally arise. Do player I has a strategic advantage to cheat by sending misleading information to player II? In particular, how should be player II's reaction to the information sent by player I?

The remainder of the paper is organized as follows: Section 2 presents the signaling game model description. The game theoretic analysis is given in Section 3. Numerical results are provided in Section 4 and Section 5 concludes the paper.

2. GAME MODEL DESCRIPTION

Under the proposed scenario, player I only knows his channel gain (H_1) without any information about the channel gain (H_2) of player II and vice-versa whereas player I – moving first– is considered more informed than the other one (player II). Users leverage the reciprocity principle to estimate the reverse link channel from the BS. The channel state is referred to as "good" (G) if the channel gain is high with probability π_G, and "bad" (B) if the channel gain is low with the distribution π_B such that $\pi_G + \pi_B = 1$. This is particularly suitable setting when mobile users are either "aggressive" (transmitting with high power level) or "peaceful" (transmitting with low power level) like in [13, 7]. It is further known that when the channel state is good the mobile user uses a low power otherwise he uses high power level. At a first stage, based on the received information from the BS, player I observes his channel state. If the channel state is good (corresponding to the higher channel gain H_G), he decides to use the lower power level (P_L^1) otherwise if the channel state is bad (corresponding to the lower channel gain H_B) he uses the higher power level (P_H^1). At a second stage, player I sends a signal to player II who, based on the received signal from player I, decides to use the higher power level (P_H^2) or the lower power level (P_L^2) depending on his belief in the received signal.

Formally, this can be written as following

- Player I has four pure *signaling* strategies: $q_i q_j$ for $i, j = 1, 2$ where q_i stands for the signal sent if he observes H_G and q_j stands for the signal sent if he observes H_B. Let S_1 be the set of these strategies, i.e., $S_1 = \{q_1 q_1, q_1 q_2, q_2 q_1, q_2 q_2\}$.

- Player II has also four pure *response* strategies: $p_l p_k$ for $l, k = 1, 2$. The strategy $p_l p_k$ means that player II chooses p_l when he observes signal q_i from player I and p_k otherwise. Similarly, let S_2 be the set of player II strategies defined as $S_2 = \{p_1 p_1, p_1 p_2, p_2 p_1, p_2 p_2\}$.

3. SIGNALING GAME MODEL ANALYSIS

3.1 The matrix game

To ease the understanding of the aforementioned problem formulation, we transform this game in a 4×4 matrix game. The matrix presents all possible actions of the both players. We thus have $q_i = P_L^1$ if $i = 1$ and $q_i = P_H^1$ if $i = 2$ for

player I and $p_l = P_L^2$ if $l = 1$ and $p_l = P_H^2$ if $l = 2$ for player II. For example, the element $q_1 q_1$ means that the good and bad states are selected by player I with the same lower power level P_L^1. Similarly, $p_1 p_1$ means that the good and bad states are chosen by player II with the same power level P_L^2. Formally, a signal is designed as follows:

$$\epsilon(H_1, i, j) = \left\{ \begin{array}{ll} s_i, & if \ H_1 = H_G; \\ s_j, & if \ H_1 = H_B. \end{array} \right.$$

s_i and s_j are computed as function of player I's belief, namely

$$s_i = \left\{ \begin{array}{ll} 1, & if \ b_1 \in [0, th_1] \ and \ H_1 = H_G; \\ 2, & if \ b_1 \in [th_1, 1] \ and \ H_1 = H_G. \end{array} \right.$$

$$s_j = \left\{ \begin{array}{ll} 1, & if \ b_1 \in [0, th_1] \ and \ H_1 = H_B; \\ 2, & if \ b_1 \in [th_1, 1] \ and \ H_1 = H_B. \end{array} \right.$$

where b_1, respectively and th_1, is the *belief*, respectively the *threshold belief* of player I. The threshold belief is defined as the level under which the signal is considered as accurate. Otherwise, the signal is considered as biased. Accordingly, player I decides to send a signal to player II as follows

- If the channel state of player I is good (i.e., $H_1 = H_G$) and his belief is lower than th_1 (i.e., $b_1 \in [0, th_1]$), then he will send an accurate signal. This means that $\epsilon(H_1, i, j) = 1$ which corresponds to the low power level. Otherwise, if his belief is higher than th_1 (i.e., $b_1 \in [th_1, 1]$), player I will send a biased signal. In this case $\epsilon(H_1, i, j) = 2$ which corresponds to the high power level.

- If the channel state of player I is bad (i.e., $H_1 = H_B$) and its belief is lower than th_1 (i.e., $b_1 \in [0, th_1]$), then he will send an accurate signal, i.e., $\epsilon(H_1, i, j) = 2$ which corresponds to the high power level. Otherwise, when player I's belief is higher than th_1, he will send an accurate information. In this case $\epsilon(H_1, i, j) = 1$ which corresponds to the low power level.

We can now define player II's belief about the received signal as

$$s(b_2) = \left\{ \begin{array}{ll} 1, & if \ b_2 \in [0, th_2], \ \text{player II does not believe}; \\ 2, & if \ b_2 \in [th_2, 1], \ \text{player II believes}. \end{array} \right.$$

where b_2, respectively th_2, is the *belief*, respectively the *threshold belief* of player II. Then, player II's *response* based on the received signal and his belief is given by

$$\phi(H_1, H_2, i, j, l, k) = \left\{ \begin{array}{ll} r_k, & if \ H_2 = H_G; \\ r_l, & if \ H_2 = H_B. \end{array} \right.$$

where r_l and r_k are computed as function of player II's belief:

$$r_k = \left\{ \begin{array}{ll} 2, & if \ \epsilon(H_1, i, j) = 1, \ s(b_2) = 1 \ and \ H_2 = H_G; \\ 1, & if \ \epsilon(H_1, i, j) = 2, \ s(b_2) = 1 \ and \ H_2 = H_G; \\ 1, & if \ \epsilon(H_1, i, j) = 1, \ s(b_2) = 2 \ and \ H_2 = H_G; \\ 2, & if \ \epsilon(H_1, i, j) = 2, \ s(b_2) = 2 \ and \ H_2 = H_G. \end{array} \right.$$

$$r_l = \left\{ \begin{array}{ll} 2, & if \ \epsilon(H_1, i, j) = 1, \ s(b_2) = 1 \ and \ H_2 = H_B; \\ 2, & if \ \epsilon(H_1, i, j) = 2, \ s(b_2) = 1 \ and \ H_2 = H_B; \\ 1, & if \ \epsilon(H_1, i, j) = 1, \ s(b_2) = 2 \ and \ H_2 = H_B; \\ 1, & if \ \epsilon(H_1, i, j) = 2, \ s(b_2) = 2 \ and \ H_2 = H_B. \end{array} \right.$$

Based on the received signal and his belief in that signal, player II decides to use the strategy $p_k p_l$.

The utility of player I depends on his decision (strategy $q_i q_j$) and the amount of interferences caused by player II. This is given by

$$\mathbf{U}^1(q_i q_j, p_k p_l) = \mathbb{E}^1_{th}(q_i q_j, p_k p_l) - C_1(q_i q_j), \qquad (1)$$

where $C_1(q_i q_j) = \mu(\pi_G q_i + \pi_B q_j)$ is the energy cost for strategy $q_i q_j$. The expected throughput of player I is given by

$$
\begin{aligned}
\mathbb{E}^1_{th}(q_i q_j, p_k p_l) \;=\; & \pi_G \Big[\pi_G \log\big(1 + \frac{q_i |H_G|^2}{\sigma^2 + p_{\phi(H_G, H_G, i, j, l, k)} |H_G|^2}\big) \\
& + \; \pi_B \log\big(1 + \frac{q_i |H_G|^2}{\sigma^2 + p_{\phi(H_G, H_B, i, j, l, k)} |H_B|^2}\big)\Big] \\
& + \; \pi_B \Big[\pi_G \log\big(1 + \frac{q_j |H_B|^2}{\sigma^2 + p_{\phi(H_B, H_G, i, j, l, k)} |H_G|^2}\big) \\
& + \; \pi_B \log\big(1 + \frac{q_j |H_B|^2}{\sigma^2 + p_{\phi(H_B, H_B, i, j, l, k)} |H_B|^2}\big)\Big].
\end{aligned}
$$

The utility of player II depending on his decision (strategy) and the received signal is given by

$$\mathbf{U}^2(q_i q_j, p_k p_l) = \mathbb{E}^2_{th}(q_i q_j, p_k p_l) - C_2(p_k p_l), \qquad (2)$$

where $C_2(p_k p_l) = \mu(\pi_G p_k + \pi_B p_l)$ is the expected cost of player II. The expected throughput of player II is

$$
\begin{aligned}
\mathbb{E}^2_{th}(q_i q_j, p_k p_l) \;=\; & \pi_G \Big[\pi_G \log\big(1 + \frac{p_{\phi(H_G, H_G, i, j, l, k)} |H_G|^2}{\sigma^2 + q_{\epsilon(H_G, i, j)} |H_G|^2}\big) \\
& + \; \pi_B \log\big(1 + \frac{p_{\phi(H_G, H_B, i, j, l, k)} |H_B|^2}{\sigma^2 + q_{\epsilon(H_G, i, j)} |H_G|^2}\big)\Big] \\
& + \; \pi_B \Big[\pi_G \log\big(1 + \frac{p_{\phi(H_B, H_G, i, j, l, k)} |H_G|^2}{\sigma^2 + q_{\epsilon(H_B, i, j)} |H_B|^2}\big) \\
& + \; \pi_B \log\big(1 + \frac{p_{\phi(H_B, H_B, i, j, l, k)} |H_B|^2}{\sigma^2 + q_{\epsilon(H_B, i, j)} |H_B|^2}\big)\Big].
\end{aligned}
$$

3.2 Equilibrium Strategies

We recall that player I sends a signal (strategy $q_i q_j \in S_1$) to player II who uses this signal to choose his strategy in S_2. At equilibrium, player II responds by a strategy $p_k^* p_l^* \in S_2$ that maximizes his expected utility.

The Nash equilibrium for such a game is a pair of strategies $(q_i^* q_j^*, p_k^* p_l^*)$ such that each player uses the best response in a non-cooperative way, given the probability distribution of the quality of their respective channel H.

LEMMA 1. *At any equilibrium point $(q_i^* q_j^*, p_k^* p_l^*)$ in the game, player I's strategy $q_i q_j \in S_1$ are function of the best response of player II, namely*

$$\underset{p_k p_l \in S_2}{\arg\max} \, U^2(q_i q_j, p_k p_l).$$

The equilibrium point $(q_i^* q_j^*, p_k^* p_l^*)$ $(i, j, k, l = 1, 2)$ are computed as follows

1. *Player I's best response is given by*

$$\underset{q_i q_j \in S_1}{\arg\max} \sum_{p_k p_l \in S_2} \mathbf{U}^1(q_i q_j, p_k p_l) = \sum_{p_k p_l \in S_2} \mathbf{U}^1(q_i^* q_j^*, p_k p_l).$$

2. *Player II's best response is given by*

$$\underset{p_k p_l \in S_2}{\arg\max} \sum_{q_i q_j \in S_1} \mathbf{U}^2(q_i q_j, p_k p_l) = \sum_{q_i q_j \in S_1} \mathbf{U}^2(q_i q_j, p_k^* p_l^*).$$
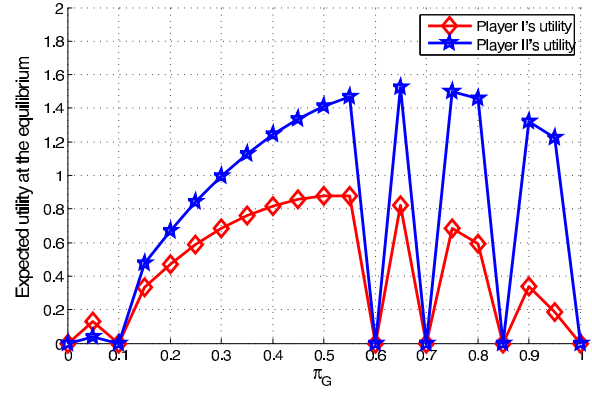

Figure 1: **Expected utility at the equilibrium as function of the probability to have a good channel state π_G when player I sends an accurate signal and player II does not believe in the received signal.**

4. NUMERICAL RESULTS

We consider the following parameters for all simulation results: $th_1 = th_2 = 0.5$, $\mu = 0.01$, $|H_G| = 100$, $|H_B| = 10$, $P_L^1 = P_L^2 = 10$ mW, $P_H^1 = 80$ mW, $P_H^2 = 100$ mW.

4.1 Impact of an accurate signal

Let us first consider the situation when the signal sent to player II is accurate. This means that player I's belief (b_1) –for his both channel states (good and bad)– is in the range $[0, th_1]$. We plot in Figure 1 the expected utilities of player I and II in function of the probability to have a good channel state π_G. We analyze both cases when player II does not believe and believe in the received signal. This translates to a belief b_2 in the range $[0, th_2]$, respectively $[th_2, 1]$.

Figure 1 depicts the expected utility for $b_2 = 0.1 < th_2$ which means that player II does not believe in the received signal. When the probability to have a good channel state π_G is less than 0.1, player I's expected utility is better than player II's expected utility whereas when π_G is higher than 0.1, player I's expected utility is worse than player II's expected utility since player II picks up a strategy by considering the received signal as inaccurate resulting in a better utility than the one of player I. Moreover when π_G increases, users tend to use a high power level which translates here into a better expected utility for player II at the expand of increasing interference for player I.

Figure 2 depicts the expected utility for $b_2 = 0.6 > th_2$ which means that player II considers the received signal as accurate. In this case, player I's expected utility is always better than player II's expected utility This is due to the fact that in this case player II picks up a strategy by following the received signal resulting in a worse utility than the one of player I.

Accordingly, we can conclude that, when player I sends an accurate signal, in order to guaranty a better expected utility that the one of player I, player II should not believe in the received signal and the probability to have a good channel must be higher than 0.1.

In Figure 3, we plot mixed equilibria when player II does not believe in the received signal. When $\pi_G < 0.1$, player II (resp. player I) chooses the strategy $p_1 p_1$ (resp. $q_1 q_1$) with probability 1 (i.e., power levels used in the good and
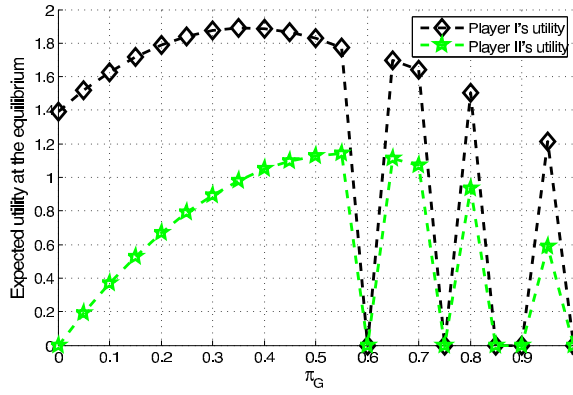
Figure 2: Expected utility at the equilibrium as function of the probability to have a good channel state π_G when player I sends an accurate signal and player II believes in the received signal.
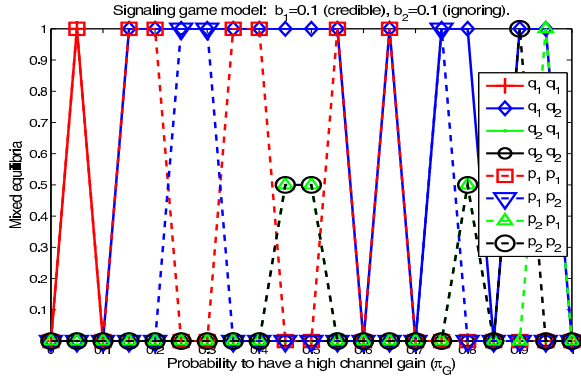

Figure 3: Mixed equilibria as function of π_G when player II does not believe in the received signal.
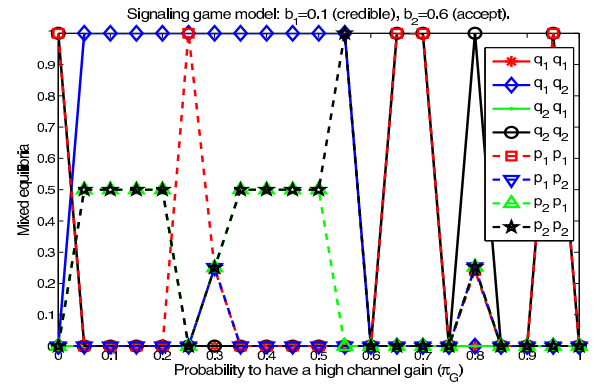

Figure 4: Mixed equilibria as function of π_G when player II believes in the received signal.

Figure 5: Expected utility at the equilibrium as function of the probability to have a good channel state π_G when player I sends a biased signal and player II does not believe in the received signal.

the bad states are P_L^i and P_H^i for $i = 1, 2$). For $\pi_G > 0.1$ player I switches to another strategy $q_1 q_2$ with probability 1 whereas player II transits between strategy $p_1 p_1$ and $p_1 p_2$ until $\pi_G = 0.6$. For $\pi_G > 0.6$, player II switches to $p_2 p_2$ and $p_2 p_1$. We also remark here that when the probability to have a good channel state is equal 0.45 and 0.55, player I chooses strategy $q_1 q_2$ with probability 1 and player II chooses strategy $p_2 p_1$ and $p_2 p_2$ with probability equal 0.5. When player II believes in the received signal we have a completely different situation. As can be shown in Figure 4 when $\pi_G < 0.6$, player I chooses strategy $q_1 q_2$ with probability 1. This means that the lower power level is chosen for the good state and the higher power level for the bad state. As player II considers the signal as credible he decides to use with probability 0.5 the strategies $p_2 p_1$ and $p_2 p_2$ for $\pi_G \in [0.05, 0.2]$ and $\pi_G \in [0.35, 0.5]$. He switches to strategy $p_1 p_1$ when $\pi_G = 0.25, 0.65, 0.7, 0.95$ with probability 1 whereas player I switches to another strategy $q_2 q_2$ with probability 1.

4.2 Impact of a biased signal

We present in Figure 5 the expected utilities for player I and player II in function of the probability to have a good channel state when the signal sent is biased and player II does not believe in the signal. For $\pi_G < 0.35$, player II's utility is higher than player I's utility at the equilibrium while

for $\pi_G > 0.35$, player I's utility at the equilibrium becomes higher than player II's utility. From Figure 6 we observe that player I always outperforms player II at the equilibrium. Here, one can conclude that, when player I sends a misleading signal, in order to obtain a better expected utility than player I at the equilibrium, player II should not believe in the received signal and the probability to have a good channel must be less than 0.35.

5. CONCLUSION

In this paper, we have introduced a signaling game approach for power control in wireless networks in which the signal sent by player I to player II may be accurate or misleading. In particular, we have showed that, at the equilibrium, player I always performs better than player II except in situations where player II does not believe in the received signal either the received signal is accurate or misleading.
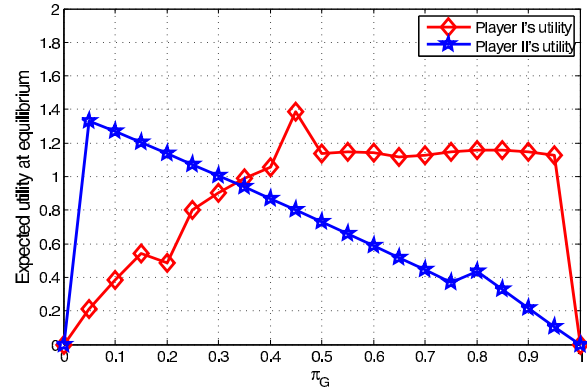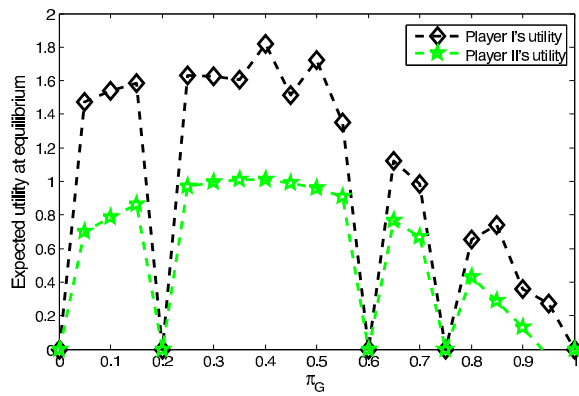
Figure 6: Expected utility at the equilibrium as function of the probability to have a good channel state π_G when player I sends a biased signal and player II believes in the received signal.

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