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► **To cite this version:**

| Nazim Fatès. A guided tour of of asynchronous cellular automata. 2014. hal-00908373v2

HAL Id: hal-00908373

<https://inria.hal.science/hal-00908373v2>

Preprint submitted on 14 Jan 2014 (v2), last revised 26 Aug 2014 (v5)

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A guided tour of asynchronous cellular automata

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January 14, 2014

Abstract

Research on asynchronous cellular automata has received a great amount of attention these last years and has turned to a thriving field. We survey the recent research that has been carried out on this topic and present a wide state of the art where computing and modelling issues are both represented.

foreword: This article is a corrected and extended version of the invited paper that appeared in the proceedings of Automata'13, 19th International Workshop on Cellular Automata and Discrete Complex Systems, LNCS 8155, 2013, p. 15-30, doi:10.1007/978-3-642-40867-0_2 (Ref. [38] in the bibliography.)

1 Introduction

Cellular automata (CA) are mathematical objects that were initially studied by von Neumann and Ulam to study the properties of self-reproduction of the living organisms with a simple “mechanical” tool [125]. Since then, they have been studied with three main motivations:

- They represent a model of spatially-extended, distributed and homogeneous computing systems.
- They are dynamical systems with a regular structure where time, space and states are discrete, yet they provide examples on how to generate complex behaviour from a simple description of the components.
- They are models of the numerous complex systems seen in Nature, in which properties of self-organisation and robustness are central.

Far from being exclusive of each other, these three motivations represent three ways to tackle the “complexity question”: how does complexity emerge from the interaction of simple components? This question have given raise to many interpretations and its richness has given birth to a scientific field called “complex systems”. Inside this field, cellular automata occupy a special position as they can be regarded as the class of models that have the simplest structure. Indeed, the essence of this simplicity lies in the use of a *local rule*, which dictates that the updating of the components of the system, the cells, should be done according to only a partial knowledge of the current state of the system. The simplicity of cellular automaton also lies in the regular arrangement of the cells and in the usually small amount of states that is used to define the cells’ behaviour.

Another important part of the definition of cellular automata regards their update: in their original definition, they are updated *synchronously*, that is, all the cells change their state simultaneously at discrete time steps. The global state of the system at a given time is thus obtained from an information that is gathered simultaneously for all cells and, for each cell, simultaneously from all its neighbours.

Making this hypothesis has many advantages: with this type of updating, the system is deterministic and simple to define. The use of a common time that is shared by all cells allows one to describe analytically the behaviour of the dynamical system or, when such a description can not be obtained, to formally state limits to the predictability of a system (undecidability results are the most commonly studied limits). From the simulation point of view, the simulation of a cellular automaton on a *sequential* machine only requires the use of a buffer (or an additional dimension if one needs to remember all the global states that are taken by the system).

In spite of these manifest advantages, there are reasons why a *pure synchrony hypothesis* needs to be questioned:

- When cellular automata are considered as a model of a massively parallel computing device, it is legitimate to ask if it is possible to operate the device without the need to share a clock signal. Various advantages can be expected from the removal of a central clock : increase of the speed of computations, economy of energy, simplicity of design, etc.
- When cellular automata are studied as dynamical systems, the transformation from a purely synchronous system to an asynchronous one may affect some, but not all, of its properties. What are the properties that will be affected by this change and what are those which remain robust to the perturbation of the updating scheme ? How can we interpret this sensitivity or robustness of the system to changes of its definition?
- When cellular automata represent a model of a natural phenomenon, the modeller may examine the necessity to use a global clock to synchronise the transitions of a model. This question is far from being simple and it is not simply possible to state that “there is no global clock in Nature to synchronise the transitions”. Indeed, it can be objected that a model is a simplified representation of a phenomenon and does not account for all the details of “reality”. Moreover, abandoning the use of global clock generally gives raise to a new dilemma: how can we then explain the existence of some kind synchronisation in Nature? This synchronisation might well result from an external phenomenon (e.g., the daily varying light of the sun) or it may be an effect that emerges from the numerous interactions between the components (e.g., coupled oscillators), but the question still remains to explain how the synchronising mechanisms are themselves synchronised!

All these questions raise difficult problems and a wide variety of paths have been taken to tackle them. The field of asynchronous cellular automata has attracted the interest of many authors and has progressively changed from a “marginal” to a “respected” question. This increase in the scientific production has reached a point where it is now difficult to follow all the contributions that appear in the literature.

The purpose of this survey is to introduce the readers to this quite diversified “landscape”, trying as much as possible to cover the various “sites” that constitute this landscape. It should not be understood as an objective description of the field but rather as a “guided tour”. Like all guided tours, it will take us from place to place according to somewhat arbitrary choices made by one “guide”... It is therefore important not to bear in mind that the exposure of the ideas can only be brief and can by no means serve as a substitute to reading the texts mentioned *themselves*.

We thus hope that readers that are unfamiliar with the topic will find landmarks for their orientations while those which are more familiar with cellular automata will find it useful to have a number of references gathered in a specific survey. We insist that such a project can only be imperfect and the author will

be grateful to all the corrections, indications, remarks, and suggestions that will be given to him to improve this text.

2 Computing with asynchronous cellular automata

We begin our guided tour by considering the contributions that deal with the question as to what can be computed by asynchronous CA and how to compute with these objects.

2.1 Simulation of (a)synchronous CAs by (a)synchronous CAs

Nakamura was one of the first authors to investigate the computation abilities of asynchronous cellular automata [77]. He described several techniques to construct a universal rule and showed how to simulate a given q -state deterministic CA with an asynchronous rule that has the same neighbourhood and whose state space is extended to $3q^2$ states (see also Lipton et al. [66], Toffoli [119] and Nehaniv [79] for similar constructions). The construction relies on the idea that if a cell is updated, it will then wait the neighbouring cells to “catch up” and makes the next transition only when all its neighbours are up to date ; additionally, it keeps its old state available for the neighbouring cells in order for them to perform the “right” transitions. This construction was later improved by the use of only $q^2 + 2q$ states by Lee, Peper et al. [63, 84].

Peper et al. also proposed to consider the case where a cell can “activate” their neighbouring cells and showed that the cost in the number of states for the simulation of q -state rule could be realised with a rule having $\mathcal{O}(q\sqrt{q})$ states [85].

Other discussions on the universality of asynchronous CA are found in the study by Takada et al., in which many important arguments and useful references can be found [117]. In particular, authors present a result showing the existence of an asynchronous, rotation-symmetric rule with 15 states and von Neumann neighbourhood that has the property of universal construction and computation.

An alternative point of view to the problem of simulating synchronous CA by an asynchronous ones was given by Golze who added an additional dimension to the system [50]. This solution simplifies the problem as there is no longer the need to save the previous and the current state in order to achieve correct computations. Another advantage of having an additional dimension is that it becomes possible to read one state of the synchronous simulated system (guest) on the asynchronous simulating system (host): it simply corresponds to reading a line (or a hyperplane) of the host. This technique, called “global synchronisation”, is presented as a means to solve various problems, such as the Firing Squad Synchronisation Problem, which would not be solvable without this requirement. However, it can be noted that this technique can be interpreted as the “deployment” of Nakamura’s technique on an additional dimension. Reciprocally, one can also see Nakamura’s technique as the “compressed” version of

Gloze’s solution, where only the necessary information is retained.

The case where asynchronous computations have to be made with stochastic *and* asynchronous components was tackled by Wang [127]. Unfortunately, this author does not position his work with regard to the previous contributions (Nakamura, Golze) and it is difficult to see if this proposition significantly differs from the previous achievements.

An original way to simulate a universal Turing machine by the means of a fully asynchronous CA has also been proposed by Dennunzio et al. [29]. Authors introduce the notion of “scattered strict simulation” in which they tolerate that only a subset of cells is used to perform the simulation. They find that asynchrony induces a quadratic slowdown compared to the speed of the simulated Turing machine.

The possibility to simulate an arbitrary rule by a universal asynchronous simulator was re-examined by Worsch [128]. This author extended Golze’s results by tackling a large scope of updating policies: *purely asynchronous* (no restriction on the set of cells) to update, α -asynchronous, *N-independent* (where two neighbouring cells are never updated at the same time), and fully asynchronous. He showed that for each such policy, there is a universal rule (the host) that can “simulate”, in a particular sense, any other guest.

2.2 Computations and order-independence

Gács tackled the question of whether the evolution of an asynchronous system could be independent of the order of updating [48]. He showed that although this property was undecidable, there exists a sufficient condition to verify this independence.

This question was re-examined by Mortveit, Macauley et al., who studied in which cases repetitions of sequential updates on Elementary Cellular Automata (ECA) could produce a set of periodic points that would be independent of the updating order [69, 70, 68]. Their results rely on the analysis of the property of what we could call “non-overlapping influences” : if two cells c and c' are such that the neighbourhood of c and c' do not overlap (that is, have no cell in common), it does not matter whether c is updated before c' , or c' before c , or both of them are updated at the same time. This conducted the authors to present a list of 104 ECA which display such an update independence. Their work also uses an original representation of ECA that differs from the classical Wolfram code and that could prove useful for future analysis of asynchronous systems. (Another notation is presented in Ref. [33, 42]).

We also point out that Vielhaber has designed a formal framework in which the computations of functions on finite binary rings ($\mathbb{Z}/n\mathbb{Z}$) are made not by changing the local rule but by a proper use of the order of updating on a *fixed* rule [124]. In particular, he showed that ECA 57 with periodic boundary conditions was a rule especially adapted for such a purpose.

2.3 Models of concurrency

Among the early references that can be found on asynchronous CA, Priese wrote a note where he considers (two-dimensional) cellular automata as a particular case of asynchronous rewriting systems (called Thue-systems) and widens the scope by considering also the case where more than one cell may be re-written at a time (the overlapping problem) [88]. He uses his construction to show how to build asynchronous circuits which are equivalent to asynchronous concurrent Petri nets.

Following this path, Zielonka examined how asynchronous CA could be used to describe the situations of concurrency that arise in distributed systems [25]. Pighizzini clarified the computing abilities of Zielonka's models [87] and the problem of how to determinise non-deterministic Büchi asynchronous cellular automata was solved by Muscholl [76]. Droste generalised to pomsets the original notion of Zielonka's asynchronous mappings [31]; these questions were later re-investigated by Kuske [32, 58, 59].

With similar preoccupations, Hagiya et al. used formal methods from logic to verify the properties of some rewriting systems, showing the links between their approach and (a)synchronous CA [53].

3 Experimental approaches

Cellular automata are known for their capacity to challenge the capacity of making general predictions about their behaviours. Classifying “classical” cellular automata has been an important topic of research and is far from being a closed question (see e.g. the recent survey by Martínez [73]). It then be thought at first that classifying asynchronous CA is then a daunting task because of the additional complexity that is induced by the asynchronous updating. In fact, this is only partially true as in many cases the asynchrony may “break” the complexity of a rule and render it more simple to study. We now discuss the experimental approaches that have been adopted to gain insight into the classification of asynchronous CA.

3.1 Numerical simulations

In 1984, while the first systematic explorations of sets of simple rules were carried out, Ingerson and Buvel carried out a pioneering work where they could show that the behaviour of simple rules could be totally disrupted by simple modifications of the updating [19]. Most importantly, they questioned to which extent was the behaviour of a rule the consequence of the local rule and to which extent it was due to the updating scheme.

This question was taken back by Bersini and Detours, who explored the difference of behaviour of the Game of Life and closely related asynchronous variants [13]. Their main observation was the existence of a “stabilising effect” of asynchronous updating but they based their statements on small-size experiments, with grids no larger than 20 by 20 cells. With such lattice sizes, they

were able to observe that the Game of Life with asynchronous updating may “freeze” on some fixed-point pattern that has a labyrinth aspect. More recent work [14, 36] (see details below) has demonstrated that it was not possible to infer the large-size behaviour from these experiments and that the stabilising effect was intimately linked to finite-size effects of the numerical experiments.

Schönfisch and de Roos gave a decisive impulse to the research on asynchronous CA by comparing various updating schemes and by exhibiting clear examples to show that the effect of these schemes could alter significantly the behaviour of a CA [106]. They distinguished *time-driven* and *step-driven* updating methods, depending on whether the updating of a cell is triggered by the determination of an event given by a continuous time proper to each cell or by a global clock which would send signals to all or part of the cells.

Note that it may be thought at first sight that “time-driven” methods provide a more realistic simulation framework since the updating signal is not artificially shared among cells. As remarked by the authors themselves, this is not true since it is easy to build a “step-driven” method that emulates a “time-driven” method: as the update is sequential, the only relevant information for the evolution of the updating is the order of updating, and a correspondence between the two schemes can thus be established. Moreover, the hypothesis of sequentiality of events which states that no two updating events can happen at the same time is not necessarily more realistic. Indeed, this hypothesis would be relevant only in the case where the transitions between states are *instantaneous*. As this hypothesis is far from being valid in many cases (especially in biological systems), a certain “degree of synchrony” between cells should thus be assumed. As this degree is difficult to measure, the problem is not so much about choosing the “right” model of updating but rather to estimate the robustness of model, that is, the degree to which it is sensitive or not to changes of its updating scheme.

This idea lead Fatès and Morvan to examine how the 256 Elementary cellular automata reacted to an updating where the degree of synchrony was varied [41]. This updating scheme they used, named simply α -asynchronism¹ consists in updating each cell with probability α , the *synchrony rate*, and leaving the state of the cells unchanged otherwise. In order to quantify the observations, the authors used the value of the asymptotic density as a means to estimate changes in the behaviour of the system. This parameter was considered as a first means to detect changes in the behaviour: a strong variation of the asymptotic density indicates that the system has undergone a transformation while an absence of variation does not say anything on the stability of the system as other modifications than the density may occur.

The experimental results revealed that no direct relationship could be drawn between the asynchronous behaviour and the “complexity” of the synchronous behaviour (e.g., the informal Wolfram classes). The rules were then classified

¹ Note that the terms α -asynchronism and α -synchronism have been both used and are both relevant: the confusion comes from the fact that α is the name of the scheme and the synchrony rate. We use here the term α -asynchronism, as it is the form that was first proposed and which has been adopted by various authors such as Regnault, Correia, Worsch, Fukš, etc.

into four qualitative sets according to their responses to the variation of the synchrony rate α : (a) continuous variation of the behaviour (e.g. ECA 232), (b) discontinuity around $\alpha = 1$ (e.g. ECA 2 or 110), (c) phase transition for a critical value $\alpha_c < 1$ (e.g. ECA 50), and (d) non-regular behaviour (e.g. ECA 184).

An interesting development on the work of asynchronism concerns how it mixes with traditional noise, that is, on randomness imposed on the *state* of the cells that compose the automaton. Early references that address this question are given by a paper by Gharavi and Anantharam, where a well-known result of Toom [49] is revisited, and the work of Kanada, which tackles ECA rules [57]. Mamei et al. also considered this question for studying “complex decentralised pervasive computing systems” that are models as two-dimensional binary CA [71]. The most recent development is by Silva and Correia, who have given a detailed account on robust or sensitive some ECA can be to asynchronism combined with noise. Interestingly, they propose to evaluate the robustness according to the difference patterns and they thus introduce a sampling compensation in order to cope with less frequent updates [110, 111].

Bandini et al. considered various possibilities to define an asynchronous updating scheme, and presented a formal description of these possibilities. They tested their effects on one-dimensional binary rules where the local function depends only on two neighbours (also called “radius-1/2” rules) [8].

The case of asynchronous CA simulated on a non-regular topology was tackled by Baetens et al., who examined an asynchronous updating with cells linked by the frontiers of a Voronoi tessellation [7].

3.2 Phase transitions

Blok and Bergersen were the first authors to identify the change that occurs in the Game of Life when cells are updated with a given probability [14]. They used α -asynchronism (without naming it so) and showed the existence of a *qualitative* transition from a “static” behaviour, where the system would settle on fixed points, to a “living” behaviour, where the system evolves by forming labyrinth-like patterns that do not fixate. These authors identified that this change of behaviour could be described by a second-order phase transition, that is, the continuous variation of the probability of update induces a continuous variation of the behaviour, but for there is a critical value for which the variation is brutal and which separate the two qualitatively different behaviours (the phases). This phase transition was shown to belong to the directed percolation universality class.

Fatès identified that similar phenomena occurred in Elementary Cellular Automata and that 7 out of the 88 “minimal” rules also displayed phase transitions. Using a different method from the one of Blok and Bergersen, it was shown that the density follows a power-law decay for the critical synchrony rate and that the system behaved according to the predictions of the directed percolation universality class [34]. A unique rule, namely ECA 178, was shown to belong to the PC/DP2 universality class, a class very similar to DP but where the states

0 and 1 play a symmetric role.

The phase transition occurring in the Game of life was also re-examined in detail, especially by studying how this phenomenon was affected by perturbations of the topology [35, 36]. The main finding was that this phase transition was strongly dependent on the regularity of the grid and that it was progressively “washed out” as links between cells were removed.

Concerning the probabilistic analysis of these phenomena, Regnault carried out a pioneering work by explaining in detail how the asynchronous minority rule displayed various types of behaviour was used [91, 93] or even when more general graphs were considered [99]. A simple puzzling observation is that the minority rule will settle out on a checkerboard or on a stripe-like pattern depending on whether the rule is defined with the von Neumann or the Moore neighbourhood. To our knowledge, there is no mathematical explanation of this empirical observation.

Remark that two different complementary views exist on phase transitions: the most common way of describing a phase transition is to establish that for an *infinite* system, a qualitative difference of behaviour occurs for an infinitesimal variation of the control parameter. This second approach was adopted by Regnault who could (partially) prove that for *finite systems*, this phase transition corresponds to a variation of the convergence time from a linear to a polynomial function of the system’s size [92].

3.3 Coalescence

A curious phenomenon was remarked where two cellular automata are coupled in the following way: the two systems are started from different initial conditions but at each time step the same updates are applied on the two systems. It was then remarked that in some cases, it was possible to observe a rapid “coalesce”, that is, the two systems take the same state and then evolve with the same trajectory (as the same sites are updated). This phenomenon is in some cases easy to understand, as when the coalescence occurs on an attractive fixed point, but, as it was observed for ECA 46 for instance [41], it is much more puzzling when the coalescence happens on a non-fixed-point region of the state space. From a more pragmatic point of view, the following interpretation of the phenomenon can be given: there are asynchronous systems whose evolution is rapidly becomes governed by the random number generator that dictates the updates, and not by the initial condition.

Rouquier and Morvan studied systematically the coalescence phenomenon for the 256 ECA [96, 98]. Their study revealed that it was possible to observe that some ECA would always coalesce, while other would never coalesce, and that there existed some rules which displayed a phase transition between a coalescing and non-coalescing behaviour, depending on the synchrony rate. It is an open problem to explain analytically the non-trivial cases of rapid coalescence. It is also interesting to compare these results with those obtained by other methods of coupling (see e.g. [103, 97]).

4 Analytical approaches

We can now examine how the intuitions gained by the numerical simulations have given rise to various types of analysis of the behaviour of asynchronous CA with a stochastic updating. It is interesting to remark that this section is presented separated from the previous one, analysis does *not* simply follow the observations with numerical simulations, but there is rather a *joint* movement of going from simulations to analysis and back. (This co-development is not necessarily done by the same authors of course.) In general, we can state that analytical results are relatively rare but they are gaining momentum as more techniques from the probability theory are being developed for the specific case of CA.

4.1 Markov chains analysis and classifications

Agapie et al. conducted one of the first analytical studies of asynchronous CA using Markov chain theory [4]. They focused on several models of finite cellular automata with fully asynchronous updating. However, as far as we could understand, their analysis was limited to a specific case where the local rule was stochastic, totalistic, symmetric with respect to an exchange of 0s and 1s, and with positive rates (the probability to reach each state is strictly positive). It is worth noting that the number of borders of a configuration is a central parameter in this analysis and that this parameter is also found in various other approaches (see below).

One of the first analytical results of classification were given by Fatès et al. who analysed the doubly-quiescent ECA (that is, rules with two quiescent states) under fully asynchronous dynamics [42] (only one cell is updated randomly at each time step). In this study, the 64 rules considered are classified according to their worst expected convergence time to fixed point, which falls into the following classes : logarithmic time, linear time, quadratic time, exponential time and non-converging rules. (Note that in the classes here are given with a “rescaled time scale” where one step corresponds to as many random updates as there are cells in the finite ring.)

These results were later extended by Regnault et al. tackling the case of α -asynchronous updating [43] and by Chassaing and Gerin, who examined what was the continuous limit of the processes when the grid was made infinite [23].

Fatès and Gerin also examined how to classify the two-dimensional totalistic CA with fully asynchronous updating [40]. They proposed a partial classification of the 64 rules and an analysis of the convergence of some well-known rules. Among the interesting phenomena remarked, they exhibited a list of rules, which showed an “erratic” behaviour: the question was to determine if these rules were exhibiting a non-converging behaviour or a “metastable” behaviour, that is, if a (long) random sequence of updates could drive the system to a stable state. By adapting techniques from automatic planning, Hoffmann et al. could solve this problem for a specific rule and showed that it converged to a fixed point in (at most) exponential time [54].

Readers interested by this topic can refer to a recent synthesis paper for a more detailed account of the open problems that regard the classification of asynchronous CA with regard to their time of convergence [39].

4.2 Detailed analysis of the asymptotic densities

Fuk s and Skelton gave an exact calculation of the asymptotic density of eight simple ECA rules with α -asynchronous updating [47]. They considered infinite systems where the initial condition was generated by an a Bernoulli measure and determined how the asymptotic density varies as a function of the initial density (that is, the parameter of the Bernoulli measure). Such exact results are generally rather difficult to obtain for the deterministic ECA and it is interesting to note that the stochastic nature of the system allows one to use different tools to make predictions.

Following this direction of research, Fuk s and Fat s considered a development of Gutowitz’s “local structure theory”: contrary to a classical mean-field approach where only cells from the neighbourhood are considered, correlations of order 2 or larger are taken into account to try to predict the asymptotic density of the system [46]. It was shown that such an approach can be useful to detect phase transitions. The limit of this approach is that the critical synchrony rate where the transitions appears remains difficult to find with this approach: for some rules, even approximation with nine cells can not predict precisely the position of the critical threshold that separates the active and inactive phases.

4.3 Reversibility

Links between the notions of asynchrony and reversibility were examined by Das et al. [104, 27], first by considering the case where the updating sequence (or “update pattern”) could be set freely. This work was recently extended to the case of fully asynchronous updating and a classification of the ECA rules was proposed: the recurrent rules, the irreversible, and strongly irreversible [108]. Wacker and Worsch also examined the question of reversibility of asynchronous CA [126], but took a different point of view. In their work, a rule is said to be reversible if there is another rule whose state-transition graph is the “inverse” of the original. Interestingly, the results presented on ECA are not far from those found in Ref. [108]. This question remains currently widely open.

4.4 m-asynchronous CA and their topological properties

Manzoni examined how the dynamical properties of CA, such as injectivity, surjectivity, permutivity, etc., could be re-defined and studied in the asynchronous updating context [72].

Taking note of the fact that the asynchronous updating of a CA could does not need to be homogeneous in space, Dennunzio et al. developed the notion of m-asynchronous cellular automata, which generalise the probabilistic updating methods used so far [30].

For more details on this line of research, we recommend to consult the recent survey paper by Formenti where more details and examples can be found [45].

4.5 Asynchronous information transmission

While most approaches of asynchronism studied so far are based on the dichotomy updated versus not updated, Bouré et al. defined a model of asynchrony which considers imperfect communications between neighbours [17, 15]. This approach is declined in two versions, called β - and γ -asynchronism, which respectively consider stochastic failures of the communication of a state to the whole neighbourhood or to each neighbour independently.

Among the various phenomena obtained with these two types of asynchronism, the most intriguing phenomena is the disappearance of some, but not all, of the phase transitions that were obtained with the α -asynchronism. More precisely, ECA 6, 38 and 134 do not show any transition for β - and γ -synchronism while ECA 58 gives an even more puzzling case as it does show a phase transition for α - and β -asynchronism but *not* for γ -asynchronism. It is an open problem to understand the origin of such radical differences of responses to the rate of failures.

Experiments also displayed that in some cases, some quantities can be conserved with one model of asynchronism only (e.g., ECA 50 has some parity conservation with β -asynchronism but not with α -asynchronism). This underlines the necessity to continue to propose various perturbations of the “classical” CA in order to examine how their properties are dependent on the updating method that is chosen.

5 Modelling with asynchronous CA

While the studies seen so far mainly tackle the question of asynchronous CA for itself, there is a number of authors who designed asynchronous rules for a specific goal such as: finding new algorithms, developing new types of computing devices, modelling various natural or artificial complex systems, etc.

In fact, this section would necessitate a whole independent survey as there are many uses of asynchronous CA where the asynchrony of the updating is not even explicitly mentioned (most multi-agent systems are updated sequentially). We will thus keep this part short and mention only a few entry points to the topic. We concentrate on the papers where the question of the updating is explicitly mentioned and discussed.

5.1 Problem solving

Tomassini and Venzi [120], Capcarrere [21] and Nehaniv [78] have studied how asynchronous CA solve problems such as the density classification or the global synchronisation. Readers interested in this issue are referred to a study by

Vanneschi and Mauri, in which an enlightening discussion on these various contributions is found and where the authors present findings of robust and generic rules [123].

Suzudo examined the use of genetic algorithms to find mass-conservative asynchronous CA that would generate non-trivial patterns [115, 116]. He classified these patterns into three categories: checkerboards, stripes and sand-like. Although in this work asynchronism is mainly used to ensure mass conservation, it appears also as a useful means of generating regular patterns out of randomness, a task that is known to be very difficult in the synchronous setting (see e.g. [37]).

Beigy and Meybodi investigated how to make asynchronous CA “learn” and presented applications of their work for patterns generation and control of cellular mobile networks [11].

It is also worth mentioning that Lee et al. designed some asynchronous models of self-reproduction [61].

5.2 Asynchronous circuits

Another major field of research on asynchronous CA was developed by Peper, Lee and their collaborators. In their constructions, asynchronous computations are realised with particles that follow Brownian movements and which interact through special “gates” [3, 2, 62, 86]. These constructions result in delay-insensitive circuits that are Turing universal (see e.g. [60, 65] and references therein).

Recently, Schneider and Worsch presented a 3-state CA that uses Moore neighbourhood which can simulate any delay-insensitive circuit [105] and Lee et al. presented a generalisation of their work in the context of number-conserving cellular automata [64].

5.3 Game theory and Ecology

A good example of such a work is given by Huberman and Glance, who challenged the validity of the simulations of spatially-extended models of the Prisoner’s dilemma on the basis that asynchronous and synchronous updating lead to drastically opposed conclusions [56]. This question was re-examined by Newth and Cornforth who considered various update methods and showed that asynchronism could also lead to the observation of new cooperative phenomena not seen in the synchronous setting [81]. (See also Ref. [80] for a non-spatially-extended version of the problem.)

Grilo and Correia also considered this problem but instead of focusing only on a comparison of fully synchronous versus fully asynchronous, they used α -asynchronous updating to explore a wide range of degrees of synchrony [51, 52]. They also examined the role played the regularity of the network of cells. Their studies reveal that the changes induced by smooth variations of the synchrony rate may affect brutally the level of cooperation in the system, a behaviour

that is strongly reminiscent of the second-order phase transition seen in binary systems (see Sec. 3.2).

Saif and Gade also investigated this issue and found that there was a first order transition between a regime with an all-defector state to a mixed state [101]. All these works share in common the conclusion that many of the previously observed equilibrium states are mere artifacts of a synchronous updating on a regular lattice.

Ruxton and Saravia have discussed the importance of the ordering in the context of ecological modelling, studying a stochastic model of colonisation of an environment by a species [100]. They argue in favour of adapting the updating scheme to the physical reality of the system that is modelled. The authors also emphasise the need to describe precisely the updating scheme that is used in order to facilitate the reproducibility of the experiments. These arguments come to strengthen the need for studying in detail the “emergence phenomena” that are seen in Ecology and question whether the predictions of the models can be observed in “real-life” systems [94, 22].

5.4 Synchronising systems

Another interesting biological example is given by Messinger et al., who investigated the link between emergence of synchrony and the simultaneous opening and closing stomatal arrays in plants [74].

In Physics, Radicchi et al. studied how simulations of an Ising spin system would be dependent on the synchrony rate and also detected phase transitions [89]. (Note that the update rule is itself stochastic). Like Correia et al., the authors emphasise the fact that neither totally synchronous nor totally asynchronous updating is fully relevant for modelling natural systems.

5.5 Other related models

On the simulation side, Overeinder and Sloot were among the first to examine how to deal with the simulation on asynchronous CA on distributed systems [83]. Bandman and other authors studied how to simulate chemical systems with asynchronous CA [10, 109]. Hoseini et al. made an implementation of asynchronous CA with FPGAs [55]. They propose a particular design of the FPGA in order to construct a “conformal computer”, that is, a computer made of physical cells “arrayed on large thin flexible substrates or sheets. Sheets may be cut, joined, bent, and stacked to conform to the physical and computational needs of an application”.

On the applications side, we mention the work of Bandini et al., who used asynchronous CA with memory for the design of an illumination facility [9] and the work of Minoofam et al., where asynchronous CA produce calligraphic patterns in the Arabic Kufic style [75]. (Unfortunately, this paper lacks precision on the model that is used).

6 Asynchronism in other discrete models of complex systems

We end this guided tour on an opening on the issue of asynchrony in the models of computation that have a structure similar to cellular automata. In fact, this topic is again so wide that it will not be possible to fully cover it and we will be obliged to indicate only a few recent papers and entry points to the topic.

6.1 Links with multi-agent systems

One first proposition to link the updating in multi-agent systems and cellular automata was made by Cornforth et al., but the models they studied are in fact standard asynchronous cellular automata [26]. The question of how to “translate” a multi-agent system with sequential updating into a synchronous CA was tackled by Spicher et al. [112]. In order to model the displacement of the agents, they proposed to operate with “transactional cellular automata” where the movements of particles occur with the help of synchronous communications between neighbouring cells. One positive effect of turning to the synchronous CA is to remove the spurious effects that could be linked to a particular updating order (the authors give the example of a model of Diffusion-Limited-Aggregation).

The link between large-scale multi-agent systems and asynchronous cellular automata was also examined by Tošić [122]. This author argues that the structure of cellular automata needs to be modified in several aspects, among which it should be made asynchronous, in order to serve as a basis for modelling large groups of interacting agents.

An alternative approach to model (discrete) multi-agent systems was proposed by Chevrier and Fatès, who studied the dynamics of a simple multi-turmite systems, also known as multiple Langton’s ants. Their formalism, inspired by cellular automata, captures the possibility to have *synchronous* interacting agents [24]. The difficulty relies in describing how to solving conflicts that occur when two or more agents simultaneously want to modify the environment. The solution relies on a framework invented by Ferber and Muller called *influence-reaction* [44]. Belgacem and Fatès later extended this work by considering a wider range of updating procedures and discovered some phenomena (e.g., gliders) that resisted variations in the updating choices [12].

6.2 Lattice-gas cellular automata

Bouré et al. designed a first version of an asynchronous Lattice-Gas Cellular Automaton (LGCA) [16]. Their proposition can be seen as a bridge between cell-based updating and agent-based updating. In this model, movements of particles are defined explicitly, like in multi-agents, but the updating is made cell by cell, like in classical cellular automata. Various responses to asynchrony are observed depending on the patterns on which the system stabilises: while the synchronous system may display strange patterns such as checkerboards [18],

these patterns disappear with an asynchronous updating. (It is an open problem to know if an infinitesimal amount of asynchrony is sufficient to destroy this pattern.) This construction needs however to be continued as there is to date no agreement on what an asynchronous LGCA should be, in particular on how to respect both the exclusion rule and the locality of the model.

6.3 Automata networks, neural networks and other models

The effect of asynchronous updating in genetic regulatory networks has also been investigated by many authors. Aracena et al. introduce a labelled directed graph that allows to determine to which extent to deterministic update schedules are equivalent [5]. Sené et al. [28], and Noual [82] examine the robustness of the system under the variation of updating schemes and this perturbation is coupled with various topological modifications of the network such as adding or removing links in the graph or changing boundary conditions.

The question of the effect of a synchronous updating in neural networks has been discussed by Taouali et al. [118]. In particular, the authors introduce an interesting distinction between the use of (a)synchronous updating at the modelling level and at the implementation level. Stark discussed the computing abilities of a computing medium formed out of non-regularly placed cells which obey asynchronous updating (“amorphous computing”) [114, 113].

To end this tour, we mention that various works studied the differences between synchronous and asynchronous updating in coupled map lattices [67, 95, 1]. Similarly, the effects of the updating in the Asymmetric Exclusion Process (ASEP) have been studied by Rajewsky et al. [90]. Tomita et al. have studied asynchronous graph-rewriting systems and showed how to make such systems simulate their synchronous counterparts [121].

7 Closing words

This guided tour allowed us to be acquainted with the various contributions that regard the question of asynchronism in cellular automata. As we have seen, asynchrony is a *negative* property that does not in itself specify any system: there are plenty of types of manners to make the updating of a CA asynchronous and all of them are equally valid. It thus necessary to continue studying how to define various asynchronous schemes.

Among the interesting open questions, there is a need to clarify the notion of *simulation* of an (a)synchronous system by another (a)synchronous one. This can be done according to the *possibility* to have the same transitions in the host and the guest or, it can be defined according of *probability distributions* that such a property is seen (see Ref. [6] for wider context). As a consequence, the meaning of “fully asynchronous” (only one cell updated at each time step) is different depending on the point of view adopted by the authors (compare e.g.

Ref. [128] and Ref. [42] where one cell is chosen *uniformly at random* among a *finite* subset of cells updated at each time step).

Moreover, in a probabilistic setting, the fact that the set of cells is finite is important and can *not* be readily generalised to infinite sets. Indeed, if for instance the set of cells considered is \mathbb{Z} , choosing one cell uniformly at random at each time step assigns a null probability to each cell to be chosen! This is why a passage to the limit needs to be done with a model that has a *continuous time* and the mathematical model that accurately describes a (stochastic) fully asynchronous updating is called an *interacting particle system*. (See e.g. [37, 20] for examples where such systems are used for solving the Density classification problem in two dimensions.) In general, it would be useful to clarify the similarities and differences between the two types of systems.

It is also important to clarify the position of asynchronous CA into the wider field of stochastic CA. Indeed, asynchrony can not be considered as mere type of noise as, for instance, the addition of asynchrony to a deterministic model does not change its fixed points. However, many of the surprising phenomena, such as the existence of second-order phase transitions controlled by the synchrony rate, need to find some highlights from the field of stochastic cellular automata. Note that here too asynchrony holds a particular position: if it is intuitive to understand why an epidemic brutally percolates when the probability of contamination increases, it is much more difficult to understand why simple rules such as ECA 50 do or do not converge to an all-0 fixed point depending on the synchrony rate of the cells...

As far as *simple* asynchronous cellular automata are concerned, the main challenge is to continue to explore their dynamical properties with a *joint work* of analytical approach and numerical simulations. We find it rather amazing that it is still an open question to determine the average converging time of some binary rules [39]. Finding the appropriate parameters to quantify the robustness or the sensitivity of the CA rules to asynchrony is a major current challenge.

Last, but not least, we point out the crucial need for a careful use of time in cellular automata simulation of complex systems. There is to date very few experimental work dedicated to validating some models of asynchrony and dismissing some others. As we mentioned earlier, the no-global-clock argument (“Nature does not possess a clock to synchronise the transitions”) can not be received directly and be taken alone as a valid objection to the use of synchronous models. Instead, it would be wiser to consider that studying *one* model of asynchrony is not sufficient and that to build novel models of “Natural computing” [107], the robustness of cellular automata rules should be tested against a large panel of updating schemes.

Acknowledgements

May all the persons that have contributed to the elaboration of this text receive my sincere expression of gratefulness. The author knows how much he owes to them and, in echo to Heraclitus' word that "the invisible harmony is greater than the apparent one" (DK B54), he hopes that they will pardon him for not being named here. (Saying thank you properly is always a delicate question.)

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