



HAL
open science

A guided tour of of asynchronous cellular automata

Nazim Fatès

► **To cite this version:**

| Nazim Fatès. A guided tour of of asynchronous cellular automata. 2013. hal-00908373v1

HAL Id: hal-00908373

<https://inria.hal.science/hal-00908373v1>

Preprint submitted on 22 Nov 2013 (v1), last revised 26 Aug 2014 (v5)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A guided tour of asynchronous cellular automata

Nazim Fatès

`nazim.fates@loria.fr`

Inria Nancy Grand-Est, LORIA UMR 7503

F-54 600, Villers-lès-Nancy, France

November 22, 2013

Abstract

Research on asynchronous cellular automata has received a great amount of attention these last years and has turned to a thriving field. We survey the recent research that has been carried out on this topic and present a wide state of the art where computing and modelling issues are both represented.

foreword: This article is a corrected and extended version of the invited paper that appeared in the proceedings of Automata'13, 19th International Workshop on Cellular Automata and Discrete Complex Systems, LNCS 8155, 2013, p. 15-30, doi:10.1007/978-3-642-40867-0_2.

1 Introduction

Research on asynchronous cellular automata has gained momentum in the last decade as it can be seen from the increase in the number of publications dedicated to this topic or the organisation of scientific meetings dedicated to this topic.

Cellular automata are discrete dynamical systems that were initially constructed by von Neumann and Ulam to study self-reproduction from a logical and mechanistic point of view [99]. The interest of cellular automata stems from the simplicity of their structure: they are constituted of a collection of simple automata, the *cells*, that are spatially arranged on a grid and which evolve according to a *local rule* which gives the new state of a cell according to its previous state and the previous states of its neighbours. Classically, cellular automata are updated synchronously, that is, all cells change their state simultaneously at discrete time steps and the global state of the system at a given time is obtained from the information gained on the state of the system at the previous time. This hypothesis has many advantages: with this type of updating, the system is deterministic and simple to define. The use of a common shared time allows one to simulate such systems very easily and, in some cases, to derive analytical

results on the behaviour of the rule. There are however some reasons why this hypothesis of perfect synchrony could be questioned:

- When cellular automata represent a model of a natural system, one should examine if there is a need of a global clock that synchronises the transitions of the model. In particular, is this clock an external phenomenon (e.g., the daily varying light of the sun) or is it an effect that emerges from the numerous interactions between the components?
- When cellular automata are considered as a model of a massively parallel computing device, then, operating such a device without having to distribute the signal of a clock could have many advantages such as an increase in the speed of computations, economy of energy, simplicity of design, etc.

These statements however do not unequivocally plead for the use of the asynchronous models as opposed to their synchronous counterparts. Indeed, as soon as the question of asynchrony is raised, numerous other questions have to be answered ; for instance: What type of asynchronism should be used? What are the properties of the system that are acquired or lost when turned from synchronous to asynchronous ? How should we study the new system (analytically, experimentally, etc.) ?

The purpose of this short review is to introduce the readers to the wide landscape of asynchronism in cellular automata. As a first version destined to the audience of AUTOMATA'13, and like all guided tours, it is not meant to be exhaustive nor fully objective, but rather to introduce some entry points and discussions on this wide topic. The author of the present text will be grateful to all indications or corrections that will be given to him. Interested readers are encouraged to check for updated versions of this text¹.

2 Early works

Nakamura was one of the first authors to investigate the computation abilities of asynchronous cellular automata (ACA) [61]. He described several techniques to construct universal ACA and showed how to simulate a given q -state deterministic CA with an ACA of the same neighbourhood whose state space is extended to $3q^2$ states (see also Toffoli [94] and Nehaniv [63]).

The construction relies on the idea that if a cell is updated, it will then wait the neighbouring cells to “catch up” and makes the next transition only when all its neighbours are up to date ; additionally, it keeps its old state available for the neighbouring cells in order for them to perform the “right” transitions. This construction was later improved by the use of only $q^2 + 2q$ states by Lee, Peper et al. [50, 67].

Priese wrote a note where he considers (two-dimensional) cellular automata as a particular case of asynchronous rewriting systems (semi-Thue-systems) and

¹ See <http://hal.inria.fr/hal-00845623> or visit the author’s web page.

widens the scope by considering also the case where more than one cell may be re-written at a time (the overlapping problem) [70]. He uses his construction to show how to build asynchronous circuits which are equivalent to asynchronous concurrent Petri nets.

Following this path, Zielonka examined how asynchronous CA could be used to describe the situations of concurrency that arise in distributed systems [19]. Pighizzini clarified the computing abilities of Zielonka's models [69] and the problem of how to determinise non-deterministic Büchi asynchronous cellular automata was solved by Muscholl [60]. Droste generalised to pomsets the original notion of Zielonka's asynchronous mappings [25]; these questions were later re-investigated by Kuske [26, 45, 46].

Readers wishing to learn more on the universality of asynchronous CA may refer to the study by Takada et al., in which many important arguments and useful references can be found [92].

3 Numerical simulations and experimental studies

At the same time when the systematic exploration of complete cellular spaces (such as Elementary Cellular Automata) were carried out, authors have examined the effect of asynchronism by the means of numerical simulation. Ingerson and Buvel carried out a pioneering work where they could show that the behaviour of simple rules could be overwhelmed by simple modifications [14]. Most importantly, they questioned to which extent was the behaviour of a rule the consequence of the local rule and to which extent it was due to the updating scheme.

Bersini and Detours explored the difference between the Game of Life and closely related models with various updating schemes [10]. They observed a “stabilising effect” of asynchronous updating. Their views were based on small-size experiments, with grids no larger than 20*20 cells. With such lattice sizes, they were able to observe that the Game of Life with asynchronous updating may “freeze” on some fixed-point pattern that has a labyrinth aspect.

Schönfisch and de Roos gave a decisive impulse to the research on ACA by comparing various updating schemes and by exhibiting clear examples to show that the effect of these schemes could alter significantly the behaviour of a CA [82]. They distinguish time-driven and step-driven updating methods, depending on whether the updating of a cell is triggered by the determination of an event given by a continuous time proper to each cell or by a global clock which would send signals to all or part of the cells.

Note that it may be believed at first sight that “time-driven” methods provide a more realistic simulation framework since the updating signal is not artificially shared among cells. As shown by the authors themselves, this is not true since it is easy to build a “step-driven” method that emulates a “time-driven” method: as the update is sequential, the only relevant information for

the evolution of the updating is the order of updating, and a correspondence between the two schemes can thus be established. Moreover, as various authors tend to believe, even the hypothesis of sequentiality of events which states that no two updating events can happen at the same time is not necessarily realistic. Indeed, this hypothesis would be relevant only in the case where the transitions between states are instantaneous. As this hypothesis is far from being valid in many cases (especially in biological systems), a certain “degree of synchrony” between cells should thus be assumed. As this degree is difficult to measure, the problem is not so much about choosing the “right” model of updating but rather to estimate the robustness of model, that is, the degree to which it is sensitive or not to changes of its updating scheme.

This idea lead Fatès and Morvan to examine how the 256 Elementary cellular automata (ECA) reacted to an updating where the degree of synchrony was varied [33]. This updating scheme they used, named simply α -asynchronism² consists in updating each cell with probability α , the *synchrony rate*, and leaving the state of the cells unchanged otherwise. In order to quantify the observations, the authors used the value of the asymptotic density as a means to estimate changes in the behaviour of the system. This parameter was considered as a first approximation to detect changes in the behaviour: a strong variation of the asymptotic density indicates that the system has undergone a transformation while an absence of variation does not say anything on the stability of the system as other modifications than the density may occur.

Despite its simplicity, this approach revealed that no direct relationship between the asynchronous behaviour and the “complexity” of the synchronous behaviour (Wolfram classes) existed. Four qualitative responses to variations were remarked:

1. continuous variation of the behaviour (e.g. ECA 232),
2. discontinuity around $\alpha = 1$ (e.g. ECA 2 or 110),
3. phase transition for a critical value $\alpha_c < 1$ (e.g. ECA 50),
4. non-regular behaviour (e.g. ECA 184).

An interesting development on the work of asynchronism concerns how it mixes with traditional noise, that is, on randomness imposed on the *state* of the cells that compose the automaton. Early references that address this question are a paper by Gharavi and Anantharam, where they study a well-known result of Toom [39], and the work of Kanada, which tackles ECA rules [44]. Mamei et al. also considered this question for studying “complex decentralised pervasive computing systems” that are models as two-dimensional binary CA [56]. The most recent development is by Silva and Correia, who have given a detailed account on robust or sensitive some ECA can be to asynchronism combined

² Note that the terms α -asynchronism and α -synchronism have been both used and are both relevant: the confusion comes from the fact that α is the name of the scheme and the synchrony rate. We use here the term α -asynchronism, as it is the form that was first proposed and which has been adopted by various authors such as Regnault, Correia, Worsch, Fuks, etc.

with noise. Interestingly, they propose to evaluate the robustness according to the difference patterns and they thus introduce a sampling compensation in order to cope with less frequent updates [85, 86].

The case of asynchronous CA simulated on a non-regular topology was tackled by Baetens et al., who examined an asynchronous updating with cells linked by the frontiers of a Voronoi tessellation [5].

4 Phase transitions

Blok and Bergersen showed that the Game of Life was subject to a phase transition when it was updated asynchronously [11]. They used α -asynchronism and showed the existence of a transition from a “static” behaviour, where the system would settle on fixed points, to a “living” behaviour where the system evolves by forming labyrinth-like patterns. These authors identified that the change of behaviour was a second-order phase transition that belongs to the directed percolation universality class.

Fatès identified that directed percolation was also the “signature” displayed by seven of the minimal ECA rules [27]. A special rule, namely ECA 178, was shown to belong to the PC/DP2 universality class, a class very similar to DP but where the each state 0 and 1 play a symmetric role.

The phase transition occurring in the Game of life was also re-examined in detail, especially by studying how this phenomenon was affected by random perturbations of the topology of the CA [28, 29]. The main finding was that this phase transition was strongly dependent on the regularity of the grid and that it was progressively “washed out” as links between cells were progressively removed.

Another puzzling phenomenon was remarked [33]: two systems, started from different initial conditions, but evolving with the same updating sequence, could rapidly “coalesce” into the same state. This is not particularly surprising per se except in the case where this coalescence phenomenon occurs even before the system stabilises on a fixed point. Rouquier and Morvan studied systematically the coalescence phenomenon for the 256 ECA [76]. Their study revealed that it was possible to observe that some ECA would always coalesce, while other would never coalesce, and that there existed some rules which displayed a phase transition between a coalescing and non-coalescing behaviour, depending on the synchrony rate.

5 Analytical results

Compared to the experimental work devoted to asynchronous CA, analytical results are relatively rare, but they are gaining momentum as more probabilistic techniques are being developed.

One of the first analytical results of a systematic classification were given by analysing the ECA with two quiescent states under fully asynchronous dy-

namics [34], that is when only one cell is updated randomly at each time step. These results were later extended by Regnault et al. tackling the case of α -asynchronous updating [35] and by Chassaing and Gerin, who examined what was continuous limit of the processes when the grid was made infinite [17].

Fatès and Gerin also examined how to classify the two-dimensional totalistic CA with fully asynchronous updating [32]. They proposed a partial classification of the 64 rules and an analysis of the convergence of some well-known rules. Among the interesting phenomena remarked, they exhibited a list of rules, which showed an “erratic” behaviour: the question was to determine if these rules were exhibiting a non-converging behaviour or a “metastable” behaviour, that is, if a (long) random sequence of updates could drive the system to a stable state. By using techniques from automatic planning, Hoffmann et al. could solve this problem for one rule and showed that it converged to a fixed point in (at most) exponential time [41]. (See also Ref. [31]).

Links between the notions of asynchrony and reversibility were examined by Das et al. [80, 21], first by considering the case where the updating sequence (or “update pattern”) could be set freely. This work was recently extended to the case of fully asynchronous updating and a classification of the ECA rules was proposed: the recurrent rules, the irreversible, and strongly irreversible [83]. Wacker and Worsch also examined the question of reversibility of ACA [100], but took a different point of view. In their work, a rule is said to be reversible if there is another rule whose state-transition graph is the “inverse” of the original. Interestingly, the results presented on ECA are not far from those found in Ref. [83]. This question remains currently widely open.

Concerning the analysis of phase transition phenomena, Regnault carried out a pioneering work by analysing in detail how the asynchronous minority rule displayed various types of behaviour, depending on whether the von Neumann and Moore neighbourhood was used [74] or even when more general graphs were considered [77]. Two different complementary views exist on phase transitions: the most common way of describing a phase transition is to establish that for an *infinite* system, a qualitative difference of behaviour occurs for an infinitesimal variation of the control parameter. This second approach was adopted by Regnault who could (partially) prove that for *finite systems*, this phase transition corresponds to a variation of the convergence time from a linear to a polynomial function of the system’s size [73].

Fukš and Skelton analysed three simple α -asynchronous rules [37]. They considered infinite systems with an initial Bernoulli measure as an initial condition and determined how the asymptotic density varies as a function of the initial density (that is, the parameter of the Bernoulli measure). Such results are generally rather difficult to obtain for the deterministic ECA.

6 Definitions of ACA and their mathematical properties

Gács was one of the first authors to investigate the question of whether the evolution of an asynchronous system could be independent of the order of updating [38]. He showed that although this property was undecidable, there existed a sufficient condition to verify this independence.

This question was re-examined by Mortveit, Macauley et al., who studied in which cases repetitions of sequential updates on Elementary Cellular Automata could produce a set of periodic points that would be independent of the updating order [54, 55, 53]. They obtained a list of 104 ECA which display such an independence and, interestingly, proposed a new representation of ECA that differs from the classical Wolfram code and that could prove useful for future analysis of asynchronous systems. (Another alternative notation is presented in Ref. [34]).

The possibility to simulate asynchronous rules by a universal asynchronous simulator was examined by Worsch [101], who proved that there was an α -asynchronous host that could simulate any α -asynchronous guest that would be updated with the same synchrony rate. This result can be even generalised to other types of asynchronism such as fully asynchronous updating or to an even wider variety of updating called “purely asynchronous updating”.

Manzoni examined how the dynamical properties of CA, such as injectivity, surjectivity, permutivity, etc., could be re-defined and studied in the asynchronous updating context [57]. An extended definition of asynchronous CA based on the use of probability measures was proposed by Formenti et al. [24]. Dennunzio et al. have proposed an original way to simulate a universal Turing machine by the means of a fully asynchronous CA [23]. They introduce the notion of “scattered strict simulation” in which they tolerate that only a subset of cells is used to perform the simulation. They find that asynchrony induces a quadratic slowdown compared to the speed of the simulated Turing machine.

Regarding the various possibilities to define asynchronous updating schemes, Bandini et al. presented a formal description and analysis of various schemes and tested their effects on one-dimensional binary rules where the local function depends only on two neighbours (also called “radius-1/2” rules) [6].

Finally, we want to mention the extension of asynchronism proposed by Bouré et al.: while most approaches of asynchronism studied so far are based on the dichotomy updated / not updated, this approach named β - and γ -synchronism, models imperfect communication of states between neighbours [13]. The main finding of the authors is that if many previously observed phenomena, such as the existence of phase transitions that depend on the synchrony rate, are reproduced, some unexpected phenomena also arise such as the conservation of additional quantities (ECA 50) and the unexpected disappearance of some phase transitions. This underlines the necessity to continue to propose various perturbations of the “classical” CA in order to examine how their properties are dependent on various hypotheses such as a perfect updating.

7 Computing with ACA

Tomassini and Venzi [95], Capcarrere [15] and Nehaniv [62] have studied how interesting asynchronous cellular automata may be used to solve problems such as the density classification or the global synchronisation. Readers interested in this issue are referred to a study by Vanneschi and Mauri, in which an enlightening discussion on these various contributions is found and where the authors present findings of robust and generic rules [97].

Suzudo examined the use of genetic algorithms to find mass-conservative ACA that would generate non-trivial patterns [90, 91]. He classified these patterns into three categories: checkerboards, stripes and sand-like. Although in this work asynchronism is mainly used to ensure mass conservation, it appears also as a useful means of generating regular patterns out of randomness, a task that is known to be very difficult in the synchronous setting (see e.g. [30]).

Another major field of research on asynchronous CA was developed by Peper, Lee and their collaborators. In their constructions, asynchronous computations are realised with particles that follow Brownian movements and which interact through special “gates” [3, 2, 49, 68]. These constructions result in delay-insensitive circuits that are Turing universal (see e.g. [47, 51] and references therein). Recently, Schneider and Worsch presented a 3-state CA that uses Moore neighbourhood which can simulate any delay-insensitive circuit [81]. It is also worth mentioning that these techniques also allowed the design of asynchronous models of self-reproduction [48].

In an original line of thought, Vielhaber has designed a formal framework in which computations are made by a proper use of the order of updating [98]. In particular, he showed that ECA 57 with periodic boundary conditions was a rule especially adapted for such a purpose.

8 Modelling with ACA

We are now close to end this guided tour and it is time to mention the work devoted to modelling physical or artificial systems with asynchronous cellular automata. While this part could be much more developed, we will keep it as short as possible and mention only a few entry points to the topic. The reason is that there is a huge number of works which employ asynchronous cellular automata, often without even mentioning the fact explicitly. We will thus try to concentrate only on some papers where the question of the updating is explicitly mentioned.

A good example is given by Huberman and Glance, who challenged the validity of the simulations of spatially-extended models of the Prisoner’s dilemma on the basis that asynchronous and synchronous updating lead to drastically opposed conclusions [43]. This question was re-examined by Newth and Cornforth [64], and Grilo and Correia [40], who used α -asynchronous updating to explore a wide range of degrees of synchrony. Saif and Gade also investigated this issue and found that there was a first order transition between a regime

with an all-defector state to a mixed state [79]. All these works share in common the conclusion that many of the previously observed equilibrium states are mere artifacts of a synchronous updating on a regular lattice.

Another interesting biological example is given by Messinger et al., who investigated the link between emergence of synchrony and the simultaneous opening and closing stomatal arrays in plants [58].

In physics, Radicchi et al. [71] studied how simulations of an Ising spin system would be dependent on the synchrony rate and also detected phase transitions. (Note that contrary to the previously seen cases [27, 74, 29], the update rule is here stochastic). Like Correia et al., the authors emphasise the fact that neither totally synchronous nor totally asynchronous updating is fully relevant for modelling natural systems.

Ruxton and Saravia have discussed the importance of the ordering in the context of ecological modelling, studying a stochastic model of colonisation of an environment by a species [78]. They argue in favour of adapting the updating scheme to the physical reality of the system that is modelled. The authors also emphasise the need to describe precisely the updating scheme that is used in order to facilitate the reproducibility of the experiments. An idea that is also developed by Caron-Lormier et al. at the same time [16].

On the simulation side, Overeinder and Sloot were among the first to examine how to deal with the simulation on ACA on distributed systems [66]. Bandman and other authors studied how to simulate chemical systems with asynchronous CA [8, 84]. Hoseini et al. made an implementation of asynchronous CA with FPGAs [42]. They propose a particular design of the FPGA in order to construct a “conformal computer”, that is, a computer made of physical cells “arrayed on large thin flexible substrates or sheets. Sheets may be cut, joined, bent, and stacked to conform to the physical and computational needs of an application” [42].

On the applications side, we mention the work of Bandini et al., who used asynchronous CA with memory for the design of an illumination facility [7] and the work of Minoofam et al., who applied asynchronous CA for producing calligraphic patterns in the Arabic language (Kufic style) [59].

9 Asynchronism in similar models

We end this guided tour on an opening on the issue of asynchrony in the models of computation that have a structure similar to cellular automata. In fact, this topic is again so wide that it will not be possible to fully cover it and we will be obliged to indicate only a few recent papers and entry points to the topic.

One first proposition to link the updating in multi-agent systems and cellular automata was made by Cornforth et al., but the models they studied are in fact standard asynchronous cellular automata [20]. The question of how to “translate” a multi-agent system with sequential updating into a synchronous CA was tackled by Spicher et al. [87]. In order to model the displacement of the agents, they proposed to operate with “transactional cellular automata” where

the movements of particles occur with the help of synchronous communications between neighbouring cells. One positive effect of turning to the synchronous CA is to remove the spurious effects that could be linked to a particular updating order (the authors give the example of a model of Diffusion-Limited-Aggregation).

The link between large-scale multi-agent systems and asynchronous cellular automata was also examined by Tošić [96]. This author argues that the structure of cellular automata needs to be modified in several aspects, among which it should be made asynchronous, in order to serve as a basis for modelling large groups of interacting agents.

An alternative approach to model (discrete) multi-agent systems was proposed by Chevrier and Fatès, who studied the dynamics of a simple multi-turmite systems, also known as multiple Langton’s ants. Their formalism, inspired by cellular automata, captures the possibility to have *synchronous* interacting agents [18]. The difficulty relies in describing how to solving conflicts that occur when two or more agents simultaneously want to modify the environment. The solution relies on a framework invented by Ferber and Muller called influence-reaction [36]. Belgacem and Fatès later extended this work by considering a wider range of updating procedures and discovered some phenomena (e.g., gliders) that resisted variations in the updating choices [9].

Bouré et al. designed a first version of an asynchronous Lattice-Gas Cellular Automaton (LGCA) [12]. Their proposition can be seen as a bridge between cell-based updating and agent-based updating. In this model, movements of particles are defined explicitly, like in multi-agents, but the updating is made cell by cell, like in classical cellular automata. Various responses to asynchrony are observed depending on the patterns on which the system stabilises: stripes or clusters are robust while checkerboards, a somewhat paradoxical pattern, are shown not to be robust. This construction needs however to be completed as there is to date no agreement on what an asynchronous LGCA should be, in particular on how to respect the locality of the model in the asynchronous setting.

The effect of asynchronous updating in genetic regulatory networks has also been investigated by many authors. Aracena et al. introduce a labeled directed graph that allows to determine to which extent to deterministic update schedules are equivalent [4]. Sené et al. [22], and Noual [65] examine the robustness of the system under the variation of updating schemes and this perturbation is coupled with various topological modifications of the network such as adding or removing links in the graph or changing boundary conditions.

The question of the effect of a synchronous updating in neural networks has been discussed by Taouali et al. [93]. In particular, the authors introduce an interesting distinction between the use of (a)synchronous updating at the modelling level and at the implementation level. Stark discussed the computing abilities of a computing medium formed out of non-regularly placed cells which obey asynchronous updating (“amorphous computing”) [89, 88].

To end this tour, we mention that various works studied the differences between synchronous and asynchronous updating in coupled map lattices [52, 75, 1]. Similarly, the effects of the updating in the Asymmetric Exclusion Process

(ASEP) have been studied by Rajewsky et al. [72].

References

- [1] Guillermo Abramson and Damián H. Zanette. Globally coupled maps with asynchronous updating. *Physical Review E*, 58:4454–4460, 1998.
- [2] Susumu Adachi, Ferdinand Peper, and Jia Lee. Computation by asynchronously updating cellular automata. *Journal of Statistical Physics*, 114(1-2):261–289, 2004.
- [3] Susumu Adachi, Ferdinand Peper, and Jia Lee. Universality of hexagonal asynchronous totalistic cellular automata. In Peter Sloot, Bastien Chopard, and Alfons Hoekstra, editors, *Proceedings of ACRI'04*, volume 3305 of *Lecture Notes in Computer Science*, pages 91–100. Springer Berlin Heidelberg, 2004.
- [4] Julio Aracena, Eric Goles, Andrés Moreira, and Lilian Salinas. On the robustness of update schedules in boolean networks. *Biosystems*, 97(1):1–8, 2009.
- [5] Jan M. Baetens, Pieter Van der Weeën, and Bernard De Baets. Effect of asynchronous updating on the stability of cellular automata. *Chaos, Solitons & Fractals*, 45(4):383–394, 2012.
- [6] Stefania Bandini, Andrea Bonomi, and Giuseppe Vizzari. An analysis of different types and effects of asynchronicity in cellular automata update schemes. *Natural Computing*, 11(2):277–287, 2012.
- [7] Stefania Bandini, Andrea Bonomi, Giuseppe Vizzari, and Vito Acconci. An asynchronous cellular automata-based adaptive illumination facility. In Roberto Serra and Rita Cucchiara, editors, *Proceedings of AI*IA 2009: Emergent Perspectives in Artificial Intelligence*, volume 5883 of *Lecture Notes in Computer Science*, pages 405–415. Springer Berlin Heidelberg, 2009.
- [8] Olga L. Bandman. Parallel composition of asynchronous cellular automata simulating reaction diffusion processes. In Stefania Bandini, Sara Manzoni, Hiroshi Umeo, and Giuseppe Vizzari, editors, *Proceedings of ACRI'10*, volume 6350 of *Lecture Notes in Computer Science*, pages 395–398. Springer, 2010.
- [9] Selma Belgacem and Nazim Fatès. Robustness of multi-agent models: the example of collaboration between turmites with synchronous and asynchronous updating. *Complex Systems*, 21(3):165–182, 2012.
- [10] Hugues Bersini and Vincent Detours. Asynchrony induces stability in cellular automata based models. In Rodney A. Brooks and Pattie Maes,

- editors, *Proceedings of the 4th International Workshop on the Synthesis and Simulation of Living Systems ArtificialLifeIV*, pages 382–387. MIT Press, 1994.
- [11] Hendrik J. Blok and Birger Bergersen. Synchronous versus asynchronous updating in the “game of life”. *Physical Review E*, 59:3876–9, 1999.
 - [12] Olivier Bouré, Nazim Fatès, and Vincent Chevrier. First steps on asynchronous lattice-gas models with an application to a swarming rule. In Georgios Sirakoulis and Stefania Bandini, editors, *Proceedings of ACRI’12*, volume 7495 of *Lecture Notes in Computer Science*, pages 633–642. Springer Berlin Heidelberg, 2012. Long version to appear in *Natural computing*.
 - [13] Olivier Bouré, Nazim Fatès, and Vincent Chevrier. Probing robustness of cellular automata through variations of asynchronous updating. *Natural Computing*, 11:553–564, 2012.
 - [14] Buvel, R.L. and Ingerson, T.E. Structure in asynchronous cellular automata. *Physica D*, 1:59–68, 1984.
 - [15] Mathieu Capcarrere. Evolution of asynchronous cellular automata. In Juan Julián Merelo Guervós, Panagiotis Adamidis, Hans-Georg Beyer, Hans-Paul Schwefel, and Jos´-Luis Fernández-Villacañas, editors, *Proceedings of Parallel Problem Solving from Nature PPSN VII*, volume 2439 of *Lecture Notes in Computer Science*, pages 903–912. Springer Berlin Heidelberg, 2002.
 - [16] Geoffrey Caron-Lormier, Roger W. Humphry, David A. Bohan, Cathy Hawes, and Pernille Thorbek. Asynchronous and synchronous updating in individual-based models. *Ecological Modelling*, 212(34):522 – 527, 2008.
 - [17] Philippe Chassaing and Lucas Gerin. Asynchronous cellular automata and brownian motion. In *DMTCS Proceedings of AofA’07*, volume AH, pages 385–402, 2007.
 - [18] Vincent Chevrier and Nazim Fatès. How important are updating schemes in multi-agent systems? an illustration on a multi-turmite model. In *Proceedings of AAMAS ’10*, pages 533–540, Richland, SC, 2010. International Foundation for Autonomous Agents and Multiagent Systems.
 - [19] Robert Cori, Yves Métivier, and Wieslaw Zielonka. Asynchronous mappings and asynchronous cellular automata. *Information and Computation*, 106(2):159–202, 1993.
 - [20] David Cornforth, David G. Green, and David Newth. Ordered asynchronous processes in multi-agent systems. *Physica D*, 204(12):70 – 82, 2005.

- [21] Sukanta Das, Anindita Sarkar, and Biplab K. Sikdar. Synthesis of reversible asynchronous cellular automata for pattern generation with specific hamming distance. In Georgios Ch. Sirakoulis and Stefania Bandini, editors, *Proceedings of ACRI'12*, volume 7495 of *Lecture Notes in Computer Science*, pages 643–652. Springer, 2012.
- [22] Jacques Demongeot, Eric Goles, Michel Morvan, Mathilde Noul, and Sylvain Sené. Attraction basins as gauges of robustness against boundary conditions in biological complex systems. *PLoS One*, 5(8):e11793, 2010.
- [23] Alberto Dennunzio, Enrico Formenti, and Luca Manzoni. Computing issues of asynchronous CA. *Fundamenta Informaticae*, 120(2):165–180, 2012.
- [24] Alberto Dennunzio, Enrico Formenti, Luca Manzoni, and Giancarlo Mauri. m-asynchronous cellular automata. In GeorgiosCh. Sirakoulis and Stefania Bandini, editors, *Proceedings of ACRI'12*, volume 7495 of *Lecture Notes in Computer Science*, pages 653–662. Springer Berlin Heidelberg, 2012. Extended version to appear in *Natural Computing*.
- [25] Manfred Droste and Paul Gastin. Asynchronous cellular automata for pomsets without auto-concurrency. In Ugo Montanari and Vladimiro Sassone, editors, *Proceedings of CONCUR'96*, volume 1119 of *Lecture Notes in Computer Science*, pages 627–638. Springer Berlin Heidelberg, 1996.
- [26] Manfred Droste, Paul Gastin, and Dietrich Kuske. Asynchronous cellular automata for pomsets. *Theoretical Computer Science*, 247(1-2):1–38, 2000.
- [27] Nazim Fatès. Asynchronism induces second order phase transitions in elementary cellular automata. *Journal of Cellular Automata*, 4(1):21–38, 2009.
- [28] Nazim Fatès. Critical phenomena in cellular automata: perturbing the update, the transitions, the topology. *Acta Physica Polonica B - Proceedings Supplement*, 3(2):315–325, 2010.
- [29] Nazim Fatès. Does *life* resist asynchrony? In Andrew Adamatzky, editor, *Game of Life Cellular Automata*, pages 257–274. Springer London, 2010.
- [30] Nazim Fatès. A note on the density classification problem in two dimensions. In *Exploratory papers presented at Automata 2012*, La Marana, Corse, France, 2012.
- [31] Nazim Fatès. A guided tour of asynchronous cellular automata. In Jarkko Kari, Martin Kutrib, and Andreas Malcher, editors, *Automata*, volume 8155 of *Lecture Notes in Computer Science*, pages 15–30. Springer, 2013.
- [32] Nazim Fatès and Lucas Gerin. Examples of fast and slow convergence of 2D asynchronous cellular systems. *Journal of Cellular Automata*, 4(4):323–337, 2009.

- [33] Nazim Fatès and Michel Morvan. An experimental study of robustness to asynchronism for elementary cellular automata. *Complex Systems*, 16:1–27, 2005.
- [34] Nazim Fatès, Michel Morvan, Nicolas Schabanel, and Eric Thierry. Fully asynchronous behavior of double-quiescent elementary cellular automata. *Theoretical Computer Science*, 362:1–16, 2006.
- [35] Nazim Fatès, Damien Regnault, Nicolas Schabanel, and Eric Thierry. Asynchronous behavior of double-quiescent elementary cellular automata. In José R. Correa, Alejandro Hevia, and Marcos A. Kiwi, editors, *Proceedings of LATIN 2006*, volume 3887 of *Lecture Notes in Computer Science*, pages 455–466. Springer, 2006.
- [36] Jacques Ferber and Jean-Pierre Müller. Influences and reaction : a model of situated multiagent systems. In *Proceedings of the 2nd International Conference on Multi-agent Systems*, pages 72–79, 1996.
- [37] Henryk Fukś and Andrew Skelton. Orbits of the bernoulli measure in single-transition asynchronous cellular automata. In Nazim Fatès, Eric Goles, Alejandro Maass, and Ivan Rapaport, editors, *Proceedings of Automata '11*, Discrete Mathematics and Theoretical Computer Science Proceedings, pages 95–112. DMTCS, 2011.
- [38] Péter Gács. Deterministic computations whose history is independent of the order of asynchronous updating. *CoRR*, cs.DC/0101026, 2001.
- [39] Reza Gharavi and Venkat Anantharam. Effect of noise on long-term memory in cellular automata with asynchronous delays between the processors. *Complex Systems*, 6(3):287–300, 1992.
- [40] Carlos Grilo and Luís Correia. Effects of asynchronism on evolutionary games. *Journal of Theoretical Biology*, 269(1):109–122, 2011.
- [41] Jörg Hoffmann, Nazim Fatès, and Héctor Palacios. Brothers in arms? on AI planning and cellular automata. In Helder Coelho, Rudi Studer, and Michael Wooldridge, editors, *Proceedings of ECAI 2010*, volume 215 of *Frontiers in Artificial Intelligence and Applications*, pages 223–228. IOS Press, 2010.
- [42] Mariam Hoseini, Zhou Tan, Chao You, and Mark Pavicic. Design of a reconfigurable pulsed quad-cell for cellular-automata-based conformal computing. *International Journal of Reconfigurable Computing*, 7907:Article ID 352428, 2010.
- [43] Bernardo A. Huberman and Natalie Glance. Evolutionary games and computer simulations. *Proceedings of the National Academy of Sciences, USA*, 90:7716–7718, 1993.

- [44] Yasusi Kanada. The effects of randomness in asynchronous 1d cellular automata (poster). *Artificial Life IV*, 1994.
- [45] Dietrich Kuske. Emptiness is decidable for asynchronous cellular machines. In Catuscia Palamidessi, editor, *Proceedings of CONCUR 2000 Concurrency Theory*, volume 1877 of *Lecture Notes in Computer Science*, pages 536–551. Springer Berlin Heidelberg, 2000.
- [46] Dietrich Kuske. Weighted asynchronous cellular automata. *Theoretical Computer Science*, 374(1-3):127–148, 2007.
- [47] Jia Lee. A simple model of asynchronous cellular automata exploiting fluctuation. *Journal of Cellular Automata*, 6(4-5):341–352, 2011.
- [48] Jia Lee, Susumu Adachi, and Ferdinand Peper. Reliable self-replicating machines in asynchronous cellular automata. *Artificial Life*, 13(4):397–413, 2007.
- [49] Jia Lee, Susumu Adachi, Ferdinand Peper, and Shinro Mashiko. Delay-insensitive computation in asynchronous cellular automata. *Journal of Computer and System Sciences*, 70(2):201–220, 2005.
- [50] Jia Lee, Susumu Adachi, Ferdinand Peper, and Kenichi Morita. Asynchronous game of life. *Physica D*, 194(34):369 – 384, 2004.
- [51] Jia Lee and Qing-Sheng Zhu. A direct proof of turing universality of delay-insensitive circuits. *International Journal of Unconventional Computing*, 8(2):107–118, 2012.
- [52] Erik D. Lumer Lumer and Grégoire Nicolis. Synchronous versus asynchronous dynamics in spatially distributed systems. *Physica D: Nonlinear Phenomena*, 71(4):440 – 452, 1994.
- [53] Matthew Macauley, Jon McCammond, and Henning Mortveit. Dynamics groups of asynchronous cellular automata. *Journal of Algebraic Combinatorics*, 33(1):11–35, 2011.
- [54] Matthew Macauley, Jon McCammond, and Henning S. Mortveit. Order independence in asynchronous cellular automata. *Journal of Cellular Automata*, 3(1):37–56, 2008.
- [55] Matthew Macauley and Henning S. Mortveit. Coxeter groups and asynchronous cellular automata. In Stefania Bandini, Sara Manzoni, Hiroshi Umeo, and Giuseppe Vizzari, editors, *Proceedings of ACR10*, volume 6350 of *Lecture Notes in Computer Science*, pages 409–418. Springer, 2010.
- [56] Marco Mamei, Andrea Roli, and Franco Zambonelli. Emergence and control of macro-spatial structures in perturbed cellular automata, and implications for pervasive computing systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part A*, 35(3):337–348, 2005.

- [57] Luca Manzoni. Asynchronous cellular automata and dynamical properties. *Natural Computing*, 11(2):269–276, 2012.
- [58] Susanna M. Messinger, Keith A. Mott, and David Peak. Task-performing dynamics in irregular, biomimetic networks: Research articles. *Complexity*, 12(6):14–21, 2007.
- [59] Seyyed Amir Hadi Minoofam and Azam Bastanfard. Square kufic pattern formation by asynchronous cellular automata. In Stefania Bandini, Sara Manzoni, Hiroshi Umeo, and Giuseppe Vizzari, editors, *Proceedings of ACRI'10*, volume 6350 of *Lecture Notes in Computer Science*, pages 79–82. Springer Berlin Heidelberg, 2010.
- [60] Anca Muscholl. On the complementation of bchi asynchronous cellular automata. In Serge Abiteboul and Eli Shamir, editors, *Automata, Languages and Programming*, volume 820 of *Lecture Notes in Computer Science*, pages 142–153. Springer Berlin Heidelberg, 1994.
- [61] Katsuo Nakamura. Asynchronous cellular automata and their computational ability. *Systems, Computers, Controls*, 5(5):58–66, 1974.
- [62] Chrystopher L. Nehaniv. Evolution in asynchronous cellular automata. In *Proceedings of the eighth international conference on Artificial life*, pages 65–73. MIT Press, 2003.
- [63] Chrystopher L. Nehaniv. Asynchronous automata networks can emulate any synchronous automata network. *International Journal of Algebra and Computation*, 14(5-6):719–739, 2004.
- [64] David Newth and David Cornforth. Asynchronous spatial evolutionary games. *Biosystems*, 95(2):120–129, 2009.
- [65] Mathilde Noual. Synchronism vs asynchronism in boolean networks. *CoRR*, abs/1104.4039, 2011.
- [66] Benno J. Overeinder and Peter M. A. Sloot. Application of time warp to parallel simulations with asynchronous cellular automata. In *Proceedings of the 1993 European Simulation Symposium*, pages 397–402, 1993.
- [67] Ferdinand Peper, Susumu Adachi, and Jia Lee. Variations on the game of life. In Andrew Adamatzky, editor, *Game of Life Cellular Automata*, pages 235–255. Springer London, 2010.
- [68] Ferdinand Peper, Jia Lee, and Teijiro Isokawa. Brownian cellular automata. *Journal of Cellular Automata*, 5(3):185–206, 2010.
- [69] Giovanni Pighizzini. Asynchronous automata versus asynchronous cellular automata. *Theoretical Computer Science*, 132(2):179–207, 1994.
- [70] Lutz Priese. A note on asynchronous cellular automata. *Journal of Computer and System Sciences*, 17(2):237–252, 1978.

- [71] Filippo Radicchi, Daniele Vilone, and Hildegard Meyer-Ortmanns. Phase transition between synchronous and asynchronous updating algorithms. *Journal of Statistical Physics*, 129(3):593–603, 2007.
- [72] N. Rajewsky, L. Santen, A. Schadschneider, and M. Schreckenberg. The asymmetric exclusion process: Comparison of update procedures. *Journal of Statistical Physics*, 92(1-2):151–194, 1998.
- [73] Damien Regnault. Proof of a phase transition in probabilistic cellular automata. In Marie-Pierre Béal and Olivier Carton, editors, *Proceedings of Developments in Language Theory*, volume 7907 of *Lecture Notes in Computer Science*, pages 433–444. Springer, 2013.
- [74] Damien Regnault, Nicolas Schabanel, and Eric Thierry. Progresses in the analysis of stochastic 2D cellular automata: A study of asynchronous 2D minority. *Theoretical Computer Science*, 410(47-49):4844–4855, 2009.
- [75] Juri Rolf, Tomas Bohr, and Mogens H. Jensen. Directed percolation universality in asynchronous evolution of spatiotemporal intermittency. *Physical Review E*, 57(3):R2503–R2506, 1998.
- [76] Jean-Baptiste Rouquier and Michel Morvan. Coalescing cellular automata: Synchronization by common random source for asynchronous updating. *Journal of Cellular Automata*, 4(1):55–78, 2009.
- [77] Jean-Baptiste Rouquier, Damien Regnault, and Éric Thierry. Stochastic minority on graphs. *Theoretical Computer Science*, 412(30):3947 – 3963, 2011.
- [78] Graeme Ruxton and Leonardo Saravia. The need for biological realism in the updating of cellular automata models. *Ecological Modelling*, 107(2):105–112, 1998.
- [79] M. Ali Saif and Prashant M. Gade. The prisoner’s dilemma with semi-synchronous updates: evidence for a first-order phase transition. *Journal of Statistical Mechanics: Theory and Experiment*, 2009(7):P07023, 2009.
- [80] Anindita Sarkar, Anindita Mukherjee, and Sukanta Das. Reversibility in asynchronous cellular automata. *Complex Systems*, 21(1):71, 2012.
- [81] Oliver Schneider and Thomas Worsch. A 3-state asynchronous ca for the simulation of delay-insensitive circuits. In Georgios Ch. Sirakoulis and Stefania Bandini, editors, *Proceedings of ACRI’12*, volume 7495 of *Lecture Notes in Computer Science*, pages 565–574. Springer, 2012.
- [82] Birgitt Schönfisch and André de Roos. Synchronous and asynchronous updating in cellular automata. *BioSystems*, 51:123–143, 1999.
- [83] Biswanath Sethi, Nazim Fatès, and Sukanta Das. Reversibility of Elementary Cellular Automata Under Fully Asynchronous Update. submitted work.

- [84] Anastasia Sharifulina and Vladimir Elokhin. Simulation of heterogeneous catalytic reaction by asynchronous cellular automata on multicomputer. In *Proceedings of the 11th international conference on Parallel computing technologies*, PaCT'11, pages 204–209, Berlin, Heidelberg, 2011. Springer-Verlag.
- [85] Fernando Silva and Luís Correia. A study of stochastic noise and asynchronism in elementary cellular automata. In Georgios Ch. Sirakoulis and Stefania Bandini, editors, *Proceedings of ACRI 2012*, pages 679–688. Springer, 2012.
- [86] Fernando Silva and Lus Correia. An experimental study of noise and asynchrony in elementary cellular automata with sampling compensation. *Natural Computing*, pages 1–16, 2013.
- [87] Antoine Spicher, Nazim Fatès, and Olivier Simonin. Translating discrete multi-agents systems into cellular automata: Application to diffusion-limited aggregation. In Joaquim Filipe, Ana Fred, and Bernadette Sharp, editors, *Agents and Artificial Intelligence*, volume 67 of *Communications in Computer and Information Science*, pages 270–282. Springer Berlin Heidelberg, 2010.
- [88] W. Richard Stark. Amorphous computing: examples, mathematics and theory. *Natural Computing*, 12(3):377–392, 2013.
- [89] W. Richard Stark and William H. Hughes. Asynchronous, irregular automata nets: the path not taken. *BioSystems*, 55:107–117, 2000.
- [90] Tomoaki Suzudo. Searching for pattern-forming asynchronous cellular automata - an evolutionary approach. In Peter M. A. Sloot, Bastien Chopard, and Alfons G. Hoekstra, editors, *Proceedings of ACRI'04*, volume 3305 of *Lecture Notes in Computer Science*, pages 151–160. Springer, 2004.
- [91] Tomoaki Suzudo. Spatial pattern formation in asynchronous cellular automata with mass conservation. *Physica A: Statistical Mechanics and its Applications*, 343:185 – 200, 2004.
- [92] Yousuke Takada, Teijiro Isokawa, Ferdinand Peper, and Nobuyuki Matsui. Construction universality in purely asynchronous cellular automata. *Journal of Computer and System Sciences*, 72(8):1368 – 1385, 2006.
- [93] Wahiba Taouali, Thierry Viéville, Nicolas P. Rougier, and Frédéric Alexandre. No clock to rule them all. *Journal of Physiology-Paris*, 105(13):83 – 90, 2011.
- [94] Tommaso Toffoli. Integration of the phase-difference relations in asynchronous sequential networks. In Giorgio Ausiello and Corrado Böhm, editors, *Proceedings of the Fifth Colloquium on Automata, Languages and*

Programming, volume 62 of *Lecture Notes in Computer Science*, pages 457–463. Springer Berlin Heidelberg, 1978.

- [95] Marco Tomassini and Mattias Venzi. Artificially evolved asynchronous cellular automata for the density task. In Stefania Bandini, Bastien Chopard, and Marco Tomassini, editors, *Proceedings of ACRI 2002*, volume 2493 of *Lecture Notes in Computer Science*, pages 44–55. Springer, 2002.
- [96] Predrag T. Tošić. On modeling large-scale multi-agent systems with parallel, sequential and genuinely asynchronous cellular automata. *Acta Physica Polonica B - Proceedings Supplement*, 4(2):217–235, 2011.
- [97] Leonardo Vanneschi and Giancarlo Mauri. A study on learning robustness using asynchronous 1D cellular automata rules. *Natural Computing*, 11(2):289–302, 2012.
- [98] Michael Vielhaber. Computation of functions on n bits by asynchronous clocking of cellular automata. *Natural Computing*, 12(3):307–322, 2013.
- [99] John von Neumann. *Theory of self-reproducing automata*. University of Illinois press Urbana, 1966. A. Burks (editor).
- [100] Simon Wacker and Thomas Worsch. Phase space invertible asynchronous cellular automata. In Enrico Formenti, editor, *Proceedings of AUTOMATA & JAC*, volume 90 of *EPTCS*, pages 236–254, 2012.
- [101] Thomas Worsch. (intrinsically?) universal asynchronous CA. In Georgios Ch. Sirakoulis and Stefania Bandini, editors, *Proceedings of ACRI 2012*, *Lecture Notes in Computer Science*, pages 689–698. Springer, 2012.