Is there something like "modellability"? - Reflections on the robustness of discrete models of complex systems

Nazim Fatès

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Nazim Fatès
nazim.fates@loria.fr

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Abstract

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In his seminal paper *On computable numbers with an application to the Entscheidungsproblem* of 1936, Turing opened the way to consider computability by showing the limits of computation. In this talk we will ask if there is anything that could be called ”modellability” and would be analogous to computability. To this end, we propose to examine the dynamical behaviour of various cellular automata and multi-agent systems and endeavour to evaluate the robustness of these systems to various perturbations.

1 About the robustness of the most simple rules

α-synchronous CA

One easy way to obtain a probabilistic cellular automata (CA) is to alter the global transition function and to consider *α-synchronous CA* in which cells are updated according to a random local choice with probability α, the *synchrony rate*. This means that for each cell, independently, the transition results in applying the rule with probability α and staying in the same state with probability $1 - \alpha$.

A first experimental study of such CA was carried out on the set of the 88 minimal ECA. This study revealed that there is no straightforward links between the classical synchronous behaviour of the rules and their robustness. In particular, there seems to be no obvious correlation between the well-known empirical classification of Wolfram and the classes of robustness that can be defined.

Fully asynchronous CA

In order to start studying CA with an analytical approach, one has to start with the most simple systems. Fully asynchronous CA are defined as applying one cell chosen randomly and uniformly at each time step. In other word, the choice of the cell is memoryless and the local transition function is applied on only one cell, while the state of the other cells is left unchanged. Note that the definition only applies for finite set of cells ($|\mathcal{L}| < \infty$) and can not be extended directly for infinite CA. A first classification of the rules with two quiescent states (that is, such that $f(0,0,0) = 0$ and $f(1,1,1) = 1$) was proposed. The classes are defined with respect to the worst average expected convergence time to a fixed point. The interesting point is that there does exist strong correlation between the behaviour of a CA and its time of convergence (more exactly to the scaling time of convergence). It is still an open problem to extend this classification to the 88 ECAs or to the two-dimensional rules. See Ref. [Fat13] for a recent synthesis on this question.

So at this point the question we face is : what is the origin of this difficulty to understand the robustness of such simple models of complex systems?

Is it because of the non-existence of objects, entities that carry out the information from one place to the other, like in most "physical models" ? In cellular automata, transmission of information is "abstract" and even when we do α-asynchronous updating, we still assume that each cells reads simultaneously all the states of the neighbours. For an systematic numerical study of the relaxation of this hypothesis, see the work on so-called β- and γ- asynchronism by Bouré et al. [BFC12].

2 Are multi-agent systems "different" ?

We will now consider different models of multi-agent systems. The first one, the multi-turmite model, is the most closely related to cellular automata. The second one is a model of the swarming phenomenon ("collective motion" is currently a fashionable topic). The third one shows how a swarm of virtual amoebae may exploit an active environment in order to realise a decentralised gathering.

Multi-turmite model (Langton's ants)

Langton ants, or turmites (name given by Dewdney), are one of the most simple multi-agent system where the local rule is simple and the global dynamics complex ; they were initially introduced by Langton to study "artificial life". We refer to the work of Gajardo et al. for the study of the dynamics of a single ant. When we take the simultaneous presence of multiple ants on the grid, very few works are found in literature. We propose a way to model such a simple multi-agent system as a discrete dynamical system [CF09]. We show that even for the synchronous update, various updating schemes can be defined, depending on the way the rules are interpreted. We present examples where starting from the same initial conditions with two turmites, various updating schemes lead to *qualitatively* different evolutions. Novel observations are made such as the formation of cyclic behaviours, infinitely extending patterns, deadlocks, etc.

The case of synchronous vs. asynchronous updating was also tackled recently. This requires to extend the formalism and also leads to interesting new observations such as the existence of gliders that are robust to a change from synchronous to sequential (deterministic) updating [BF12].

In this case, we see that it is difficult to find robust phenomena. Hence the question, is it because the system is somewhat "too discrete" (but the majority rule *is* robust) ? because it is deterministic ? etc.

LGCA model of swarming

Swarming refers to the capacity of for huge ensembles of animals to form a coherent group and move together. This phenomenon is often seen with birds (starlings), fishes, amoebae, etc. We are interested in re-visiting a model by A. Deutsch in order to assess its robustness. The model is simple enough as it has only two parameters : ρ, the initial density and σ, the sensitivity to control to which extent a particle may align with its neighbours. The main studied phenomenon is the existence of a phase transition from a disorder state where no pattern appears to an order state where particles tend to align and form diagonal stripes. However, a more extensive studies reveals that there exist (at least) new phenomena : (a) if the sensitivity σ is made sufficiently high, one no longer observes stripes but rather clusters, (b) if the density of particles ρ is high enough, we observe an astounding new behaviour as particles tend to *anti-align* instead of aligning! [BFC13b]. This patterns disappears with an asynchronous updating of the LGCA, while other patterns seem robust, at least up to a certain degree [BFC13a]. It is an open problem to characterise the nature of the phase transitions between patterns, in particular to determine if the transition from order to disorder is continuous or not.

3 Discussion

It is now time to come back to our three initial question: "Is there something like modellability ?" Clearly, the time has not yet come where an analogue to computability can be defined in the context of modelling ; however, there seems to be the need for it... In the context of cellular automata and simple reactive multi-agent systems, we asked the following three questions:

How can we think about the limits of modelling ?

We examined some examples of simple models where a simple variation, namely the updating scheme, often resulted in qualitative change of behaviour. The case of binary cellular automata shows that this change is not systematic and that there exist rules which are robust. The study focuses on cases of sensibility as this challenges the validity of our models of complex systems? This leads us to question: what is a good model for *synchrony* in such models?

Do the limits shown by Turing also apply to the models of physical situations?

In many models of "natural" systems, which are stochastic, the undecidability results do not apply. Nevertheless, we see very puzzling phenomena by numerical simulations and there are precise cases as with non-equilibrium phase transitions (e.g. directed percolation) where we know that analytical results are hard to obtain.

What advantages can we expect from the examination of the robustness of discrete models ?

Showing the *limits* allows us to replace models in a restricted context of validity. One phenomenon becomes valid only in the region where it is robust. In the other regions, new phenomena can be observed. These new phenomena are often interesting by themselves. An image to conclude about broadening the perspective: the difference between the *Mona Lisa* by da Vinci and the *Marriage at Cana* by Veronese...

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