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1 **Sensitivity of Ocean-Atmosphere Coupled Models to the Coupling**
2 **Method : Example of Tropical Cyclone Erica**

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ABSTRACT

7
8 In this paper, the sensitivity of Atmospheric and Oceanic Coupled Models (AOCMs) to
9 the coupling method is investigated. We propose the adaptation of a Schwarz-like domain
10 decomposition method to AOCMs. We show that the iterative process of the method ensures
11 consistency of the coupled solution across the air-sea interface, contrarily to usual *ad-hoc*
12 algorithmic approaches. The latter are equivalent to only one iteration of a Schwarz-like
13 iterative method, which does not provide a converged state. It is generally assumed that
14 this lack of consistency does not affect significantly the physical properties of the solution.
15 The relevancy of this statement is first assessed in a simplified problem, then in the realistic
16 application of a mesoscale atmospheric model (WRF) coupled with a regional oceanic model
17 (ROMS) to simulate the genesis and propagation of tropical cyclone Erica. Sensitivity tests
18 to the coupling method are carried out in an ensemble approach. We show that with a
19 mathematically consistent coupling the spread of the ensemble is reduced, suggesting that
20 there is room for further improvements in the formulation of AOCMs at a mathematical and
21 numerical level.

1. Introduction

a. Context

Many applications in global/regional oceanography and meteorology require Atmospheric and Oceanic Coupled Models (AOCMs) which account for important air-sea feedbacks. Separate integrations of the oceanic and atmospheric numerical models in forced mode (i.e. without feedback from one component to the other) may be satisfactory for numerous applications and process studies (e.g. Marchesiello et al. 2003; Colas et al. 2011; Lemarié et al. 2012). However, two-way coupling is essential for analyzing energetic and complex phenomena like tropical cyclones (e.g. Bao et al. 2000; Jullien et al. 2014), eastern boundary upwellings (e.g. Perlin et al. 2007; Capet et al. 2008), and more generally climate trends (e.g. Large and Danabasoglu 2006; Terray et al. 2011). In that case, connecting the two model solutions at the air-sea interface is an arduous task which jointly raises mathematical, physical and computational issues.

Besides the oceanic and atmospheric models, the formulation of AOCMs is based on several components: a parameterization of the turbulent air-sea fluxes, a coupling infrastructure, and a coupling algorithm. Several parameterizations for surface atmospheric flow dynamics under oceanic influence have been designed. Those parameterizations are derived at a semi-empirical level and are based on field and laboratory experiments designed to carefully tune the parameter values (Fairall et al. 2003; Large 2006). As for computational issues, numerous coupling softwares were developed in the last decade (Hill et al. 2004; Joppich and Kürschner 2006; Redler et al. 2010). Those tools are needed to handle message passing, synchronization in time and regridding procedures (i.e., interpolation/extrapolation) between the computational grids. The last ingredient, of numerical nature, in the design of an AOCM is a *consistent* coupling algorithm; this is the main subject of this paper. The notion of *consistency* associated with the ocean atmosphere coupling problem will be clarified below in Sec. 1b.

48 It is known that coupled model solutions exhibit a strong sensitivity to model parameters
49 (Bengtsson 1999; McWilliams 2007). More specifically, sensitivity to perturbations in the
50 initial conditions (Ploshay and Anderson 2002), coupling frequency (Lebeaupin Brossier et al.
51 2009; Terray et al. 2011; Masson et al. 2012), and to air-sea flux formulation (Lebeaupin Brossier
52 et al. 2008) have been reported, in addition to the sensitivity that is inherent to each com-
53 ponent of the coupled system (Tribbia and Baumhefner 1988). This is arguably a source of
54 concern when it comes to assess the AOCMs solution. Uncertainties in the specification of
55 air-sea fluxes is strongly related to the empirical nature of atmospheric surface layer param-
56 eterizations resulting from the extreme complexity of the physical processes that we wish
57 to represent. In general, a given parameterization is designed and tuned for a particular
58 geographical region or a particular range of static stability. A lot of efforts are being di-
59 rected toward improving physical parameterizations because these are usually considered as
60 the major source of errors in AOCMs. However, it is arguably crucial to keep working on
61 other model developments and identify other possible sources of error/deficiency. Prelimi-
62 nary studies of the fifth phase of the Coupled Model Comparison Project (CMIP5) do not
63 report significant improvements in biases in present-day climate compared to CMIP3 (e.g.
64 Roehrig et al. 2013; Sen Gupta et al. 2013). This suggests that finer horizontal resolution
65 is not sufficient to cure the aforementioned problems. There is clearly a need for a more
66 complete understanding of what goes on in numerical coupled models, notably through a
67 finer consideration of numerical methods and modeling assumptions. To our knowledge,
68 no systematic sensitivity study of the algorithmic aspects of air-sea coupling has been re-
69 ported. We investigate this sensitivity in the present paper, starting with the definition of
70 the coupling problem.

71 *b. Coupling Problem*

72 We symbolically describe the oceanic and atmospheric circulation models by partial
73 differential operators \mathcal{L}_{oce} and \mathcal{L}_{atm} corresponding to the systems of equations solved by

74 numerical models. Traditionally, we consider the primitive equations in the ocean and
75 the fully-compressible Euler equations in the atmosphere. On the computational domain
76 $\Omega = \Omega_{\text{atm}} \cup \Omega_{\text{oce}}$ (with external boundaries $\partial\Omega_{\text{atm}}^{\text{ext}}$ and $\partial\Omega_{\text{oce}}^{\text{ext}}$, see Fig. 1), the integration over
77 a time period $[0, \mathcal{T}]$ corresponds to

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}}, & \text{in } \Omega_{\text{atm}} \times [0, \mathcal{T}], \\ \mathcal{B}_{\text{atm}} \mathbf{U}^a = g_{\text{atm}}, & \text{in } \partial\Omega_{\text{atm}}^{\text{ext}} \times [0, \mathcal{T}], \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = \mathbf{F}_{\text{oa}}(\mathbf{U}^o, \mathbf{U}^a, \mathcal{R}), & \text{on } \Gamma \times [0, \mathcal{T}], \end{cases} \quad (1)$$

78

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}}, & \text{in } \Omega_{\text{oce}} \times [0, \mathcal{T}], \\ \mathcal{B}_{\text{oce}} \mathbf{U}^o = g_{\text{oce}}, & \text{in } \partial\Omega_{\text{oce}}^{\text{ext}} \times [0, \mathcal{T}], \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = \mathbf{F}_{\text{oa}}(\mathbf{U}^o, \mathbf{U}^a, \mathcal{R}), & \text{on } \Gamma \times [0, \mathcal{T}], \end{cases} \quad (2)$$

79 with appropriate boundary conditions provided through the boundary operators \mathcal{B}_{oce} and
80 \mathcal{B}_{atm} (the initialization of coupled models is an open problem that is beyond the scope here).
81 In (1) and (2), $\mathbf{U}^a = (\mathbf{u}_h^a, T^a)^t$ and $\mathbf{U}^o = (\mathbf{u}_h^o, T^o)^t$ are the state variables with \mathbf{u}_h the
82 horizontal velocity and T the (potential) temperature, f_{atm} and f_{oce} are forcing terms. For
83 the sake of simplicity, we focus on temperature and momentum, we do not include salinity
84 and humidity in the continuous formulation of the problem (it is however included in the
85 practical application discussed in Sec. 5). The oceanic domain Ω_{oce} and the atmospheric
86 domain Ω_{atm} have a common interface Γ (Fig. 1). \mathbf{F}_{oa} is a function allowing the computation
87 of air-sea fluxes. This function, generally based on the atmospheric surface layer similarity
88 theory (e.g. Large 2006), depends on \mathbf{U}^a and \mathbf{U}^o in the vicinity of the air-sea interface (as
89 shown in Fig. 1), and on a set of non-turbulent radiative fluxes \mathcal{R} .

90 In (1)-(2), the interface operators are defined as

$$\mathcal{F}_{\text{atm}\bullet} = \rho^a \mathbf{K}^a \partial_z \bullet, \quad \mathcal{F}_{\text{oce}\bullet} = \rho^o \mathbf{K}^o \partial_z \bullet,$$

91 where z is positive upward, ρ^a and ρ^o are the densities of the fluids, and

$$\mathbf{K}^a = \begin{pmatrix} K_m^a \\ K_m^a \\ c_p^a K_t^a \end{pmatrix}, \quad \mathbf{K}^o = \begin{pmatrix} K_m^o \\ K_m^o \\ c_p^o K_t^o \end{pmatrix}.$$

92 We note K_m^a and K_m^o the eddy viscosities, and K_t^a and K_t^o the eddy diffusivities. The c_p terms
93 correspond to the specific heat of the fluid.

94 In forced mode, \mathbf{U}^o (resp. \mathbf{U}^a and \mathcal{R}) in (1) (resp. (2)) is provided offline by existing
95 satellite-based or reanalysis products. In coupled mode, both models are run either simulta-
96 neously or successively on the same space-time interval. In this case, the consistency required
97 at the air-sea interface is the continuity of vertical fluxes¹

$$\begin{aligned} \rho^a K_m^a \partial_z \mathbf{u}_h^a &= \rho^o K_m^o \partial_z \mathbf{u}_h^o &= \boldsymbol{\tau} & \text{on } \Gamma \times [0, \mathcal{T}] \\ \rho^a c_p^a K_t^a \partial_z T^a &= \rho^o c_p^o K_t^o \partial_z T^o &= Q_{\text{net}} = \mathcal{R} + Q_S & \text{on } \Gamma \times [0, \mathcal{T}] \end{aligned}$$

98 where the surface wind stress $\boldsymbol{\tau}$ and the sensible heat flux Q_S (consequently the net heat flux
99 Q_{net}) are computed using the function $\mathbf{F}_{\text{oa}} = (\boldsymbol{\tau}, Q_{\text{net}})^t$ previously introduced. The turbulent
100 air-sea fluxes are given by a parameterization of the atmospheric surface layer. They usually
101 take the form

$$\boldsymbol{\tau} = \rho^a C_D \|\Delta \mathbf{U}\| \Delta \mathbf{U}, \quad Q_S = \rho^a c_p^a C_H \|\Delta \mathbf{U}\| \Delta T, \quad (3)$$

102 where C_D and C_H are exchange coefficients that depend on surface roughness and local
103 stability. $\Delta \mathbf{U}$ (resp. ΔT) correspond to the velocity (resp. temperature) jump across the
104 air-sea interface which is defined, in a bulk way, as the region between the lowest vertical
105 level in the atmospheric model and the shallowest vertical level in the oceanic model.

106 At this point we have defined all the necessary notations to formulate the coupled problem

¹Here, we assume wind-wave equilibrium, i.e., the atmospheric momentum flux to the wave field is immediately transferred to the ocean through wave breaking. The addition of a wave component to the coupling system would act as a low-pass filter to air-sea exchanges (see Sec.3a and Sec.6)

107 under study:

Find \mathbf{U}^a and \mathbf{U}^o that satisfy

$$\left\{ \begin{array}{ll} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, \mathcal{T}] \\ \mathcal{B}_{\text{atm}} \mathbf{U}^a = g_{\text{atm}} & \text{in } \partial\Omega_{\text{atm}}^{\text{ext}} \times [0, \mathcal{T}] \\ \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, \mathcal{T}] \\ \mathcal{B}_{\text{oce}} \mathbf{U}^o = g_{\text{oce}} & \text{in } \partial\Omega_{\text{oce}}^{\text{ext}} \times [0, \mathcal{T}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = \mathcal{F}_{\text{oce}} \mathbf{U}^o = \mathbf{F}_{\text{oa}}(\mathbf{U}^o, \mathbf{U}^a, \mathcal{R}) & \text{on } \Gamma \times [0, \mathcal{T}] \end{array} \right. \quad (4)$$

108 for given initial and boundary conditions. It is worth mentioning that Lions et al. (1995)
 109 proved the existence of a global weak solution to problem (4) with interface conditions given
 110 by (3), considering that \mathcal{L}_{atm} and \mathcal{L}_{oce} are the primitive equations of the atmosphere and
 111 the ocean.

112 This paper is organized as follows. The existing coupling methods currently in use to solve
 113 (4) are presented in Sec. 2. In Sec. 3, we provide critical comments about these methods
 114 and we introduce a new theoretical framework to solve the coupling problem. In Sec. 4
 115 we present the formulation of our AOCM based on the Weather Research and Forecasting
 116 (WRF, Skamarock and Klemp 2008) model and the Regional Oceanic Modeling System
 117 (ROMS, Shchepetkin and McWilliams 2005). In Sec. 5 we assess the sensitivity of this
 118 model to the algorithmic approach for the simulation of a tropical cyclone. We show that
 119 a mathematically consistent algorithm leads to reduced stochasticity, i.e., less sensitivity of
 120 the coupled solution to perturbations in the initial condition and coupling frequency. We
 121 summarize and discuss our findings in Sec. 6.

122 2. A Classification of Existing Methods

123 Coupling methods can be categorized in two different groups : 1-way algorithms, when
 124 one model receive informations but does not send anything back to the other (e.g. Muller
 125 et al. 2007), and 2-way algorithms, when a feedback is introduced. We focus here on the

126 latter. The usual coupling strategies can be found in Bryan et al. (1996) mostly for global
 127 configurations and in Bao et al. (2000) and Perlin et al. (2007) for regional high-resolution
 128 studies. A first algorithmic approach used for global problems is based on the exchange of
 129 averaged-in-time fluxes between the models (usually referred to as *asynchronous* coupling)
 130 whereas a second one deals with instantaneous fluxes (usually referred to as *synchronous*
 131 coupling). In this section, we expose the key differences between the two strategies.

132 *a. Asynchronous Coupling by Time Windows (Based on Mean Fluxes)*

133 Asynchronous coupling is the strategy used in most climate models of the IPCC². For
 134 this method, the total simulation time $[0, \mathcal{T}]$ is split into M smallest time windows $[t_i, t_{i+1}]$,
 135 i.e. $[0, \mathcal{T}] = \cup_{i=1}^M [t_i, t_{i+1}]$. The length of those time windows are typically between 1 hour
 136 and 1 day depending on target applications and the need to resolve or not the diurnal cycle.
 137 On a given time window, atmospheric and oceanic models only exchange time-averaged
 138 quantities. It has the advantage of requiring few communications between models. Noting
 139 $\langle \cdot \rangle_i$ a temporal average on the time window $[t_i, t_{i+1}]$, the coupling algorithm is

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^{\text{a}} = \mathbf{F}_{\text{oa}}(\langle \mathbf{U}^{\text{o}} \rangle_{i-1}, \mathbf{U}^{\text{a}}, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases} \quad (5)$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^{\text{o}} = \langle \mathcal{F}_{\text{atm}} \mathbf{U}^{\text{a}} \rangle_i & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

140 This algorithm is schematically described in Fig. 2. First the atmospheric model is advanced
 141 from t_i to t_{i+1} using the averaged ocean state computed on the previous time window. Then,
 142 the fluxes used to force the atmospheric model are averaged and applied to the oceanic model
 143 on the same time interval (the fluxes are generally piecewise constant on each time window).
 144 This methodology ensures that over a time interval $[t_i, t_{i+1}]$ both models are forced by the

²Intergovernmental Panel on Climate Change

145 exact same mean fluxes, implying a strict conservation of the quantities. Indeed, in both
 146 models the time integral of surface fluxes over the simulation equals $\langle \mathcal{F}_{\text{atm}} \mathbf{U}^a \rangle_{[0, \tau]}$.

147 However, the solution of algorithm (5) is not rigorously solution of the original problem
 148 (4) because there is a synchronicity issue. The modification of the oceanic state \mathbf{U}^o on
 149 $[t_i, t_{i+1}]$ is not provided to the atmospheric component on the proper time interval $[t_i, t_{i+1}]$
 150 but only on $[t_{i+1}, t_{i+2}]$. It is usually assumed that the loss of synchronization does not
 151 significantly impair the coupled solution, but we will show on the contrary that algorithm
 152 (5) leads to significant numerical errors (Sec. 3b).

153 *b. Synchronous Coupling at the Time Step Level (Based on Instantaneous Fluxes)*

154 In nature, the ocean and the atmosphere continuously exchange fluxes on scales ranging
 155 from global to micro scales. Therefore, proper coupling frequency between numerical models
 156 should be as small as possible, typically the largest time step between the oceanic and
 157 the atmospheric model. In this regard, a relevant algorithm would consist in exchanging
 158 instantaneous fluxes. Let us consider that the oceanic time step Δt_o is such that $\Delta t_o = N \Delta t_a$.
 159 The corresponding algorithm reads

$$\begin{cases} \mathcal{L}_{\text{atm}} \mathbf{U}^a = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_i + N \Delta t_a] \\ \mathcal{F}_{\text{atm}} \mathbf{U}^a = \mathbf{F}_{\text{oa}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t), \mathcal{R}(t)) & \text{on } \Gamma \times [t_i, t_i + N \Delta t_a] \end{cases} \quad (6)$$

$$\begin{cases} \mathcal{L}_{\text{oce}} \mathbf{U}^o = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_i + \Delta t_o] \\ \mathcal{F}_{\text{oce}} \mathbf{U}^o = \mathbf{F}_{\text{oa}}(\mathbf{U}^o(t_i), \mathbf{U}^a(t_i), \mathcal{R}(t_i)) & \text{on } \Gamma \times [t_i, t_i + \Delta t_o] \end{cases}$$

160 The oceanic and atmospheric components are integrated forward for a time period corre-
 161 sponding to Δt_o (or equivalently $N \Delta t_a$). Data exchange of instantaneous values is then
 162 performed and model integration continues for another Δt_o period of time. This process is
 163 repeated until the final forecast time (Fig. 3). In (6) the oceanic component receives instan-
 164 taneous values from the atmosphere but integrated values can also be considered between

165 t_i and $t_i + N\Delta t_a$ to avoid aliasing errors (Fig. 3,b). Both choices raise either conservation,
166 aliasing or synchronization problems. In addition, algorithm (6) is difficult to implement effi-
167 ciently from computational and numerical view points. Code communications are extremely
168 frequent and time integration schemes must be carefully considered for consistent interfacial
169 conditions.

170 At first glance, (6) may appear as a solution of the full problem (4). This is, however,
171 formally true only in the limit $\Delta t_o, \Delta t_a \rightarrow 0$. Indeed, in most numerical models vertical
172 diffusion is treated implicitly-in-time; air-sea fluxes are thus provided at time $t_i + \Delta t$ rather
173 than t_i as in (6). As for algorithm (5), the loss of mathematical consistency due to numerical
174 implementation may lead to significant errors (Sec. 3b).

175 **3. Numerical and Physical Considerations**

176 In the previous section, we showed that the numerical methods generally used to solve
177 the coupled problem (4) are not fully satisfying from a mathematical point of view. In
178 this section, we specify further the ocean-atmosphere coupling problem based on physical
179 considerations, then we propose an alternative method to solve it.

180 *a. Physical Constraints*

181 Coupling methods used in the context of AOCMs differ from the methods usually used for
182 multi-physics (fluid-fluid) coupling. However, the latter can not be easily applied to AOCMs
183 because they often assume a full representation of all the spatio-temporal scales involved
184 in the problem. The oceanic and atmospheric variability is spectrally broad band across
185 all scales from micro $\mathcal{O}(10^{-3} \text{ m}, 10^{-3} \text{ s})$ to global $\mathcal{O}(10^7 \text{ m}, 10^{10} \text{ s})$ scales. AOCMs contain
186 essential parameterization to account for unresolved processes. The fluxes exchanged by
187 the models are not the result of a discrete derivative in the vicinity of the air-sea interface
188 but are given by atmospheric surface layer parameterizations based on the so-called bulk

189 aerodynamic formulae. Bulk formulations are symbolically represented by the function F_{oa}
190 in (4). They are defined and calibrated semi-empirically using measurements averaged in
191 time over about an hour or more, and for a restricted range of stability values (Large 2006).
192 There is little knowledge and observations of air-sea fluxes at high temporal scales (see
193 discussion in Danabasoglu et al. (2006), Sec. 2) and the sign of air-sea fluxes is uncertain on
194 time scales less than 10 minutes. In addition, high frequency physical processes are associated
195 with the wavy boundary layer that requires specific time-dependent equations to be solved
196 (Bao et al. 2000). The wavy boundary layer tends to act as a low-pass filter on air-sea
197 exchanges. Therefore and for all reasons mentioned above, we consider mean hourly fluxes
198 preferable when using bulk formulations (see Large 2006, for a discussion). A time-averaging
199 procedure, acting as a smoother of smallest time scales, is thus adopted in algorithm (5).
200 We consider that algorithm (6) is relevant only if additional physical processes predominant
201 on short time-scales are explicitly addressed in the flux computation.

202 *b. Numerical Considerations*

203 We showed in Sec. 2 that usual coupling methods used in AOCMs are not entirely
204 satisfactory with respect to consistency, conservation or synchronization. An illustration of
205 numerical errors is given here based on a simplified one-dimensional coupling problem with
206 an interface at $z = 0$:

$$\left\{ \begin{array}{ll} \partial_t q_2 - \partial_z(\nu_2 \partial_z q_2) = f_2, & \text{in }]0, L_2[\times [0, \mathcal{T}], \\ q_2(L_2, t) = 0, & t \in [0, \mathcal{T}], \\ \nu_2 \partial_z q_2(0, t) = \nu_1 \partial_z q_1(0, t), & t \in [0, \mathcal{T}], \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{ll} \partial_t q_1 - \partial_z(\nu_1 \partial_z q_1) = f_1, & \text{in }]-L_1, 0[\times [0, \mathcal{T}], \\ q_1(-L_1, t) = 0, & t \in [0, \mathcal{T}], \\ q_1(0, t) = q_2(0, t), & t \in [0, \mathcal{T}]. \end{array} \right. \quad (8)$$

208 The complete setup for this simplified test-case is described in App. A, and a thorough
209 mathematical and numerical study of this problem can be found in Lemarié et al. (2013b,c).

210 We impose continuity of tracer q and its vertical flux at the interface. Figure 4 (left panel)
211 shows the ℓ_2 -norm of errors associated with synchronous and asynchronous methods. Note
212 that we chose a testcase forced with right hand sides f_1 and f_2 which are identical whatever
213 the coupling method. That is the reason why the coupling error does not grow significantly
214 with time. It must be clear that setting $f_1 = f_2 = 0$ in (7-8) would lead to much larger
215 errors constantly growing with time.

216 Based on purely numerical arguments, the synchronous method is preferable over the
217 asynchronous one as it leads to much smaller error. However, because the synchronous
218 method does not satisfy the aforementioned physical constraints (Sec. 3a) for coupling and
219 also because of computational burden, we strive to propose an alternative approach which
220 would theoretically prove more consistent.

221 *c. Global in Time Schwarz Method*

222 The Schwarz-like domain decomposition methods (see Gander 2008, for a review) are
223 widely used for coupling problems with different physics and/or different numerical treat-
224 ment. Originally introduced for stationary problems, those methods have been recently
225 extended to time-dependent problems to provide a global-in-time Schwarz method, a.k.a.
226 Schwarz waveform relaxation (e.g. Gander and Halpern 2007). The idea is to separate the
227 original problem on $\Omega = \Omega_{\text{atm}} \cup \Omega_{\text{oce}}$ into subproblems on Ω_{atm} and Ω_{oce} , which can be solved
228 separately. An iterative process is then applied to achieve convergence to the solution of
229 the original problem. The main drawback of this approach is the iterative procedure which
230 increases the computational cost of coupling, especially when convergence is slow (note that
231 there is currently an active research aiming at optimizing the convergence speed of Schwarz-
232 like methods; see discussion in Sec. 6). These methods have already been applied to oceanic
233 models for improving Poisson and Helmholtz solvers, open boundary specification and nest-
234 ing techniques (Debreu and Blayo 1998; Blayo and Debreu 2006; Cailleau et al. 2008). They
235 also seem well suited to our ocean-atmosphere coupling problem (4). Using the notations

introduced previously, the iterative algorithm on a time window $[t_i, t_{i+1}]$ can be written as follows (for a given initial condition at $t = t_i$) :

$$\begin{aligned} & \text{Loop over } k \text{ until convergence} \\ & \left\{ \begin{array}{ll} \mathcal{L}_{\text{atm}} \mathbf{U}_k^{\text{a}} = f_{\text{atm}}, & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}_k^{\text{a}} = \mathbf{F}_{\text{oa}}(\mathbf{U}_{k-1}^{\text{o}}, \mathbf{U}_k^{\text{a}}, \mathcal{R}_k), & \text{on } \Gamma \times [t_i, t_{i+1}] \\ \mathcal{L}_{\text{oce}} \mathbf{U}_k^{\text{o}} = f_{\text{oce}}, & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}_k^{\text{o}} = \mathcal{F}_{\text{atm}} \mathbf{U}_k^{\text{a}}, & \text{on } \Gamma \times [t_i, t_{i+1}] \end{array} \right. \quad (9) \end{aligned}$$

where the subscript k denotes the iteration number. The first guess $\mathbf{U}_{k=0}^{\text{o}}$ on $\Gamma \times [t_i, t_{i+1}]$ is generally taken from the converged solution on the previous time window $[t_{i-1}, t_i]$. The two models, at each iteration, are run successively: this is the so called *multiplicative* form of the algorithm. If the condition $\mathcal{F}_{\text{oce}} \mathbf{U}_k^{\text{o}} = \mathcal{F}_{\text{atm}} \mathbf{U}_k^{\text{a}}$ is replaced by $\mathcal{F}_{\text{oce}} \mathbf{U}_k^{\text{o}} = \mathcal{F}_{\text{atm}} \mathbf{U}_{k-1}^{\text{a}}$ both models can be run in parallel over the whole time window $[t_i, t_{i+1}]$: this is the *parallel* form of the algorithm. When convergence is reached, this algorithm gives the exact solution to (4). Note that with algorithm (9) the solution is independent of the size of time window $[t_i, t_{i+1}]$, as opposed to the asynchronous coupling method (akin to performing only one iteration of the Schwarz algorithm). However, for physical constraints on high-frequency treatment mentioned in Sec. 3a, algorithm (9) should be modified to include time-averaging of the quantities near the air-sea interface as in (5):

$$\begin{aligned} & \text{Loop over } k \text{ until convergence} \\ & \left\{ \begin{array}{ll} \mathcal{L}_{\text{atm}} \mathbf{U}_k^{\text{a}} = f_{\text{atm}}, & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \mathbf{U}_k^{\text{a}} = \mathbf{F}_{\text{oa}}(\langle \mathbf{U}_{k-1}^{\text{o}} \rangle_i, \mathbf{U}_k^{\text{a}}, \mathcal{R}_k), & \text{on } \Gamma \times [t_i, t_{i+1}] \\ \mathcal{L}_{\text{oce}} \mathbf{U}_k^{\text{o}} = f_{\text{oce}}, & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \mathbf{U}_k^{\text{o}} = \langle \mathcal{F}_{\text{atm}} \mathbf{U}_k^{\text{a}} \rangle_i, & \text{on } \Gamma \times [t_i, t_{i+1}] \end{array} \right. \quad (10) \end{aligned}$$

Figure 4 (right panel) shows that numerical errors using algorithm (10) are significantly reduced compared to the asynchronous method (5). Moreover, we numerically checked on a simple test case that algorithm (10) converges.

253 We have so far discussed numerous subtleties of numerical and physical nature involved
254 in the design of AOCMs. Before proceeding to a real-case study, we draw a few remarks
255 based on our survey of the algorithmic aspects of ocean-atmosphere coupling. Looking at
256 (5) and (10), it appears that the asynchronous coupling method currently in use in global
257 climate models corresponds to one iteration of the *multiplicative* form of a global-in-time
258 Schwarz algorithm (Lemarié 2008). In this respect, asynchronous coupling is mathemati-
259 cally inconsistent because it does not give the solution of the coupling problem (4) but an
260 approximation. It can also easily be shown that the synchronous coupling (6) is equivalent
261 to one iteration of a local-in-time Schwarz algorithm (Cai and Sarkis 1998). As described
262 above, it is possible to satisfy the required consistency at the expense of an iterative process.

263 To avoid the burden of the iteration process associated with Schwarz methods, a mono-
264 lithic scheme (i.e. a single model including both the oceanic and atmospheric physics) may
265 be used, but that would impose the ocean and atmosphere to be advanced on the same
266 horizontal grid with the same time-step. This is unnatural in the case of space-time multi-
267 physics problems. It must be clear that the Schwarz method does not interfere with the
268 time-scale of physical processes. Convergence of the Schwarz method does not imply any
269 instantaneous physical adjustment, e.g. of the oceanic boundary layer to the overlying at-
270 mospheric conditions. Its mode of action is on the mathematical consistency of the solution
271 at the air-sea interface. It should not be confused with other iterative methods sometimes
272 used in convective adjustment schemes or bulk formulations.

273 When using an iterative method, there are intertwined concerns : the computational
274 cost and the convergence speed. The present paper is a preliminary study where attention is
275 given to the relevancy of consistent methods, rather than their computational cost. However,
276 one important question that first needs addressing is whether the Schwarz method actually
277 converges in a practical application. We address this question in the next section by applying
278 the *multiplicative Schwarz algorithm* to model the genesis and movement of tropical cyclone

279 (TC) Erica in the southwest Pacific ocean during March 2003.

280 4. Model description and Experimental Setup

281 In this section we briefly present the numerical models composing our AOCM and our
282 coupling strategy based on algorithm (10) and applied to tropical cyclone Erica.

283 a. Oceanic Model

284 The numerical oceanic model is ROMS (Shchepetkin and McWilliams 2005) in its AGRIF-
285 IRD³ version, see Shchepetkin and McWilliams (2009) for a description of the various ROMS
286 kernels. ROMS is a split-explicit time stepping, hydrostatic, Boussinesq, free-surface prim-
287 itive equation model specifically designed for regional applications. ROMS equations are
288 formulated using a generalized terrain-following σ -coordinate, that can be configured to en-
289 hance resolution in the surface boundary layer at the air-sea interface. Our experiment is
290 setup in a configuration with four open boundaries, 50 σ -coordinate levels (grid parame-
291 ters are chosen to ensure that the surface grid-box depth is at most 1 m), a horizontal
292 grid with $\Delta x = 1/3^\circ$, and $\Delta t = 1800\text{s}$. The domain roughly extends from 6°S to 25°S in
293 latitude and from 142.5°E to 172.5°E in longitude (Fig. 5). Note that special care must
294 be given to the numerical schemes for tracer transport to properly simulate this area of
295 complex bathymetry (Marchesiello et al. 2009). Vertical mixing of tracers and momentum
296 to predict $K_m^o(z)$ and $K_t^o(z)$ is done with the KPP parameterization (Large et al. 1994).
297 The coupled simulations are conducted without any flux correction scheme nor sea-surface
298 temperature or salinity restoring. Model initialization results from a ten-year spin-up sim-
299 ulation forced by climatological atmospheric fluxes provided by the Comprehensive Ocean-
300 Atmosphere Dataset (COADS) fields and by the QuikSCAT Climatology of Ocean Winds

³Adaptive Grid Refinement in Fortran-Institut de Recherche pour le Développement

<http://roms.mpl.ird.fr/>

301 (QuikCOW). The oceanic boundary conditions are interpolated from a climatology based
302 on the SODA reanalysis (the forcing fields are constructed using the Romstools utilities,
303 Penven et al. (2008)). Note that the spin-up simulation develops large intrinsic variability at
304 the mesoscale that is uncorrelated at this scale with the actual motion (no data assimilation
305 techniques were used). Our goal here is to document the solution sensitivity to the coupling
306 methodology rather than provide the best possible hindcast.

307 *b. Atmospheric Model*

308 The atmospheric model used in our experiment is the WRF-ARW⁴ solver (Skamarock and
309 Klemp 2008). WRF integrates the fully compressible nonhydrostatic Euler equations for-
310 mulated using a terrain-following mass vertical coordinate. The model grid has a horizontal
311 resolution of $1/3^\circ$ with 31 vertical levels and the time step is 180s. The meteorological data
312 used for model initialization and boundary conditions are the NCEP2 reanalysis⁵. Note
313 that there was no need for bogus injection to initialize the generation of TC Erica, as is
314 sometimes done for cyclone studies, because the initial perturbation was captured by the
315 NCEP2 reanalysis. The physical options used for the present study are the WSM3 (WRF
316 Single-Moment 3-class) scheme for microphysics, the Rapid Radiative Transfer Model for
317 longwave radiation, the Dudhia shortwave radiation scheme, the 5-layer thermal-diffusion
318 land-surface model, and the Betts-Miller-Janjic cumulus parameterization. The Planetary
319 Boundary Layer scheme used to compute $K_m^a(z)$ and $K_t^a(z)$ is a non-local K-profile scheme
320 (the Yonsei University (YSU) scheme, Hong et al. (2006)).

321 Similar WRF and ROMS configurations were used in an uncoupled mode in Jourdain
322 et al. (2011) and Jullien et al. (2012) respectively to study the statistics of cyclonic activity
323 over the South-Pacific, and the oceanic response to tropical cyclones. A following ROMS-

⁴Advanced Research WRF, <http://www.mmm.ucar.edu/wrf/users/>

⁵National Center for Environmental Prediction, [http://nomad3.ncep.noaa.gov/ncep_data/index.](http://nomad3.ncep.noaa.gov/ncep_data/index.html)

324 WRF coupled study using our coupling method has recently been addressed (Jullien et al.
325 2014). The present study is focused on coupling rather than TC science.

326 *c. Model Coupling Strategy*

327 We implement a global-in-time Schwarz method to couple WRF and ROMS. In practice,
328 the different steps corresponding to algorithm (10) on a given time window $[t_i, t_{i+1}]$ and for
329 iteration k are :

- 330 i. Compute the atmospheric solution from t_i to t_{i+1} using \mathbf{U}_{k-1}^o (or $\mathbf{U}_{[t_{i-1}, t_i]}^o$ for $k = 1$)
- 331 ii. Send averaged air-sea fluxes $\langle \mathcal{F}_{\text{atm}} \mathbf{U}_k^a \rangle_i$ on $[t_i, t_{i+1}]$ to the oceanic model
- 332 iii. Integrate the oceanic model on the same time period (using $\mathcal{F}_{\text{atm}} \mathbf{U}_k^a(t_i)$ and $\langle \mathcal{F}_{\text{atm}} \mathbf{U}_k^a \rangle_i$,
333 fluxes from t_i to t_{i+1} are linearly reconstructed; see Fig. 6)
- 334 iv. Send the newly computed \mathbf{U}_k^o to the atmospheric model.

335 Steps 1 to 4 are iteratively applied until convergence (or until a fixed number of iterations
336 is attained). On each time-window $[t_i, t_{i+1}]$ the initial condition at $t = t_i$ corresponds to
337 the converged solution from the previous time-window $[t_{i-1}, t_i]$. As mentioned earlier, one
338 sequence of steps 1 to 4, without iterating, corresponds to (5).

339 In our practical implementation, the turbulent components of air-sea fluxes are computed
340 in the WRF surface layer scheme based on a classical similarity theory which uses stability
341 functions from Paulson (1970), Dyer and Hicks (1970), and Webb (1970) to compute the
342 surface transfer coefficients for heat, moisture, and momentum. For our experiments we
343 consider the same horizontal grids in WRF and ROMS. This is not a limitation of our method
344 but a choice to ensure that the focus is on coupling rather than interpolation/extrapolation
345 errors.

346 *d. Experimental Setup*

347 Erica, a category 4 cyclone (on the Australia and Fiji scale), was generated in March
348 2003 off Australia and reached New Caledonia a few days later causing human and material
349 damage on the island. This event was poorly forecast as various attempts gave a totally
350 different track than observed. As a result the local alert system failed. Cyclone track
351 prediction has improved in the last decade but large uncertainties remain, in part due to the
352 neglect of ocean and atmosphere coupling (Bender et al. 1993). Latent heat release to the
353 atmosphere is the source of cyclone intensity but TC-induced cooling of the surface ocean
354 is a powerful negative feedback affecting both intensity and trajectory (Jullien et al. 2012,
355 2014). Representing the correct two-way interaction between sea surface temperature (SST)
356 and tropical cyclones is thus of primary importance to cyclogenesis.

357 The present study is designed to check the viability of the Schwarz method in a strong
358 coupling case and to address two important questions, namely the convergence properties
359 of the method in a fully realistic framework and the impact of the coupling method on the
360 coupled solution. Our analysis focuses on the inter-comparison between coupling strategies
361 rather than the direct comparison with observations. The coupled model is not optimally
362 tuned for that. The default bulk coefficients in WRF are not set for extreme events and wave
363 age effects are neglected in the absence of a coupled wave model. In addition, TC intensity is
364 underestimated here as horizontal grid spacing of more than 2-3 km cannot explicitly solve
365 eyewall dynamics (including vortex Rossby waves and mesovortices) with consequences on
366 the intensity of ascent surrounding the eye, warming of the core and associated cyclonic
367 intensity (Gentry and Lackmann 2009; Hill and Lackmann 2009). Nevertheless, our simula-
368 tions of Erica are realistic enough for the task at hand as it reproduces the observed intensity
369 and track with some accuracy (see Fig. 10).

371 Let us now describe the ensemble design. We restrict our model comparison to that
 372 between the asynchronous (5) and the Schwarz (10) methods. The synchronous coupling
 373 method (at the time-step level) is cumbersome and tedious to implement. In addition, as
 374 explained earlier, current parameterizations are inadequate for this type of coupling.

375 AOCMs tends to exhibit strong sensitivity to model parameters like the initial condition
 376 and/or the coupling frequency (Sec. 1). To check whether part of this sensitivity is related
 377 to coupling consistency errors (rather than stochasticity), we design two ensembles of 18
 378 members, one for each coupling method. The 18 members are generated through pertur-
 379 bations of initial conditions and coupling frequency. We consider time windows of 3 hours
 380 and 6 hours, and three different initial conditions are chosen for the atmospheric and the
 381 oceanic models. The atmospheric model is started either from Feb. 28, March 1 or March
 382 2, corresponding to distinct synoptic setting (Fig. 7). As for the oceanic model, three initial
 383 conditions for the month of March are selected from the 10 year climatological spin-up. All
 384 coupled simulations cover the duration of cyclonic event, ending in March 16, 2003. For
 385 the ensemble integrated using the Schwarz method we systematically proceed to $M = 9$
 386 iterations. This is done to avoid choosing an arbitrary stopping criterion, and to keep the
 387 computational cost at a reasonable level.

388 5. Numerical results

389 *a. Convergence Properties*

390 First, it is natural to check the behavior of the coupled solution with respect to the
 391 iterations when the Schwarz method is used. To do so, we introduce a convergence rate R_k
 392 using the sea surface temperature $\bar{T}^0(z = 0)$ averaged in time on a given time window. R_k is
 393 defined as the ratio between the ℓ_2 -norm of the error at two successive iterations, considering

394 that the solution at iterate $k = M$ is the true solution of the problem :

$$R_k = \frac{\|e_k\|_2}{\|e_{k-1}\|_2} = \frac{\|\bar{T}_k^o(z=0) - \bar{T}_M^o(z=0)\|_2}{\|\bar{T}_{k-1}^o(z=0) - \bar{T}_M^o(z=0)\|_2}, \quad \|e_k\|_2 = \sqrt{\sum_{i=1}^{n_x \times n_y} |(\bar{T}_k^o)_i(z=0) - (\bar{T}_M^o)_i(z=0)|^2}$$

395 with $\|\cdot\|_2$ the ℓ_2 -norm, and n_x (resp. n_y) the number of grid points in the zonal (resp.
 396 meridional) direction. For the algorithm to converge R_k must remain smaller than unity, and
 397 the smaller R_k the faster the convergence. In this study, it turned out that four iterations
 398 were usually sufficient for convergence (Fig. 8). However, the convergence speed of the
 399 method should depend on the model configuration and grid resolution. Finer resolution
 400 would lead to more energetic nonlinear effects which may impair the convergence speed of
 401 the method. These complications are left for a future study.

402 From a theoretical point of view, it is straightforward to show that Schwarz methods,
 403 and more generally iterative methods, are slower to converge in presence of low frequencies
 404 in the error as long as the problem under consideration is only weakly nonlinear. We can
 405 thus anticipate that the main differences in the coupled solutions integrated with Schwarz
 406 and asynchronous methods occur at low frequencies. Figure 9 shows the temporal spectrum

$$\text{Sp}_\omega \{ \|\mathbf{u}_h^a(z=10\text{ m})_{k=M}\| - \|\mathbf{u}_h^a(z=10\text{ m})_{\text{asyn}}\| \}$$

407 of the difference between the coupling methods (\mathbf{u}_h^a being the solution obtained with
 408 the asynchronous method, i.e. one iteration only), with ω the frequency. As expected,
 409 the effect of iterations is primarily visible for low time-frequencies, which suggests that
 410 the Schwarz method would have a significant impact on large-scale and climate processes.
 411 Note that it is theoretically possible to design a Schwarz method which could converge
 412 equally fast at all frequencies (this is the so-called *Optimized Schwarz Method*). However, this
 413 method is tractable only for relatively simple academic problems and is not straightforward
 414 to generalize to the ocean-atmosphere coupling problem (see discussion in Sec. 6).

415 *b. Ensemble Spread*

416 We checked that the Schwarz method converges at least in one realistic application of
417 our coupled model system. Let us now illustrate the impact of the method on the robustness
418 of the coupled solutions. Our analysis will focus on the cyclone track and intensity of each
419 member of the two ensembles corresponding to the two coupling algorithms. We identify the
420 cyclone track using minimum pressure at the first vertical level of the atmospheric model.
421 The track envelop shown in Fig. 11 approximately follows the observed track with motion
422 speed and underlying sea surface temperature in the same range (Fig. 10). It is striking that
423 the dispersion of trajectories is significantly smaller for the ensemble based on the Schwarz
424 method. In this case, the maximum standard deviation is 164 km and the standard deviation
425 is around 92 km compared to respectively 212 km and 125 km with the asynchronous method.
426 Note that the track spread in both ensembles tend to reduce when approaching the south-
427 east corner of the domain, presumably because all simulations share identical boundary
428 conditions.

429 The same behavior as for cyclone track is manifest in time histories of the maximum
430 surface wind speed (not shown), with a maximum deviation of 6.2 m s^{-1} for the asynchronous
431 method compared to 3.7 m s^{-1} for the Schwarz method (3.7 m s^{-1} versus 2.2 m s^{-1} for the
432 standard deviation). Therefore, two very important features of tropical cyclone events,
433 namely their trajectory and intensity, are more robustly simulated when improving the
434 consistency of coupling method.

435 **6. Discussion and Conclusion**

436 We have emphasized in this paper the key role of the coupling algorithm in the design of
437 an atmospheric and oceanic coupled model. Very popular coupling methods used in regional
438 and global climate models do not provide the exact solution to the ocean-atmosphere coupling
439 problem (4), but an approximation. We introduced a natural and non-intrusive method to

440 solve this problem and we showed its relevance. This method called Global-in-Time Schwarz
441 Method is based on an iterative process. It can easily be shown that the usual synchronous
442 (6) and asynchronous (5) methods prominent in climate models correspond to only one
443 iteration of a Schwarz algorithm. However, the iterative procedure can substantially increase
444 the computational cost of the AOCM (by a factor corresponding to the number of iterations
445 to reach convergence).

446 A realistic application is produced by an ensemble of simulations through perturbations
447 of the initial conditions and coupling frequency of a regional AOCM. The results suggest
448 that part of the model sensitivity to perturbed parameters can be attributed to inaccuracies
449 in the coupling method. Specifically, coupling inconsistencies can spuriously increase the
450 physical stochasticity of atmospheric and oceanic events (materialized here by the ensemble
451 spread). For our particular case, three iterations of the Schwarz method are sufficient to
452 improve the coupled solutions with respect to the ensemble spread. We show that the
453 iterative process should significantly modify the low-frequency component of the solution,
454 whereas high-frequencies converge very rapidly. Therefore, we anticipate that the iterative
455 procedure would be particularly relevant to long-term climate studies. However, it remains
456 to be seen how robust is the convergence regardless of the model formulation, particularly
457 the boundary layer parameterizations at the air-sea interface. In this regard, the Schwarz
458 algorithm can also be used as diagnostic tool to assess consistency between atmospheric and
459 oceanic boundary layer parameterizations: two schemes could be recognized as *consistent* if
460 they lead to a converging Schwarz algorithm.

461 For some applications, the computational cost of the Schwarz algorithm may be unaf-
462 fordable and one would thus proceed to only one iteration of the method, thereby retrieving
463 the asynchronous method. Based on our study and on our earlier theoretical work (Lemarié
464 et al. 2013a,b,c), we can formulate some recommendations when the asynchronous coupling
465 method is used :

- 466 • The coupling frequency (i.e. the size of time window between two exchanges) can be

467 adequately set to minimize coupling errors and that of surface flux parameterizations.
468 Because the Schwarz algorithm converges slowly at low-frequency, the shorter the time
469 window the smaller the coupling error. However, because of large uncertainties in the
470 specification of surface fluxes, the larger the time window the smaller the parameteri-
471 zation error. With those constraints and the requirement of a proper representation of
472 the diurnal cycle, a coupling frequency between 1 and 3 hours would provide a good
473 compromise. In addition, it is generally observed that the wave field acts as a low-pass
474 filter on air-sea exchanges, with a cutoff around 2 hours, which suggests that a time
475 window of 2 hours may be considered as a relevant choice.

476 • Lemarié et al. (2013b,c) show that the mathematical type of interface condition has
477 great impact on the convergence speed of the Schwarz algorithm. In the case of a
478 diffusion problem, replacing Dirichlet-Neumann⁶ conditions by a Robin (a.k.a. Fourier)
479 interface condition (i.e., a linear combination of Dirichlet and a Neumann conditions)
480 can dramatically improve the convergence speed if the weights in the linear combination
481 are properly chosen. Even in the case of one single iteration of the method (i.e., the
482 asynchronous method) coupling errors can be reduced by simply working on the type
483 of interface condition. However, the theoretical problem is not straightforward for
484 AOCMs because of bulk formulations. It is a work in progress.

⁶A Dirichlet condition amounts to specifying a value in the interface, whereas a Neumann condition amounts to specifying a flux.

485 7. Figures and tables

486 *a. Figures*

487 *b. Tables*

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Test Problem: One-Dimensional Diffusion Problem

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In this section, we compare different coupling methods using a simple test case, which is assumed representative of an ocean-atmosphere type problem. To do so, we define two subdomains $\Omega_1 =] - L_1, 0[$ and $\Omega_2 =]0, L_2[$ with $L_1 = L_2 = 250$ m. In this example, we consider the one dimensional diffusion equation of a scalar quantity q

$$\partial_t q - \partial_z (\nu \partial_z q) = f, \quad (\text{A1})$$

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with ν a diffusion coefficient such that $\nu = \nu_1$ in Ω_1 and $\nu = \nu_2$ in Ω_2 . We choose $\nu_1 \neq \nu_2$ to model the heterogeneous physical properties between the two subdomains. Uniqueness of solutions for the coupling problem is obtained with Dirichlet-Neumann conditions at $z = 0$ (i.e. we require the equality of the subproblems solutions and of their fluxes). The corresponding coupling problem reads

$$\left\{ \begin{array}{ll} \partial_t q_2 - \partial_z (\nu_2 \partial_z q_2) = f_2, & \text{in }]0, L_2[\times]0, \mathcal{T}[, \\ q_2(L_2, t) = \tilde{q}_2(t), & t \in [0, \mathcal{T}], \\ \nu_2 \partial_z q_2(0, t) = \nu_1 \partial_z q_1(0, t), & t \in [0, \mathcal{T}], \end{array} \right. \quad (\text{A2})$$

506

$$\left\{ \begin{array}{ll} \partial_t q_1 - \partial_z (\nu_1 \partial_z q_1) = f_1, & \text{in }] - L_1, 0[\times]0, \mathcal{T}[, \\ q_1(-L_1, t) = \tilde{q}_1(t), & t \in [0, \mathcal{T}], \\ q_1(0, t) = q_2(0, t), & t \in [0, \mathcal{T}], \end{array} \right. \quad (\text{A3})$$

507

for a given initial condition. We pose the problem to get analytical solutions of the form

$$q_2(z, t) = \frac{q_0}{8} \left[3 - \exp\left(-\frac{z}{\alpha_2}\right) \right] \left[1 + \cos^2\left(\frac{\pi t}{T}\right) \right]$$

508

$$q_1(z, t) = \frac{q_0}{8} \left[1 + \exp\left(\frac{z}{\alpha_1}\right) \right] \left[1 + \cos^2\left(\frac{\pi t}{T}\right) \right]$$

509 where the condition $\nu_2\alpha_1 = \nu_1\alpha_2$ is sufficient to ensure the proper regularity of the solution
510 across the interface at $z = 0$. To obtain such analytical solutions, we set the right hand sides
511 f_1 and f_2

$$f_1(z, t) = -\frac{q_0}{8} \left\{ \left(\frac{\pi}{T} \sin \frac{2\pi t}{T} \right) \left[1 + \exp \left(\frac{z}{\alpha_1} \right) \right] + \frac{\nu_1}{\alpha_1^2} \exp \left(\frac{z}{\alpha_1} \right) \left[1 + \cos^2 \left(\frac{\pi t}{T} \right) \right] \right\},$$

$$f_2(z, t) = -\frac{q_0}{8} \left\{ \left(\frac{\pi}{T} \sin \frac{2\pi t}{T} \right) \left[3 - \exp \left(-\frac{z}{\alpha_2} \right) \right] - \frac{\nu_2}{\alpha_2^2} \exp \left(-\frac{z}{\alpha_2} \right) \left[1 + \cos^2 \left(\frac{\pi t}{T} \right) \right] \right\}.$$

512
513 Equation (A1) is discretized using a backward Euler scheme in time and a second-order
514 scheme on a staggered grid in space. The parameter values are $\alpha_1 = 50$, $\alpha_2 = 10$, $q_0 = 15$,
515 $T_0 = 24 \times 3600$ s, $\nu_1 = 1$ m s⁻¹, $\nu_2 = 0.2$ m s⁻¹, $\Delta z = 1$ m, $\Delta t = 900$ s, and the total
516 simulation time is $\mathcal{T} = 2000$ days. To numerically solve the coupling problem (A2-A3),
517 we use the six different methods described in the paper : the asynchronous method with
518 instantaneous and averaged fluxes (the fluxes are reconstructed using a linear conservative
519 scheme), the global-in-time Schwarz method with instantaneous and averaged fluxes, the
520 synchronous method and the local-in-time Schwarz method. The global-in-time Schwarz
521 method and the asynchronous method are integrated with time windows of 6 hours. As
522 expected, the Schwarz methods (local or global in time) with instantaneous fluxes provide
523 strictly the same solution $q^*(z, t)(z \in \Omega_1 \cup \Omega_2, t \in [0, \mathcal{T}])$ which is used as a reference to
524 compute the ℓ_2 -norm of the error for other methods

$$\|\varepsilon(t_i)\|_2 = \sqrt{\sum_{k=1}^{k=N} |q(z_k, t_i) - q^*(z_k, t_i)|^2},$$

525 with N the number of grid points between $-L_1$ and L_2 . The time evolution of $\|\varepsilon\|_2$ is
526 represented in Figure 4, and discussed in Sec. 3b and 3c.

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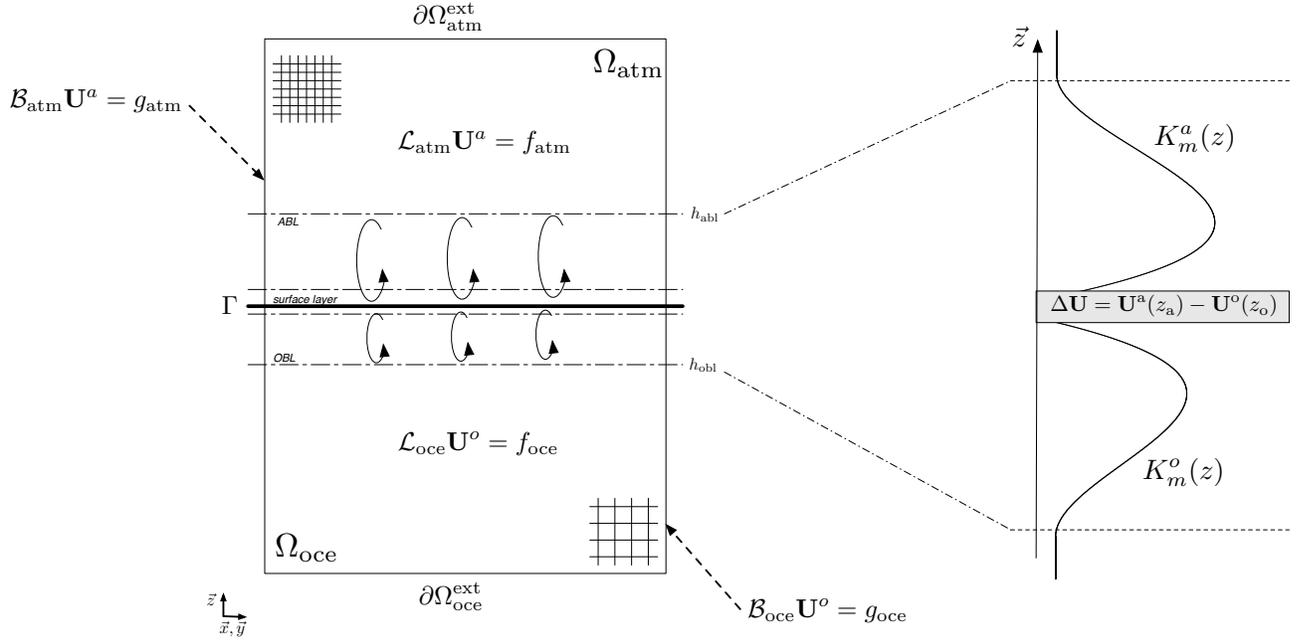


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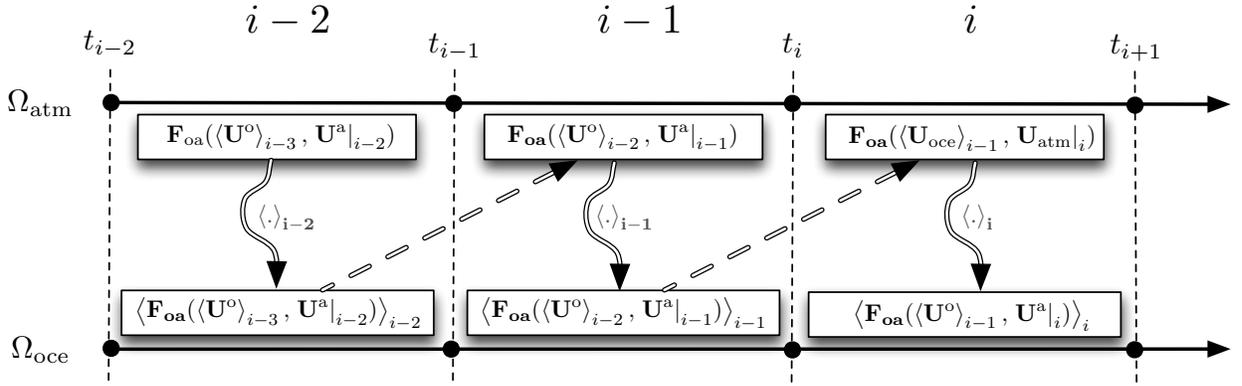


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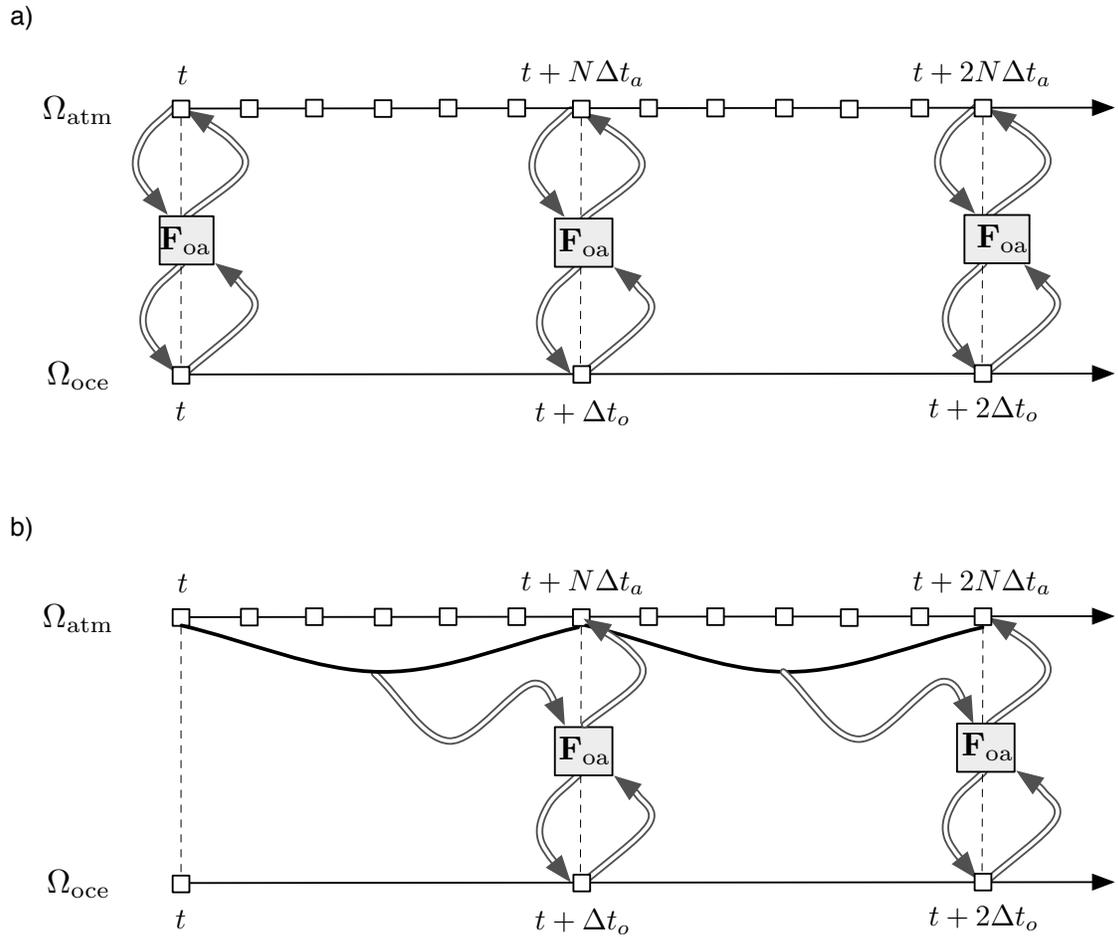


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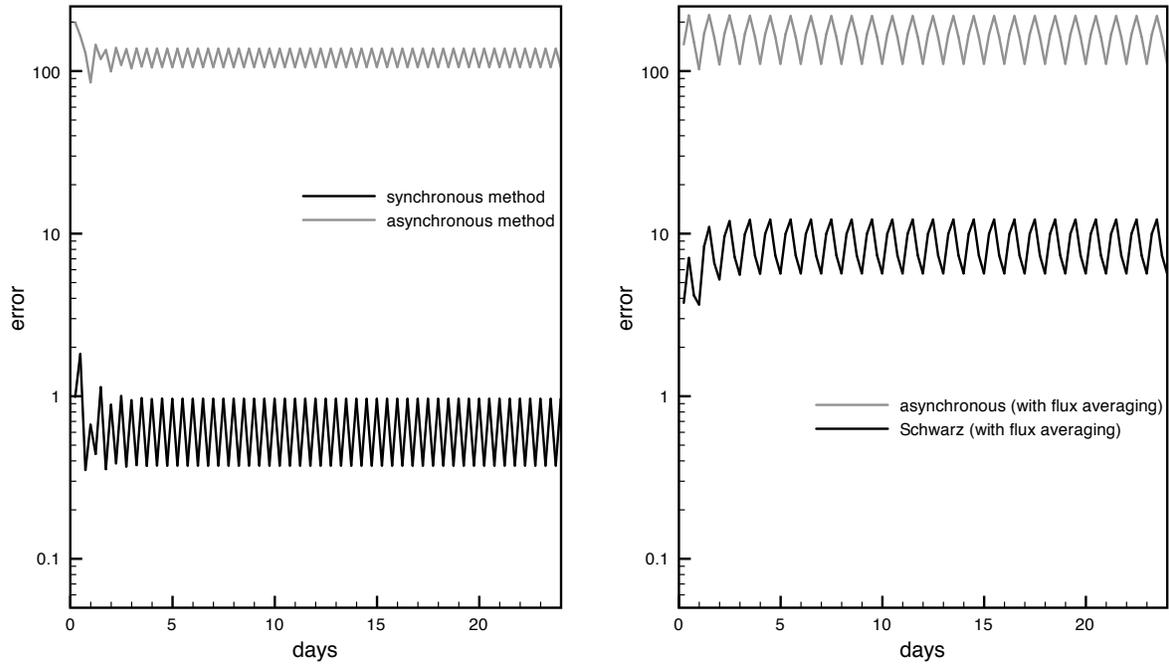


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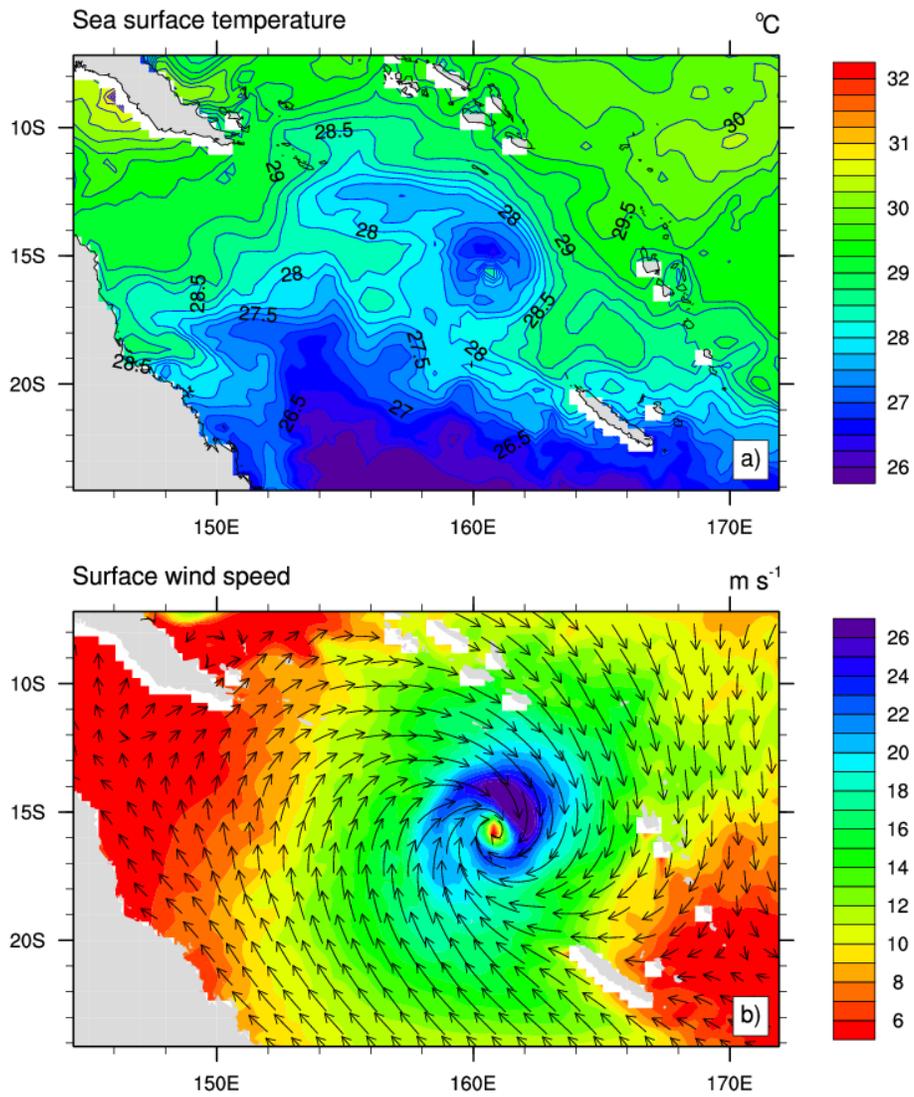


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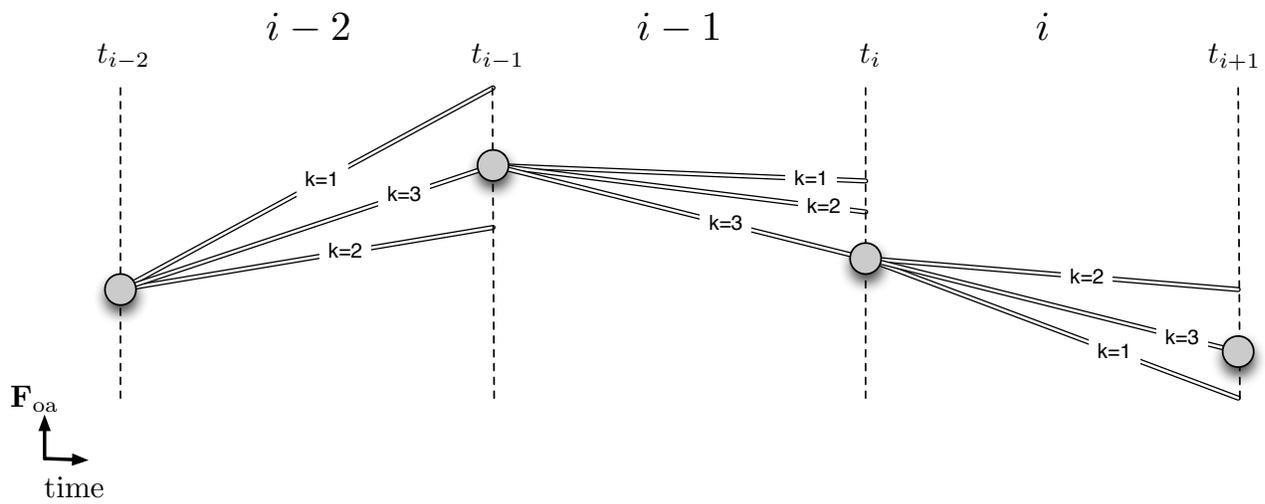


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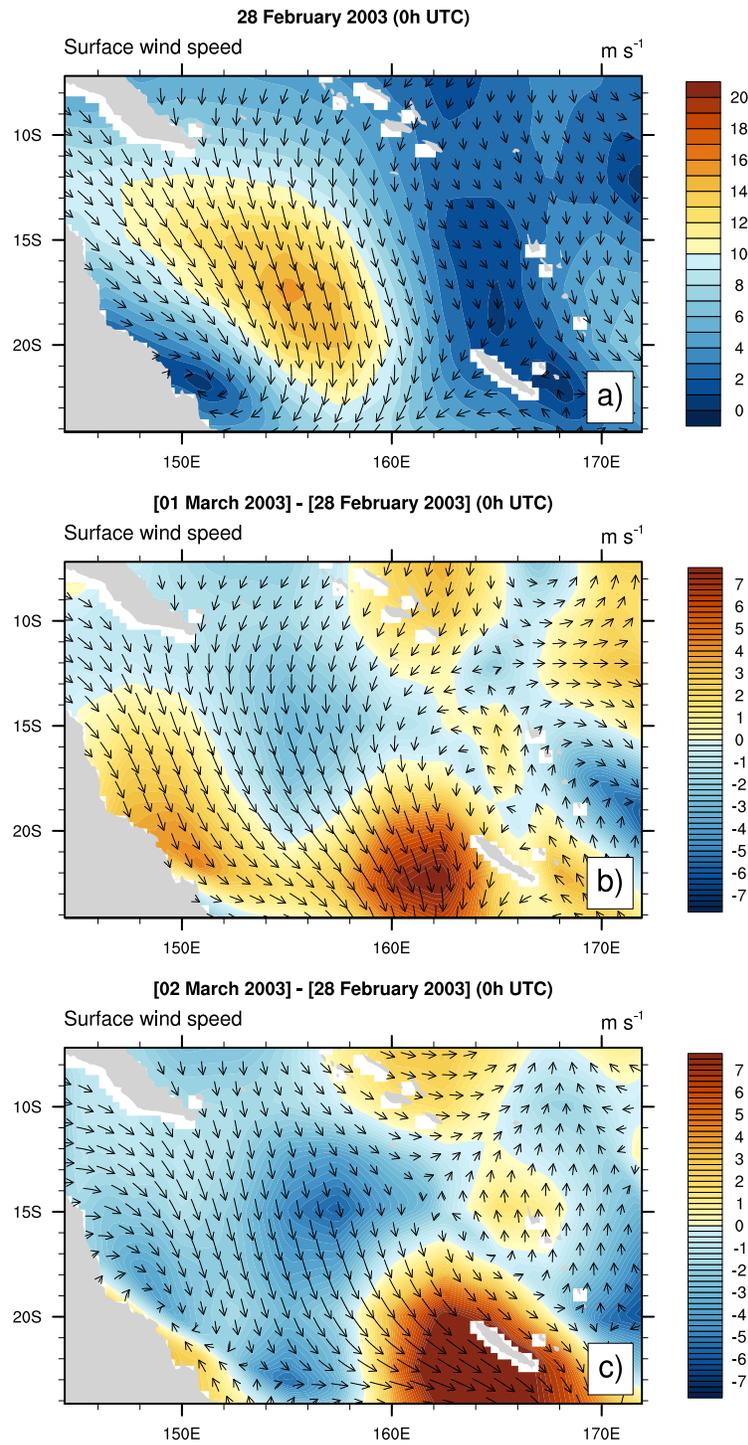


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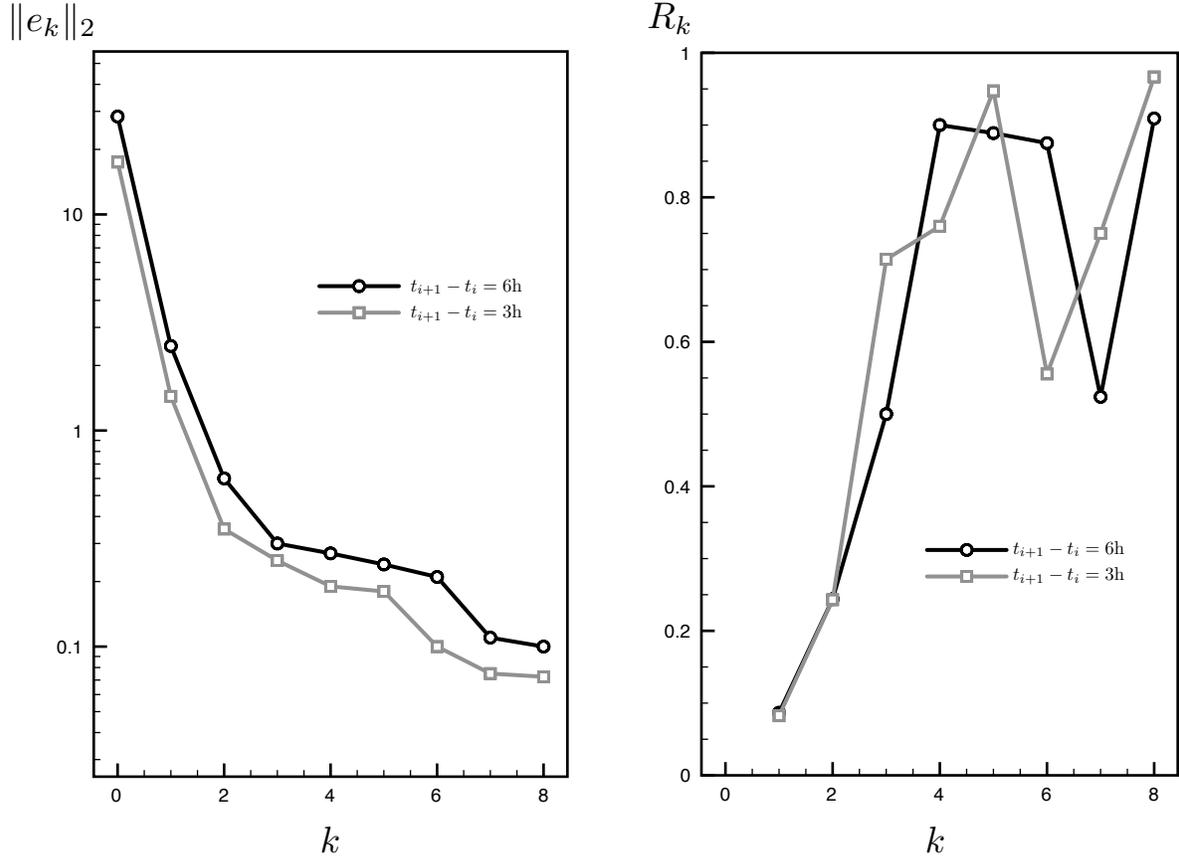


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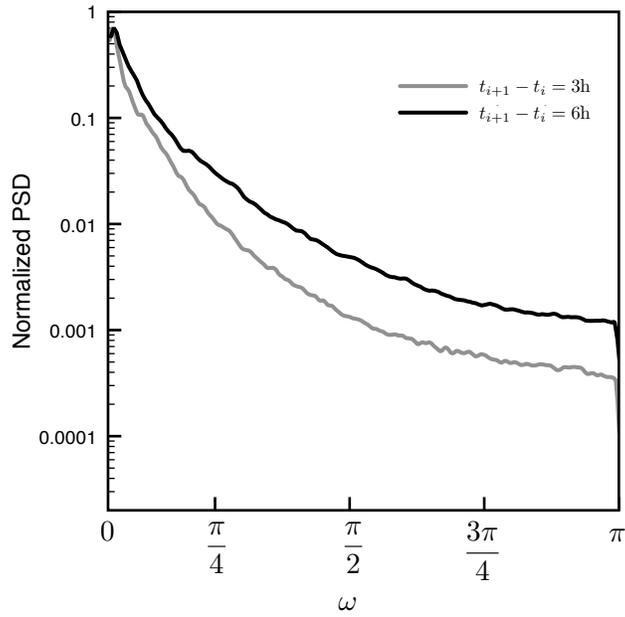


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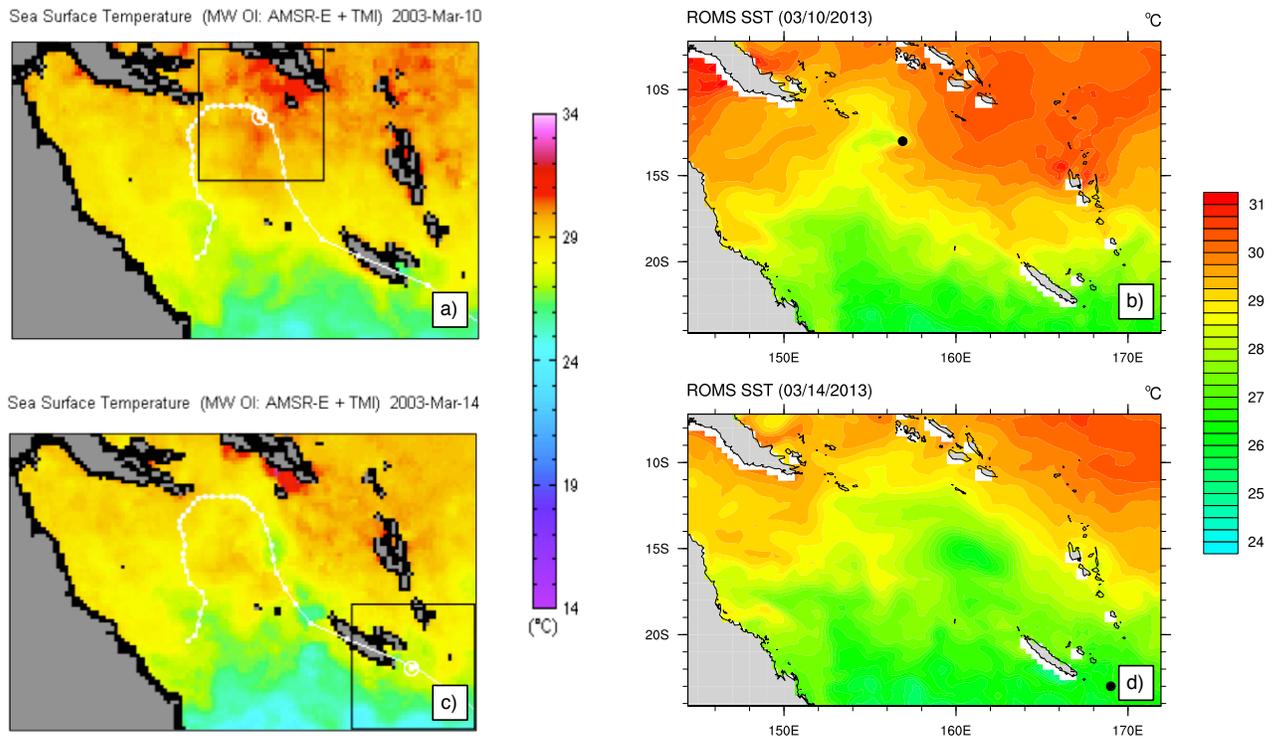


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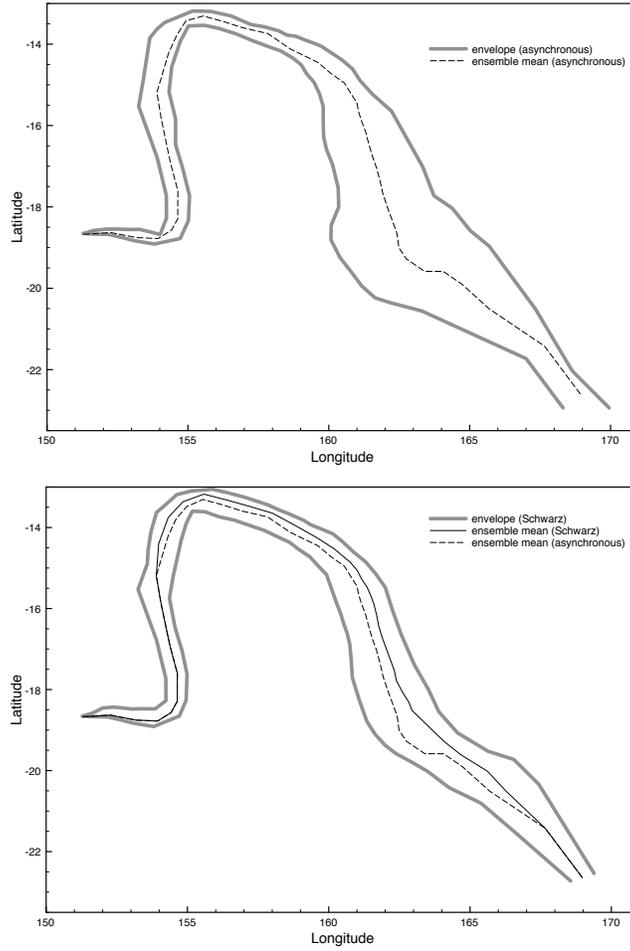


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