Mathematical modeling of soil strains growth at batch culture

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Abstract

A mathematical model, validated on experimental data aiming at describing and predicting microbial growth on an essential limited substrate in batch cultures is proposed. This model takes into account viable cell growth, substrate consumption, cell mortality, nonviable cell accumulation in the culture medium and partial dead cell recycling into substrate. A detailed analysis is carried out. Least squares method is used to identify model parameters.

Material & Methods

The substrate solution was prepared by dissolving glucose (as the only carbon source) in the medium culture (Bergersen medium) at the required concentration. The pH was adjusted to 6.8 due to the high biodegradation efficiency of the strains at this value. Both the medium and the equipment were sterilized before inoculation. Then, the strains previously isolated from soils, were inoculated into the culture medium. The temperature was controlled at 28C and the culture was automatically and continuously shaken. The experiments were carried out in triplicate. The growth of miocroorganisms on different glucose concentrations was monitored by manual optical density measurements in a spectrometer at 600 nm, where a linear relationship between optical density and concentration of viable and nonviable cells was assumed.

Modeling and identification of the specific growth rate

Assuming there is initially no dead cell, and that the natural mortality is negligible during the exponential phase, we calculated the regression coefficient in the exponential phase for each glucose concentration. The growth rate of all strains were then identified as functions of glucose concentrations. The best-fit results based on the Haldane equation

$$\mu(s) = \frac{\mu_{max} s}{k_s + s + \frac{s^2}{k_i}}$$

were obtained using the least squares method and and are presented hereafter where μ_{max} is the maximum growth rate, k_s is the saturation constant and k_i is the inhibition coefficient.

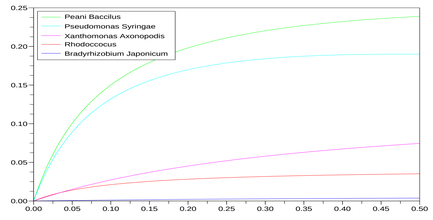


Figure 2: Specific growth rates

Mathematical model

Let s, x and x_d denote the substrate, the viable cells and the nonviable cells concentrations, respectively. The model can be written as :

$$\begin{cases} \dot{s} = -\dfrac{\mu(s)}{k} x + \lambda(1-\delta) m x, \\ \dot{x} = (\mu(s) - m) x, \qquad \text{such that} \quad \dfrac{1}{k} > \lambda(1-\delta) \\ \dot{x}_d = \delta m x, \end{cases} \tag{1}$$

$$s(0) = s_0 > 0, \quad x(0) = x_0 > 0 \quad \text{and} \quad x_d(0) = 0.$$

1. The solution of system (1) has positive, bounded components and thus is defined for every positive time t.
2. $\{(s, x, x_d) \in \mathbb{R}^3_+ / ks + x + \dfrac{1 - k\lambda(1-\delta)}{\delta} x_d = ks_0 + x_0\}$ is positively invariant under (1).
3. Equilibrium points of system (1) have the following form $A^* = (s^*, 0, x_d^*)$ such that $ks^* + \dfrac{1 - k\lambda(1-\delta)}{\delta} x_d^* = ks_0 + x_0$.

   There exists two values s_1^* and s_2^* such $0 \leq s_1^* < s_2^*$ satisfying $\mu(s_1^*) = \mu(s_2^*) = m$.
4. If $s^* < s_1^*$ or $s_2^* < s^*$, then the equilibrium point A^* is semi-stable .
5. If $s_1^* < s^* < s_2^*$, the equilibrium point A^* is unstable.
6. The equilibrium points set $\Gamma = \left\{(s_1^*, 0, x_d^*); \quad x_d^* \geq 0\right\} \cup \left\{(s_2^*, 0, x_d^*); \quad x_d^* \geq 0\right\}$ separate semi-stable equilibrium points set and unstable equilibrium points set one.

Parameter identification

Available measurements were the substrate concentration and the optical density, modeled as a linear combination of viable and nonviable cells concentrations

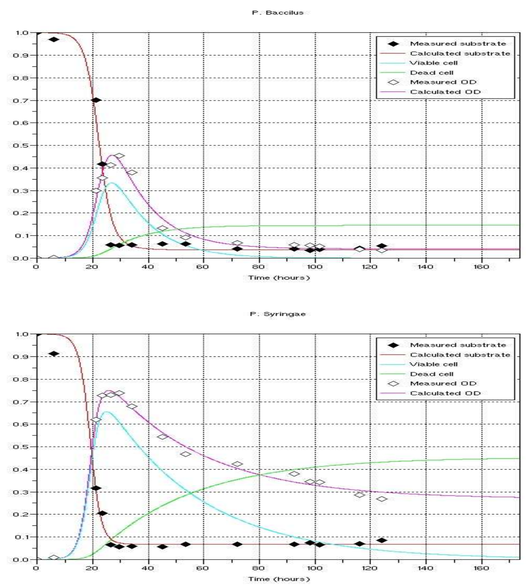
$$y = \begin{pmatrix} s \\ \gamma_1 x + \gamma_2 x_d \end{pmatrix}.$$

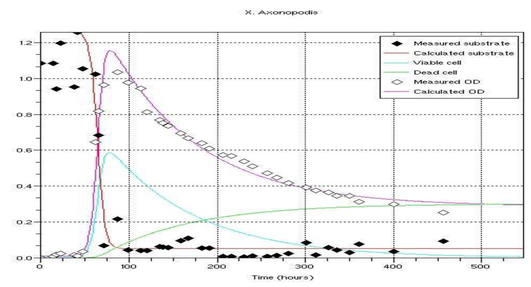
The least squares method was used to calculate model parameters by minimizing the following criterion :

$$J = \frac{\sigma_1}{2} \sum_{i=1}^{n} \Big(s(t_i) - s_i\Big)^2 + \frac{\sigma_2}{2} \sum_{i=1}^{n} \Big(\gamma_1\, x(t_i) + \gamma_2\, x_d(t) - z_i\Big)^2$$

where t_i is the time variable, s_i is the substrate concentration and z_i is the optical density. $s(t_i), x(t_i)$ and $x_d(t_i)$ are the substrate, the viable and the nonviable cells concentrations simulated using the model at time $t_i, i = 1, ..., n$ while σ_1 and σ_2 are weighting coefficients.

Results





Conclusions and Perspectives

We have developed and validated simple models in revisiting the way the optical density is modeled. Perspectives include co-cultures modeling in nonhomogeneous media.

References

[1] S.J. Pirt (1975) Principles of microbe and cell cultivation. *Blackwell Scientific Publications*, London.

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