

PROOF OF LEMMA 1 OF THE PAPER THE GEOMETRY OF SOUND-SOURCE LOCALIZATION USING NON-COPLANAR MICROPHONE ARRAYS

Xavier Alameda-Pineda^{1,2}, Radu Horaud¹ and Bernard Mourrain³

¹ INRIA Grenoble Rhône-Alpes, ² Université de Grenoble, ³ INRIA Sophia Antipolis, France

In this annex we present the formal proof of the lemma 1 enounced in the paper.

1. INTRODUCTION

We first recall the assumptions on the propagation model. The sound wave is emitted at S and is assumed to travel through the direct-path at a constant speed until the M microphones, placed at $\{M_1, \dots, M_M\} = \mathcal{M}$. Hence, the time of arrival is $t_m = \|S - M_m\|/\nu$, where ν is the sound speed and $\|\cdot\|$ is the Euclidean norm. Consequently the time difference of arrival or time delay between the m^{th} and the n^{th} microphones is:

$$t_{m,n}(S) = t_n - t_m = \frac{\|S - M_n\| - \|S - M_m\|}{\nu}. \quad (1)$$

2. PROOF OF LEMMA 1

Lemma 1 *The set of sound-source locations $S \in \mathbb{R}^N$ satisfying $t_{m,n}(S) = \hat{t}_{m,n}$ is:*

- (i). empty if $|\hat{t}_{m,n}| > t_{m,n}^*$, where $t_{m,n}^* = \|M_m - M_n\|/\nu$,
- (ii). the half line $L_{m,n}^{\text{MAX}}$ (or $L_{m,n}^{\text{MIN}}$), if $\hat{t}_{m,n} = t_{m,n}^*$ (or if $\hat{t}_{m,n} = -t_{m,n}^*$), where $L_{m,n}^{\text{MAX}} = \{H_{m,n} + \mu V_{m,n}\}$, $L_{m,n}^{\text{MIN}} = \{H_{m,n} - \mu V_{m,n}\}$, $\mu \geq 1/2$, $H_{m,n} = (M_m + M_n)/2$ and $V_{m,n} = M_m - M_n$,
- (iii). the hyperplane passing by $H_{m,n}$ perpendicular to $V_{m,n}$, if $\hat{t}_{m,n} = 0$ or
- (iv). one sheet of a two-sheet hyperboloid with foci M_m and M_n for other values of $\hat{t}_{m,n}$.

Proof: It is easy to see $-t_{m,n}^* \leq t_{m,n}(S) \leq t_{m,n}^*$, $\forall S \in \mathbb{R}^N$, which proves (i). (ii) is proven by rewriting $S = H_{m,n} + \mu_1 V_{m,n} + \sum_{k=2}^N \mu_k W_k$, where $(V_{m,n}, W_2, \dots, W_N)$ is an orthogonal basis of \mathbb{R}^N , and deriving with respect to the μ_i 's. In order to prove (iii) and (iv) and without loss of generality, we can assume $M_m = e_1$, $M_n = -e_1$ and $\nu = 1$. Equation (1) rewrites:

$$(\hat{t}_{m,n})^2 + 4x_1 = -2\hat{t}_{m,n} \left((x_1 + 1)^2 + \sum_{k=2}^N x_k^2 \right)^{\frac{1}{2}}, \quad (2)$$

where $(x_1, \dots, x_N)^T$ are the coordinates of S . By squaring the previous equation we obtain:

$$a(4 - a) + 4a \sum_{k=2}^N x_k^2 - 4(4 - a)x_1^2 = 0, \quad (3)$$

where $a = (\hat{t}_{m,n})^2$. Notice that if $\hat{t}_{m,n} = 0$ we get $x_1 = 0$, which corresponds to the statement in (iii). For the rest of values of a , that is $0 < a < (t_{m,n}^*)^2 = 4$, equation (3) represents a two-sheet hyperboloid, since all coefficients are strictly positive except the one of x_1^2 . Because $0 < a < 4$, we can rewrite (3) as:

$$x_1^2 = \frac{a(4 - a) + 4a \sum_{k=2}^N x_k^2}{4(4 - a)}. \quad (4)$$

We observe that the set of solutions of (3) can be split into two subsets $S_{m,n}^+$ and $S_{m,n}^-$ parametrized by (x_2, \dots, x_N) , corresponding to the two solutions for x_1 of equation (4). These two sets are the two sheets of the hyperboloid defined in (3). Moreover, one can easily verify that $t_{m,n}(S_{m,n}^+) = -t_{m,n}(S_{m,n}^-)$, so either $t_{m,n}(S_{m,n}^+) = \hat{t}_{m,n}$ or $t_{m,n}(S_{m,n}^-) = \hat{t}_{m,n}$, but not both. Hence the set of points S satisfying $t_{m,n}(S) = \hat{t}_{m,n}$ is either $S_{m,n}^+$ or $S_{m,n}^-$, so one sheet of a two-sheet hyperboloid. \square

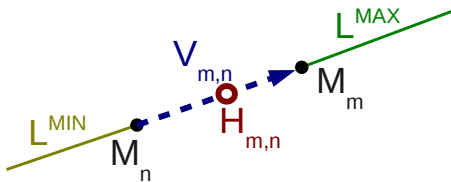


Figure 1: Geometry associated with the two microphone case, located at M_m and M_n (see Lemma 1). $H_{m,n}$ is the mid-point of the the microphones (in red) and $V_{m,n}$ the vector $M_m - M_n$ (in dashed-blue). $L_{m,n}^{\text{MAX}}$ and $L_{m,n}^{\text{MIN}}$ are the two half lines represented in green and yellow respectively.