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Tree Automata and Constraints: a brief survey

Emmanuel Filiot Florent Jacquemard Sophie Tison

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Dagstuhl Seminar 13192
Tree Transducers and Formal Methods

Tree Automata and non-linearity

Non-linearity phenomena are not captured by tree automata, e.g. :

- ▶ The set of ground instances of a non-linear term is not recognizable.
- ▶ The class of recognizable tree languages is not closed under non-linear homomorphisms.

How to enrich transitions by equality and disequality constraints ?

Two main ideas

- ▶ Add **local** tests, e.g. to capture the set of instances of $f(g(x), x)$
Why: pattern matching, rewriting, representing finitely image by non-linear transformations of recognizable tree languages (type checking?), ...
When: from 80's to now
- ▶ Add **global** tests, e.g. to capture "all the subterms rooted by f are different".
Why: integrity constraints (XML), non linear query languages, ...
When: more recently

Leitmotiv: keeping as far as possible good closure and decision properties.

Tree Automata with Local Constraints

Equality and Disequality Constraints

An **equality constraint** (resp. a **disequality constraint**) is a predicate on $T(\Sigma)$ written $\pi = \pi'$ (resp. $\pi \neq \pi'$) where $\pi, \pi' \in \{1, \dots, k\}^*$.

Such a predicate is satisfied on a tree t , which we write $t \models \pi = \pi'$, if $\pi, \pi' \in \mathcal{Post}$ and $\models t|_{\pi} = t|_{\pi'}$ (resp. $\pi \neq \pi'$ is satisfied on t if $\pi = \pi'$ is not satisfied on t).

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These constraints are indeed **unlabeled path constraints**. More **general disequality constraints** have been defined by H. Seidl and A. Reuß (LPAR 2010, FOSSACS 2012).

Automata with Equality and Disequality Constraints

The general class

An **automaton with equality and disequality constraints** is a tuple (Q, Σ, Q_f, Δ) where

- ▶ Σ is a finite ranked alphabet
- ▶ Q is a finite set of states
- ▶ Q_f is a subset of Q of final states
- ▶ Δ is a set of transition rules of the form:

$$f(q_1, \dots, q_n) \xrightarrow{c} q$$

where $f \in \Sigma$, $q_1, \dots, q_n, q \in Q$, and c is a Boolean combination of equality and disequality constraints.

Automata with Equality and Disequality Constraints

The general class

- ▶ The move relation $\rightarrow_{\mathcal{A}}$ is defined by:
 $t \rightarrow_{\mathcal{A}} t'$ if and only

$$t = C[f(q_1(u_1), \dots, q_n(u_n))],$$

$$t' = C[q(f(u_1, \dots, u_n))]$$

$$f(q_1, \dots, q_n) \xrightarrow{c} q \in \Delta,$$

$$f(u_1, \dots, u_n) \models c.$$

- ▶ $\rightarrow_{\mathcal{A}}^*$ is the reflexive and transitive closure of $\rightarrow_{\mathcal{A}}$.
- ▶ \mathcal{A} **accepts** a tree $t \in T(\Sigma)$ if $t \rightarrow_{\mathcal{A}}^* q$ for some final state q .
- ▶ The **language accepted**, or **recognized**, is the set $L(\mathcal{A})$ of trees $t \in T(\Sigma)$ accepted by \mathcal{A} .

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- ▶ Class with only equalities defined in 1981 (Dauchet & Mongy)

Automata with Equality and Disequality Constraints

Example 1

- ▶ $Q = \{q\}$,
- ▶ $\Sigma = \{f, a\}$
- ▶ final states: $\{q\}$
- ▶ Δ consists of the following rules:

$$r_1 : \quad a \rightarrow q$$
$$r_2 : \quad f(q, q) \xrightarrow{1=2} q$$

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The automaton recognizes the set of balanced trees.

Automata with Equality and Disequality Constraints

Example 2

- ▶ $Q = (\{q, q_g, q_{fin}\},$
- ▶ $\Sigma = \{f, g, a\}$
- ▶ final states: $\{q_{fin}\}$
- ▶ Δ consists of the following rules:

$$r_1 : \quad a \rightarrow q$$

$$r_2 : \quad f(q, q) \rightarrow q$$

$$r_3 : \quad f(q_g, q) \xrightarrow{11=2} q_{fin}$$

$$r_4 : \quad g(q) \rightarrow q_g$$

$$r_5 : \quad g(q) \rightarrow q$$

Automata with Equality and Disequality Constraints

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$$r_4 : \quad g(q) \rightarrow q_g$$

$$r_5 : \quad g(q) \rightarrow q$$

The automaton recognizes the ground instances of $f(g(x), x)$.

Automata with Equality and Disequality Constraints

Example 3

- ▶ $Q = (\{q\},$
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Automata with Equality and Disequality Constraints

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The automaton recognizes "grids".

Automata with Equality and Disequality Constraints

Properties

- ▶ Can be determinized (and completed) (exponential blow up)

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- ▶ Emptiness is undecidable for this class -e.g. use Post problem or previous encoding of grids -: undecidable even with only equalities between cousins.
- ▶ capture homomorphic images of recognizable languages (only equality constraints are needed) and images by bottom-up transducers.

Undecidability comes in some sense from superposition of (equality) tests. To get a decidable class, we can try to limit this superposition...

Automata with Constraints

Restrictions

- ▶ restrict the form of the tests
- ▶ limit the number of equality tests
- ▶ limit the superposition of equality tests

Automata with Constraints between brothers

Constraints are reduced to constraints between brothers (siblings)

Allowed constraints are $i = j$ or $i \neq j$.

$f(q_1, q_2, q_3) \rightarrow_{1=2, 1 \neq 3}$ allowed, $f(q_1, q_2) \rightarrow_{1=21}$ forbidden.

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- ▶ stable under boolean operations.
- ▶ Emptiness *EXPTIME*-complete but polynomial in the deterministic case.

Proof for "normal" TA: compute by fix-point the set of reachable states and check it contains a final state. Does it work here?

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The unranked case: W. Karianto and C. Löding, ICALP 2007.

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- ▶ $f(q, q) \rightarrow q, 1 \neq 2$: We need to count, but we just have to count up to maximal arity.
The unranked case: W. Karianto and C. Löding, ICALP 2007.
- ▶ Recognizability is decidable.

Reduction automata

Limit the number of equality tests along a path

A **reduction automaton** \mathcal{A} is an automaton with equality and disequality constraints with an ordering on the states such that for each rule $f(q_1, \dots, q_n) \xrightarrow{c} q$,

- ▶ q is maximal in q, q_1, \dots, q_n
- ▶ q is a strict upper bound of q_1, \dots, q_n if c contains an equality constraint.

Idea: Bound the number of equalities tested along a path.
Defined/studied by Dauchet, Caron, Coquidé, Comon and Jacquemard (94 and following)

Reduction automata

An example

How to define the set of ground instances of $f(f(x, x), f(x, y))$?

Reduction automata

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► $\Delta =$

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$$f(q_f, q) \rightarrow q_f$$

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$$f(q_f, q_f) \rightarrow q_f$$

▶ $q < q_f < q_{fin}$

Reduction automata

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$$f(q, q_f) \rightarrow q_f$$

$$f(q_f, q_f) \rightarrow q_f$$

▶ $q < q_f < q_{fin}$

▶ q_{fin} final state

Reduction automata

An example

How to define the set of ground instances of $f(f(x, x), f(x, y))$
with $\Sigma = \{a, f\}$.

Deterministic version:

$\Delta =$

$$\begin{array}{ll} a \rightarrow q & f(q, q) \rightarrow q_f \\ f(q, q_f) \rightarrow q_f & f(q_f, q) \rightarrow q_f \\ f(q_f, q_f) \xrightarrow{11=12 \wedge 11=21} q_{fin} & f(q_f, q_f) \xrightarrow{11 \neq 12 \vee 11 \neq 21} q_f \end{array}$$

Reduction automata

Properties

- ▶ closed under union and intersection

Reduction automata

Properties

- ▶ closed under union and intersection
- ▶ closed under complement for deterministic automata

Reduction automata

Properties

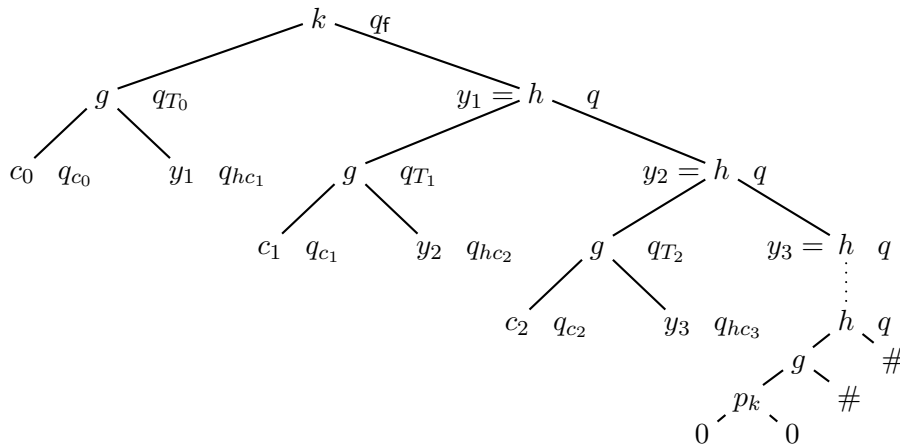
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- ▶ closed under complement for deterministic automata
- ▶ emptiness decidable for **deterministic** automata (M. Dauchet and alt. 1993)

Reduction automata

Properties

- ▶ closed under union and intersection
- ▶ closed under complement for deterministic automata
- ▶ emptiness decidable for **deterministic** automata (M. Dauchet and alt. 1993)
- ▶ emptiness **undecidable** for non deterministic automata (F. Jacquemard and alt. 2008)

Deciding emptiness "needs" determinism



Automata with only one kind of tests

- ▶ TA_{\neq} : automata with only disequality tests
 - ▶ closed under union and intersection
 - ▶ emptiness

Automata with only one kind of tests

- ▶ TA_{\neq} : automata with only disequality tests
 - ▶ closed under union and intersection
 - ▶ emptiness decidable

Automata with only one kind of tests

- ▶ TA_{\neq} : automata with only disequality tests
 - ▶ closed under union and intersection
 - ▶ emptiness decidable in EXPTIME (Comon & Jacquemard, 97)
- ▶ $TA_{=}$: automata with only equality tests
 - ▶ closed under union and intersection
 - ▶ emptiness

Automata with only one kind of tests

- ▶ TA_{\neq} : automata with only disequality tests
 - ▶ closed under union and intersection
 - ▶ emptiness decidable in EXPTIME (Comon & Jacquemard, 97)
- ▶ $TA_{=}$: automata with only equality tests
 - ▶ closed under union and intersection
 - ▶ emptiness undecidable

Automata with only one kind of tests

- ▶ The complement of a $TA_{=}$ is a TA_{\neq} .

Automata with only one kind of tests

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Sketch of the (effective) proof: a state reached by t in the automaton for the complement of L corresponds to the set of all states that t can't reach in the automaton for L . (folklore construction, formalized by Creus & alt. STOC 2010).

Automata with only one kind of tests

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Universality is decidable for $TA_=$, undecidable for TA_{\neq} .

The homomorphism Problem

Hom-problem

Instance A recognizable language R , an homomorphism H .

Answer “yes” if and only if $H(R)$ is recognizable

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Is it decidable?

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Remark: If H is shallow -i.e the images of symbols are shallow-,

The homomorphism Problem

Hom-problem

Instance A recognizable language R , an homomorphism H .

Answer “yes” if and only if $H(R)$ is recognizable

Is it decidable?

Remark: If H is shallow -i.e the images of symbols are shallow-, $H(R)$ is recognizable by an automaton with tests between brothers. as recognizability is decidable for language recognizable by an automaton with tests between brothers: the homomorphism problem is decidable for shallow homomorphisms.

The homomorphism Problem

Proposition

The HOM–problem is decidable (Godoy & alt, STOC 2010) and EXPTIME-Complete (Creus & alt., LICS 2012)

- ▶ TA_{Hom} : equality test without superposition, no disequality tests: $t \xrightarrow{c} q$ where $t \in \mathcal{T}(\Sigma, Q)$, and c is a Boolean combination of equality constraints for positions of t labeled by states.
 - ▶ capture homomorphic images of recognizable tree languages
 - ▶ emptiness, finiteness, recognizability are decidable for this class.
- ▶ $TA_{Hom, \neq}$: equality and disequality tests, no superposition of equality tests : emptiness is decidable

Application to transducers?

- ▶ The image of a recognizable tree language by a bottom-up tree transducer can be represented finitely by a TA_{Hom} .
- ▶ E.g., we can decide whether $T_1(L_1) \subset T_2(L_2)$, L_1, L_2 recognizable tree languages, T_1, T_2 bottom-up tree transducers.

What about global constraints?

Tree Automata with Global Constraints

Local Constraints

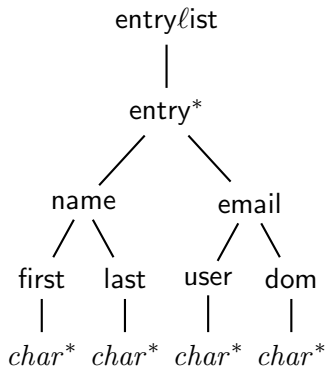
- ▶ non linear pattern matching
- ▶ non-linear transformation
- ▶ term rewriting, reachability analysis

Global Constraints

- ▶ tree structured data processing
- ▶ schema = type + integrity constraints

Type Definition for XML Data

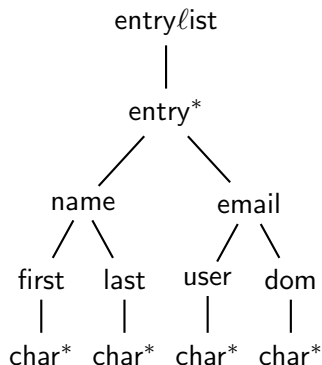
DTD



Tree Automaton

$\text{entry}(p_n, p_m)$	\rightarrow	p_e
empty	\rightarrow	p_{el}
$\text{entrylist}(p_e, p_{el})$	\rightarrow	p_{el}
<hr/>		
$\text{name}(p_f, p_l)$	\rightarrow	p_n
$\text{first}(p)$	\rightarrow	p_f
$\text{last}(p)$	\rightarrow	p_l
<hr/>		
$\text{email}(p_u, p_d)$	\rightarrow	p_m
$\text{user}(p)$	\rightarrow	p_u
$\text{dom}(p)$	\rightarrow	p_d
<hr/>		
$a(p)$	\rightarrow	p
		\vdots
ε	\rightarrow	p

Integrity Constraint



- ▶ email is a **key** (ID)
subtrees below email are pairwise distinct

Cannot be expressed with standard tree automata

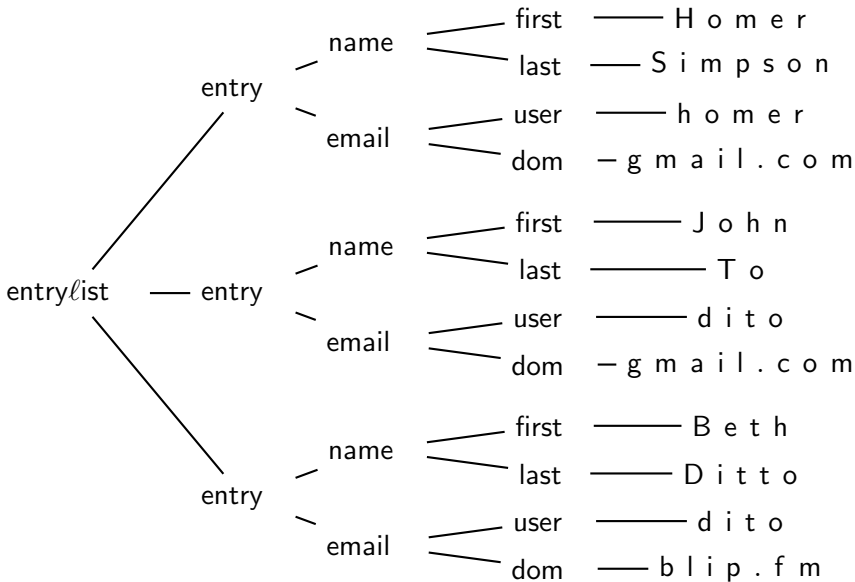
Tree Automata with Local Constraints

- ▶ equality and disequality test in transitions
- ▶ checked during computation

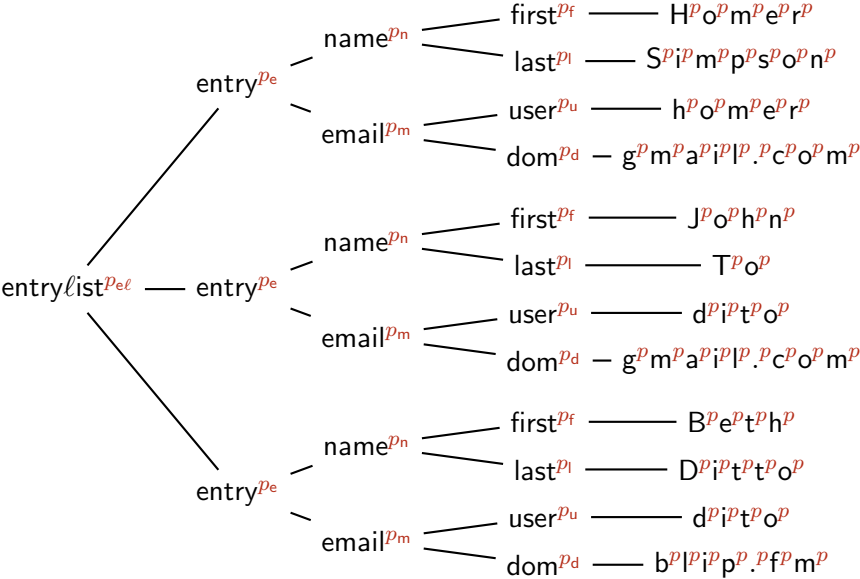
Tree Automata with Global Constraints

- ▶ one standard tree automaton + one separate constraint
- ▶ checked on run, after computation

Tree

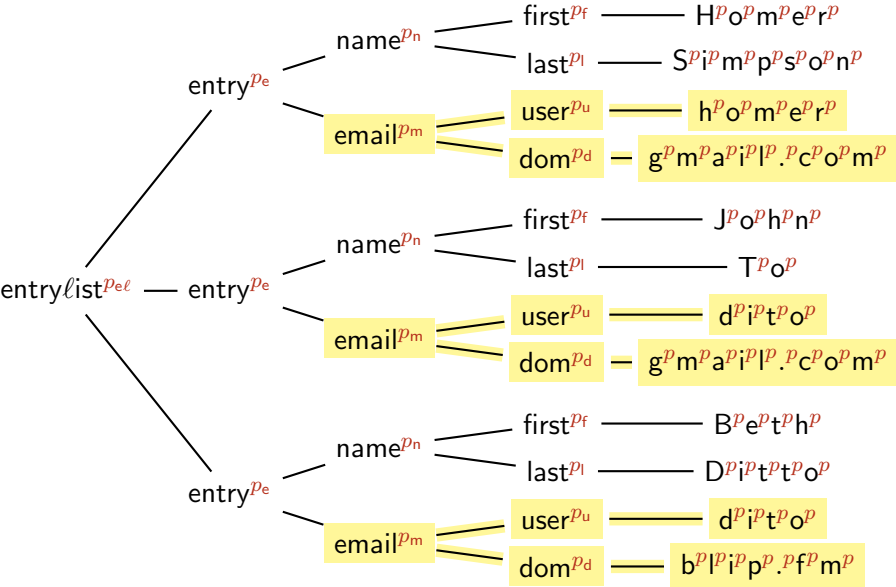


Tree Automaton Run



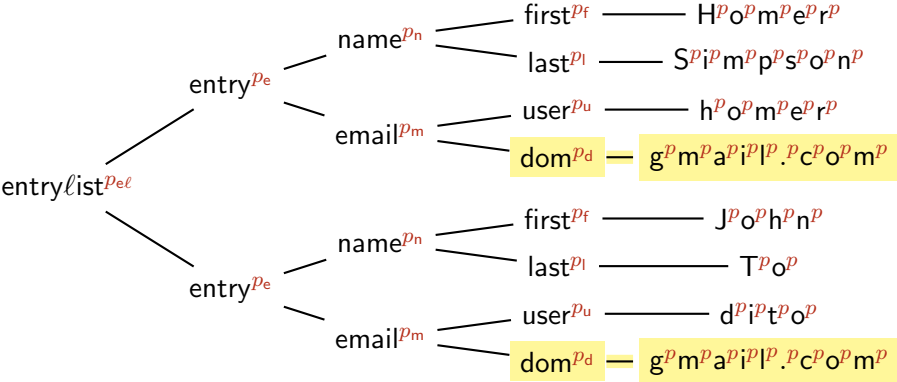
Key Constraint

email is a key: $\forall x, y p_m(x) \wedge p_m(y) \wedge x \neq y \Rightarrow t|_x \neq t|_y$



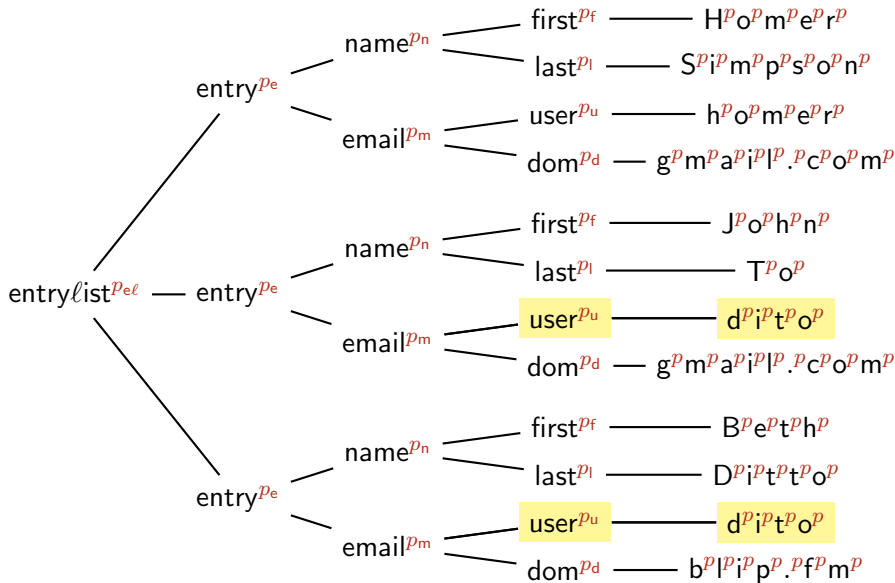
Global Equality Constraint

all domain's coincide: $\forall x, y p_d(x) \wedge p_d(y) \Rightarrow t|x_x = t|y$



Negation

user is a not a key: $\exists x, y p_u(x) \wedge p_u(y) \wedge x \neq y \wedge t|_x = t|_y$



Tree Automata with Global Constraints

A TAGC \mathcal{A} is given by

- ▶ a tree automaton $ta(\mathcal{A}) = \langle \Sigma, Q, F, \Delta \rangle$
- ▶ a constraint ϕ : FO formula interpreted on a term $t \in \mathcal{T}(\Sigma)$ and a run r of $ta(\mathcal{A})$ on t , build with predicates
 - ▶ $x = y$ (node equality)
 - ▶ $p(x)$ (the label of x in r is p)
 - ▶ $t|_x = t|_y$ (subterm equality)

A relabeling $r : dom(t) \rightarrow Q$ is a **successful run** of \mathcal{A} on t iff

- ▶ r run of $ta(\mathcal{A})$ on t
- ▶ $r(\text{root}) \in F$
- ▶ $t, r \models \phi$.

Language $\mathcal{L}(\mathcal{A}) = \{t \in \mathcal{T}(\Sigma) \mid \exists r \text{ successful run of } \mathcal{A} \text{ on } t\}$.

Constraints: Notations

type	formula
$p_1 \approx p_2$	$\forall x, y p_1(x) \wedge p_2(y) \Rightarrow t _x = t _y$
$p \approx_{\text{ref}} p$	subcase with same states
$p_1 \approx_{\text{irr}} p_2$	subcase with distinct states
$p_1 \not\approx p_2$	$\forall x, y p_1(x) \wedge p_2(y) \wedge x \neq y \Rightarrow t _x \neq t _y$
$p \not\approx_{\text{ref}} p$	subcase with same states (key)
$p_1 \not\approx_{\text{irr}} p_2$	subcase with distinct states

note that $\neg p_1 \approx p_2$ and $p_1 \not\approx p_2$ have different semantics.

notation $\text{TAGC}[\tau_1, \dots, \tau_n]$: the constraint is a Boolean combination of atomic constraints of types τ_1, \dots, τ_n .

$\text{TAGC}^\wedge[\tau_1, \dots, \tau_n]$: when constraint is positive conjunctive.

Rest of the talk

some subclasses of $\text{TAGC}[\approx, \neq]$ studied

TAGED = $\text{TAGC}^\wedge[\approx, \neq_{\text{irr}}]$ [Filiot, Talbot, Tison 2007, 08]

RTA $\equiv \text{TAGC}^\wedge[\approx_{\text{ref}}]$ [J Klay Vacher 2009, 11]

Dag Automata $\equiv \text{TAGC}^\wedge[\neq_{\text{irr}}]$ [Charatonik 1999]

$\text{TAGC}[\approx, \neq]$ [Godoy et al 2010]

decidable extensions

$\text{TAGC}[\approx, \neq, \mathbb{N}]$ arithmetic [Godoy et al 2010,12]

$\text{TACB}[\approx, \neq, \mathbb{N}]$ with local equality tests
+ modulo

open classes

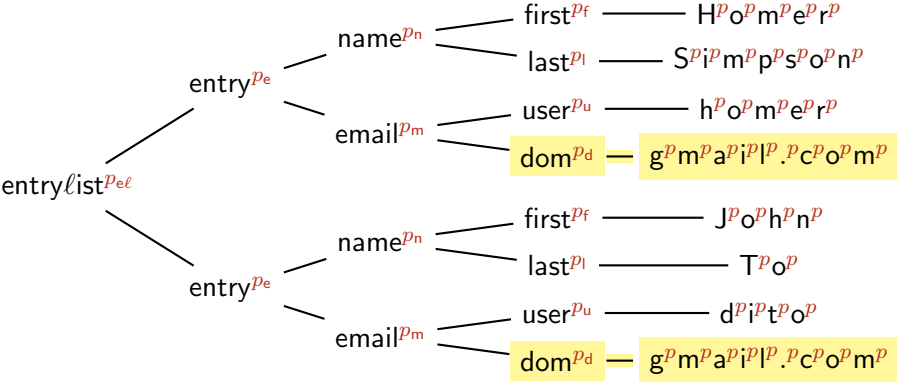
inclusion constraints

TAGED [Filiot, Talbot, Tison 2007, 08]

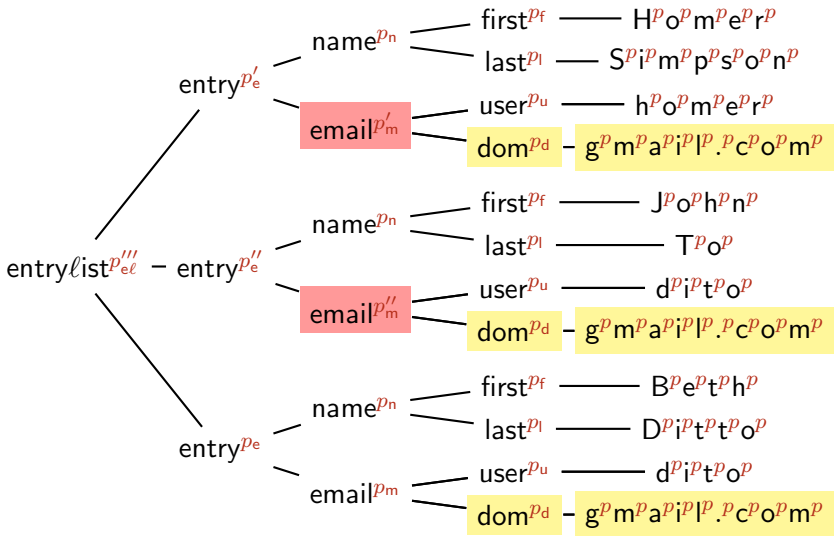
TAGED := TAGC[^][\approx , \neq_{irr}] (positive conjunctive)

decision tool for TQL spatial logic [Cardelli, Ghelli 2002].

Example: all domain's coincide: with $p_d \approx p_d$



all domain's coincide, there are at least two different email's:
 with $p_d \approx p_d \wedge p'_m \not\approx p''_m$ and $\text{email}(p_u p_d) \rightarrow p_m \mid p'_m \mid p''_m$, and
 transitions such that p'_m, p''_m occur exactly once in a successful run.



TAGED: Closure Properties

The class of TAGED- ($\text{TAGC}^{\wedge}[\approx, \not\approx_{\text{irr}}]$) languages is effectively closed under

- ▶ \cup : linear construction
disjoint union of automata and global constraints
- ▶ \cap : quadratic construction (Cartesian product)

$\mathcal{A}_1 \cap \mathcal{A}_2$	\mathcal{A}_1	\mathcal{A}_2
$Q_1 \times Q_2$	Q_1	Q_2
$\langle p_1, p_2 \rangle \approx \langle p'_1, p'_2 \rangle$	iff	$p_1 \approx p'_1$ or $p_2 \approx p'_2$
$\langle p_1, p_2 \rangle \not\approx \langle p'_1, p'_2 \rangle$	iff	$p_1 \not\approx p'_1$ or $p_2 \not\approx p'_2$

- ▶ not \neg
 $\{\text{trees with subtree } g(x, y) \mid x \neq y\}$ is in TAGED,
not its complement.
- ▶ not determinizable

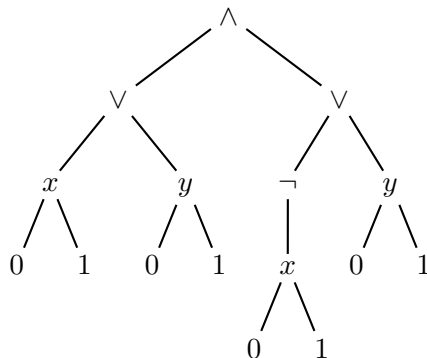
TAGED: Decision I

Membership is NP-complete

[Filiot et al 2007 CSL]

hardness: reduction of satisfiability of Boolean expressions.

Ex. $(x \vee y) \wedge (\neg x \vee y)$



$$\begin{aligned} 0 &\rightarrow q_x \mid q \\ 1 &\rightarrow q_x \mid q \\ x(q, q_x) &\rightarrow q_1 \\ x(q_x, q) &\rightarrow q_0 \\ 0 &\rightarrow q_y \mid q \\ 1 &\rightarrow q_y \mid q \\ \dots & \\ q_x \approx q_x \wedge q_y \approx q_y \end{aligned}$$

This TAGED is in $\text{TAGC}^{\wedge}[\approx_{\text{ref}}]$

TAGED: Decision II

Universality and inclusion are undecidable

Reduction of PCP into $\text{TAGC}^{\wedge}[\approx_{\text{ref}}]$ universality.

Emptiness

- ▶ EXPTIME-complete for $\text{TAGC}^{\wedge}[\approx]$
- ▶ NEXPTIME for $\text{TAGC}^{\wedge}[\not\approx_{\text{irr}}]$
- ▶ decidable for subclass of TAGED testing bounded number of $\approx, \not\approx_{\text{irr}}$ [Filiot et al 2007 CSL]
- ▶ decidable for subclass of TAGED testing bounded number of $\not\approx_{\text{irr}}$ [Filiot et al 2008 DLT]

RTA = subclass $\text{TAGC}^{\wedge}[\approx_{\text{ref}}]$ of TAGED

for data flow analysis of security protocols

\approx -constraints are sufficient for non-linear pattern matching
(bounded number of local equality tests)

Ex. for the pattern $f(x, x)$,

$$\begin{aligned} a &\rightarrow q|q_x, \\ b &\rightarrow q|q_x, \\ f(q, q) &\rightarrow q|q_x, \\ f(q_x, q_x) &\rightarrow q_f \\ q_x &\approx q_x \end{aligned}$$

successful run of \mathcal{A} on $f(f(a, b), f(a, b))$: $q_f(q_x(q, q), q_x(q, q))$.

RTA: Expressiveness

$RTA \subsetneq TAGED$

$TAGC^{\wedge}[\approx] \equiv TAGC^{\wedge}[\approx_{ref}]$

[Filiot et al 2008 DLT]

construction \rightarrow with exponential blowup.

Determinism

$TA \subsetneq DRTA \subsetneq RTA$

$\{f(x, x) \mid x \in \mathcal{T}(\Sigma)\}$ not recognizable by DRTA.

Subclass of **visibly RTA** determinizable.

Rewrite closure

(under equational specification of cryptographic operators)

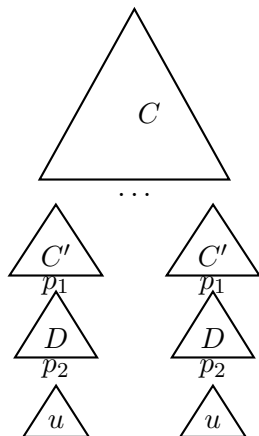
RTA: Expressiveness II

Pumping Lemma

If $t \in L(\mathcal{A})$ and $\text{depth}(t) > (|Q| + 1)|R|$,
then there exist

- ▶ a context C ,
- ▶ two contexts C' and D ,
with D non-trivial,
- ▶ a term u

such that $t = C[C'[D[u]], \dots, C'[D[u]]]$
and for all $n \geq 0$,
 $C[C'[D^n[u]], \dots, C'[D^n[u]]] \in L(\mathcal{A})$.



The set of balanced binary trees is not a RTA language.

RTA: Closure

like TAGED

▶ \cup (polynomial)

▶ \cap (exponential):

2 RTA \rightarrow 1 TAGC $^{\wedge}[\approx]$ for $\cap \rightarrow$ 1 RTA for \cap

It is a lower bound (reduction of the emptiness of intersections for tree automata).

▶ not \neg

The set of balanced binary trees is not RTA.

Its complement is recognized by

$$\begin{array}{llll} a & \rightarrow & q|q_r & f(q, q) \rightarrow q|q_r \\ f(q_r, q) & \rightarrow & q'_r & f(q'_r, q) \rightarrow q'_r \\ f(q, q'_r) & \rightarrow & q'_r & f(q, q_r) \rightarrow q' \\ f(q_r, q'_r) & \rightarrow & q_f & f(q'_r, q_r) \rightarrow q_f \\ f(q_f, q) & \rightarrow & q_f & f(q, q_f) \rightarrow q_f \end{array} \quad q_r \approx q_r$$

RTA: Decision

Decision

- ▶ membership is NP-complete
- ▶ universality, inclusion undecidable
- ▶ finiteness:
 - ▶ PTIME for RTA ($\text{TAGC}^{\wedge}[\approx_{\text{ref}}]$),
 - ▶ EXPTIME for $\text{TAGC}^{\wedge}[\approx]$.
- ▶ emptiness decidable in linear time

For emptiness: same state marking algorithm as for tree automata.

- ▶ regularity is undecidable for RTA, TAGED, $\text{TAGC}^{\wedge}[\approx]$

Global vs Local Constraints

TAGED (RTA) and automata with local constraints are orthogonal.

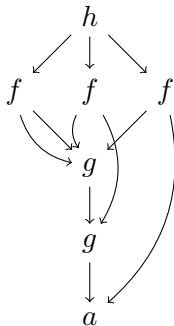
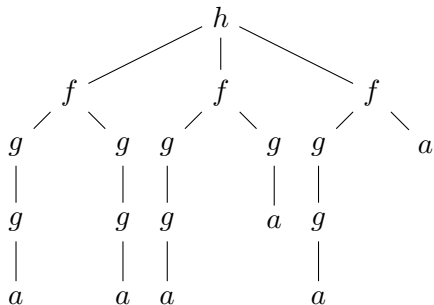
- ▶ balanced binary trees \notin TAGED
- ▶ "all domain's coincide" \notin local constraints automata.

Lets call two nodes equivalent in a successful run if they have been "successfully tested" to be equal in the run.

	# of eq. classes	cardinality of eq. classes
local constraints	unbounded	bounded
TAGEDs	bounded	unbounded

DAG Automata

DA: tree automata computing (relabeling of nodes by states) on DAGs representations of ranked trees (maximal sharing).



DAG Automata

DA: tree automata computing (relabeling of nodes by states) on DAGs representations of ranked trees (maximal sharing).

Membership is NP-complete for DA

[Charatonik 1999], [Anantharaman et al 2005]

Membership is PTIME for deterministic DA

Tree membership is PTIME complete

[Lohrey Maneth 2006]

DAG Automata and Global \approx Constraints

- ▶ **DA**: a unique state is associated to equal subtrees (same node in the DAG representation)
- ▶ **RTA**: a unique subtree is associated to every occurrence of the same state q if $q \approx q$

DA and RTA are orthogonal.

$\{f(x, x)\}$ is not recognizable by DA.

DAG Automata and Global \neq Constraints

Emptiness is NP-complete for DA

[Charatonik 1999]

[Vacher 2010 PhD]

- ▶ $DA \equiv TAGC^{\wedge}[\neq_{\text{irr}}]$
 - $\Rightarrow q_1 \neq q_2$ iff $q_1 \neq q_2$.
 - \Leftarrow states $2^Q \setminus \{P \mid \exists q_1, q_2 \in P, q_1 \neq q_2\}$

DAG Automata and Global \neq Constraints

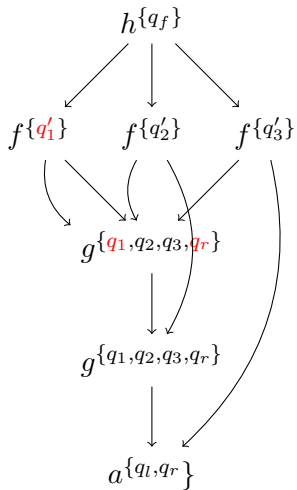
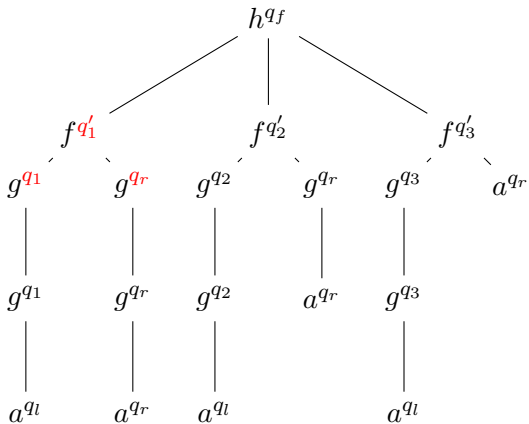
Emptiness is NP-complete for DA

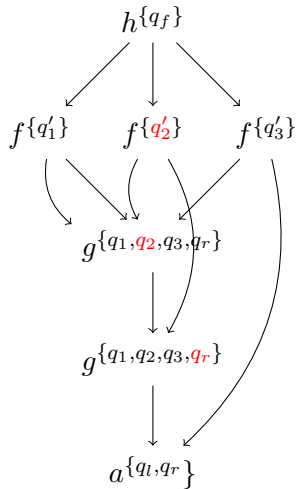
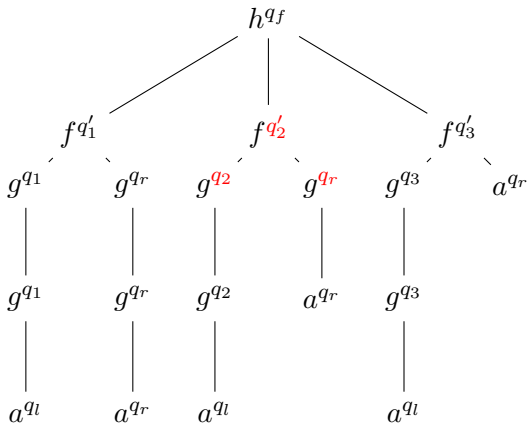
[Charatonik 1999]

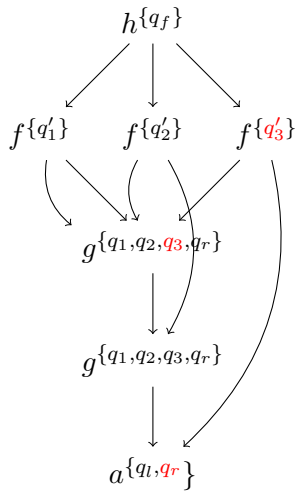
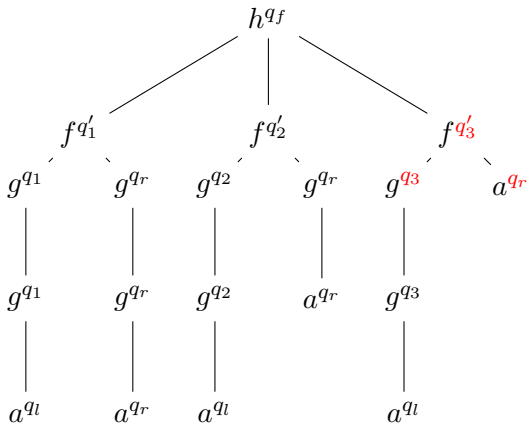
[Vacher 2010 PhD]

- ▶ $DA \equiv TAGC^{\wedge}[\neq_{\text{irr}}]$
 - $\Rightarrow q_1 \neq q_2$ iff $q_1 \neq q_2$.
 - \Leftarrow states $2^Q \setminus \{P \mid \exists q_1, q_2 \in P, q_1 \neq q_2\}$
- ▶ $TAGC^{\wedge}[\approx, \neq_{\text{irr}}] \equiv DA[\approx]$
 - $DA[\approx]: q_1 \approx q_2 := \forall x, y \in \text{dom}(d) \ q_1(x) \wedge q_2(y) \Rightarrow x = y$
- ▶ Emptiness is still NP-complete for $DA[\approx]$
- ▶ Emptiness is decidable in NEXPTIME for TAGED

Open problem: generalization to $TAGC^{\wedge}[\approx, \neq]$
(elementary upper bound for emptiness decision)







Closure and Decision Results

	TA TAGC[]	RTA TAGC [^] [\approx_{ref}]	DA TAGC [^] [$\not\approx_{\text{irr}}$]	TAGED TAGC [^] [$\approx, \not\approx_{\text{irr}}$]
\cup	PTIME	PTIME	PTIME	PTIME
\cap	PTIME	EXPTIME	not	EXPTIME
\neg	EXPTIME	not	not	not
emptiness	linear time	linear time	NP-c.	NEXPTIME-c.
membership	PTIME	NP-c.	NP-c.	NP-c.
\cap -emptiness	EXPTIME-c.	EXPTIME-c.	EXPTIME-c.	
universality	EXPTIME-c.	undec.	undec.	undec.
inclusion	EXPTIME-c.	undec.	undec.	undec.
finiteness	PTIME	EXPTIME	PTIME	

Emptiness is decidable for TAGC[\approx , $\not\approx$]

we use $\text{TAGC}[\approx, \not\approx] \equiv \text{TAGC}^\wedge[\approx, \not\approx]$

- ▶ One tree is accepted iff a tree of "small" height is accepted
- ▶ **global pumping**: replace all subtrees of height h by selected subtrees of height $< h$ while preserving all the relative \approx , $\not\approx$
- ▶ accepted tree $t \mapsto$ sequence of measures $e_0, e_1, \dots, e_{h(t)}$ st if $e_i \preceq e_j$ for $i < j$ then there exists a global pumping
- ▶ **Higman's Lemma, König's Lemma**: exists a bound B on the maximal length of sequences (for any t) without $e_i \preceq e_j$, $i < j$
- ▶ every t of height $> B$ can be reduced by a global pumping.

Decidable Extensions of TAGC[\approx , \neq]

extension to **unranked trees** immediate, using encoding into binary trees [Godoy et al 2012]

on **ranked trees**, **emptiness** is still decidable for

- ▶ TAGC[\approx , \neq] extended with local $=$ and \neq constraints between siblings, à la [Bogaert Tison 1992]
- ▶ TAGC[\approx , \neq] where \approx and \neq are interpreted modulo flat equational theories

Arithmetic Constraints

linear inequality $\sum_{q \in Q} a_q \cdot |q| \geq a$ or $\sum_{q \in Q} a_q \cdot \|q\| \geq a$, $a_q, a \in \mathbb{Z}$

for a run r on a tree t , $|q| = |r^{-1}(q)|$
 $\|q\| = |\{t|_x \mid x \in \text{dom}(t), r(x) = q\}|$

natural inequality (type ' \mathbb{N} ') when all a_q, a have the same sign

Presburger automata [Seidl et al 2003, 2008], [Dal Zilio Lugiez 2006]: count the siblings of unranked trees (*local cstr*).

- ▶ emptiness decidable in NPTIME for TAGC[$|\cdot|, \mathbb{Z}$]
- ▶ emptiness undecidable for TAGC[$\approx, |\cdot|, \mathbb{Z}$] [Godoy et al 2010]
- ▶ TAGC[$\approx, \not\approx, |\cdot|, \mathbb{N}, \|\cdot\|, \mathbb{N}$] \equiv TAGC $^\wedge$ [$\approx, \not\approx$] [id]

Monadic Second Order Logic

$\text{MSO}[+1, \approx, \not\approx, |\cdot|_{\mathbb{Z}}, \|\cdot\|_{\mathbb{Z}}]$ monadic second-order logic

- ▶ first order variables x : position in a tree
- ▶ second order variables X : finite set of positions

with predicates

$a(x)$ (x labeled by $a \in \Sigma$ in t)

$+1$ $S_{\downarrow}(x, y)$ (y child of x) and $S_{\rightarrow}(x, y)$ (y next sibling of x)

\approx $X \approx Y$ (for all $x \in X, y \in Y, t|_x = t|_y$)

$\not\approx$ $X \not\approx Y$ (for all $x \in X, y \in Y, t|_x \neq t|_y$)

$|\cdot|_{\mathbb{Z}}$ $\sum a_i \cdot |X_i| \geq a, a_i, a \in \mathbb{Z}$ ($|X_i|$ is cardinality of X_i)

$|\cdot|_{\mathbb{N}}$ when a_i, a have same sign

$\|\cdot\|_{\mathbb{Z}}$ $\sum a_i \cdot \|X_i\| \geq a$ ($\|X_i\|$ is cardinality of $\{t|_x \mid x \in X_i\}$)

$\|\cdot\|_{\mathbb{N}}$ when a_i, a have same sign

Monadic Second Order Logic: satisfiability

- ▶ $\text{MSO}[+1] \equiv$ tree automata [Thatcher Wright 1968]
- ▶ $\text{MSO}[+1, \approx]$ undecidable
- ▶ $\text{MSO}[+1, \mathbb{Z}]$ undecidable [Klaedtke Ruess 2002]

EMSO: $\exists X_1 \dots \exists X_n \phi(X_1, \dots, X_n) \wedge \psi(X_1, \dots, X_n)$ where

- ▶ $\phi(X_1, \dots, X_n)$ in $\text{MSO}[+1]$
- ▶ $\psi(X_1, \dots, X_n)$ in $\text{MSO}[+1, \approx, \neq, |\cdot|_{\mathbb{Z}}, \|\cdot\|_{\mathbb{Z}}]$, free

- ▶ $\text{EMSO}[+1, \mathbb{Z}]$ decidable [Klaedtke Ruess 2002]
- ▶ fragment of $\text{EMSO}[+1, \approx, \neq]$ decidable [Filiot et al 2008]
- ▶ $\text{EMSO}[+1, \approx, \neq, |\cdot|_{\mathbb{N}}, \|\cdot\|_{\mathbb{N}}]$ decidable [Godoy et al 2010]

Perspectives

- ▶ Combining Local/Global Constraints for **unranked ordered trees**
 - ▶ combination of TAGC[$\approx, \not\approx$]
 - ▶ with UTASC: unranked tree automata with local sibling constraints [Löding Wong 2007, 09]

- ▶ Extension of TAGC for **inclusion constraints**

$$\forall x \exists y p(x) \Rightarrow (q(y) \wedge t|_x = t|_y) \quad (x, y \in \text{positions})$$

$$\forall u \exists v p(u) \Rightarrow (q(v) \wedge u = v) \quad (u, v \in \text{subtrees})$$

- ▶ TAGC with **constraints in monadic FO** over $q(y)$ and $x = y$ interpretation in the domain of subtrees (related to automata on DAGs)

Thank you

Thank you