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Tree Automata and Constraints: a brief survey

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Dagstuhl Seminar 13192
Tree Transducers and Formal Methods
Non-linearity phenomena are non captured by tree automata, e.g.:

- The set of ground instances of a non-linear term is not recognizable.
- The class of recognizable tree languages is not closed under non-linear homomorphisms.

How to enrich transitions by equality and disequality constraints?
Two main ideas

- **Add local** tests, e.g. to capture the set of instances of $f(g(x), x)$
  Why: pattern matching, rewriting, representing finitely image by non-linear transformations of recognizable tree languages (type checking?), ...
  When: from 80’s to now

- **Add global** tests, e.g. to capture ”all the subterms rooted by $f$ are different”.
  Why: integrity constraints (XML), non linear query languages, ...
  When: more recently

*Leitmotiv: keeping as far as possible good closure and decision properties.*
Tree Automata with Local Constraints
An equality constraint (resp. a disequality constraint) is a predicate on $T(\Sigma)$ written $\pi = \pi'$ (resp. $\pi \neq \pi'$) where $\pi, \pi' \in \{1, \ldots, k\}^*$. Such a predicate is satisfied on a tree $t$, which we write $t \models \pi = \pi'$, if $\pi, \pi' \in Post$ and $\models t|_\pi = t|_{\pi'}$ (resp. $\pi \neq \pi'$ is satisfied on $t$ if $\pi = \pi'$ is not satisfied on $t$).
 Equality and Disequality Constraints

An equality constraint (resp. a disequality constraint) is a predicate on $T(\Sigma)$ written $\pi = \pi'$ (resp. $\pi \neq \pi'$) where $\pi, \pi' \in \{1, \ldots, k\}^*$. Such a predicate is satisfied on a tree $t$, which we write $t \models \pi = \pi'$, if $\pi, \pi' \in Post$ and $t|_{\pi} = t|_{\pi'}$ (resp. $\pi \neq \pi'$ is satisfied on $t$ if $\pi = \pi'$ is not satisfied on $t$).

These constraints are indeed unlabeled path constraints. More general disequality constraints have been defined by H. Seidl and A. Reuß (LPAR 2010, FOSSACS 2012).
An **automaton with equality and disequality constraints** is a tuple $(Q, \Sigma, Q_f, \Delta)$ where

- $\Sigma$ is a finite ranked alphabet
- $Q$ is a finite set of states
- $Q_f$ is a subset of $Q$ of final states
- $\Delta$ is a set of transition rules of the form:

$$f(q_1, \ldots, q_n) \xrightarrow{c} q$$

where $f \in \Sigma$, $q_1, \ldots, q_n, q \in Q$, and $c$ is a Boolean combination of equality and disequality constraints.
Automata with Equality and Disequality Constraints

The general class

- The move relation $\rightarrow_{\mathcal{A}}$ is defined by:
  \[ t \rightarrow_{\mathcal{A}} t' \text{ if and only if } \]
  \[ t = C[f(q_1(u_1), \ldots, q_n(u_n))], \]
  \[ t' = C[q(f(u_1, \ldots, u_n))] \]
  \[ f(q_1, \ldots, q_n) \xrightarrow{c} q \in \Delta, \]
  \[ f(u_1, \ldots, u_n) \models c. \]

- $\rightarrow_{\mathcal{A}}^*$ is the reflexive and transitive closure of $\rightarrow_{\mathcal{A}}$.

- $\mathcal{A}$ accepts a tree $t \in T(\Sigma)$ if $t \rightarrow_{\mathcal{A}}^* q$ for some final state $q$.

- The language accepted, or recognized, is the set $L(\mathcal{A})$ of trees $t \in T(\Sigma)$ accepted by $\mathcal{A}$. 
Automata with Equality and Disequality Constraints

The general class

- The move relation $\rightarrow_{\mathcal{A}}$ is defined by:

  $t \rightarrow_{\mathcal{A}} t'$ if and only if

  $t = C[f(q_1(u_1), \ldots, q_n(u_n))]$,  
  $t' = C[q(f(u_1, \ldots, u_n))]$  
  $f(q_1, \ldots, q_n) \rightarrow c \in \Delta$,  
  $f(u_1, \ldots, u_n) \models c$.

- $\rightarrow^*_{\mathcal{A}}$ is the reflexive and transitive closure of $\rightarrow_{\mathcal{A}}$.

- $\mathcal{A}$ accepts a tree $t \in T(\Sigma)$ if $t \rightarrow^*_{\mathcal{A}} q$ for some final state $q$.

- The language accepted, or recognized, is the set $L(\mathcal{A})$ of trees $t \in T(\Sigma)$ accepted by $\mathcal{A}$.

- Class with only equalities defined in 1981 (Dauchet & Mongy)
Automata with Equality and Disequality Constraints

Example 1

- $Q = \{q\}$,
- $\Sigma = \{f, a\}$
- final states: $\{q\}$
- $\Delta$ consists of the following rules:

\[
\begin{align*}
    r_1 : & \quad a \rightarrow q \\
    r_2 : & \quad f(q, q) \xrightarrow{1=2} q
\end{align*}
\]
Automata with Equality and Disequality Constraints

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The automaton recognizes the set of balanced trees.
Automata with Equality and Disequality Constraints

Example 2

- $Q = \{q, q_g, q_{fin}\}$,
- $\Sigma = \{f, g, a\}$
- final states: $\{q_{fin}\}$
- $\Delta$ consists of the following rules:

$$
\begin{align*}
    r_1 : & \quad a \rightarrow q \\
    r_2 : & \quad f(q, q) \rightarrow q \\
    r_3 : & \quad f(q_g, q) \xrightarrow{11=2} q_{fin} \\
    r_4 : & \quad g(q) \rightarrow q_g \\
    r_5 : & \quad g(q) \rightarrow q
\end{align*}
$$
Automata with Equality and Disequality Constraints

Example 2

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  r_3 : \quad f(q_g, q) \xrightarrow{11=2} q_{fin} \\
  r_4 : \quad g(q) \rightarrow q_g \\
  r_5 : \quad g(q) \rightarrow q 
  \]

The automaton recognizes the ground instances of $f(g(x), x)$. 
Example 3

- $Q = \{q\}$,
- $\Sigma = \{f, a\}$
- final states: $\{q\}$
- $\Delta$ consists of the following rules:

\[
\begin{align*}
\triangleleft r_1 : & \quad a \rightarrow qa \\
\triangleleft r_2 : & \quad f(qa, qa) \rightarrow q \\
\triangleleft r_3 : & \quad f(q, q) \xrightarrow{12=21} q
\end{align*}
\]
Automata with Equality and Disequality Constraints

Example 3

- $Q = \{q\}$
- $\Sigma = \{f, a\}$
- final states: $\{q\}$
- $\Delta$ consists of the following rules:

  $r_1 : \ a \rightarrow qa$
  $r_2 : \ f(qa, qa) \rightarrow q$
  $r_3 : \ f(q, q) \ \overset{12=21}{\rightarrow} \ q$

The automaton recognizes "grids".
Automata with Equality and Disequality Constraints

Properties

- Can be determinized (and completed) (exponential blow up)
Automata with Equality and Disequality Constraints

Properties

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- Closure under boolean operations
Automata with Equality and Disequality Constraints

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- Emptiness is undecidable for this class - e.g. use Post problem or previous encoding of grids -: undecidable even with only equalities between cousins.
Automata with Equality and Disequality Constraints

Properties

- Can be determinized (and completed) (exponential blow up)
- Closure under boolean operations
- Emptiness is undecidable for this class - e.g. use Post problem or previous encoding of grids -: undecidable even with only equalities between cousins.
- Capture homomorphic images of recognizable languages (only equality constraints are needed) and images by bottom-up transducers.

Undecidability comes in some sense from superposition of (equality) tests. To get a decidable class, we can try to limit this superposition...
Automata with Constraints

Restrictions

- restrict the form of the tests
- limit the number of equality tests
- limit the superposition of equality tests
Automata with Constraints between brothers

Constraints are reduced to constraints between brothers (siblings)

Allowed constraints are $i = j$ or $i \neq j$.

$f(q_1, q_2, q_3) \rightarrow_{1=2}, 1\neq 3$ allowed, $f(q_1, q_2) \rightarrow_{1=21}$ forbidden.
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- can be determinized (exponential blow up)
- stable under boolean operations.
- Emptiness $\textit{EXPTIME}$–complete but polynomial in the deterministic case.

Proof for ”normal” TA: compute by fix-point the set of reachable states and check it contains a final state. Does it work here?

- $f(q_1, q_2) \rightarrow q, 1 = 2$:
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- $f(q_1, q_2) \rightarrow q, 1 = 2$: if the automaton is not deterministic, we have to check whether there is some tree leading both to $q_1$ and $q_2$: we can determinize
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- $f(q, q) \rightarrow q, 1 \neq 2$: We need to count, but we just have to count up to maximal arity.
Automata with Constraints between brothers
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Allowed constraints are \( i = j \) or \( i \neq j \).

\[ f(q_1, q_2, q_3) \rightarrow_{1=2, \ 1\neq 3} \text{allowed, } f(q_1, q_2) \rightarrow_{1=21} \text{forbidden.} \]

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Automata with Constraints between brothers

Constraints are reduced to constraints between brothers (siblings)

- Allowed constraints are $i = j$ or $i \neq j$.
- $f(q_1, q_2, q_3) \rightarrow_{1=2}, 1 \neq 3$ allowed, $f(q_1, q_2) \rightarrow_{1=21}$ forbidden.

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- $f(q_1, q_2) \rightarrow q, 1 = 2$: if the automaton is not deterministic, we have to check whether there is some tree leading both to $q_1$ and $q_2$: we can determinize
- $f(q, q) \rightarrow q, 1 \neq 2$: We need to count, but we just have to count up to maximal arity.


- Recognizability is decidable.
A reduction automaton $\mathcal{A}$ is an automaton with equality and disequality constraints with an ordering on the states such that for each rule $f(q_1, \ldots, q_n) \xrightarrow{c} q$,

- $q$ is maximal in $q, q_1, \ldots, q_n$
- $q$ is a strict upper bound of $q_1, \ldots, q_n$ if $c$ contains an equality constraint.

Idea: Bound the number of equalities tested along a path. Defined/studied by Dauchet, Caron, Coquidé, Comon and Jacquemard (94 and following)
Reduction automata

An example

How to define the set of ground instances of $f(f(x, x), f(x, y))$?
Reduction automata
An example

How to define the set of ground instances of $f(f(x, x), f(x, y))$?

$\Delta =$

\[
\begin{align*}
a & \to q \\
f(q, q) & \to q_f \\
f(q_f, q) & \to q_f \\
f(q_f, q_f) & \overset{11=12 \land 11=21}{\longrightarrow} q_{fin}
\end{align*}
\]
Reduction automata

An example

How to define the set of ground instances of \( f(f(x, x), f(x, y)) \)?

\[
\Delta = 
\begin{align*}
  a & \rightarrow q \\
  f(q, q) & \rightarrow q_f \\
  f(q_f, q) & \rightarrow q_f \\
  f(q_f, q_f) & \xrightarrow{11=12 \land 11=21} q_{fin} \\
  f(q_f, q_f) & \rightarrow q_f
\end{align*}
\]

\[ q < q_f < q_{fin} \]
Reduction automata

An example

How to define the set of ground instances of $f(f(x, x), f(x, y))$?

$\Delta =$

\[
\begin{align*}
  a &\rightarrow q \\
  f(q, q) &\rightarrow qf \\
  f(qf, q) &\rightarrow qf \\
  f(qf, qf) &\xrightarrow{11=12 \land 11=21} qfin
\end{align*}
\]

$q < qf < qfin$

$qfin$ final state
How to define the set of ground instances of $f(f(x, x), f(x, y))$ with $\Sigma = \{a, f\}$.

Deterministic version:

$$
\Delta = 
\begin{align*}
  a & \rightarrow q \\
  f(q, q_f) & \rightarrow q_f \\
  f(q_f, q_f) & \xrightarrow{11=12 \land 11=21} q_{fin} \\
  f(q_f, q_f) & \xrightarrow{11\neq 12 \lor 11\neq 21} q_f
\end{align*}
\begin{align*}
  f(q, q) & \rightarrow q_f \\
  f(q_f, q) & \rightarrow q_f \\
  f(q_f, q_f) & \xrightarrow{11\neq 12 \lor 11\neq 21} q_f
\end{align*}
$$
Reduction automata

Properties

- closed under union and intersection
Reduction automata

Properties

- closed under union and intersection
- closed under complement for deterministic automata
Reduction automata

Properties

- closed under union and intersection
- closed under complement for deterministic automata
- emptiness decidable for deterministic automata (M. Dauchet and alt. 1993)
Reduction automata

Properties

- closed under union and intersection
- closed under complement for deterministic automata
- emptiness decidable for deterministic automata (M. Dauchet and alt. 1993)
- emptiness undecidable for non deterministic automata (F. Jacquemard and alt. 2008)
Deciding emptiness ”needs” determinism
Automata with only one kind of tests

- $TA_\neq$: automata with only disequality tests
  - closed under union and intersection
  - emptiness
Automata with only one kind of tests

- $TA\neq$: automata with only disequality tests
  - closed under union and intersection
  - emptiness decidable
Automata with only one kind of tests

- $TA_{\neq}$: automata with only disequality tests
  - closed under union and intersection
  - emptiness decidable in EXPTIME (Comon & Jacquemard, 97)

- $TA_{=}$: automata with only equality tests
  - closed under union and intersection
  - emptiness
Automata with only one kind of tests

- $TA_{\neq}$: automata with only disequality tests
  - closed under union and intersection
  - emptiness decidable in EXPTIME (Comon & Jacquemard, 97)

- $TA_{=}$: automata with only equality tests
  - closed under union and intersection
  - emptiness undecidable
Automata with only one kind of tests

- The complement of a $TA_=$ is a $TA_\neq$. 
Automata with only one kind of tests

- The complement of a $TA_\subset$ is a $TA_\not\subset$.
- The complement of a $TA_\not\subset$ is a $TA_\subset$.

Sketch of the (effective) proof: a state reached by $t$ in the automaton for the complement of $L$ corresponds to the set of all states that $t$ can’t reach in the automaton for $L$. (folklore construction, formalized by Creus & alt. STOC 2010).
Automata with only one kind of tests

- The complement of a $TA_=\neg$ is a $TA_\neq$.
- The complement of a $TA_\neq$ is a $TA_=\neg$.

Sketch of the (effective) proof: a state reached by $t$ in the automaton for the complement of $L$ corresponds to the set of all states that $t$ can’t reach in the automaton for $L$. (folklore construction, formalized by Creus & alt. STOC 2010).

Universality is decidable for $TA_=\neg$, undecidable for $TA_\neq$. 
The homomorphism Problem

*Hom*—problem

**Instance**  A recognizable language $R$, an homomorphism $H$.

**Answer**  “yes” if and only $H(R)$ is recognizable.
The homomorphism Problem

*Hom*-problem

**Instance**  A recognizable language $R$, an homomorphism $H$.

**Answer**  “yes” if and only $H(R)$ is recognizable

Is it decidable?
The homomorphism Problem
$Hom$–problem

**Instance** A recognizable language $R$, an homomorphism $H$.

**Answer** “yes” if and only $H(R)$ is recognizable

Is it decidable?

Remark: If $H$ is shallow -i.e the images of symbols are shallow-,
The homomorphism Problem

*$Hom*$-problem

**Instance** A recognizable language $R$, an homomorphism $H$.

**Answer** “yes” if and only if $H(R)$ is recognizable

Is it decidable?

**Remark:** If $H$ is shallow -i.e. the images of symbols are shallow-, $H(R)$ is recognizable by an automaton with tests between brothers. As recognizability is decidable for language recognizable by an automaton with tests between brothers: the homomorphism problem is decidable for shallow homomorphisms.
The homomorphism Problem

Proposition

The $HOM-$problem is decidable (Godoy & alt, STOC 2010) and EXPTIME-Complete (Creus & alt., LICS 2012)

$TA_{Hom}$: equality test without superposition, no disequality tests: $t \rightarrow^c q$ where $t \in T(\Sigma, Q)$, and $c$ is a Boolean combination of equality constraints for positions of $t$ labeled by states.

- capture homomorphic images of recognizable tree languages
- emptiness, finiteness, recognizability are decidable for this class.

$TA_{Hom, \neq}$: equality and disequality tests, no superposition of equality tests: emptiness is decidable
Application to transducers?

- The image of a recognizable tree language by a bottom-up tree transducer can be represented finitely by a $TA_{Hom}$.
- E.g., we can decide whether $T_1(L_1) \subset T_2(L_2)$, $L_1, L_2$ recognizable tree languages, $T_1, T_2$ bottom-up tree transducers.
What about global constraints?
Tree Automata with Global Constraints
Local Constraints

- non linear pattern matching
- non-linear transformation
- term rewriting, reachability analysis

Global Constraints

- tree structured data processing
- schema = type + integrity constraints
Type Definition for XML Data

**DTD**

- `entrylist`  
  - `entry*`  
    - `name`  
    - `email`  
      - `first`  
      - `last`  
      - `user`  
      - `dom`  
        - `char*`  
        - `char*`  
        - `char*`  
        - `char*`

**Tree Automaton**

- `entry(p_n, p_m) → p_e`
- `empty → p_el`
- `entrylist(p_e, p_el) → p_el`
- `name(p_f, p_l) → p_n`
- `first(p) → p_f`
- `last(p) → p_l`
- `email(p_u, p_d) → p_m`
- `user(p) → p_u`
- `dom(p) → p_d`
- `a(p) → p`
- `ε → p`
Integrity Constraint

- entry
- list
  - entry
  - entry
    - name
    - email
      - first
      - last
    - user
      - dom
    - char
    - char
    - char
    - char

- email is a key (ID)
- subtrees below email are pairwise distinct

Cannot be expressed with standard tree automata
Tree Automata with Local Constraints
- equality and disequality test in transitions
- checked during computation

Tree Automata with Global Constraints
- one standard tree automaton + one separate constraint
- checked on run, after computation
Tree Automaton Run

```
entry \ell \rightarrow entry \psi e

entry list \psi e \rightarrow entry \psi e

name \psi n \rightarrow first \psi f \rightarrow H p \circ m \circ e \circ r p
last \psi l \rightarrow S p i \circ m \circ p \circ s \circ o \circ n p

user \psi u \rightarrow h p \circ m \circ e \circ r p

dom \psi d \rightarrow g m a i l \cdot c o m p

entry \psi e \rightarrow entry \psi e

name \psi n \rightarrow first \psi f \rightarrow J p o h n p
last \psi l \rightarrow T p o p

user \psi u \rightarrow d p t o p

dom \psi d \rightarrow g m a i l \cdot c o m p

entry \psi e \rightarrow entry \psi e

name \psi n \rightarrow first \psi f \rightarrow B e t h p
last \psi l \rightarrow D p t t o p

user \psi u \rightarrow d p t o p

dom \psi d \rightarrow b l i p . f m p
```
Key Constraint

email is a key: $\forall x, y \ p_m(x) \land p_m(y) \land x \neq y \Rightarrow t_x \neq t_y$
Global Equality Constraint

all domain’s coincide: \( \forall x, y \ p_d(x) \land p_d(y) \Rightarrow t|_x = t|_y \)
Negation

user is a not a key: $\exists x, y \ p_u(x) \land p_u(y) \land x \neq y \land t|_x = t|_y$
A TAGC $\mathcal{A}$ is given by

- a tree automaton $ta(\mathcal{A}) = \langle \Sigma, Q, F, \Delta \rangle$
- a constraint $\phi$: FO formula interpreted on a term $t \in T(\Sigma)$ and a run $r$ of $ta(\mathcal{A})$ on $t$, build with predicates
  - $x = y$ (node equality)
  - $p(x)$ (the label of $x$ in $r$ is $p$)
  - $t|_x = t|_y$ (subterm equality)

A relabeling $r : dom(t) \rightarrow Q$ is a successful run of $\mathcal{A}$ on $t$ iff

- $r$ run of $ta(\mathcal{A})$ on $t$
- $r(root) \in F$
- $t, r \models \phi$.

Language $\mathcal{L}(\mathcal{A}) = \{t \in T(\Sigma) \mid \exists r \text{ successful run of } \mathcal{A} \text{ on } t\}$. 
## Constraints: Notations

<table>
<thead>
<tr>
<th>type</th>
<th>formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 \approx p_2 )</td>
<td>( \forall x, y ; p_1(x) \land p_2(y) \Rightarrow t</td>
</tr>
<tr>
<td>( p \approx_{\text{ref}} p )</td>
<td>subcase with same states</td>
</tr>
<tr>
<td>( p_1 \approx_{\text{irr}} p_2 )</td>
<td>subcase with distinct states</td>
</tr>
<tr>
<td>( p_1 \not\approx p_2 )</td>
<td>( \forall x, y ; p_1(x) \land p_2(y) \land x \neq y \Rightarrow t</td>
</tr>
<tr>
<td>( p \not\approx_{\text{ref}} p )</td>
<td>subcase with same states (key)</td>
</tr>
<tr>
<td>( p_1 \not\approx_{\text{irr}} p_2 )</td>
<td>subcase with distinct states</td>
</tr>
</tbody>
</table>

Note that \( \neg p_1 \approx p_2 \) and \( p_1 \not\approx p_2 \) have different semantics.

Notation \( \text{TAGC}[\tau_1, \ldots, \tau_n] \): the constraint is a Boolean combination of atomic constraints of types \( \tau_1, \ldots, \tau_n \).

\( \text{TAGC}^[\tau_1, \ldots, \tau_n] \): when constraint is positive conjunctive.
Rest of the talk

some subclasses of TAGC[≃, ⊄] studied

<table>
<thead>
<tr>
<th>Class</th>
<th>Definition</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAGED</td>
<td>TAGC^[≃, ⊄_irr]</td>
<td>[Filiot, Talbot, Tison 2007, 08]</td>
</tr>
<tr>
<td>RTA</td>
<td>TAGC^[≃_ref]</td>
<td>[J Klay Vacher 2009, 11]</td>
</tr>
<tr>
<td>Dag Automata</td>
<td>TAGC^[⊄_irr]</td>
<td>[Charatonik 1999]</td>
</tr>
<tr>
<td>TAGC[≃, ⊄]</td>
<td></td>
<td>[Godoy et al 2010]</td>
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</tbody>
</table>

decidable extensions

<table>
<thead>
<tr>
<th>Class</th>
<th>Definition</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAGC[≃, ⊄, N]</td>
<td>arithmetic</td>
<td>[Godoy et al 2010,12]</td>
</tr>
<tr>
<td>TACB[≃, ⊄, N]</td>
<td>with local equality tests + modulo</td>
<td></td>
</tr>
</tbody>
</table>

open classes

<table>
<thead>
<tr>
<th>Class</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>inclusion constraints</td>
<td></td>
</tr>
</tbody>
</table>
TAGED [Filiot, Talbot, Tison 2007, 08]

TAGED := TAGC^[[\approx, \not\approx_{irr}]] (positive conjunctive)

decision tool for TQL spatial logic [Cardelli, Ghelli 2002].

Example: all domain's coincide: with \( p_d \approx p_d \)
all domain’s coincide, there are at least two different email’s: with $p_d \approx p_d \land p'_m \not\approx p''_m$ and email$(p_u p_d) \rightarrow p_m \mid p'_m \mid p''_m$, and transitions such that $p'_m, p''_m$ occur exactly once in a successful run.
TAGED: Closure Properties

The class of TAGED- \((\text{TAGC}^\sim, \not\approx_\text{irr})\) languages is effectively closed under

- \(\cup\): linear construction
disjoint union of automata and global constraints
- \(\cap\): quadratic construction (Cartesian product)

<table>
<thead>
<tr>
<th></th>
<th>(A_1 \cap A_2)</th>
<th>(A_1)</th>
<th>(A_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_1 \times Q_2)</td>
<td>(Q_1)</td>
<td>(Q_2)</td>
<td></td>
</tr>
</tbody>
</table>

\[
\langle p_1, p_2 \rangle \approx \langle p'_1, p'_2 \rangle \iff p_1 \approx p'_1 \text{ or } p_2 \approx p'_2
\]

\[
\langle p_1, p_2 \rangle \not\approx \langle p'_1, p'_2 \rangle \iff p_1 \not\approx p'_1 \text{ or } p_2 \not\approx p'_2
\]

- not \(\neg\)
{trees with subtree \(g(x, y)\mid x \neq y\)} is in TAGED, not its complement.
- not determinizable
**TAGED: Decision I**

Membership is NP-complete [Filiot et al 2007 CSL]

Hardness: reduction of satisfiability of Boolean expressions.

Ex. \((x \lor y) \land (\neg x \lor y)\)

This TAGED is in TAGC^\[\approx_{\text{ref}}\]
Universality and inclusion are undecidable

Reduction of PCP into TAGC[^ref] universality.

Emptiness

- EXPTIME-complete for TAGC[^≈]
- NEXPTIME for TAGC[^irr]
- decidable for subclass of TAGED testing bounded number of ≈, ^irr [Filiot et al 2007 CSL]
- decidable for subclass of TAGED testing bounded number of ^irr [Filiot et al 2008 DLT]
Rigid Tree Automata

RTA = subclass TAGC^[≈_{ref}] of TAGED

for data flow analysis of security protocols

≈-constraints are sufficient for non-linear pattern matching
(bounded number of local equality tests)

Ex. for the pattern f(x, x),

\[
\begin{align*}
a & \rightarrow q|x, \\
b & \rightarrow q|x, \\
f(q, q) & \rightarrow q|x, \\
f(q_x, q_x) & \rightarrow q_f \\
q_x & \approx q_x
\end{align*}
\]

successful run of \( A \) on \( f(f(a, b), f(a, b)) \) : \( q_f(q_x(q, q), q_x(q, q)) \).
RTA: Expressiveness

\[ \text{RTA} \subsetneq \text{TAGED} \]

\[ \text{TAGC}^\approx \equiv \text{TAGC}^{\approx_{\text{ref}}} \] \[\text{[Filiot et al 2008 DLT]}\]

construction → with exponential blowup.

**Determinism**

\[ \text{TA} \subsetneq \text{DRTA} \subsetneq \text{RTA} \]

\( \{ f(x, x) \mid x \in T(\Sigma) \} \) not recognizable by DRTA.

Subclass of visibly RTA determinizable.

Rewrite closure
(under equational specification of cryptographic operators)
RTA: Expressiveness II

Pumping Lemma

If \( t \in L(A) \) and \( \text{depth}(t) > (|Q| + 1)|R| \), then there exist

- a context \( C \),
- two contexts \( C' \) and \( D \), with \( D \) non-trivial,
- a term \( u \)

such that \( t = C[C'[D[u]], \ldots, C'[D[u]]] \) and for all \( n \geq 0 \),

\[
C[C'[D^n[u]], \ldots, C'[D^n[u]]] \in L(A).
\]

The set of balanced binary trees is not a RTA language.
RTA: Closure

like TAGED

- $\cup$ (polynomial)
- $\cap$ (exponential):
  $2 \text{ RTA} \rightarrow 1 \text{ TAGC}^\ast$ for $\cap \rightarrow 1 \text{ RTA}$ for $\cap$
  It is a lower bound (reduction of the emptiness of intersections for tree automata).

- not $\neg$

The set of balanced binary trees is not RTA.
Its complement is recognized by

$$
\begin{align*}
  a & \rightarrow q|q_r \\
  f(q_r, q) & \rightarrow q'_r \\
  f(q, q'_r) & \rightarrow q_r \\
  f(q_r, q'_r) & \rightarrow q_f \\
  f(q_f, q) & \rightarrow q_f,
\end{align*}
\begin{align*}
  f(q, q) & \rightarrow q|q_r \\
  f(q', q) & \rightarrow q'_r \\
  f(q_r, q) & \rightarrow q'_r \\
  f(q_r, q_r) & \rightarrow q_f \\
  f(q, q_f) & \rightarrow q_f
\end{align*}
$$

$q_r \approx q_r$
RTA: Decision

Decision

- membership is NP-complete
- universality, inclusion undecidable
- finiteness:
  - PTIME for RTA (TAGC^[≃_{ref}]),
  - EXPTIME for TAGC^[≃].
- emptiness decidable in linear time

For emptiness: same state marking algorithm as for tree automata.

- regularity is undecidable for RTA, TAGED, TAGC^[≃]
Global vs Local Constraints

TAGED (RTA) and automata with local constraints are orthogonal.

- balanced binary trees \( \not\in \) TAGED
- ”all domain’s coincide” \( \not\in \) local constraints automata.

Lets call two nodes equivalent in a successful run if they have been “successfully tested” to be equal in the run.

<table>
<thead>
<tr>
<th></th>
<th># of eq. classes</th>
<th>cardinality of eq. classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>local constraints</td>
<td>unbounded</td>
<td>bounded</td>
</tr>
<tr>
<td>TAGEDs</td>
<td>bounded</td>
<td>unbounded</td>
</tr>
</tbody>
</table>
DAG Automata

**DA:** tree automata computing (relabeling of nodes by states) on DAGs representations of ranked trees (maximal sharing).
DAG Automata

DA: tree automata computing (relabeling of nodes by states) on DAGs representations of ranked trees (maximal sharing).

Membership is NP-complete for DA  
[Charatonik 1999], [Anantharaman et al 2005]

Membership is PTIME complete for deterministic DA  
[Lohrey Maneth 2006]

Tree membership is PTIME complete  
[Lohrey Maneth 2006]
DAG Automata and Global $\approx$ Constraints

- **DA**: a unique state is associated to equal subtrees (same node in the DAG representation)
- **RTA**: a unique subtree is associated to every occurrence of the same state $q$ if $q \approx q$

DA and RTA are orthogonal.

\{f(x, x)\} is not recognizable by DA.
DAG Automata and Global $\not= \bowtie$ Constraints

Emptiness is NP-complete for DA [Charatonik 1999]

[Vacher 2010 PhD]

- $DA \equiv TAGC^\land[\not=_{irr}]$
  
  $\Rightarrow$ $q_1 \not= q_2$ iff $q_1 \neq q_2$.
  
  $\Leftarrow$ states $2^Q \setminus \{P \mid \exists q_1, q_2 \in P, q_1 \not= q_2\}$
DAG Automata and Global \( \Join \) Constraints

Emptiness is NP-complete for DA [Charatonik 1999]

[Vacher 2010 PhD]

- \( \text{DA} \equiv \text{TAGC}^\Join[^{\Join_{irr}}] \)
  \[ \Rightarrow q_1 \not\Join q_2 \iff q_1 \neq q_2. \]
  \[ \Leftarrow \text{states} \ 2^Q \setminus \{P \mid \exists q_1, q_2 \in P, q_1 \not\Join q_2\} \]

- \( \text{TAGC}^\Join[\simeq, \Join_{irr}] \equiv \text{DA}[\simeq] \)
  \[ \text{DA}[\simeq]: \ q_1 \simeq q_2 := \forall x, y \in \text{dom}(d) \ q_1(x) \land q_2(y) \Rightarrow x = y \]

- Emptiness is still NP-complete for \( \text{DA}[\simeq] \)

- Emptiness is decidable in NEXPTIME for \( \text{TAGED} \)

Open problem: generalization to \( \text{TAGC}^\Join[\simeq, \not\Join] \)
(elementary upper bound for emptiness decision)
## Closure and Decision Results

<table>
<thead>
<tr>
<th>Operation</th>
<th>TA $\text{TAGC}[\ ]$</th>
<th>RTA $\text{TAGC}^\Delta[\approx_{\text{ref}}]$</th>
<th>DA $\text{TAGC}^\Delta[\not\approx_{\text{irr}}]$</th>
<th>TAGED $\text{TAGC}^\Delta[\approx, \not\approx_{\text{irr}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cup$</td>
<td>PTIME</td>
<td>PTIME</td>
<td>PTIME</td>
<td>PTIME</td>
</tr>
<tr>
<td>$\cap$</td>
<td>PTIME</td>
<td>EXPTIME</td>
<td>not</td>
<td>EXPTIME</td>
</tr>
<tr>
<td>$\neg$</td>
<td>EXPTIME</td>
<td>not</td>
<td>not</td>
<td>not</td>
</tr>
<tr>
<td>emptiness</td>
<td>linear time</td>
<td>linear time</td>
<td>NP-c.</td>
<td>NEXPTIME-c.</td>
</tr>
<tr>
<td>membership</td>
<td>PTIME</td>
<td>NP-c.</td>
<td>NP-c.</td>
<td>NP-c.</td>
</tr>
<tr>
<td>$\cap$-emptiness</td>
<td>EXPTIME-c.</td>
<td>EXPTIME-c.</td>
<td>EXPTIME-c.</td>
<td>EXPTIME-c.</td>
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<tr>
<td>universality</td>
<td>EXPTIME-c.</td>
<td>undec.</td>
<td>undec.</td>
<td>undec.</td>
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<tr>
<td>inclusion</td>
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<td>undec.</td>
<td>undec.</td>
<td>undec.</td>
</tr>
<tr>
<td>finiteness</td>
<td>PTIME</td>
<td>EXPTIME</td>
<td>PTIME</td>
<td></td>
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</tbody>
</table>
Emptiness is decidable for TAGC[\%approx, \!\neq] we use TAGC[\%approx, \!\neq] \equiv TAGC^\land[\%approx, \!\neq]

- One tree is accepted iff a tree of "small" height is accepted
- **global pumping**: replace all subtrees of height \( h \) by selected subtrees of height \(< h\) while preserving all the relative \( \%approx, \!\neq\)
- accepted tree \( t \mapsto \) sequence of measures \( e_0, e_1, \ldots, e_{h(t)} \) st if \( e_i \preceq e_j \) for \( i < j \) then there exists a global pumping
- **Higman’s Lemma, König’s Lemma**: exists a bound \( B \) on the maximal length of sequences (for any \( t \)) without \( e_i \preceq e_j, i < j \)
- every \( t \) of height \( > B \) can be reduced by a global pumping.
Decidable Extensions of TAGC$[\approx, \not\approx]$ 

extension to unranked trees immediate, using encoding into binary trees [Godoy et al 2012] 

on ranked trees, emptiness is still decidable for 

- TAGC$[\approx, \not\approx]$ extended with local $= \text{and} \neq$ constraints between siblings, à la [Bogaert Tison 1992] 
- TAGC$[\approx, \not\approx]$ where $\approx \text{and} \not\approx$ are interpreted modulo flat equational theories
Arithmetic Constraints

linear inequality \( \sum_{q \in Q} a_q \cdot |q| \geq a \) or \( \sum_{q \in Q} a_q \cdot \|q\| \geq a \), \( a_q, a \in \mathbb{Z} \)

for a run \( r \) on a tree \( t \),
\[
|q| = |r^{-1}(q)|
\]
\[
\|q\| = \{|t_x| \mid x \in \text{dom}(t), r(x) = q\}|
\]

natural inequality (type 'N') when all \( a_q, a \) have the same sign

Presburger automata [Seidl et al 2003, 2008], [Dal Zilio Lugiez 2006]: count the siblings of unranked trees (local cstr).

- emptiness decidable in \( \text{NPTIME} \) for \( \text{TAGC}[|.|_\mathbb{Z}] \)
- emptiness undecidable for \( \text{TAGC}[^\approx,|.|_\mathbb{Z}] \) [Godoy et al 2010]
- \( \text{TAGC}[^\approx,\not\approx,|.|_\mathbb{N},\|.|_\mathbb{N}] \equiv \text{TAGC}^\wedge[^\approx,\not\approx] \) [id]
Monadic Second Order Logic

\( \text{MSO}[+1, \approx, \not\approx, \mid \cdot \mid \mathbb{Z}, \| \cdot \| \mathbb{Z}] \) monadic second-order logic

- first order variables \( x \): position in a tree
- second order variables \( X \): finite set of positions

with predicates

\[ a(x) \quad (x \text{ labeled by } a \in \Sigma \text{ in } t) \]
\[ +1 \ S_\downarrow(x, y) \quad (y \text{ child of } x) \text{ and } S_\rightarrow(x, y) \quad (y \text{ next sibling of } x) \]
\[ \approx \ X \approx Y \quad (\text{for all } x \in X, y \in Y, t|_x = t|_y) \]
\[ \not\approx \ X \not\approx Y \quad (\text{for all } x \in X, y \in Y, t|_x \neq t|_y) \]
\[ \mid \cdot \mid \mathbb{Z} \sum a_i \cdot |X_i| \geq a, a_i, a \in \mathbb{Z} \quad (|X_i| \text{ is cardinality of } X_i) \]
\[ \mid \cdot \mid \mathbb{N} \quad \text{when } a_i, a \text{ have same sign} \]
\[ \| \cdot \| \mathbb{Z} \sum a_i \cdot \|X_i\| \geq a \quad (\|X_i\| \text{ is cardinality of } \{t|_x \mid x \in X_i\}) \]
\[ \| \cdot \| \mathbb{N} \quad \text{when } a_i, a \text{ have same sign} \]
Monadic Second Order Logic: satisfiability

- MSO[+1] \equiv \text{tree automata} [\text{Thatcher Wright 1968}]
- MSO[+1, \approx] \text{undecidable}
- MSO[+1, \mathbb{Z}] \text{undecidable} [\text{Klaedtke Ruess 2002}]

\text{EMSO: } \exists X_1 \ldots \exists X_n \phi(X_1, \ldots, X_n) \land \psi(X_1, \ldots, X_n) \text{ where}

- \phi(X_1, \ldots, X_n) \text{ in MSO}[+1]
- \psi(X_1, \ldots, X_n) \text{ in MSO}[+1, \approx, \not\approx, \mid_\mathbb{Z}, \parallel_\mathbb{Z}], \text{ free}

- EMSO[+1, \mathbb{Z}] \text{decidable} [\text{Klaedtke Ruess 2002}]
- \text{fragment of EMSO}[+1, \approx, \not\approx] \text{decidable} [\text{Filiot et al 2008}]
- EMSO[+1, \approx, \not\approx, \mid_\mathbb{N}, \parallel_\mathbb{N}] \text{decidable} [\text{Godoy et al 2010}]
Perspectives

- Combining Local/Global Constraints for unranked ordered trees
  - combination of TAGC[$\approx$, $\not\approx$]
  - with UTASC: unranked tree automata with local sibling constraints [Löding Wong 2007, 09]
- Extension of TAGC for inclusion constraints
  \[
  \forall x \exists y \ p(x) \Rightarrow (q(y) \land t\mid x = t\mid y) \quad (x, y \in \text{positions})
  \]
  \[
  \forall u \exists v \ p(u) \Rightarrow (q(v) \land u = v) \quad (u, v \in \text{subtrees})
  \]
- TAGC with constraints in monadic FO over $q(y)$ and $x = y$
  interpretation in the domain of subtrees
  (related to automata on DAGs)

Thank you
Thank you