Review of "Piecewise-Smooth Dynamical Systems: Theory and Applications by M. di Bernardo, C. Budd, A. Champneys and P. Kowalczyk; 2008"

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Book Review

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Piecewise-Smooth Dynamical Systems: Theory and Applications

by MARIO DI BERNARDO, CHRISTOPHER J. BUDD, ALAN R. CHAMPNEYS, and PIOTR KOWALCZYK

Reviewed by Bernard Brogliato

This book deals with the analysis of bifurcations and chaos in nonsmooth become a major topic in the control community [6], there remains no precise definition of what systems or more accurately, of what models are nonsmooth. Nonsmooth systems are usually represented by mathematical formalisms, such as switching systems, piecewise something systems, differential inclusions (there are various types of these depending on the properties of the set-valued right-hand-side), impulsive ordinary differential equations (ODEs), evolution variational inequalities, projected dynamical systems, complementarity dynamical systems (there are also various types of these), hybrid systems, and so on. My point here is that it soon becomes quite difficult to know precisely what a nonsmooth dynamical system is and how to classify them because nonsmooth systems are much like nonlinear systems relative to linear systems. That is, nonsmooth systems include everything that is not (called piecewise smooth by the authors) dynamical systems. Despite the fact that nonsmooth systems have smooth, which yields a set with a lot of elements. Nevertheless, most nonsmooth formalisms share the property that they are not a simple extension of smooth formalisms developed for models whose right-hand-side possesses strong differentiability properties. This observation is true for control [6], for numerical methods [2], and also for dynamical analysis, as this book shows. Roughly speaking, what happens is that all the tools that are based on linearization, such as eigenvalues, eigenspaces, continuous dependence of eigenvalues on the parameter, center manifold reduction, and Taylor series expansion, no longer work for nonsmooth systems.

For a control scientist or an applied mathematician, a reasonable path is to rely on the applications and modeling work done in fields such as mechanics, electromagnetism, and biology, where nonsmooth models have been derived. For instance, mechanical systems subject to nonsmooth contact laws (impacts, or Coulomb friction are the most common), switched electrical circuits, and Filippov systems, which are familiar to systems and control researchers. Once an application has been targeted, a specific model can be chosen for control or analysis purposes. This approach is chosen in this book, which deals with bifurcation and chaos

analysis for several classes of maps and systems, namely, C^0, discontinuous, square-root, and C^k maps, systems with C^0 nondifferentiable vector fields, Filippov systems, and vibro- impact systems. Strangely, the word “bifurcations” does not appear in the title of the book, despite the fact that this is the main topic being addressed. The general goal of this monograph is to present the peculiarities of bifurcations and chaos in such systems, in a rather detailed way, where many examples with detailed calculations, comments, and numerical results illustrate the theoretical results. Most of the theorems are stated without proof, but are illustrated through worked examples. It is clear that the authors have made a particular pedagogical effort in writing this book. The central topic, although not about feedback and control, is connected to control, and throughout the book the reader can find many notions that are widely used in control analysis, such as stability, canonical forms, relative degree, relay systems, Filippov systems, sliding modes, and Poincaré maps.

CONTENTS

The first chapter introduces the main notions that are used in the subsequent chapters, through the analysis of several examples, including impact oscillators, relay control, dry friction, dc-dc converter, and typical maps (square root and piecewise linear). Impact and stroboscopic Poincaré maps, grazing bifurcations, periodic orbits, numerical methods, limit sets, and penalized models are presented as well as experimental methods that are used to study and validate bifurcation diagrams. Readers who know the basics of bifurcation and chaos theory for smooth systems will certainly appreciate this chapter. Those who do not may first jump to the second chapter and then return to Chapter 1. Many fundamental tools and results concerning smooth systems are recalled in Section 2.1, which also gives insight into why bifurcation theory for smooth systems does not extend easily to nonsmooth ones. In particular, it is shown that a key result obtained from the implicit function theorem and used in bifurcation theory for smooth systems no longer works (Theorem 2.4). Sections 2.2 and 2.3 review various mathematical formalisms for nonsmooth systems and maps. Sections 2.4 and 2.5 are dedicated to nonsmooth systems. The definition of a bifurcation, especially a discontinuity-induced bifurcation (DIB), is given. DIBs form a family of bifurcations that are typical to nonsmooth systems and that do not exist in smooth systems. Chapter 2 ends with a section on numerical methods that are used to integrate the systems of interest. Nonsmooth systems must be time discretized and integrated with great care [2], and most of the available software packages used in systems and control are not quite adequate.

I would make two main reproaches to chapters 1 and 2. First the difference between smooth and nonsmooth (piecewise smooth in the book's terminology) systems could be clearer. Definition 2.2 introduces smooth systems of index r, and Definition 2.20 concerns piecewise-smooth systems. It is not clear, however, whether the intersection between these is empty or not. A hint is given in a remark (page 50), where one understands that smooth means with a vector field that is C^k with $k > 0$, while the rest of the book confirms that piecewise smooth concerns vector fields that are C^0 or less regular. Second, a definition of bifurcations (the DIBs) in nonsmooth systems is given (Definition 2.32) based on the property of piecewise structural stability. Other researchers have argued (for example, [3]) that this definition is not the only possible one and may sometimes not be suitable. Some words on this point would have been welcome. Another point that seems surprising is the absence of Lyapunov exponents, which seem to be a central tool widely used by other authors [4], [5]. This is certainly a deliberate choice, which can be understood since focusing "only" on DIBs already fills almost the whole book. In conclusion, despite a few details that many readers will correct themselves, chapters 1 and 2 nicely introduce and motivate the topic of the book.

The remainder of the book (chapters 3–9) is dedicated to the very detailed study of DIBs in nonsmooth maps and non-smooth systems. These chapters contain an enormous amount of information on DIBs, which clearly make this book a unique contribution to the field of bifurcations and chaos in non- smooth systems. I confess I did not have the time to read it all with sufficient care to provide an in-depth review. I will therefore focus on a few parts only. However, since the chapters are presented in the same way, with theoretical aspects soon followed by simple examples and numerical results with discussions, I believe that observations that hold for these parts are representative of the remaining parts.

Chapter 6 deals with limit-cycle bifurcations in vibro-impact systems, which are simple mechanical systems that undergo a succession of free-motion trajectories and impacts and are as such highly nonsmooth and nonlinear; these systems are sometimes called flows with collisions [6]. The chapter starts with several examples of impacting systems, along with an introduction to suitable Poincaré maps. The main point of this chapter is what happens close to grazing orbits, which are introduced in Chapter 1. Grazing trajectories touch the boundary, but with a zero normal velocity. It happens that the Poincaré maps that are associated with such grazing trajectories are piecewise smooth, linear on one side of the switching surface and of the square-root type on the other side. The square root function has an infinite gradient at zero, resulting in a stretching of the phase space in the zero limit. Intuitively, this property means that the dynamics of a system that evolves on a grazing trajectory (that is a trajectory that touches the boundary of the switching surface at one point, with a zero normal velocity) may vary in a very abrupt way when a parameter is changed, and the system undergoes a (strong) bifurcation, possibly chaos. The construction of the square-root maps is explained in detail in Section 6.2. Perhaps the presentation of this part could have been simplified by focusing directly on mechanical systems and not on an extension of them. Generally speaking, embedding non-smooth mechanical systems in a hybrid framework is, in my humble opinion, quite useless. The chapter continues with the study of grazing bifurcations for periodic trajectories and the derivation of Poincaré maps.

The chapter ends with the influence of chattering, that is, accumulations of impacts, on the bifurcation process, as well as a short section on multiple impacts, that is, several collisions occurring at the same time in a system. Here it is not clear to me how accumulations are managed numerically, since it seems that event-driven methods [2] are used to get the bifurcation diagrams. This technique must be limited to simple cases where one knows what happens after the accumulation point. Also, since chattering is in a sense a plastic impact (zero restitution coefficient $r = 0$) obtained after an infinite number of impacts, I was wondering whether the case $r = 0$ would be equivalent to the case of chattering. Chapter 8 deals with sliding mode systems. A typical example is relay systems, which have been extensively studied in the control community from the point of view of existence of limit cycles.

I will end this part of the review with a few comments on Chapter 9. Section 9.1 discusses the possible discrepancies that arise between numerical and experimental results in simple impacting systems and provides explanations for these discrepancies, such as neglected dynamics and parameter uncertainties. Sections 9.3 and 9.4 present two applications and numerical calculations, namely, rattling gears and a hydraulic damper. The objective of Section 9.3 is to show that two different models (rigid body and restitution, and compliant contact) provide the same results. What is missing here is a conclusion on what model is preferable. Certainly, the numerical integration as the stiffness becomes large can be a criterion, because one would like to avoid integrating stiff ODEs and prefer specific methods for rigid-body systems [2]. The chapter ends with examples of two-parameter sliding bifurcations in one degree-of-freedom mechanical systems with Coulomb friction, in contrast to the rest of the book, which is dedicated to one-parameter bifurcations. Chapter 9 looks like Chapter 1 in its construction and nicely closes the book.

CONCLUSIONS

This book is undoubtedly a strong contribution to the field of bifurcation and chaos analysis and more generally to the field of nonsmooth dynamical systems analysis. The authors have made a remarkable effort in mixing intricate technical developments with numerous examples, numerical results, and experimental results. It is obvious that the book is primarily intended for bifurcation and chaos specialists, for whom it will serve as a reference. Most systems and control researchers will certainly find parts of it a bit hard to read but once again the detailed examples help a lot. For beginners in the field, first reading a basic text-book on bifurcations and chaos (for instance, [7]) will certainly be quite helpful. Also, it is clear that control researchers would have appreciated a section on the control applications of bifurcations and chaos (the one that comes to my mind is the OGY method to stabilize chaotic systems on a periodic orbit, but there are more), for instance as a section of Chapter 9. The authors cannot be blamed for this omission, since feedback stabilization was not at all the primary objective of the book. Finally, the presentation, including both text and figures, is of high quality, and I found very few typos.

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