

The background features a detailed pencil sketch of a landscape on the left, showing a church with a tall steeple and rolling hills. On the right, a 3D wireframe camera frustum is superimposed over the scene, with a figure of a person standing inside it, holding a camera. The title text is centered over these elements.

# 5D COVARIANCE TRACING

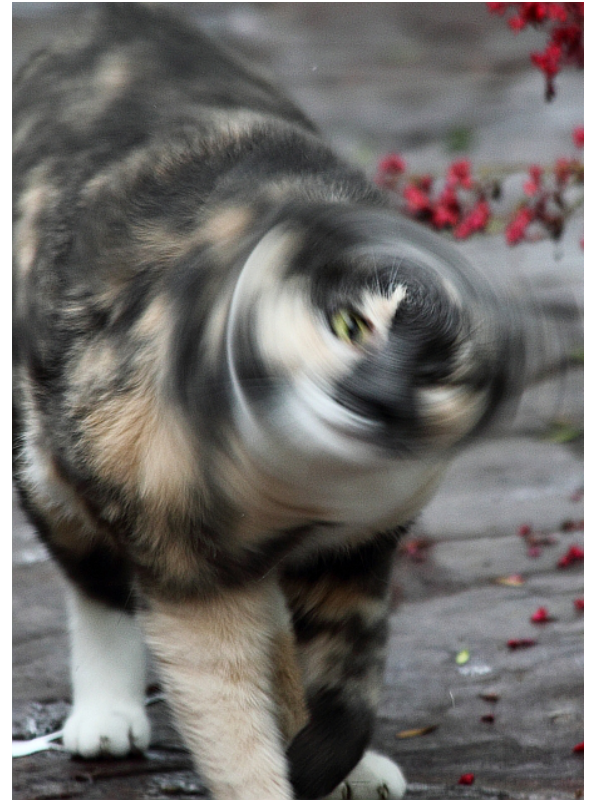
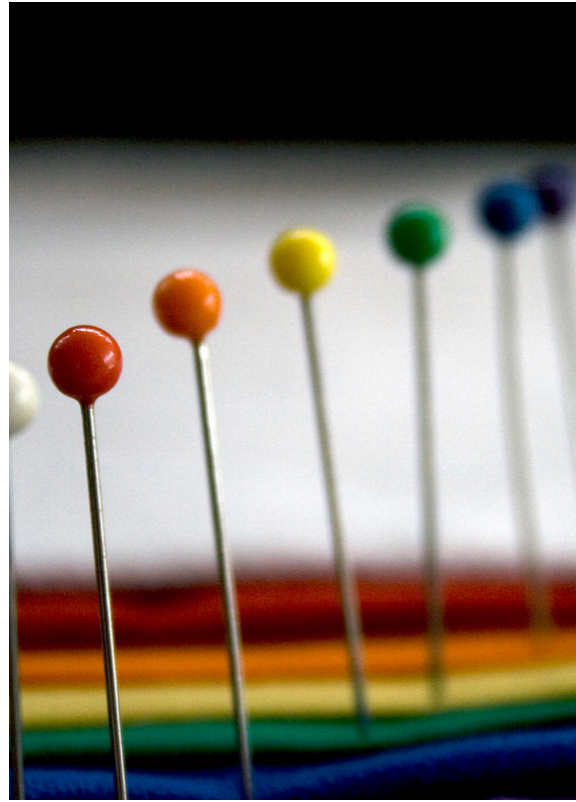
## FOR EFFICIENT DEFOCUS AND MOTION BLUR

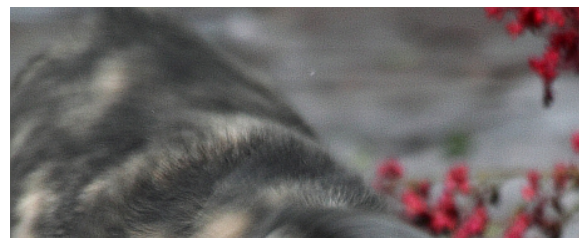
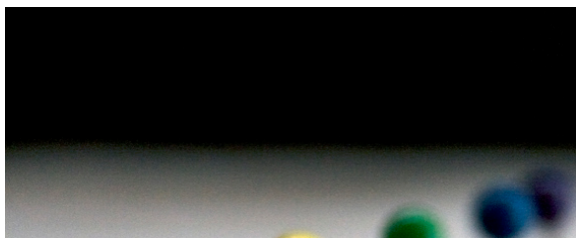
Laurent Belcour<sup>1</sup> Cyril Soler<sup>2</sup> Kartic Subr<sup>3</sup> Nicolas Holzschuch<sup>2</sup> Frédo Durand<sup>4</sup>

<sup>1</sup>Grenoble Université, <sup>2</sup>Inria, <sup>3</sup>UC London, <sup>4</sup>MIT CSAIL



**SIGGRAPH**2013

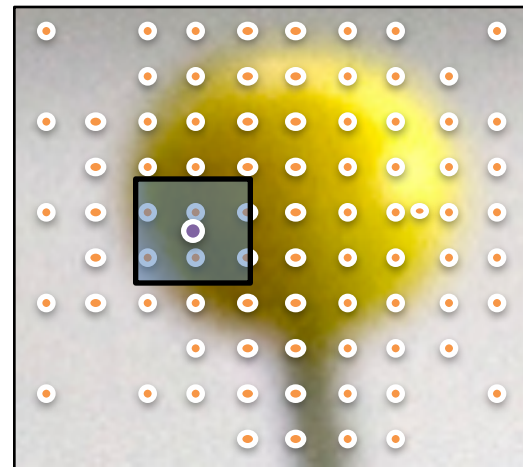
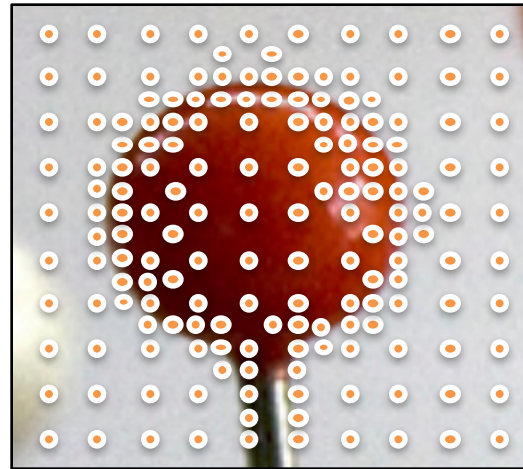
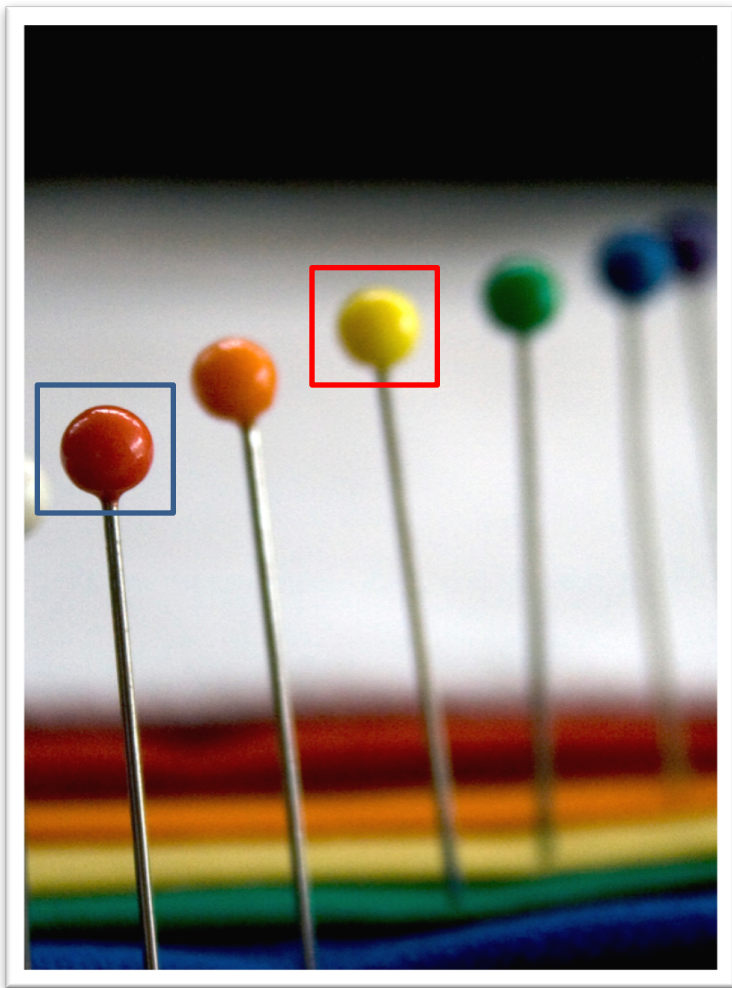




Blur is costly to simulate !









Time =  
*integration*

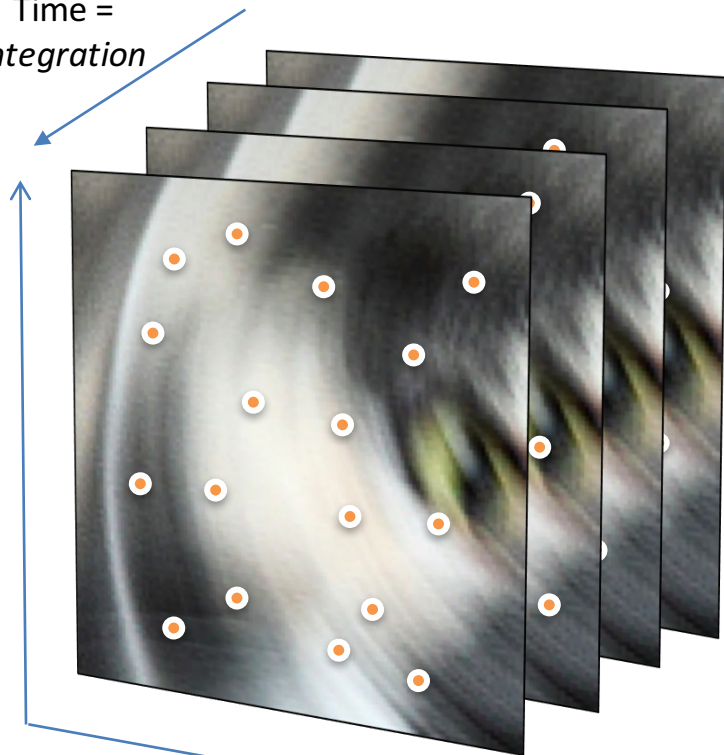
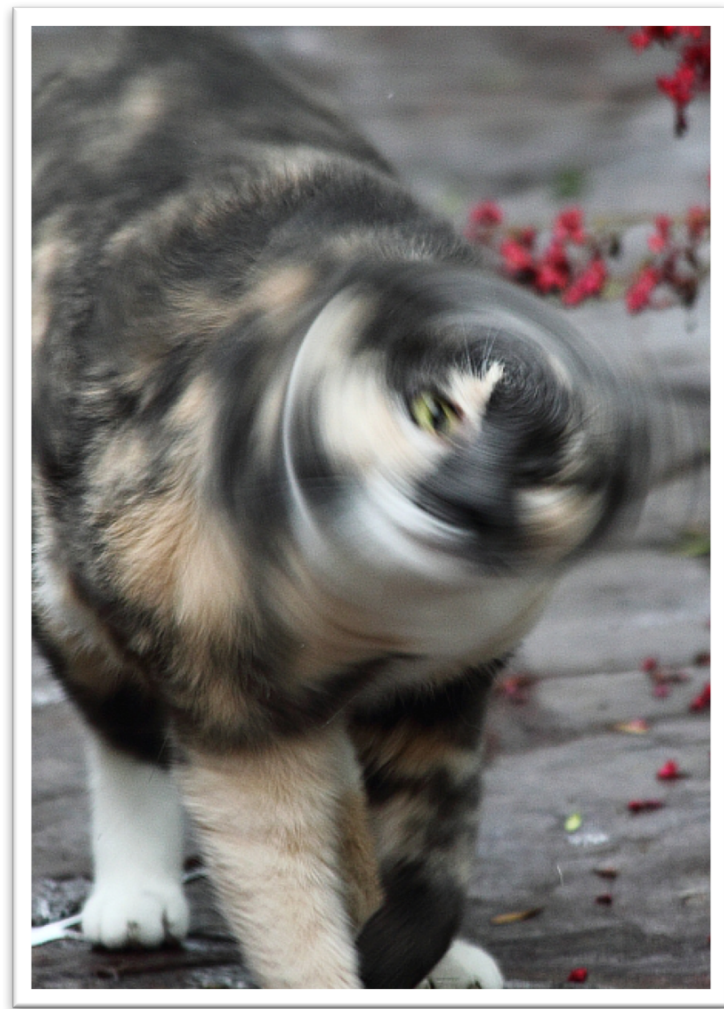


Image Space =  
*reconstruction*



# Previous works: *a posteriori*

- Image space methods
    - [Mitchell 1987], [Overbeck *et al.* 2009],
    - [Sen *et al.* 2011], [Rousselle *et al.* 2011]
  - Integration space
    - [Hachisuka *et al.* 2008]
  - Reconstruction
    - [Lehtinen *et al.* 2011], [Lehtinen *et al.* 2012]
- ✓ Easy to plug
- Require already dense sampling
  - Rely on point samples



# Previous work: *a priori*

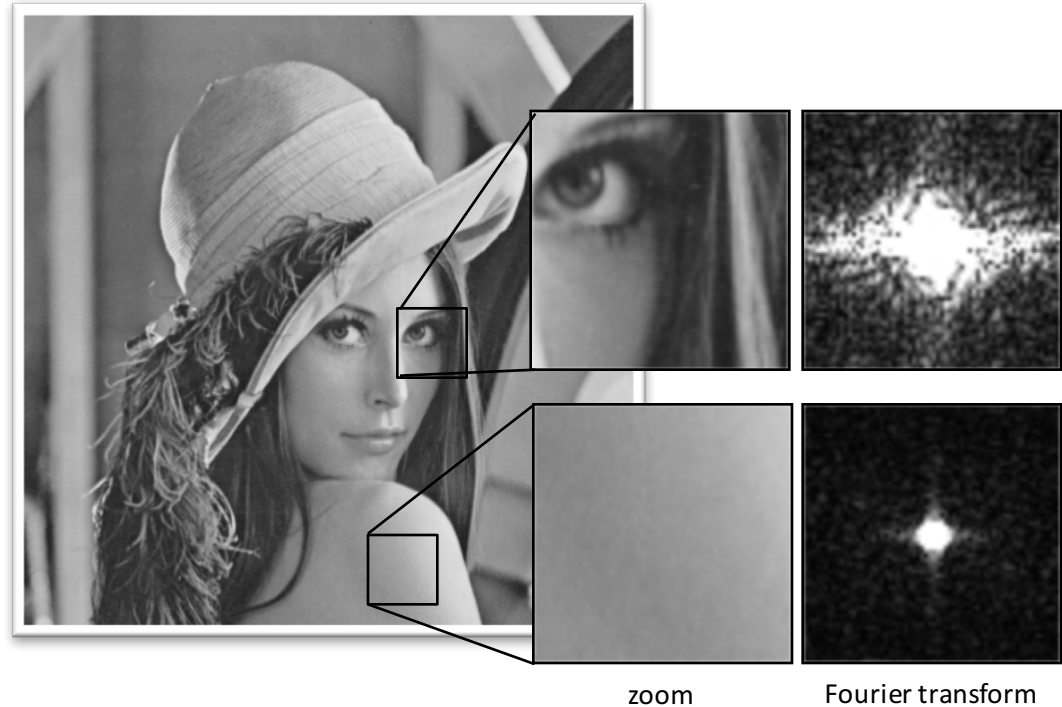
- First order analysis  
[Ramamoorthi *et al.* 2007]
- Frequency analysis  
[Durand *et al.* 2005]





# Previous work: *a priori*

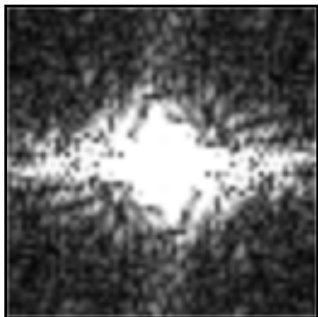
- First order analysis  
[Ramamoorthi *et al.* 2007]
- Frequency analysis  
[Durand *et al.* 2005]



# Previous work: *a priori*

## Predict full spectrum

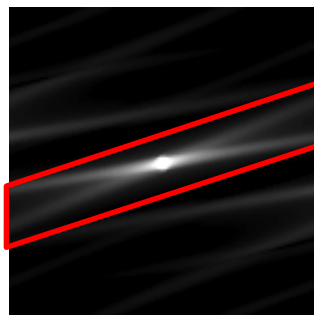
- ✓ Anisotropic information
- Unwieldy



[Soler *et al.* 2009]

## Predict bounds

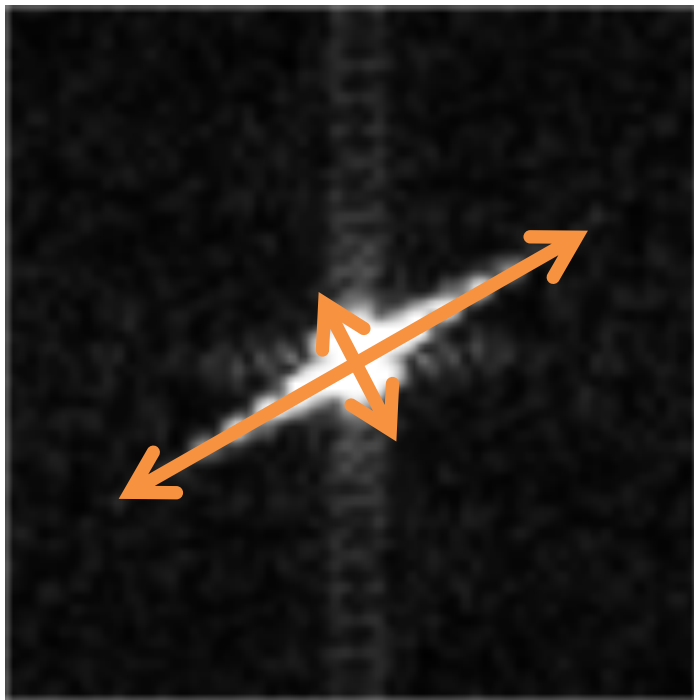
- ✓ Compact & efficient
- Special cases formula



[Egan *et al.* 2009], [Bagher *et al.* 2013],  
[Meha *et al.* 2012]

None can work with full global illumination!

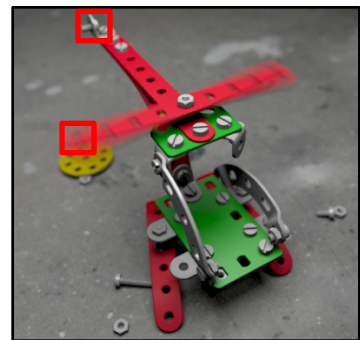
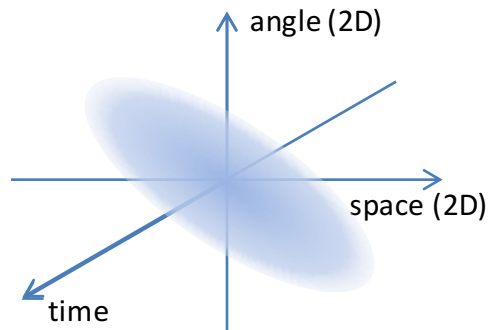
# Our idea: 5D Covariance representation





# 5D Covariance representation

- Use second moments
  - 5x5 matrix
  - Equivalent to Gaussian approx.
- Formulate all interactions
  - Analytical matrix operators
  - Gaussian approx. for reflection
- Nice properties
  - Symmetry
  - Additivity

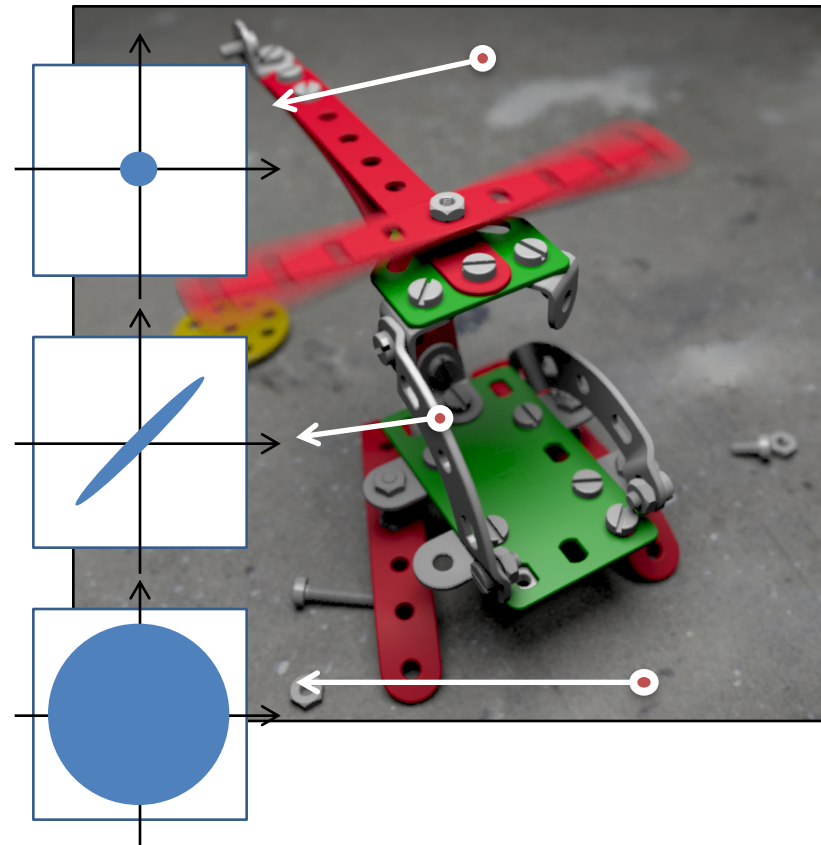


# Contributions

- Unified temporal frequency analysis
- Covariance tracing
- Adaptive sampling & reconstruction algorithm

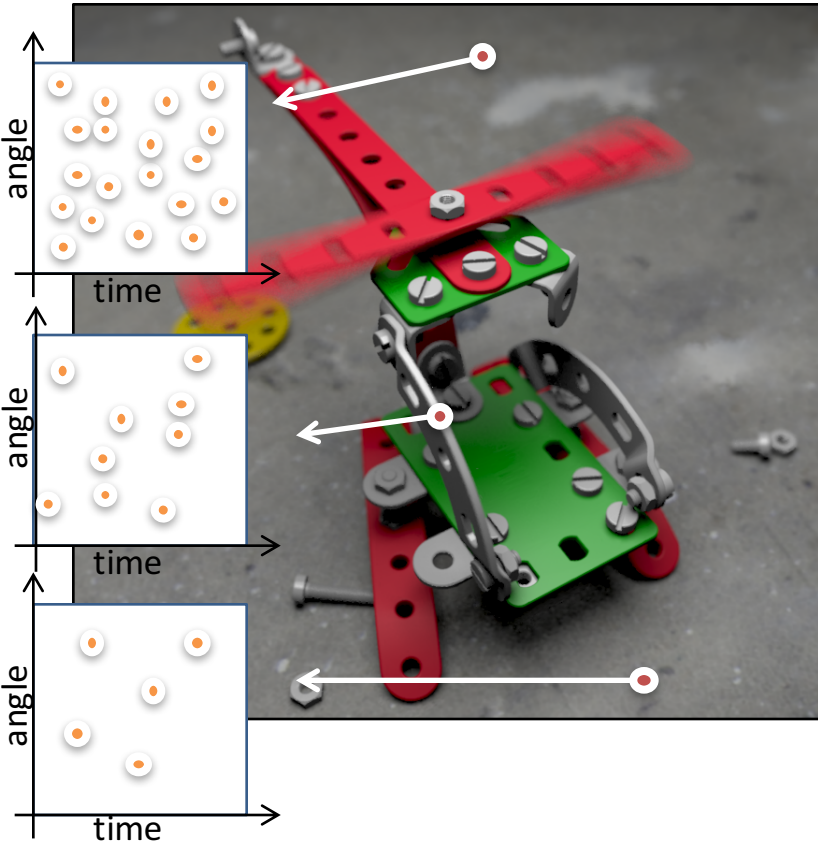
# Our algorithm

Accumulate 5D Covariance  
in screen space





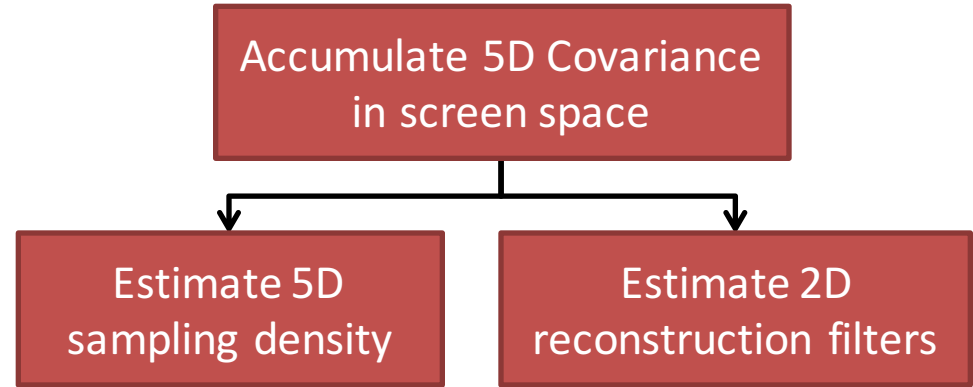
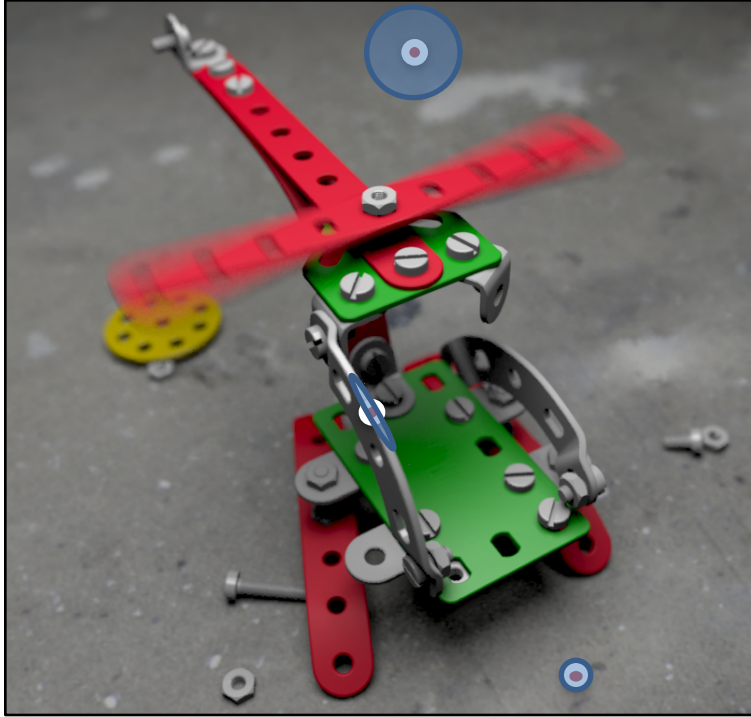
# Our algorithm



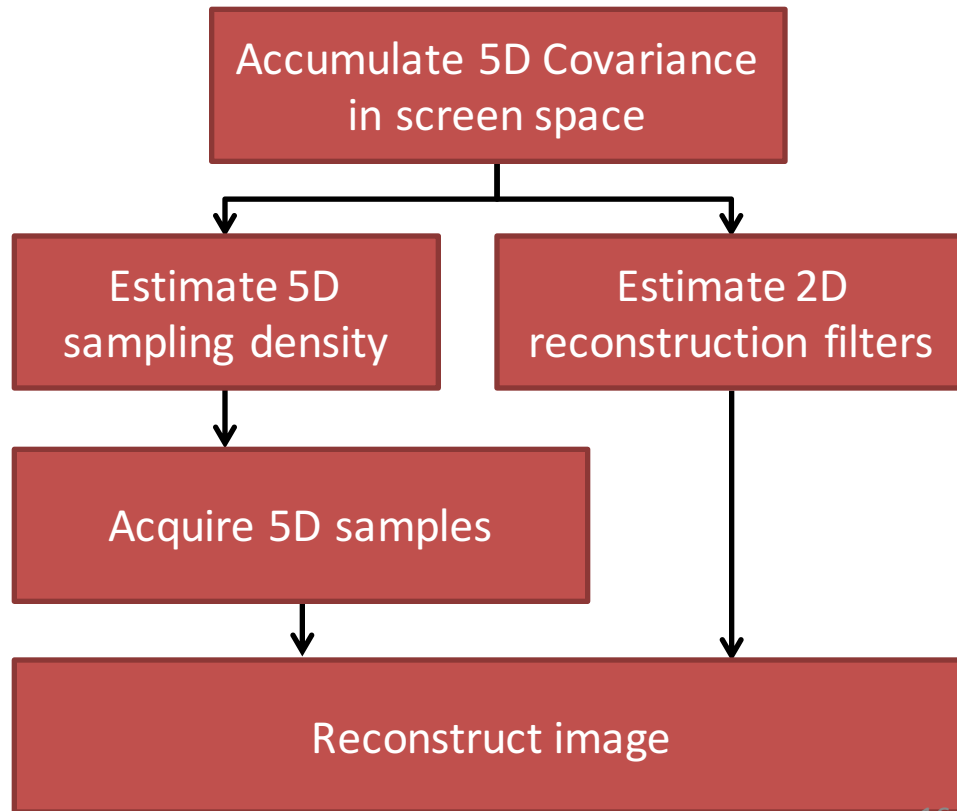
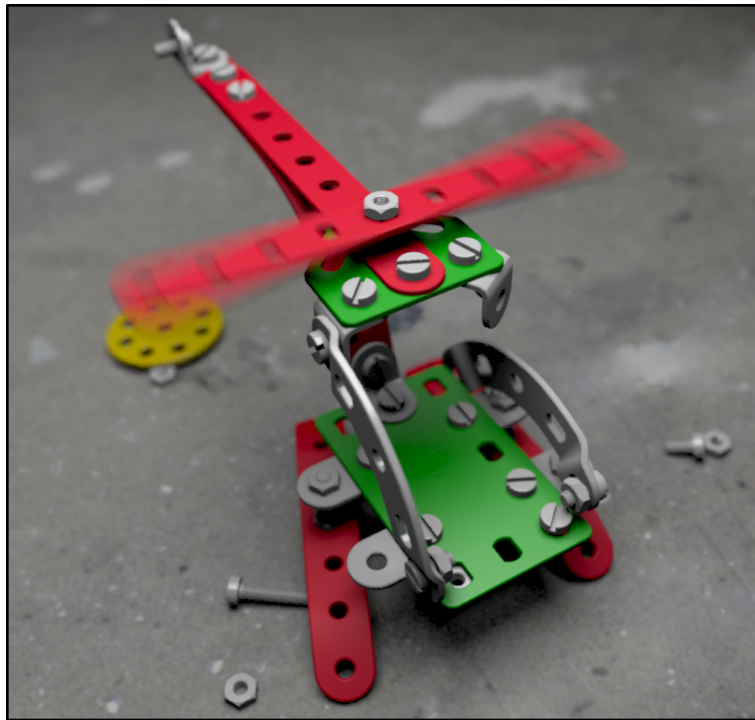
Accumulate 5D Covariance  
in screen space

Estimate 5D  
sampling density

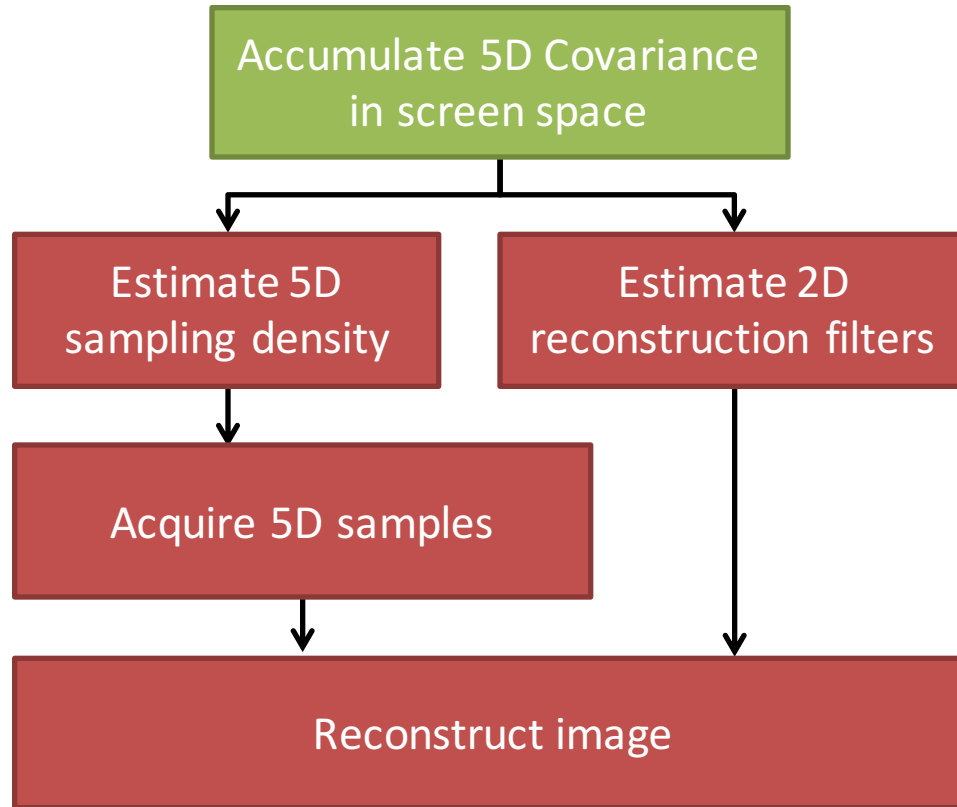
# Our algorithm



# Our algorithm

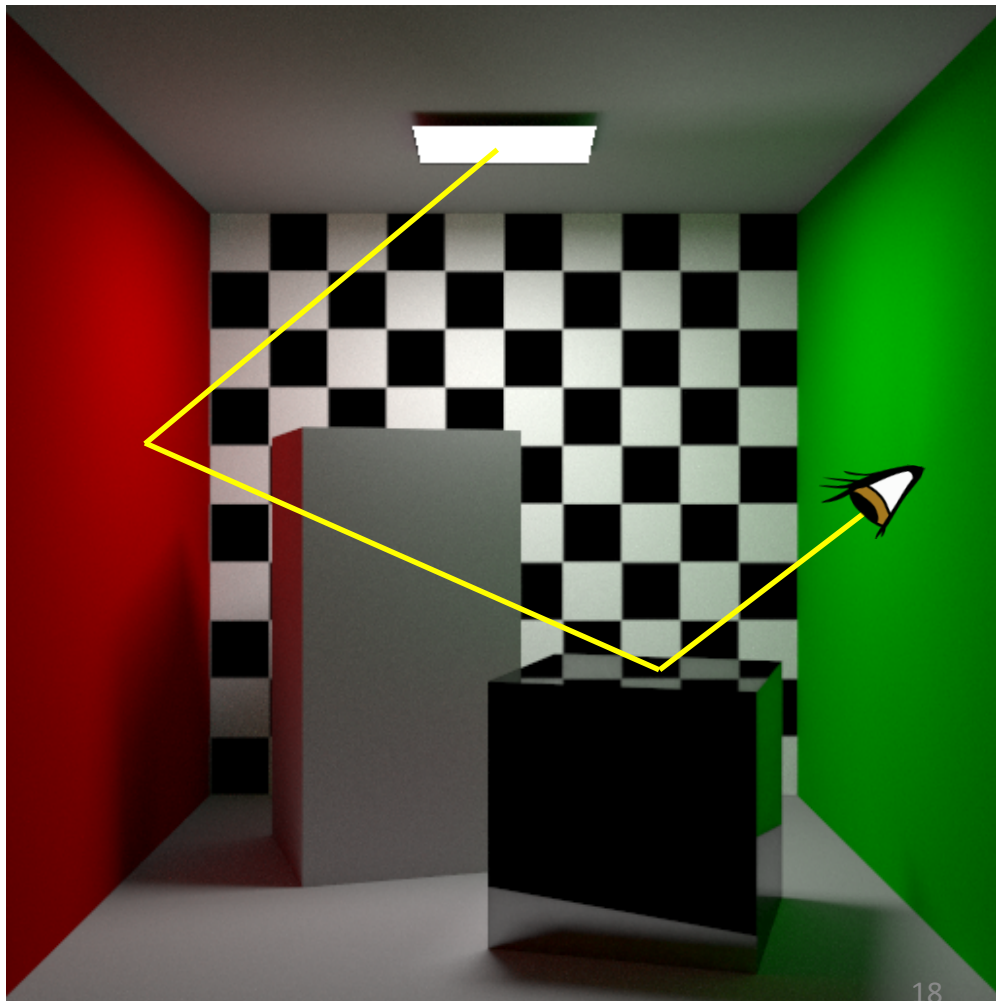




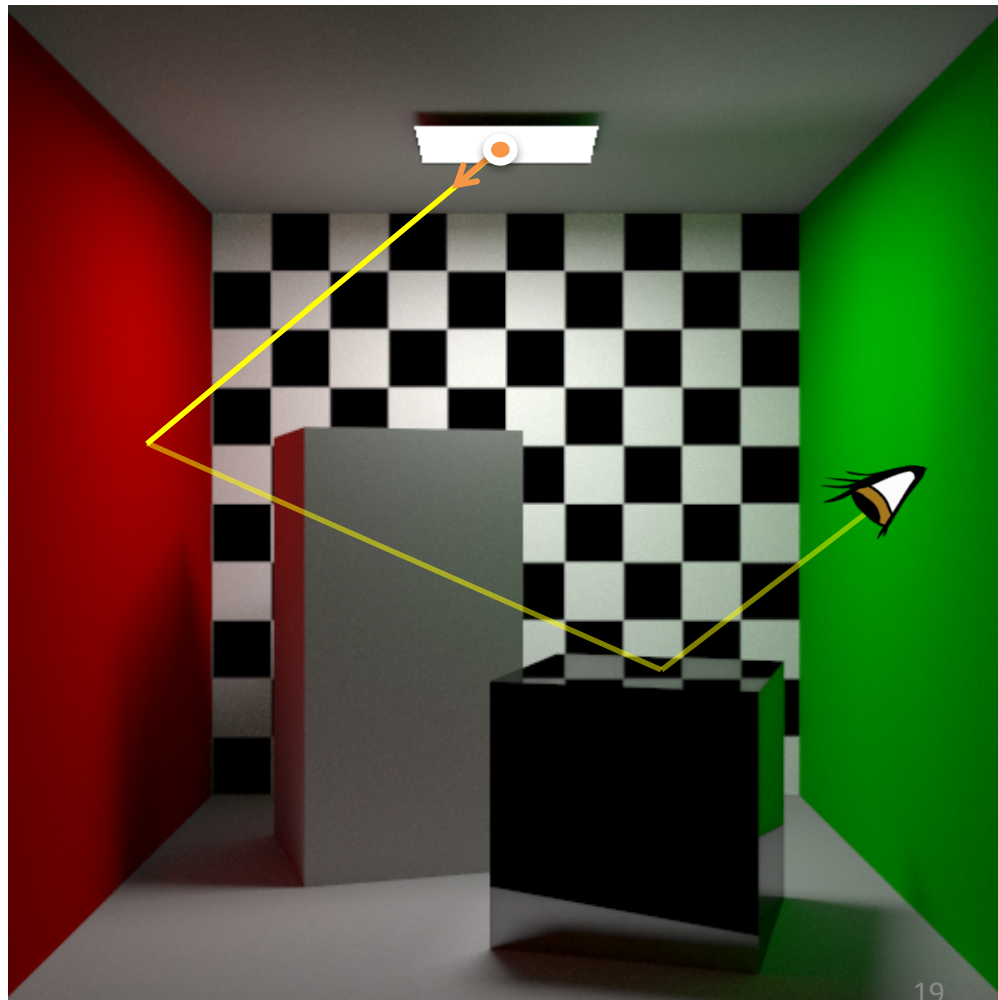
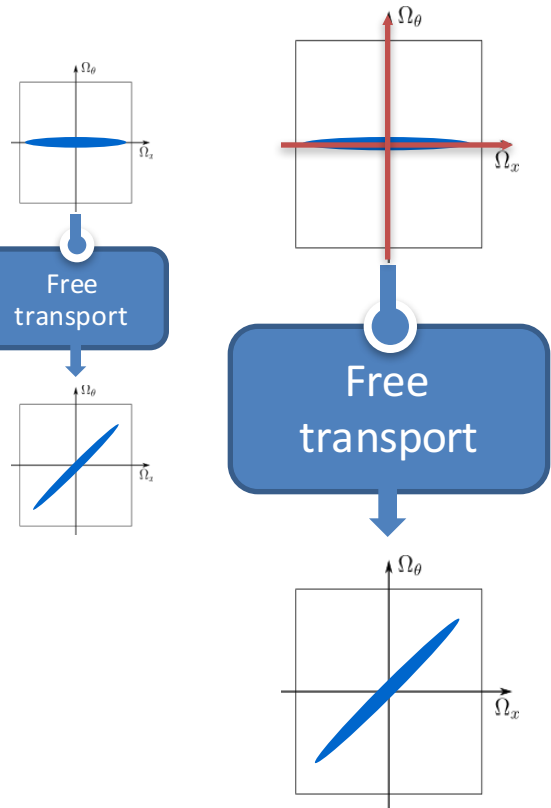


# Covariance tracing

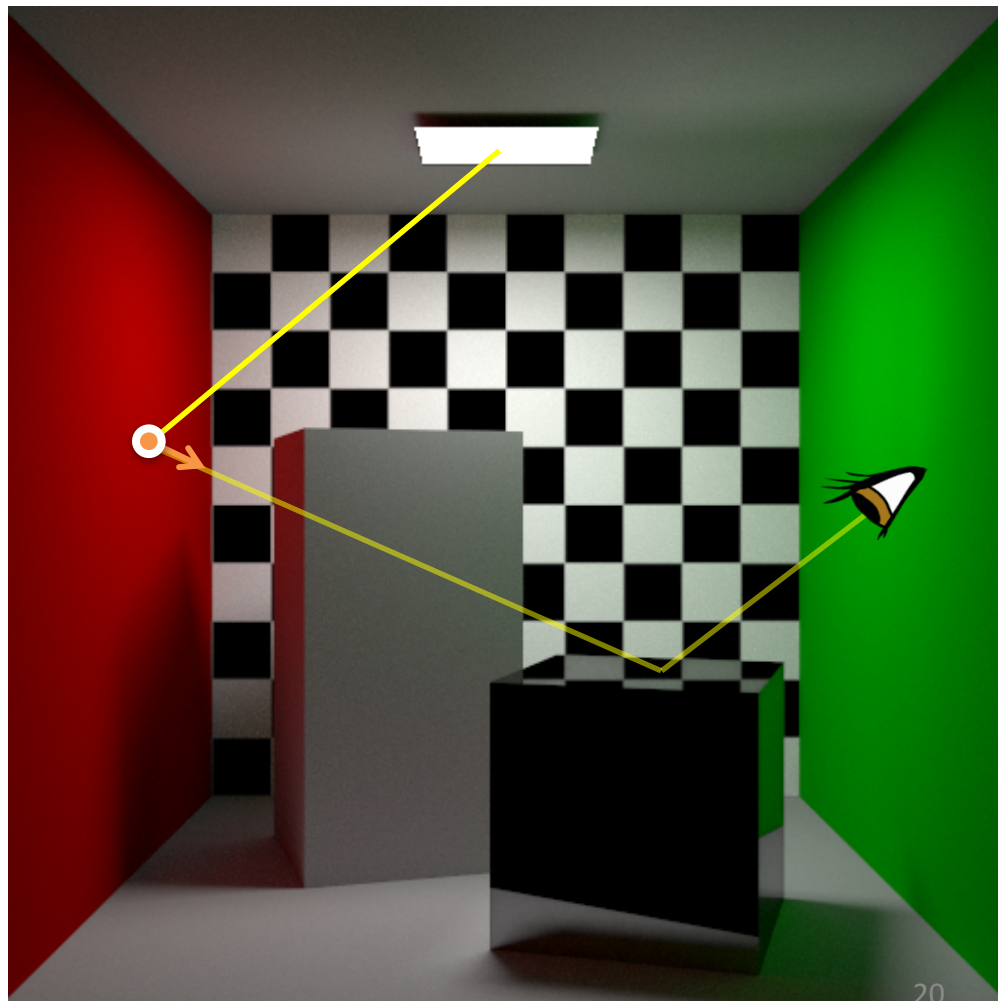
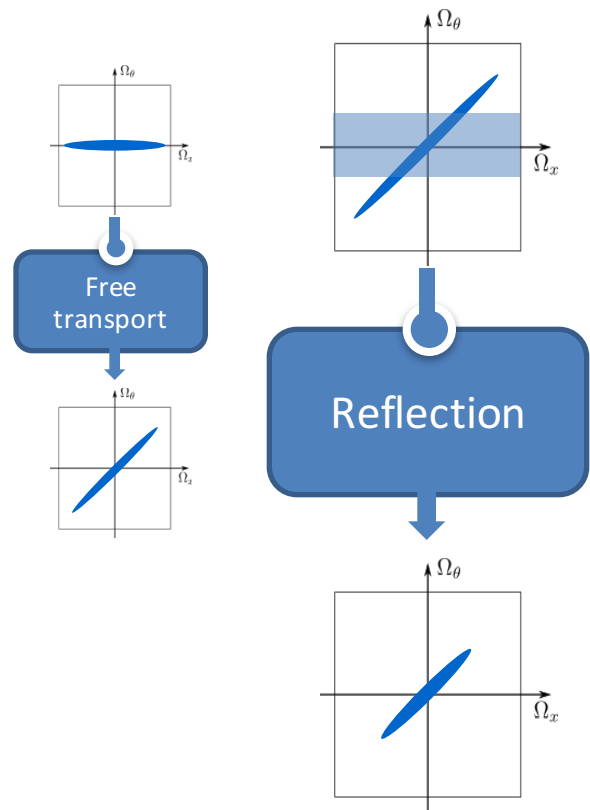
- Add information to light paths
- Update the covariance along light path
- Atomic decomposition for genericity



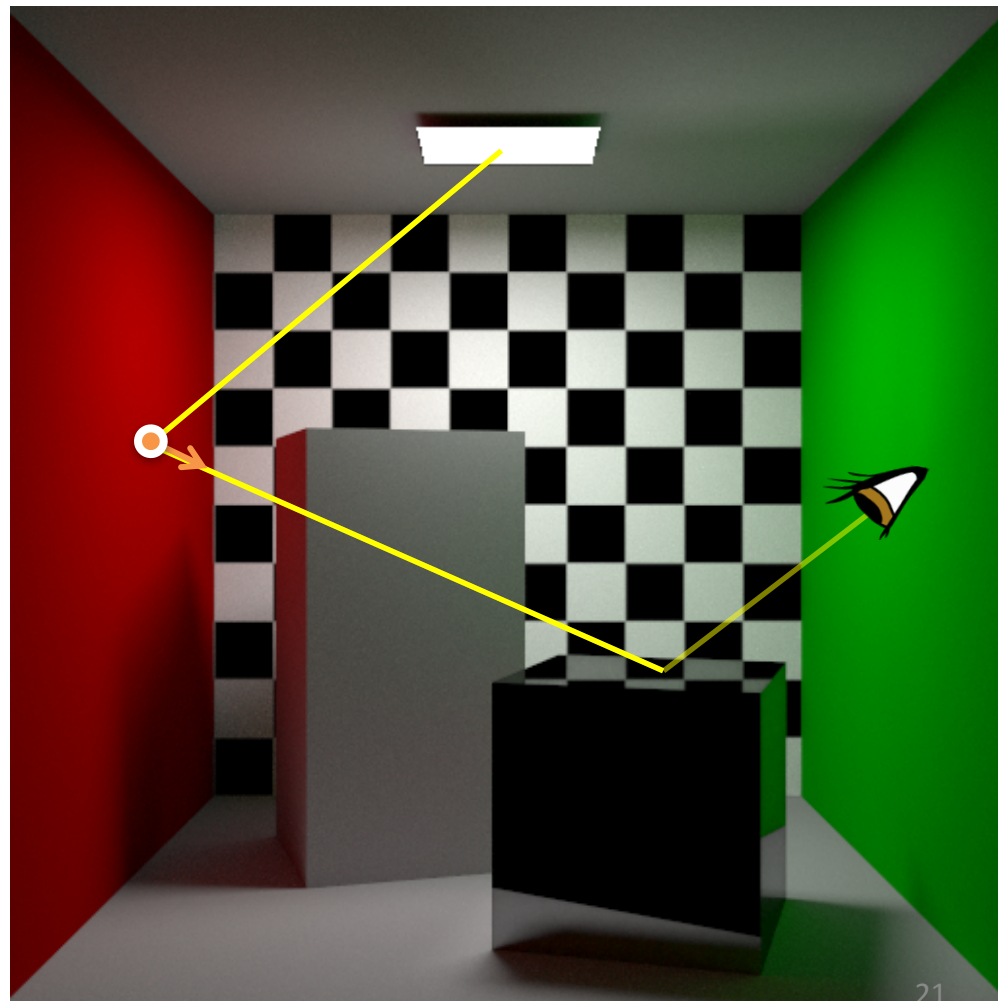
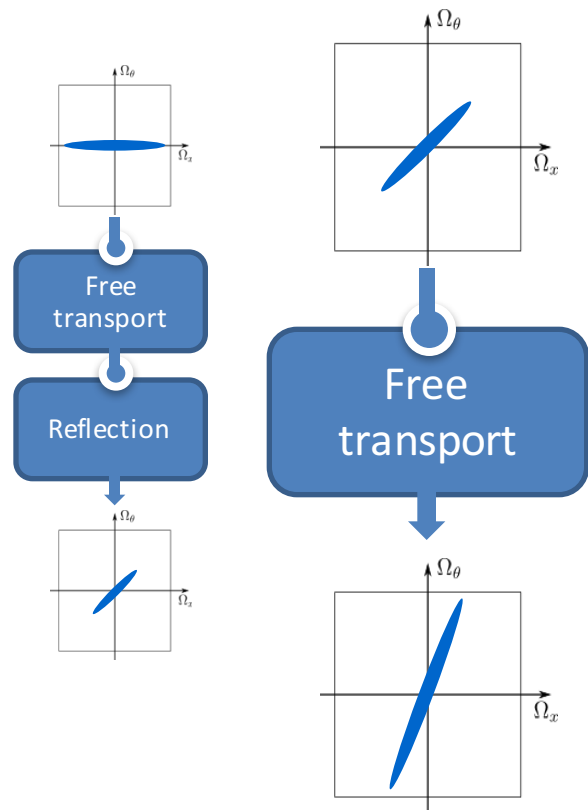
# Covariance tracing



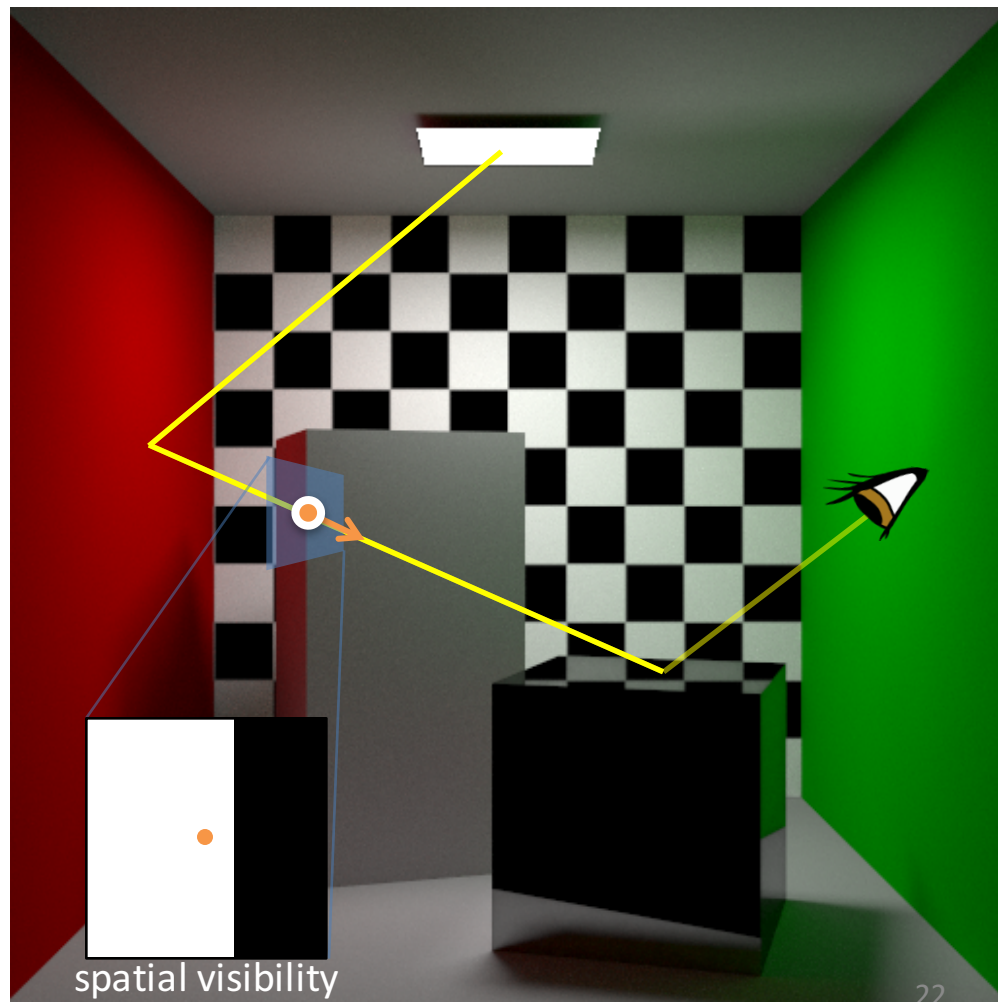
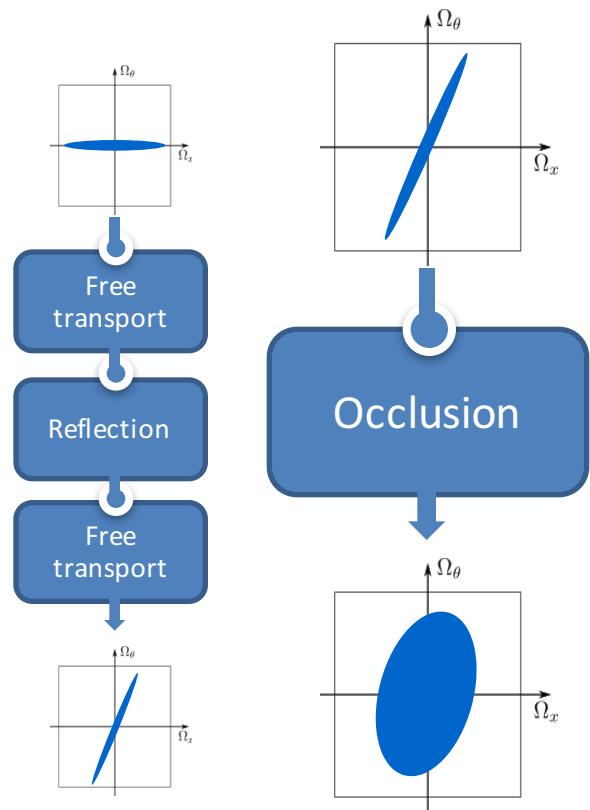
# Covariance tracing



# Covariance tracing

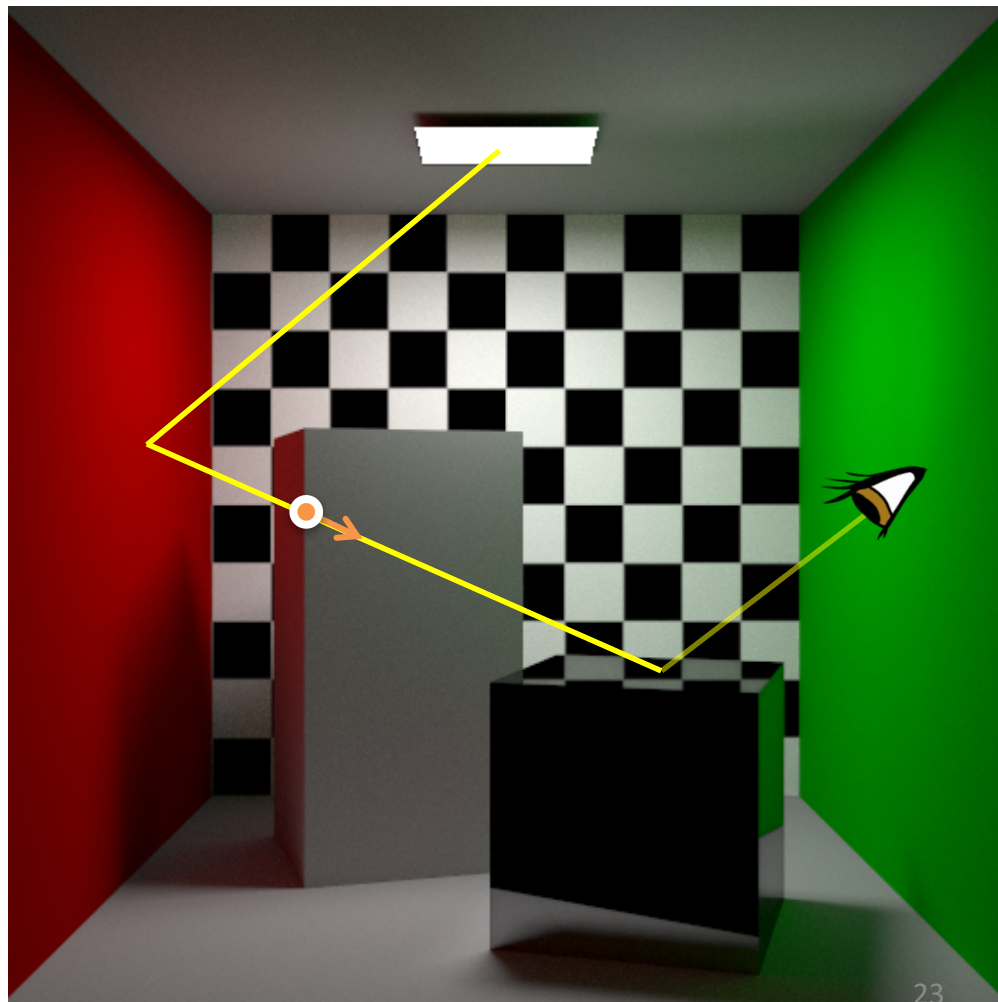
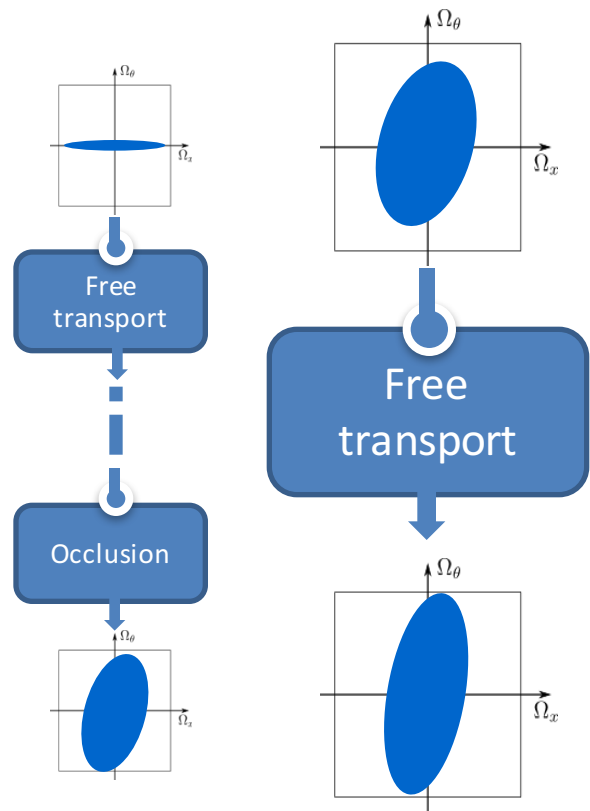


# Covariance tracing

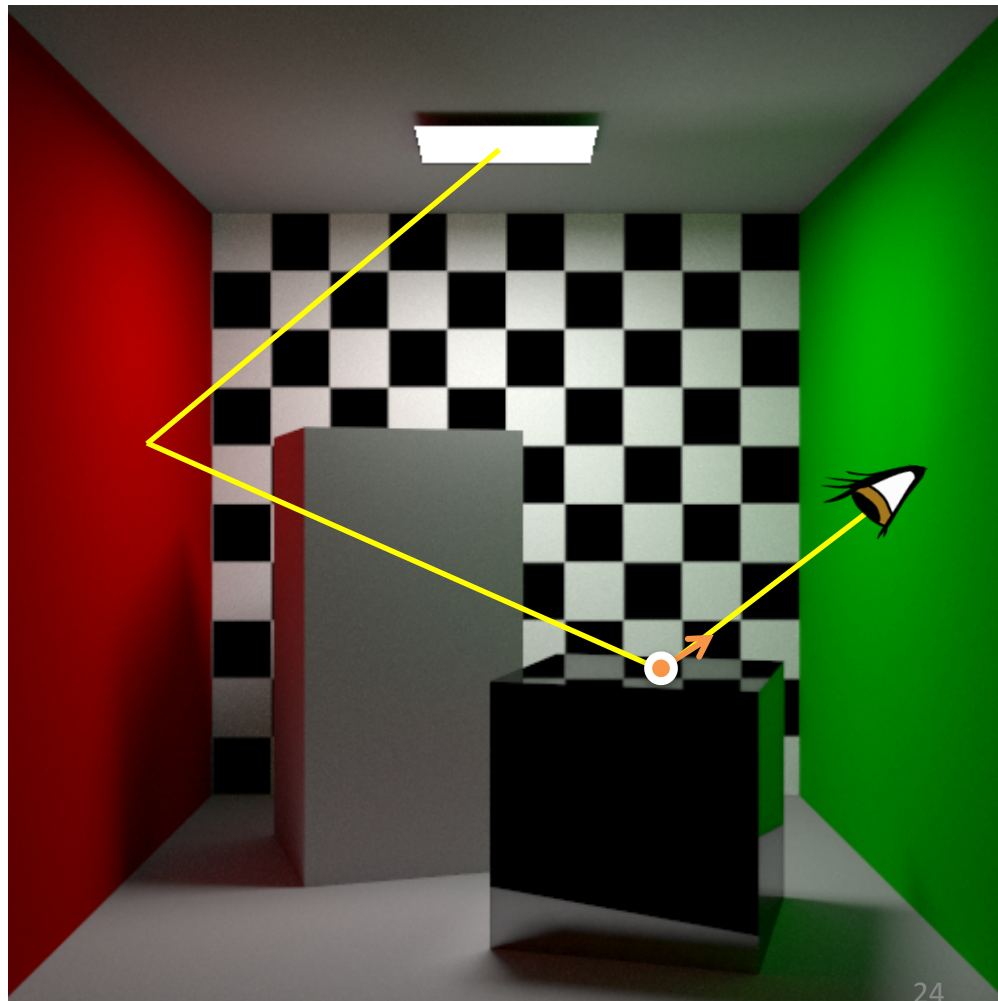
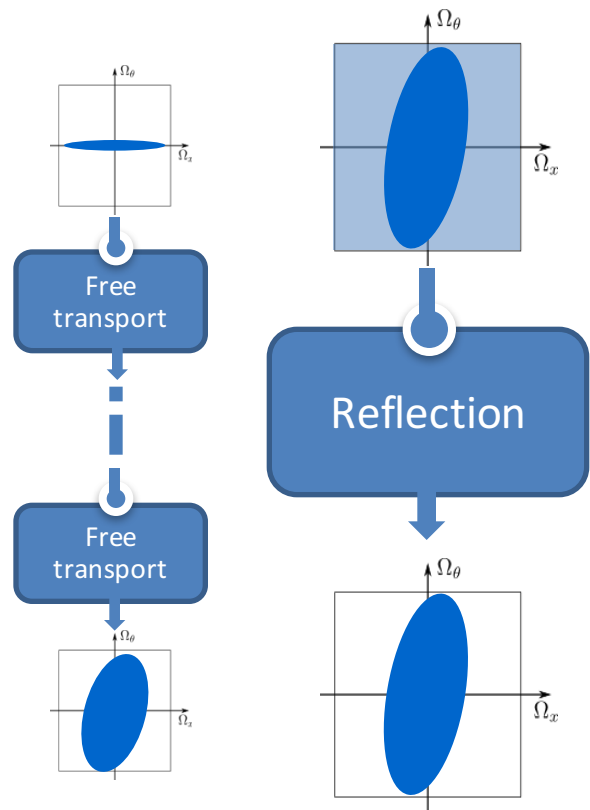




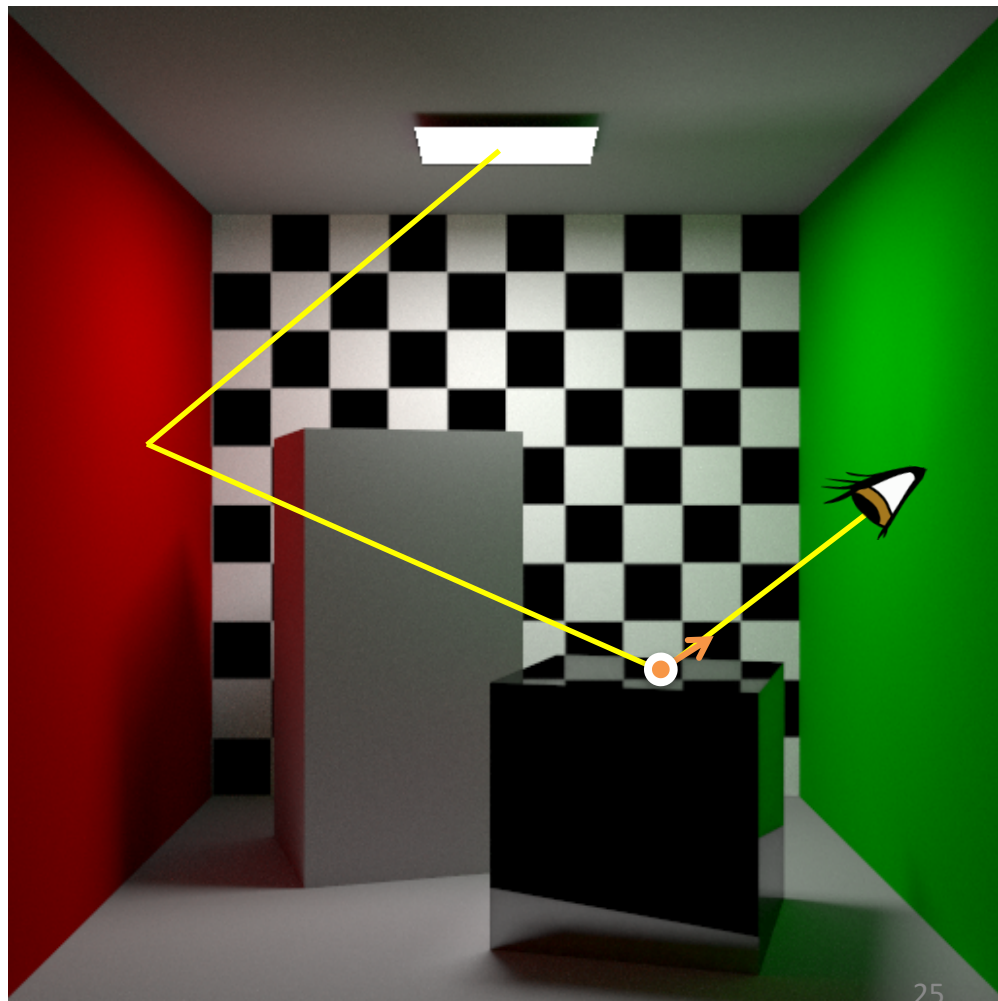
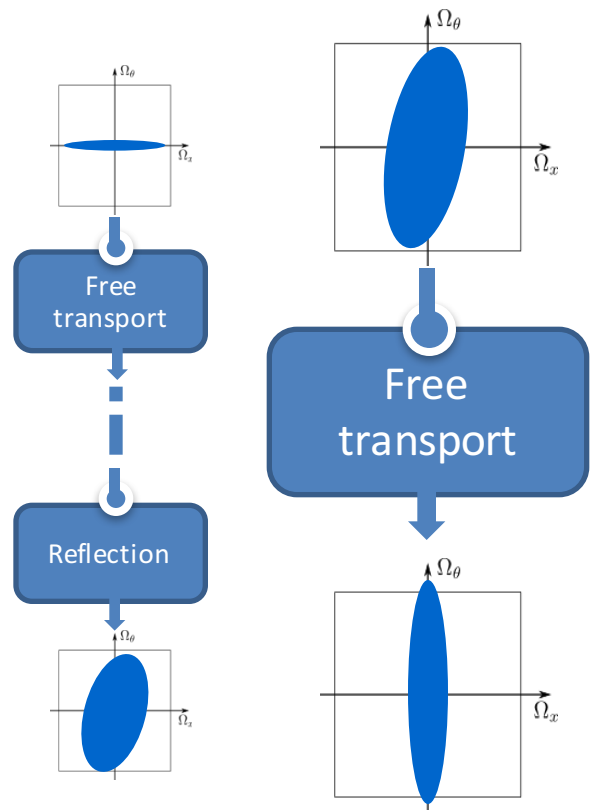
# Covariance tracing



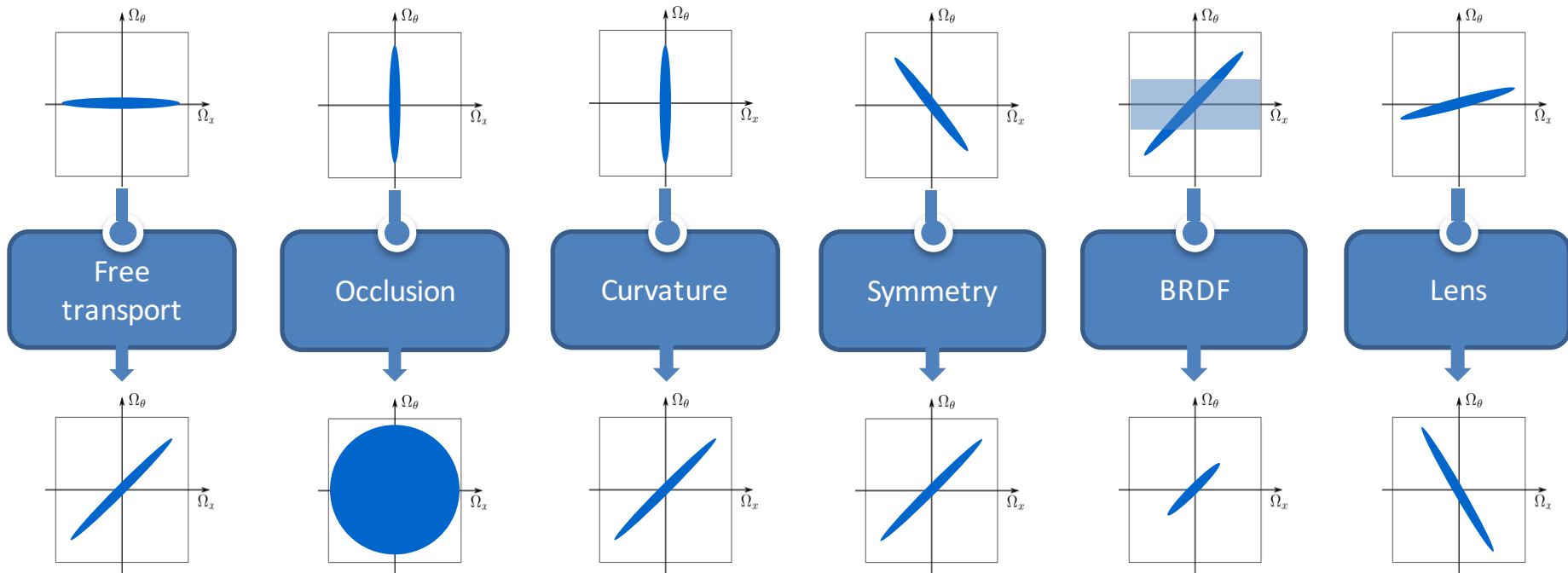
# Covariance tracing



# Covariance tracing



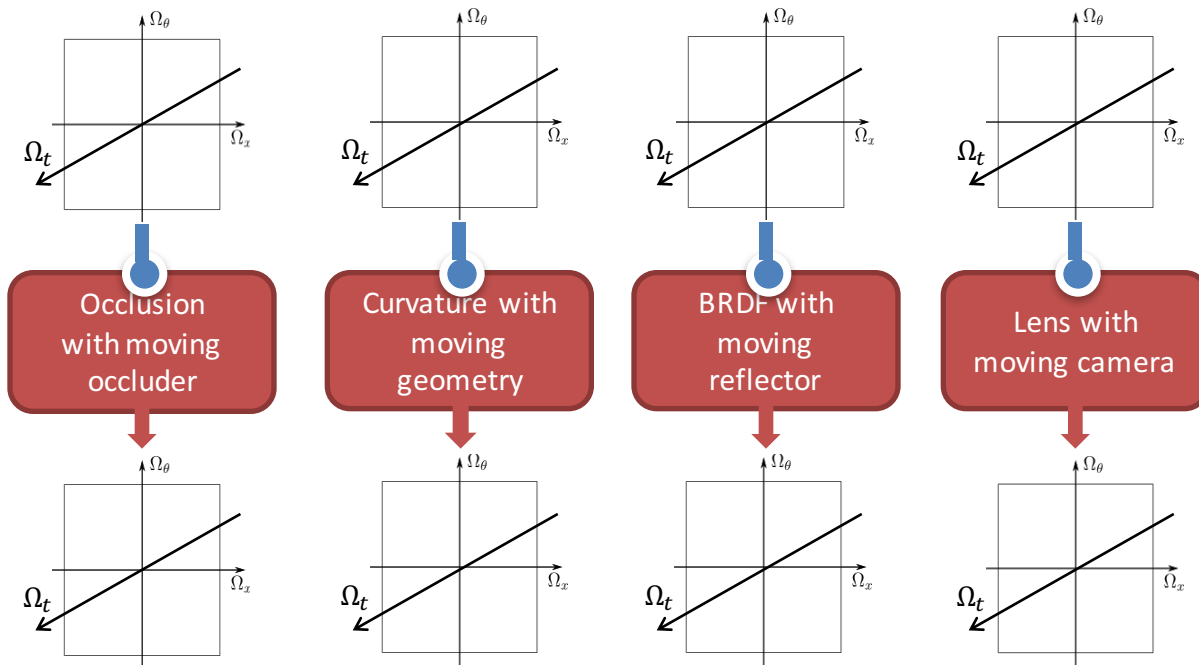
# Just a chain of operators



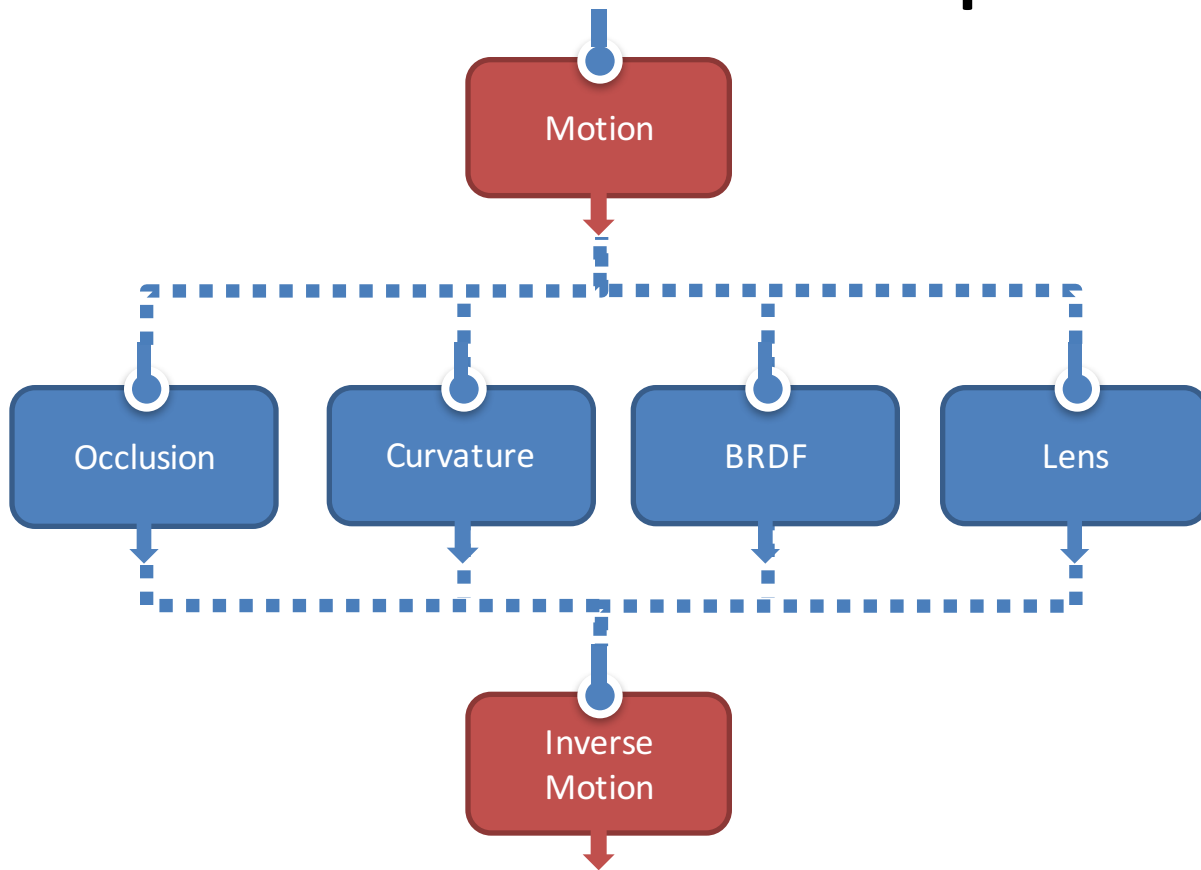




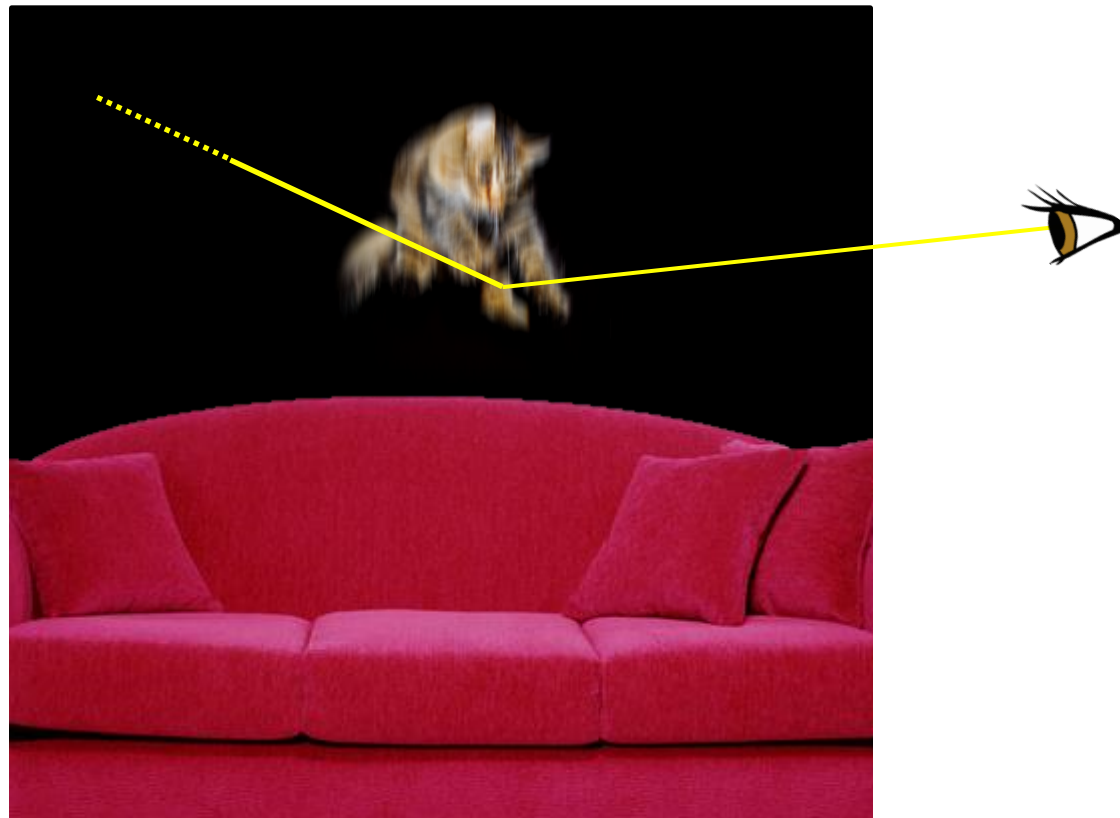
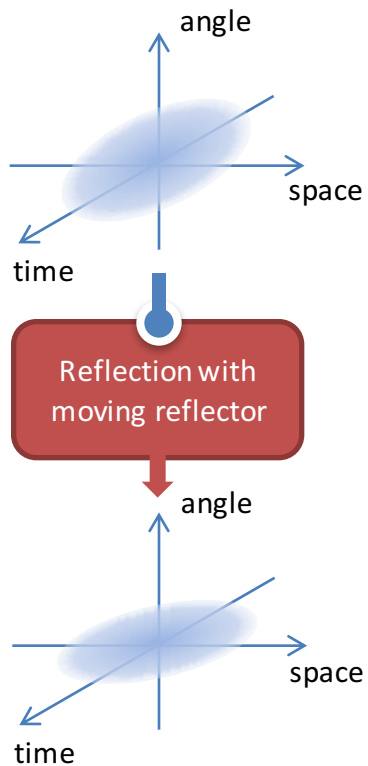
# We could rewrite all operators...



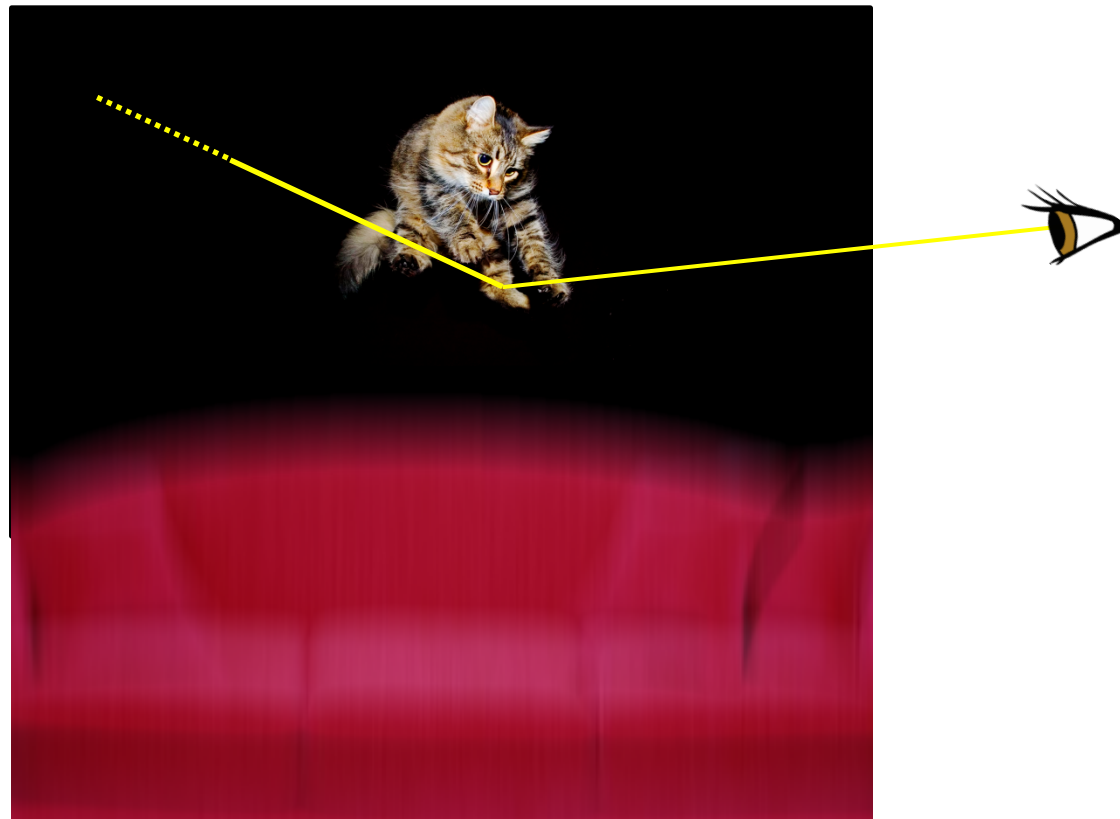
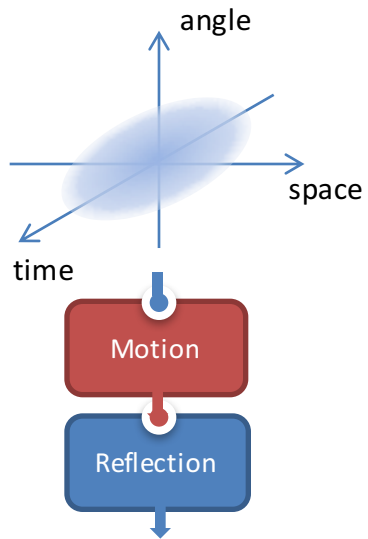
# We will not rewrite all operators!



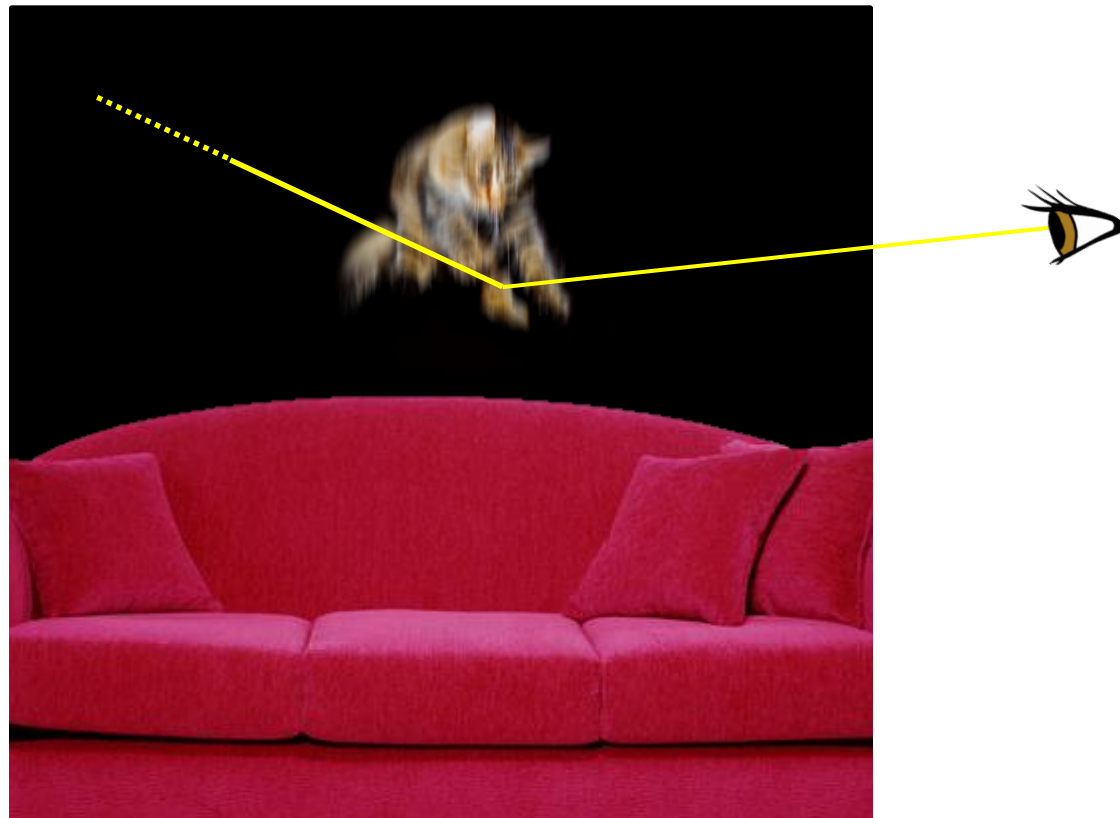
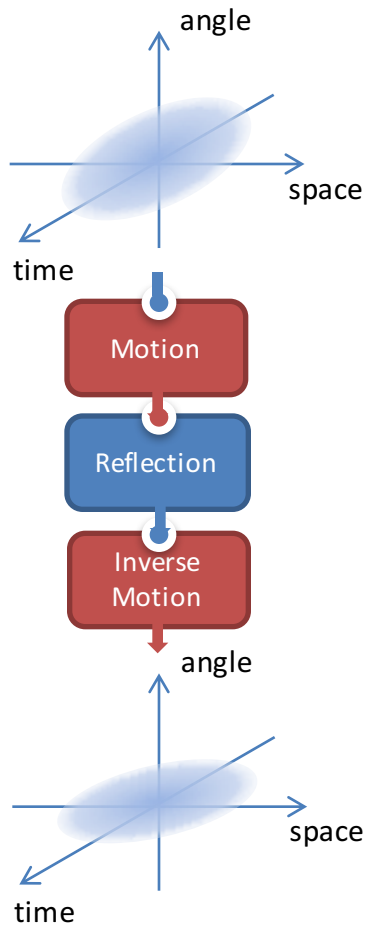
# Motion operator



# Motion operator

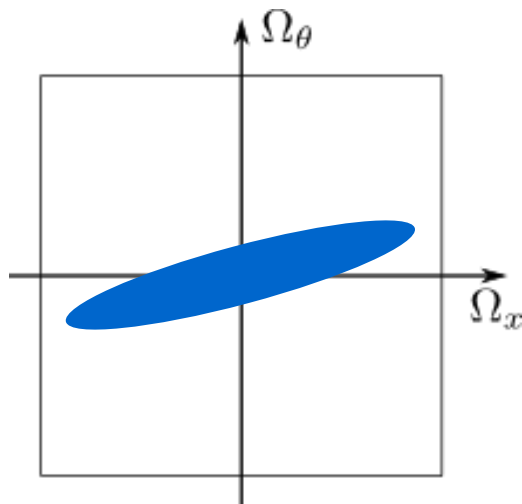
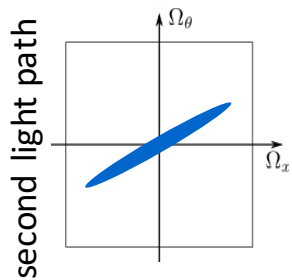
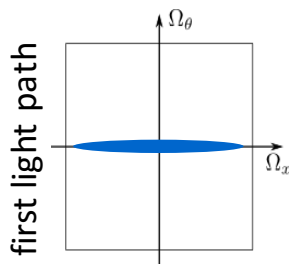


# Motion operator

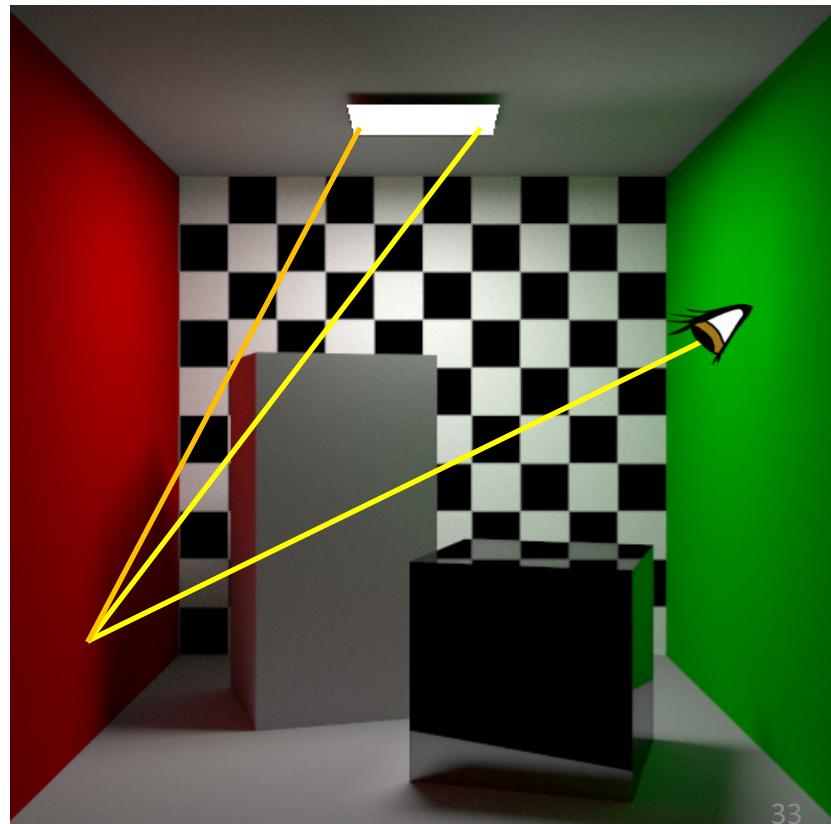


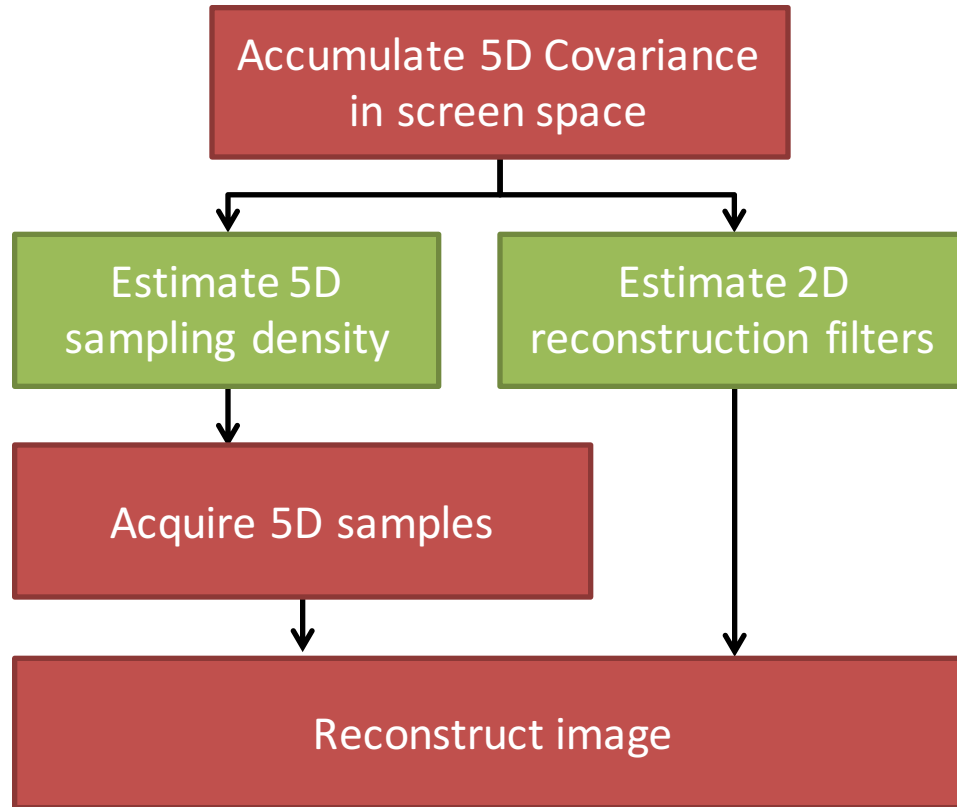


# Accumulate covariance



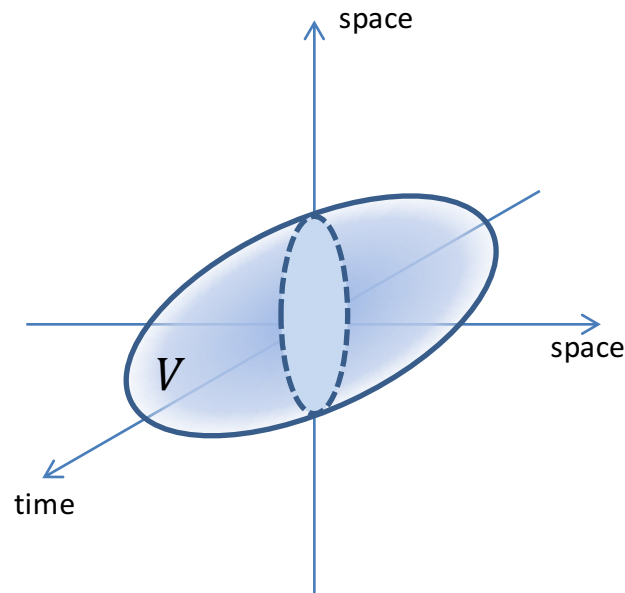
final covariance

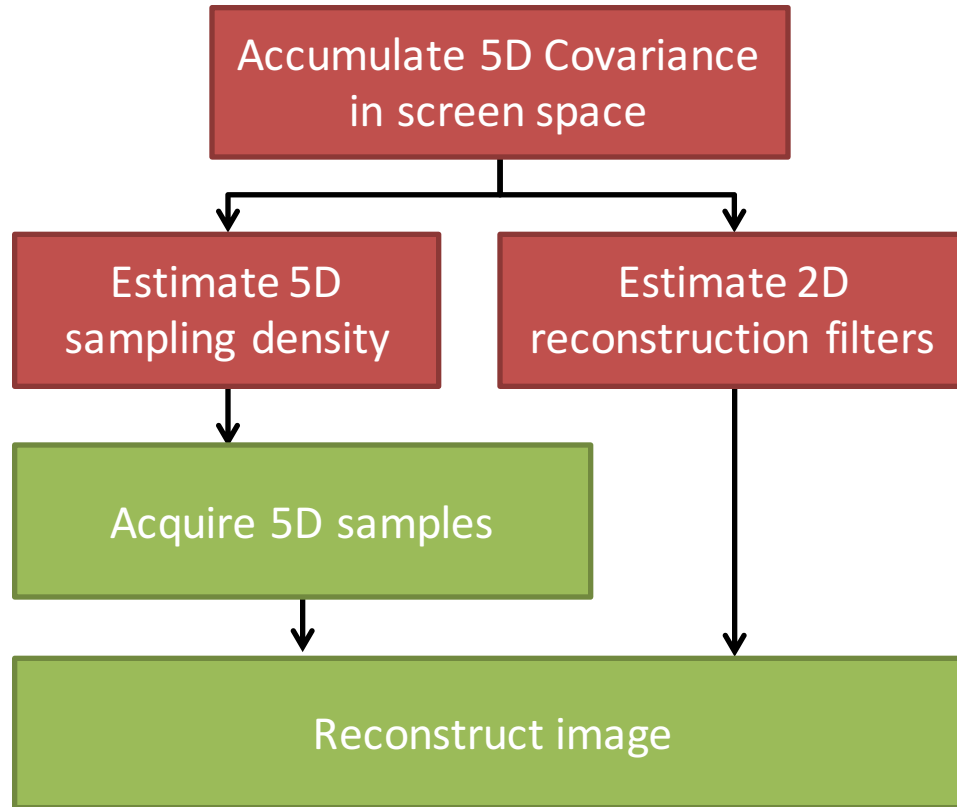




# Using covariance information

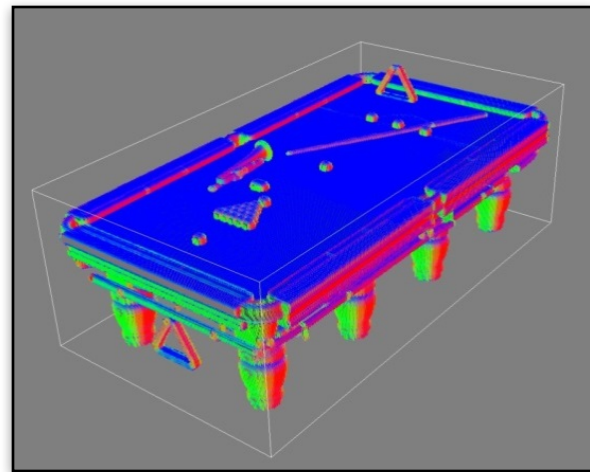
- How can we extract bandwidth ?
  - Using the volume
  - Determinant of the covariance
- How can we estimate the filter ?
  - Frequency analysis of integration [Durand 2011]
  - Slicing the equivalent Gaussian





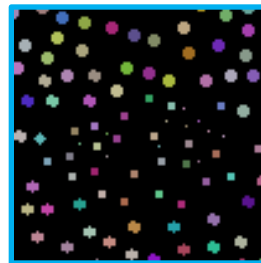
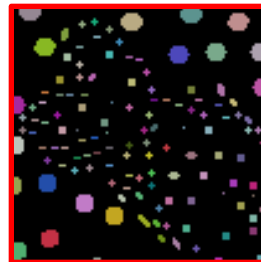
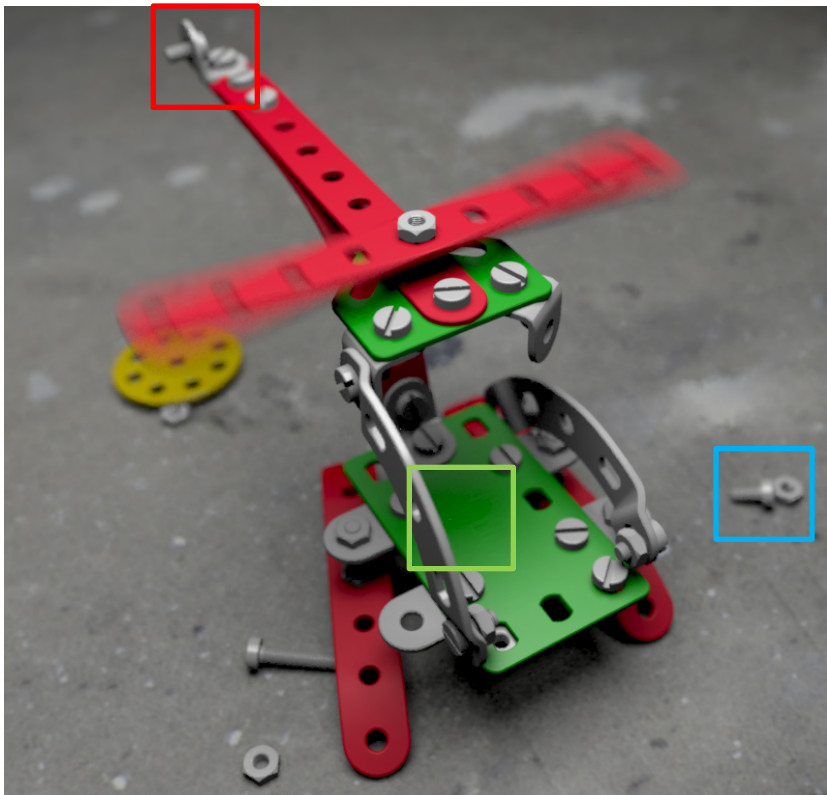
# Implementation details: occlusion

- Occlusion using a voxelized scene
- Use the 3x3 covariance of normals distribution
- Evaluate using ray marching

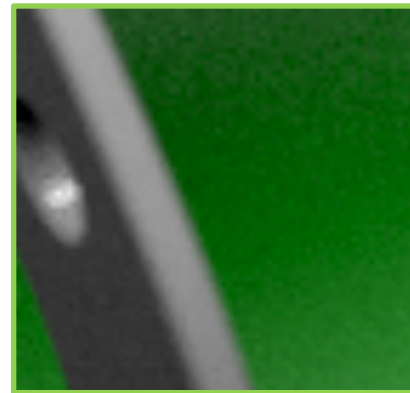




# Results: the helicopter

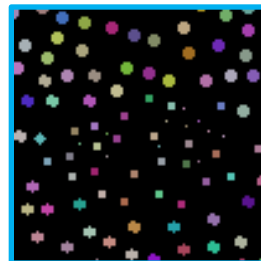
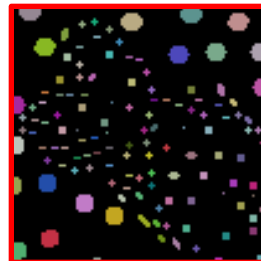
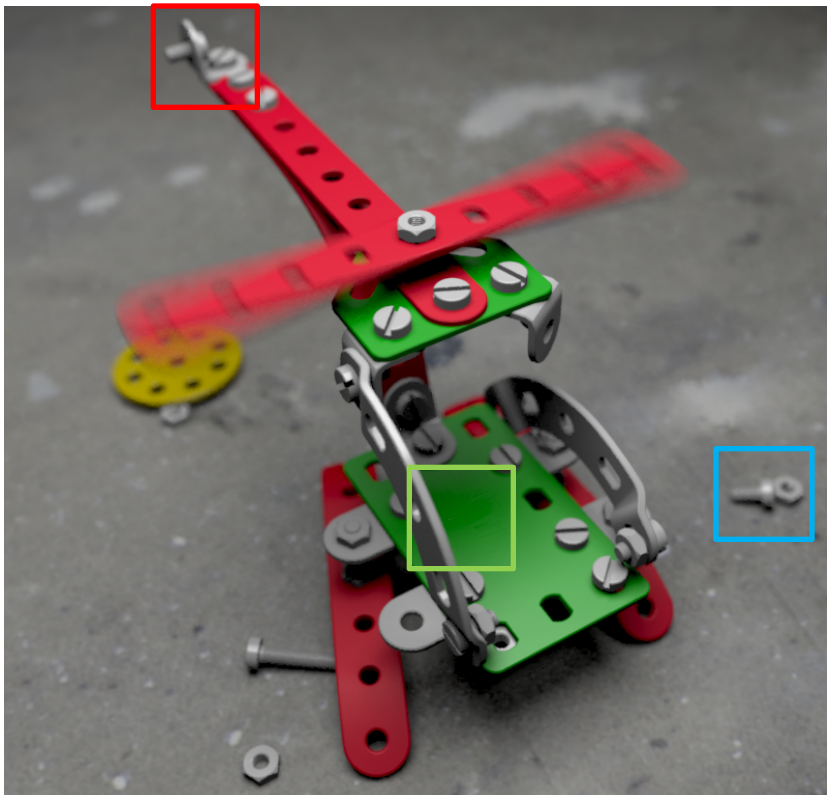


Our algorithm



Equal time Monte-Carlo $\infty$

# Results: the helicopter

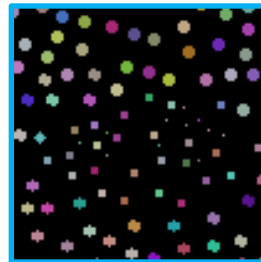
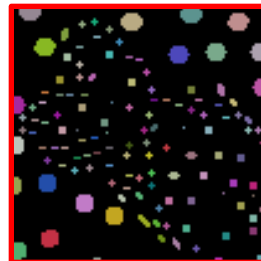
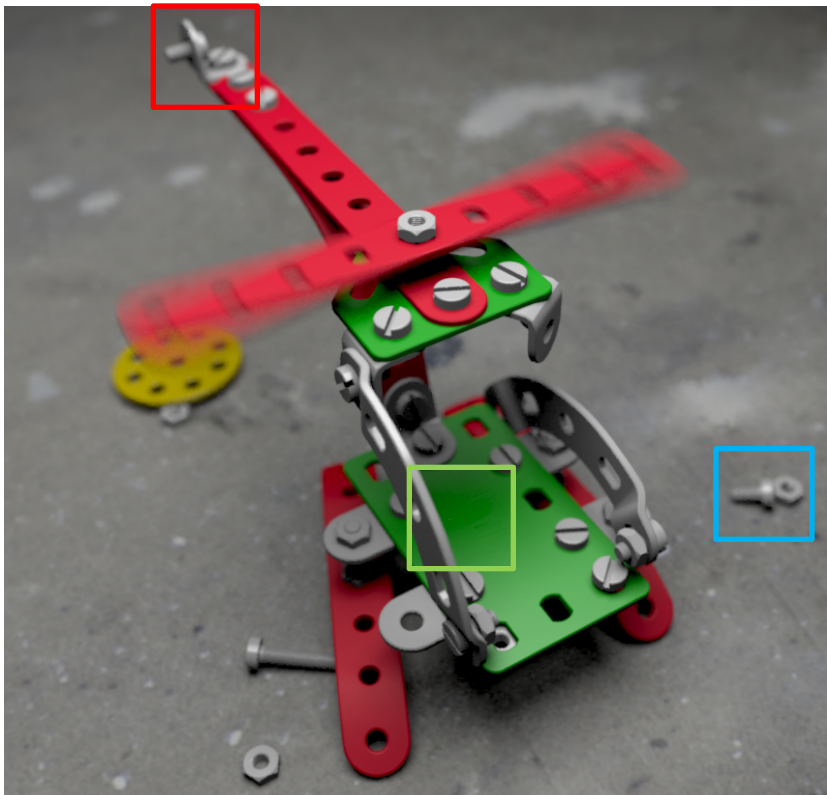


Our algorithm



Equal time Monte-Carlo

# Results: the helicopter

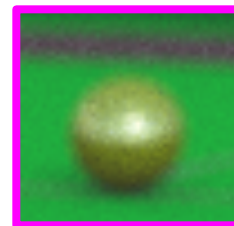
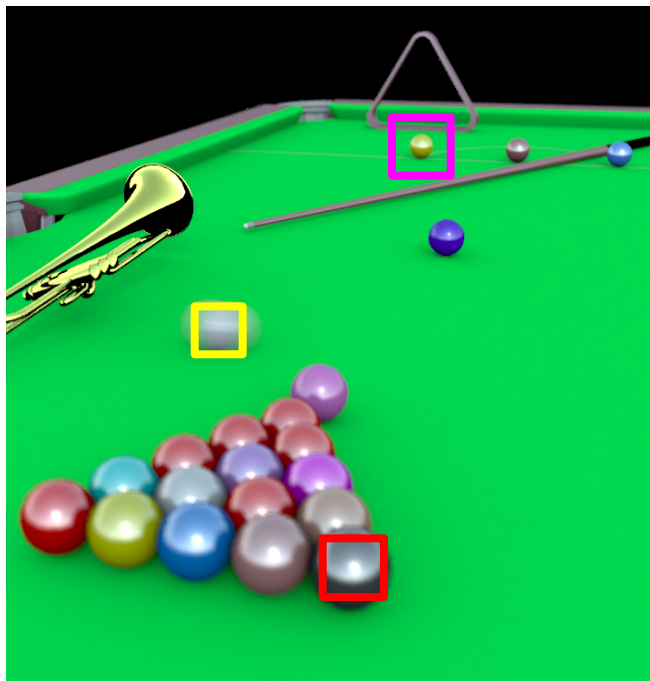


Our algorithm

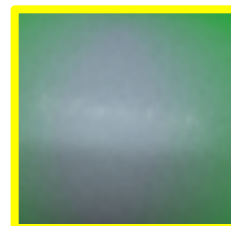


Equal time Monte-Carlo

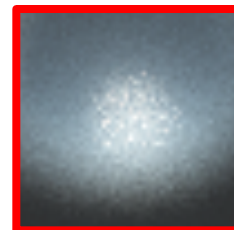
# Results: the snooker



← defocus blur



← motion blur

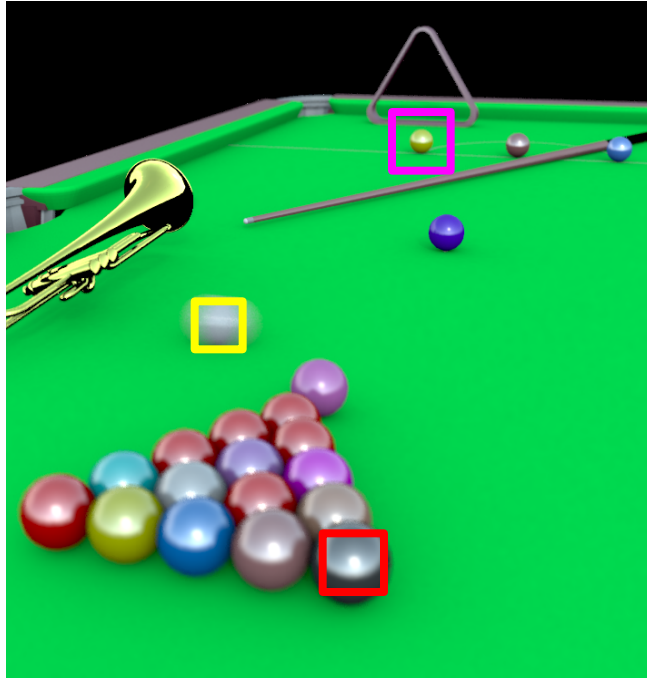


← BRDF blur

Our method

Equal-time  
Monte Carlo

# Results: the snooker



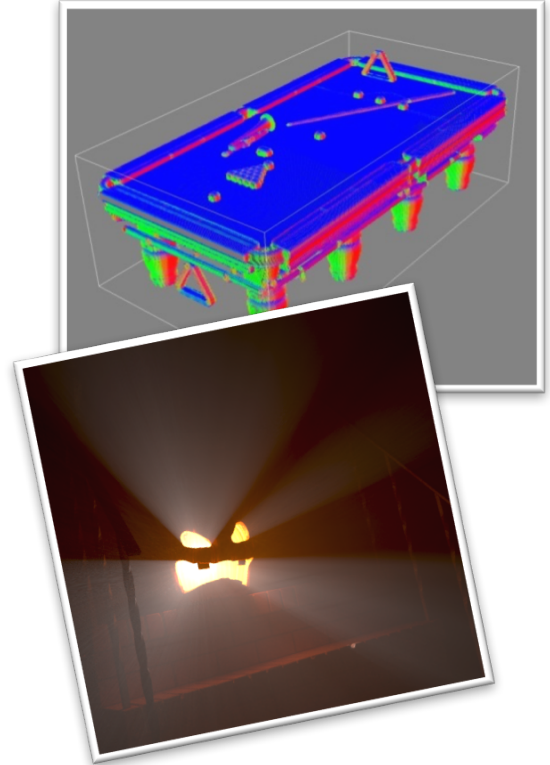
- Our method: 25min
- Eq. quality Monte Carlo: 2h25min
  - 200 light field samples per pixel
- Covariance tracing: 2min 36s
  - 10 covariance per pixel
- Reconstruction: 16s

# Conclusion

- Covariance tracing
  - Generate better light paths
  - Simple formulation
- Unified frequency analysis
  - Temporal light fields
  - No special case

# Future work

- Tracing covariance has a cost
  - Mostly due to the local occlusion query
- New operators
  - Participating media







GROUND IS MOVING!