

Additional file 1 — Theoretical details for the false positive rate

As exposed in Section “*The Bloom Data Structure index*”, the BDS index is a probabilistic data structure, that may consider a k -mer as indexed while this is not the case (*i.e.* a false positive). Here, we tried to express the false positive rate for each hash function that we defined in Section “*Particular hash functions*” and their combinations with respect to the parameter k and the number n of distinct indexed k -mers.

False positive probability for each function Assuming the base composition of the indexed and query k -mers is unbiased, we can easily compute the probability, $P_{FP}(f, k, n)$, for any query k -mer to be a false positive with one of the seven hash functions, f . This probability depends on the number of distinct k -mers sharing the same hash code. We can notice that for the balanced functions, f_1 , f_2 and f_3 , each 0 and 1 value can come from exactly 2 distinct nucleotides, thus the number of k -mers sharing the same hash code is the same for all k -mers and equals: 2^k . The probability for 2 k -mers to have distinct hash codes is then $1 - \frac{2^k}{4^k} = 1 - \frac{1}{2^k}$, and therefore the probability to have at least one k -mer among the n that are indexed sharing the same hash code is:

$$\forall i \in \{1, 2, 3\} P_{FP}(f_i, k, n) = 1 - \left(1 - \frac{1}{2^k}\right)^n \quad ((3))$$

Note that this corresponds to the false positive probability of any hash function that distributes the hash codes uniformly in a 2^k bit-array, such as those inspired of Jenkins functions, used as a comparison in Section “*Comparison with other hash functions and with a classical Bloom filter*”.

As for the unbalanced functions, since the 0 bit-value encodes only one base, the number of k -mers sharing the same hash code depends on the number of 0 in the hash code of the query. For a given query k -mer with a hash code having x 0 the above probability for functions f_4 , f_5 , f_6 and f_7 becomes: $1 - \left(1 - \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{(k-x)}\right)^n$. To obtain the probability for any k -mer, we have to sum over the different values of x the latter probability weighed by the probability for a k -mer hash code to have x 0. The composition of a given base in a k -mer of length k , assuming unbiased nucleotide composition, follows a binomial distribution, thus we get:

$$\forall i \in \{4, 5, 6, 7\} \quad P_{FP}(f_i, k, n) = \sum_{x=0}^k \binom{k}{x} a_x (1 - (1 - a_x)^n) \quad \text{with} \quad a_x = \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{k-x} \quad ((4))$$

$\binom{k}{x}$ being the binomial coefficient, ie $\binom{k}{x} = \frac{k!}{x!(k-x)!}$.

We can see in Figure 2 that balanced functions give much less false positives than unbalanced ones. This can be explained by the fact that for unbalanced functions, for a given k -mer with a “normal” composition and thus 25% of 0 in its hash-code, there are many more k -mers with the same hash-code than for a balanced

function: $3^{\frac{3k}{4}} \gg 2^k$.

FP probability for a combination of functions When combining several functions in our BDS, in order to have a false positive for a query k -mer all functions must return a false positive. As concerns the balanced function, we can easily see that for a given k -mer, we can not find another k -mer that is a false positive simultaneously for any two of these functions. In other words, there do not exist two distinct k -mers that have the same couple of hash codes with any two of these functions. This implies that the probability of having a false positive with one function does not depend on the result with another function, apart from the fact that the effective number of indexed k -mers that can be a false positive (n in equation 3) is reduced: indeed if x k -mers have the same hash code for one function, these k -mers have a null probability of having the same hash code for another function. Note that this effect can be neglected given that n is very large. Therefore the product of individual probabilities for each balanced function gives the following upper bound:

$$P_{FP}(f_1 \cap f_2 \cap f_3, k, n) \lesssim (1 - (1 - \frac{1}{2^k})^n)^3 \quad ((5))$$

Concerning the unbalanced functions, this independence property is lost, since it is possible to find a single k -mer that is a false positive for at least 2 of the unbalanced functions, or for one balanced function and at least one unbalanced. Therefore we could not figure out the theoretical false positive rate, or even an upper bound.