



**HAL**  
open science

## Extension of Analytic Signal conception on Dynamic Image Sequences

Liang Wang, Zhengjun Liu, Patrick R. Girard, Patrick Clarysse, Philippe Delachartre

► **To cite this version:**

Liang Wang, Zhengjun Liu, Patrick R. Girard, Patrick Clarysse, Philippe Delachartre. Extension of Analytic Signal conception on Dynamic Image Sequences. 9ème édition de la conférence MANifestation des JEunes Chercheurs en Sciences et Technologies de l'Information et de la Communication - MajecSTIC 2012, Nicolas Gouvy, Oct 2012, Villeneuve d'Ascq, France. hal-00780356

**HAL Id: hal-00780356**

**<https://inria.hal.science/hal-00780356>**

Submitted on 23 Jan 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Extension of Analytic Signal conception on Dynamic Image Sequences

Liang Wang<sup>1</sup>, Zhengjun Liu<sup>2</sup>, Patrick R. Girard<sup>1</sup>, Patrick Clarysse<sup>1</sup>, Philippe Delachartre<sup>1</sup>

1: Université de Lyon, CREATIS ; CNRS UMR5220 ; Inserm U1044 ; INSA-Lyon ; Université Lyon 1, France. 7 avenue Jean Capelle, 69621 Villeurbanne - France.

2: Harbin Institute of Technology, HIT-INSA Sino French Research Center for Biomedical Imaging, 150001, Harbin - China

Contact: liang.wang@creatis.insa-lyon.fr

---

### Résumé

Cet article présente un cadre algébrique et une mise en œuvre pour le signal analytique 2D+t à l'aide des biquaternions de Clifford et de la Transformée de Fourier Cliffordienne. La contribution de ce travail est de définir et réaliser le signal analytique spatio-temporel pour analyser des séquences vidéo alors que la plupart des travaux précédents ont traité du signal analytique pour les images 2D. À partir du signal analytique 2D+t que nous définissons, plusieurs paramètres physiques liés à la dimension du temps et de l'espace pourront être calculés. Nous détaillons ici la méthode de calcul numérique pour obtenir le signal analytique spatio-temporel. Dans cet article, on considère l'exemple de simulation d'une onde plane progressive et d'une onde modulée en amplitude. À partir de ces exemples, on montre le calcul algébrique et numérique du signal hyperanalytique, puis l'extraction des phases pour l'estimation de paramètres physiques tels que la fréquence instantanée, la vitesse de phase et la vitesse de groupe. Les résultats numériques obtenus permettent de valider l'approche et montrent le potentiel de la méthode pour l'estimation de paramètres physiques.

### Abstract

This paper presents an algebraic framework and an implementation for the 2D + t analytic signal by using Clifford biquaternions and Clifford Fourier transform. The contribution of this work is to define and realize the analytical signal for space-time analysis of video sequences, whereas most of the previous works are based on 2D images. It enables us to obtain the physical parameters related to time dimension from some specific cases of 2D + t analytic signal. Here, we describe the numerical calculation method to obtain the space-time analytical signal. Considering the simulation example of a progressive plane wave and an amplitude modulated wave, the calculation of algebraical and numerical hyper-analytic is introduced. The physical parameters of the phase function, such as instantaneous frequency, phase velocity and group velocity are extracted. Numerical results are given for validating the proposed method and show the potential of physical parameters estimating method.

**Mots-clés :** biquaternion de Clifford, Transformée de Fourier Clifford, signal analytique 2D+t, paramètres physiques

**Keywords:** Clifford biquaternions, Clifford Fourier transforms, 2D+t analytic signals, physical parameters

---

### 1. Introduction

During the last years, the quaternion-based method has been applied to image analysis and signal processing based on Clifford algebras [5,7]. As a mathematical tool, it has been used for motion

detection, motion estimation and tracking [1,6,9]. It's also used for edge detection of color image thanks to its multicomponent property [8].

Generally, phase information can be obtained with quaternion-based method. Some advantages of phase information are known as that [3]: (1) there is strong evidence that the human visual system makes use of local phase; (2) phase-based processing is to a large extent invariant to changes of lighting conditions; (3) the reconstruction of an image from phase information is much better than that from amplitude information. Furthermore, several works have shown previously the advantage of using phase information to analyse images and image sequences. Bülow and Sommer [2] found that a component of the quaternionic phase is sensitive to the change of a structure like a flaw in a textile. In 2006, using hypercomplex phase, Witten and Shragge [10] proposed an image disparity estimation method to find differences between subtly varying images. Woo *et al.* [11] used both the local phase information and the intensity information for ultrasound image registration. Felsberg [3] used the monogenic phase for optical flow estimation and indicated several advantages of phase information.

However, most of these developments above used the approaches to process only two-dimension images or image sequences. It means that the dimension of time was not involved when calculating the analytic signal and the phase information of analytic signal. In order to deal with this problem and to process an entire video signal at once, a  $2D + t$  analytic video signal is defined in this paper. Therefore, it has three components: time, vertical position and horizontal position. With the objective to process an entire video signal at once, a biquaternion algebraic framework is developed to express Clifford Fourier Transforms(CFT) of  $2D + t$  video analytic signals in standard form. On the specific case of a progressive plane wave, the physic parameters are found, which could be useful to estimate the motion of the progressive plane wave.

## 2. Clifford Biquaternion $2D+t$ Analytic Signal: Implementation

In  $2D + t$  dimensions, from the introduction of Clifford biquaternion Algebra in [4], we have the full algebra elements:

$$\left[ \begin{array}{cccc} 1 & i = e_2 e_3 & j = e_3 e_1 & k = e_1 e_2 \\ \epsilon = i'I = -e_1 e_2 e_3 & e_1 = \epsilon i & e_2 = \epsilon j & e_3 = \epsilon k \end{array} \right]. \quad (1)$$

### 2.1. Clifford Fourier Transform

Given a function  $f(x)$  having its value in the Clifford algebra with  $x = (x_1, x_2, x_3)$ , and let  $F(u)$  with  $u = (u_1, u_2, u_3)$  denotes the biquaternion CFT:

$$F(u) = \int_{\mathbb{R}^3} f(x) e^{-\epsilon i 2\pi u_1 x_1} e^{-\epsilon j 2\pi u_2 x_2} e^{-\epsilon k 2\pi u_3 x_3} dx_1 dx_2 dx_3. \quad (2)$$

Thus, here the function  $F(u)$  is a biquaternion of eight algebra elements that introduced in Eq. 1. The inverse CFT is given by:

$$f(x) = \int_{\mathbb{R}^3} F(u) e^{\epsilon k 2\pi u_3 x_3} e^{\epsilon j 2\pi u_2 x_2} e^{\epsilon i 2\pi u_1 x_1} du_1 du_2 du_3. \quad (3)$$

To compute the direct CFT, one proceeds in cascade integrating first with respect to  $x_1$  using a standard FFT. The second FFT (integration with respect to  $x_2$ ) is then applied on each real component of the previous complex number. Then, the third FFT (integration on  $x_3$ ) is applied on each of the resulting real components. Finally, all the components are properly displayed as a Clifford biquaternion. For the inverse CFT, one proceeds in the same way on each real component of the Clifford biquaternion using IFFT and reversing the order of integration. Due to the symmetries of the CFT of a scalar function  $f(x_1, x_2, x_3)$ , only one orthant of the Fourier space is necessary to obtain the entire Fourier space.

### 2.2. Analytic Signal

The analytic Clifford Fourier transform is defined by:

$$F_A(u) = [1 + \text{sign}(u_1)] [1 + \text{sign}(u_2)] [1 + \text{sign}(u_3)] F(u), \quad (4)$$

and the analytic signal by:

$$f_A(x) = \int_{\mathbb{R}^3} F_A(u) e^{ik_2\pi u_3 x_3} e^{ej2\pi u_2 x_2} e^{ei2\pi u_1 x_1} du_1 du_2 du_3. \quad (5)$$

### 3. Application of Analytic Signal

To put the analytic signal into application, a progressive plane wave and an amplitude modulated wave are created by cosine functions. Then their analytic video signals are calculated. Subsequently, some physic parameters from the phase of analytic video signal can be found, such as wave velocity, instantaneous frequency, phase velocity and group velocity.

#### 3.1. Example 1: progressive plane wave and its instantaneous frequency and wave velocity

Consider the progressive plane wave given by:

$$f_1(t, x, y) = \cos(\omega t + k_x x + k_y y), \quad (6)$$

where  $\omega$ ,  $k_x$  and  $k_y$  are the pulsations of the plane wave.  $t$  represents the time axis,  $x$  represents the horizontal axis, and  $y$  represents the vertical axis of a frame in an image sequence. The analytic signal is given by:

$$f_A(t, x, y) = \begin{bmatrix} \cos(\omega t + k_x x + k_y y) \\ -\cos(\omega t - k_x x + k_y y) \\ \cos(\omega t + k_x x - k_y y) \\ -\cos(\omega t - k_x x - k_y y) \end{bmatrix} + \varepsilon \begin{bmatrix} \sin(\omega t - k_x x + k_y y) \\ \sin(\omega t + k_x x + k_y y) \\ -\sin(\omega t - k_x x - k_y y) \\ -\sin(\omega t + k_x x - k_y y) \end{bmatrix}. \quad (7)$$

Based on the analytic signal given by Eq. 7, the instantaneous frequency and wave velocity can be obtained as follows:

$$\phi_1 = \text{angle}[f_A(1) + if_A(6)] = \text{atan2}(FA(1), FA(6)) = \omega t + k_x x + k_y y, \quad (8)$$

where  $FA(1)$  means the first component of  $f_A$  and  $FA(6)$  the sixth component of  $f_A$ . The  $\text{atan2}()$  function is the four quadrant arctangent that used in [2]. Then, the instantaneous frequency  $\omega$ ,  $k_x$ ,  $k_y$  are given by:

$$\omega = \frac{\partial \phi_1}{\partial t}, \quad k_x = \frac{\partial \phi_1}{\partial x}, \quad k_y = \frac{\partial \phi_1}{\partial y}, \quad (9)$$

next, the wave velocities correspond to horizontal axis  $x$  and vertical axis  $y$  are given by:

$$v_x = -\frac{\omega}{k_x}, \quad v_y = -\frac{\omega}{k_y}. \quad (10)$$

Thus, from this example, we can get the velocities of this progressive plane wave along horizontal and vertical axis separately by using its analytic signal. Thanks to the 2D+t analytic signal, the equations of these velocities in Eq. 10 are continuous with respect to the time axis.

#### 3.2. Example 2: Amplitude modulated wave and its phase velocity and group velocity

Consider the function

$$f_2(t, x, y) = \cos(\omega_1 t + k_{x1} x + k_{y1} y) + \cos(\omega_2 t + k_{x2} x + k_{y2} y), \quad (11)$$

with  $\omega_i = \Omega \pm \frac{\omega}{2}$ ,  $k_{xi} = K_x \pm \frac{k_x}{2}$ ,  $k_{yi} = K_y \pm \frac{k_y}{2}$  and  $\Omega > \omega$ ,  $K_x > k_x$ ,  $K_y > k_y$ . The analytic signal is given by:

$$f_A(t, x, y) = \begin{bmatrix} 2 \cos(\omega t + k_x x + k_y y) \cos(\Omega t + K_x x + K_y y) \\ -2 \cos(\omega t - k_x x + k_y y) \cos(\Omega t - K_x x + K_y y) \\ 2 \cos(\omega t + k_x x - k_y y) \cos(\Omega t + K_x x - K_y y) \\ -2 \cos(\omega t - k_x x - k_y y) \cos(\Omega t - K_x x - K_y y) \end{bmatrix} + \varepsilon \begin{bmatrix} 2 \cos(\omega t - k_x x + k_y y) \sin(\Omega t - K_x x + K_y y) \\ 2 \cos(\omega t + k_x x + k_y y) \sin(\Omega t + K_x x + K_y y) \\ -2 \cos(\omega t - k_x x - k_y y) \sin(\Omega t - K_x x - K_y y) \\ -2 \cos(\omega t + k_x x - k_y y) \sin(\Omega t + K_x x - K_y y) \end{bmatrix}. \quad (12)$$

Then computes

$$d = [f_A(1)]^2 + [f_A(6)]^2 = 4 \cos^2(\omega t + k_x x + k_y y) = 2\{1 + \cos [2(\omega t + k_x x + k_y y)]\}. \quad (13)$$

The phase and group velocities are obtained as follows:

$$\Phi_1 = \Omega t + K_x x + K_y y = \text{atan2}(FA(1), FA(6)), \quad (14)$$

$$\Phi_2 = \omega t + k_x x + k_y y = \frac{1}{2} \arccos\left(\frac{d}{2} - 1\right), \quad (15)$$

then the phase velocities  $v_{\phi x}, v_{\phi y}$  and group velocities  $v_{gx}, v_{gy}$  are:

$$v_{\phi x} = -\frac{\frac{\partial \Phi_1}{\partial t}}{\frac{\partial \Phi_1}{\partial x}} = -\frac{\Omega}{K_x}, \quad v_{\phi y} = -\frac{\frac{\partial \Phi_1}{\partial t}}{\frac{\partial \Phi_1}{\partial y}} = -\frac{\Omega}{K_y}, \quad (16)$$

$$v_{gx} = -\frac{\frac{\partial \Phi_2}{\partial t}}{\frac{\partial \Phi_2}{\partial x}} = -\frac{\omega}{k_x}, \quad v_{gy} = -\frac{\frac{\partial \Phi_2}{\partial t}}{\frac{\partial \Phi_2}{\partial y}} = -\frac{\omega}{k_y}. \quad (17)$$

Thus, from the analytic signal of this amplitude modulated wave, we can get the phase velocities and the group velocities along horizontal and vertical axis separately. Thanks to the 2D+t analytic signal, the equations of these velocities in Eq. 16 and Eq. 17 are continuous with respect to the time axis.

#### 4. Numerical Approximation

The numerical approximation is calculated via MatLab. For the progressive plane wave video signal  $f_1(t, x, y) = \cos(\omega t + k_x x + k_y y)$ , its analytic video signal  $f_A$  is shown in Fig. 1. The range of  $t, x, y$  is normalized into  $[0, 1]$  and the pulsation values are  $\omega = k_x = k_y = 2\pi$ . From Eq. 8, there is the numerical presentation in Fig. 2 of the phase function. Next, by Eq. 9, the instantaneous frequencies are obtained from  $\phi_1$ . For a fixed value of horizontal index  $X$  and vertical index  $Y$ ,  $\omega$  can be found. The value is equal to  $6.283 (= 2\pi)$ , which corresponds to the pulsation value of  $\omega$  of input video signal. Meanwhile, the instantaneous frequencies  $k_x$  and  $k_y$  can be obtained correctly by the same way. Fig. 3 shows the  $\omega$  value and a profile of  $\phi_1$  from Fig. 2(a). Hence, the wave velocity can be calculated by Eq. 10, i.e.,  $V_x = -\frac{\omega}{k_x} = -\frac{6.283}{6.283} = -1 \text{ m} \cdot \text{s}^{-1}$  and  $V_y = -\frac{\omega}{k_y} = -\frac{6.283}{6.283} = -1 \text{ m} \cdot \text{s}^{-1}$ . The error between the algebraical and numerical result is less than  $1 \times 10^{-13} \approx 0$ .

Considering another plane wave signal  $f_2(t, x, y) = \cos(\omega_1 t + k_{x1} x + k_{y1} y) + \cos(\omega_2 t + k_{x2} x + k_{y2} y)$ , the range of  $x, y$  is normalized into  $[-0.5, 0.5]$  and the range of  $t$  is  $[0, 2\pi/\omega_1]$ . The pulsation values are  $\omega_1 = \omega_2 = 2 \text{ rad} \cdot \text{s}^{-1}$ ,  $k_{x1} = 8 \text{ rad} \cdot \text{m}^{-1}$ ,  $k_{x2} = 4 \text{ rad} \cdot \text{m}^{-1}$ ,  $k_{y1} = 2 \text{ rad} \cdot \text{m}^{-1}$ ,  $k_{y2} = 1 \text{ rad} \cdot \text{m}^{-1}$ . As the result, the phase of the analytic signal  $\Phi_1$  with unwrap and  $\Phi_2$  without unwrap are shown in Fig. 4, which corresponds to Eq. 14. Subsequently, by partial differential, we obtain the frequency parameters  $\Omega, K_x$  and  $K_y$  of phase  $\Phi_1$ , that are presented in Fig. 5. The values of these parameters correspond correctly to the algebraical results of  $K_x, K_y$  and  $\Omega$  in Eqs. 11 and 12. Hence, the phase velocities are calculated as:  $v_{\phi x} = \Omega/K_x = 1/3 \text{ m} \cdot \text{s}^{-1}$ ,  $v_{\phi y} = \Omega/K_y = 4/3 \text{ m} \cdot \text{s}^{-1}$ . Meanwhile, we obtain also the frequency parameters  $\omega, k_x$  and  $k_y$  of phase  $\Phi_2$  shown in Fig. 6.

#### 5. Conclusion

The paper has presented a concrete algebraic framework, i.e., Clifford's biquaternions for the expression of Clifford Fourier transforms and 2D + t analytic signals. Then, we have shown firstly how to extract the instantaneous frequency and wave velocity from a progressive plane wave, and secondly how to extract the phase velocity and group velocity from an amplitude modulated wave. Finally, with the numerical approximation, we have shown that these physical parameters can be recovered correctly. Our next work is to develop the method to estimate the motion from 2D+t medical image sequences based on the phase information of image sequences.

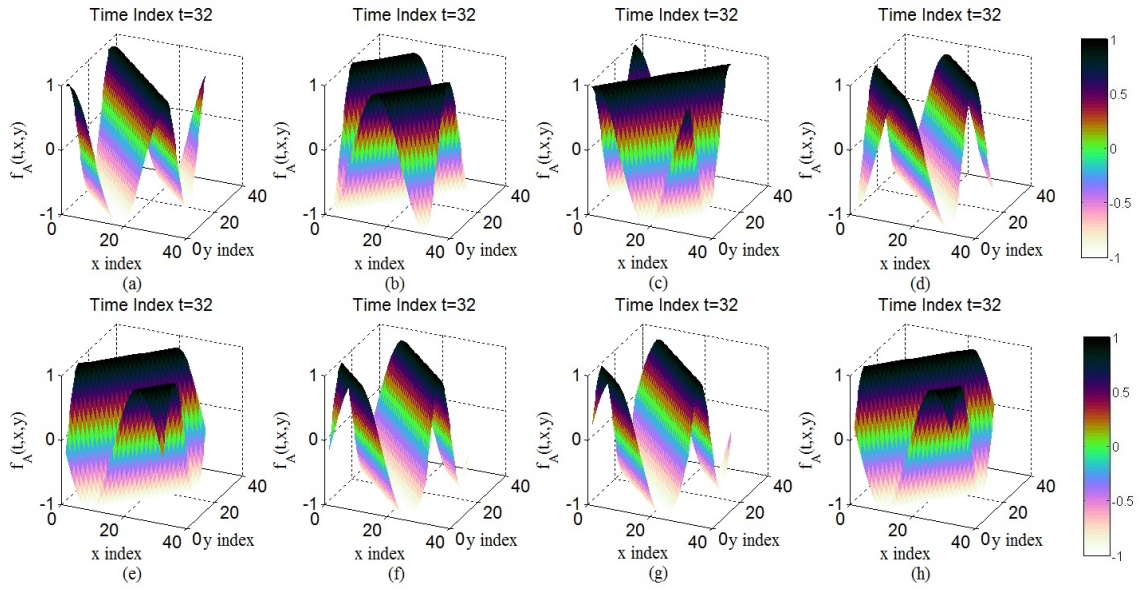


Figure 1: The eight components of the analytic video signal:  $f_A = [a, b, c, d] + \epsilon[e, f, g, h]$  at  $t$  index= 32, with the 32 sample points for each  $t$ ,  $x$  and  $y$ .

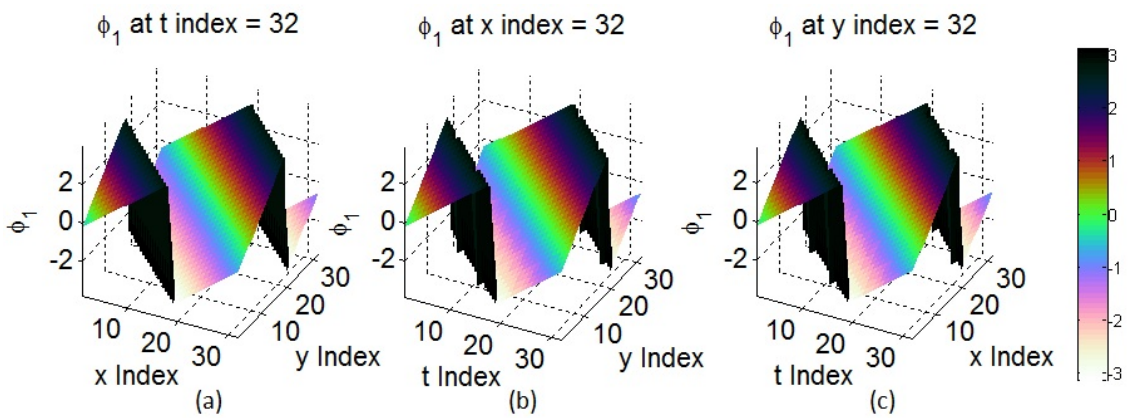


Figure 2: The phase  $\phi_1$  in 3 point of views at a fixed value of  $t$  index=32,  $x$  index=32 and  $y$  index=32 separately.

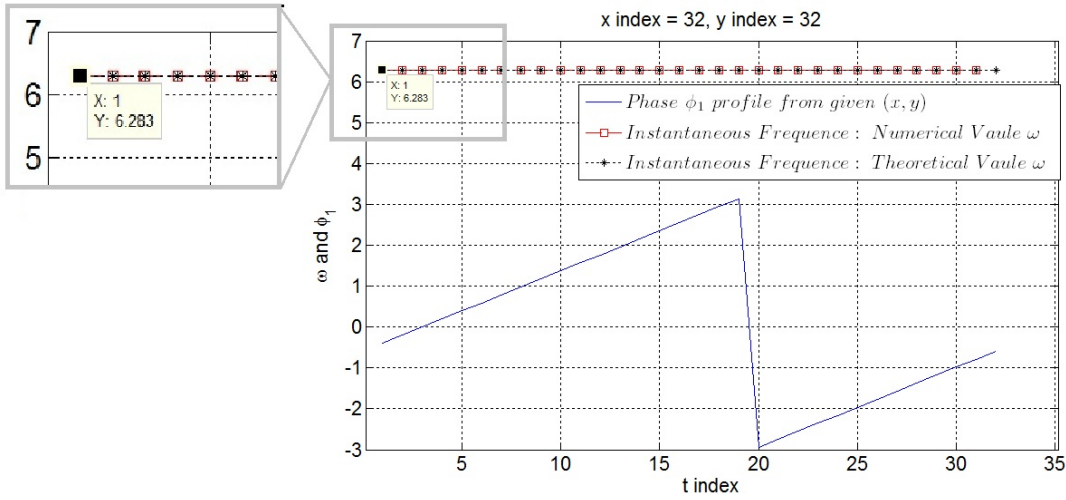


Figure 3:  $\omega$  value from a profile of  $\phi_1$  on  $x$  index=32 and  $y$  index=32. The continuous line represents the profile of  $\phi_1$ , the square symbol line is the numerical value of instantaneous frequency  $\frac{\partial \phi_1}{\partial t}$ , that equals to the algebraical value  $\omega = 2\pi$ (star symbol line).

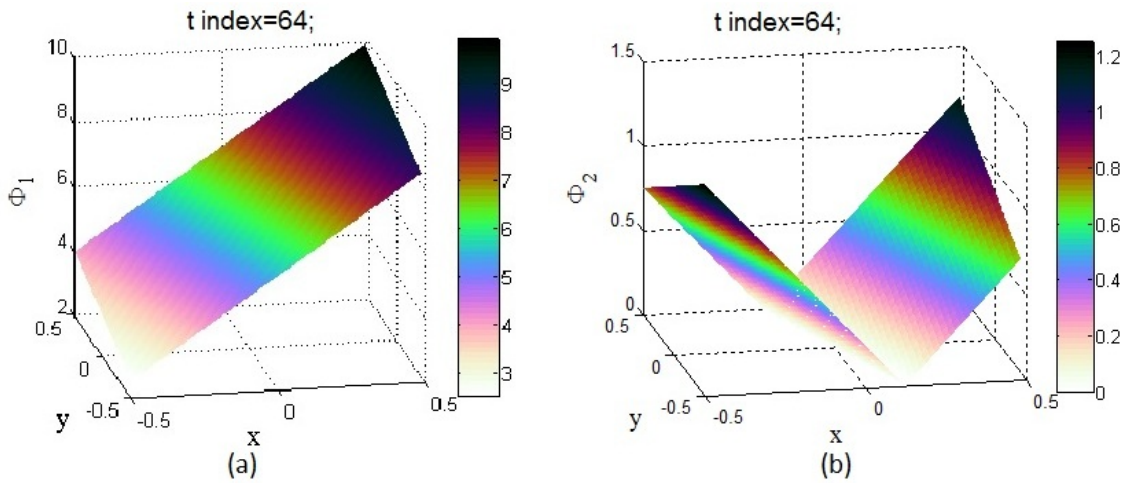


Figure 4: (a) The phase  $\Phi_1(t, x, y)$  with unwrap at  $t$  index= 64. (b) The phase  $\Phi_2(t, x, y)$  without unwrap at  $t$  index= 64.

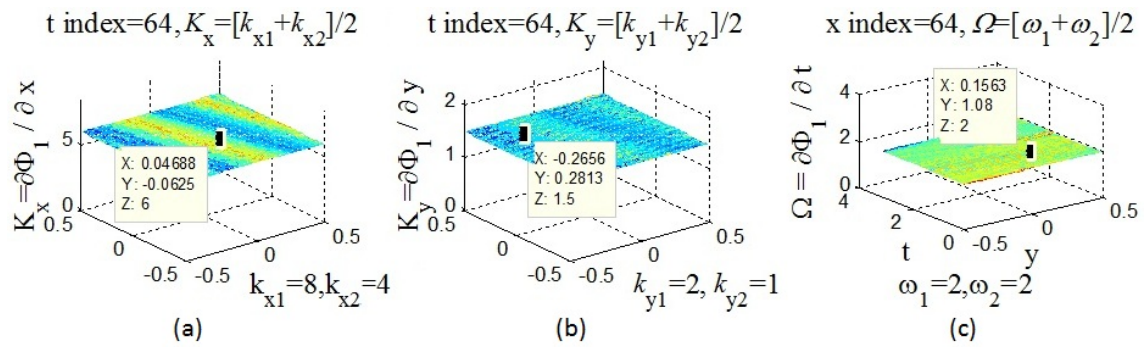


Figure 5: Three frequency parameters of  $\Phi_1$  at a given index separately: (a)  $K_x = 6 \text{ rad} \cdot \text{m}^{-1}$  at  $t$  index=64, (b)  $K_y = 1.5 \text{ rad} \cdot \text{m}^{-1}$  at  $t$  index=64, (c)  $\Omega = 2 \text{ rad} \cdot \text{s}^{-1}$  at  $x$  index=64; The values of these parameters correspond correctly to the algebraical results of  $K_x$ ,  $K_y$  and  $\Omega$  in Eqs. 12.

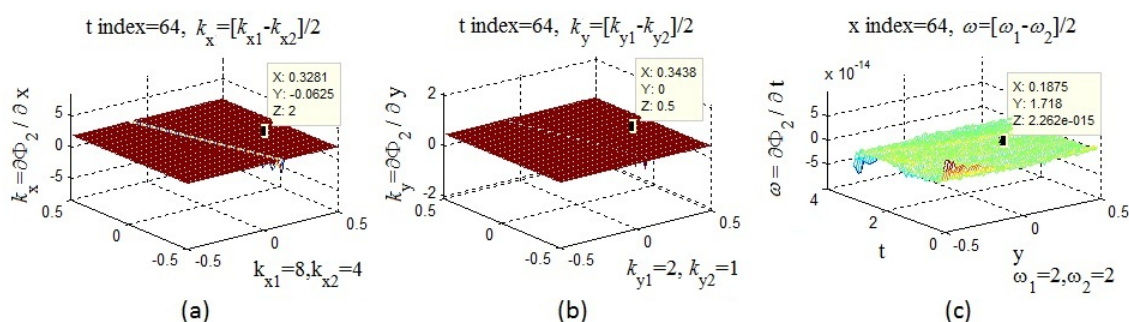


Figure 6: Three frequency parameters of  $\Phi_2$  at a given index separately: (a)  $k_x = 2 \text{ rad} \cdot \text{m}^{-1}$  at  $t \text{ index}=64$ , (b)  $k_y = 0.5 \text{ rad} \cdot \text{m}^{-1}$  at  $t \text{ index}=64$ , (c)  $\omega \in \pm 5 \times 10^{-14} \approx 0 \text{ rad} \cdot \text{s}^{-1}$  at  $x \text{ index}=64$ ; The values of these parameters correspond correctly to the algebraical results of  $k_x, k_y$  and  $\omega$  in Eqs. 12.

---

This work was supported by BQR INSA Lyon and conducted in the framework of the LabEX PRIMES (“Physics Radiobiology Medical Imaging and Simulation”).

---

## Bibliographie

1. T. Bülöw. *Hypercomplex spectral signal representations for the processing and analysis of images*. Christian-Albrechts-Universität, 1999.
2. T. Bülöw and G. Sommer. Hypercomplex signals—a novel extension of the analytic signal to the multidimensional case. *IEEE Transactions on Signal Processing*, 49(11):2844–2852, November 2001.
3. Michael Felsberg. Optical flow estimation from monogenic phase. In *Proceedings of the 1st international conference on Complex motion, IWCM’04*, page 1–13, Berlin, Heidelberg, 2007. Springer-Verlag.
4. P.R. Girard, R. Pujol, P. Clarysse, A. Marion, R. Goutte, and P. Delachartre. Analytic video (2d+t) signals by clifford fourier transforms in multiquaternion grassmann-hamilton-clifford algebras. In *9th International Conference on Clifford Algebras and Their Applications in Mathematical Physics*, Editor, K. Gürlebeck., Weimar, Germany, July 2011.
5. S.L. Hahn and K.M. Snopce. The unified theory of n-dimensional complex and hypercomplex analytic signals. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, 59(2):167–181, 2011.
6. Martin Krause and Gerald Sommer. A 3D isotropic quadrature filter for motion estimation problems. In *Visual Communications and Image Processing*, volume 5960 of *Proceedings of SPIE - The International Society for Optical Engineering*, pages 1295–1306, Beijing, China, July 2005. SPIE.
7. Nicolas Le Bihan and Stephen Sangwine. Hypercomplex analytic signals : extension of the analytic signal concept to complex signals. In *Proceedings of the 15th European Signal Processing Conference, EUSIPCO-2007*, pages A5P–H–8376, Poznan, Pologne, September 2007.
8. S. J Sangwine and T. A Ell. Colour image filters based on hypercomplex convolution. *Vision, Image and Signal Processing, IEEE Proceedings -*, 147(2):89–93, April 2000.
9. Leonardo Traversoni. Movement detection and recognition with quaternion wavelets. In *International Conference on the Applications of Computer Science and Mathematics in Architecture and Civil Engineering*, 2006.
10. Ben Witten and Jeff Shragge. Quaternion-based signal processing. In *2006 SEG Annual Meeting*, New Orleans, Louisiana, 2006. Society of Exploration Geophysicists.
11. Jonghye Woo, Byung-Woo Hong, Chang-Hong Hu, K. Kirk Shung, C.-C. Jay Kuo, and Piotr J. Slomka. Non-rigid ultrasound image registration based on intensity and local phase information. *Journal of Signal Processing Systems*, 54(1-3):33–43, 2009.