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Power Control in Random Networks: The Effect of Disorder in User Positions

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***Abstract*—Consider a wireless network of transmitter-receiver pairs. The transmitters adjust their powers to maintain a particular SINR target in the presence of interference from neighboring transmitters. In this paper we analyze the optimal power vector that may achieve this target in the presence of randomness in the network. In particular, we address the scenario where the receiver pairs may be located in one of two distinct distances from their serving transmitter base. We apply concepts from random matrix theory to evaluate the asymptotic mean optimal power per link. Our analytical results show substantial agreement with numerically generated networks, not only in one-dimensional network arrays but also in two dimensional network geometries. Notably, we observe that the optimal power in random networks does not go to infinity in a continuous fashion as in regular grids. Rather, beyond a certain point, no finite power solution exists.**

I. Introduction

Transmitted power is an important resource in wireless networks and therefore power control has been crucial since the development of legacy networks. For example, the introduction of efficient power control algorithms (both closed loop and open loop), was one of the main improvements third generation CDMA-based cellular networks brought about. Several algorithms have appeared that provably allow users to obtain e.g. a minimum SINR requirement $\text{SINR}_k \geq \gamma_k$ for link k while minimizing the total power or the power per user, subject to the feasibility of this solution, in particular the Foschini-Miljanic algorithm [1]

Of course, power optimization remains an important problem in emerging and future networks. Ad hoc networks are one such class, where substantial effort has been made to analyze their behavior, such as connectivity and transport capacity [2]–[5]. A number of works have discussed the conditions for feasibility of the optimal power vector [4], [6] under certain general conditions, without however addressing specific gains from power control. In contrast, [5] using the Laplace transform method, has calculated the effects on the interference of fading, pathloss and random erasures on the interference of a random receiver in both regular and Poisson random networks. In addition, they analyze the effects of power-control by inverting the pathloss and/or the fading coefficient of the direct link between transmitter and receiver. However, the interference from neighboring transmitters is only taken into account by considering them as an effective medium without any feedback taken into account. Hence, the effects of the power increase on a given link to other neighboring links, which also control their power in order to maintain a target SINR are neglected. A similar approach is taken by [7] who introduce a scheme to compensate partially for the fading coefficient of the direct link between transmitter and receiver, without addressing the effects on neighboring links. Power control together with scheduling becomes more important when multiple hops on the network are necessary for information to reach its destination. In a different setting, the density of WiFi networks is already high enough to create significant interference. This will also become an issue when femto-cells, a recently proposed network paradigm, will become massively deployed: due to their close proximity, neighboring femto-cells may create interference to one another [8]. Hence, when each transmitter increases its power in order to compensate for this interference, it may create domino effect of power increases, which needs to be addressed. As a result, all above situations are expected to benefit significantly from power optimization. Nevertheless, little progress has been made in finding analytic estimates of the performance of random, interference-limited networks when power control is applied [9].

In this paper we apply results from random matrix theory, [10], [11] to provide an analytic estimate of the optimal power performance for a large network in the presence of both interference and randomness. In contrast to previous works, where the effects of neighboring transmitters was taken into account in an *average* sense, we will jointly consider power control on all users, taking into account the possibility of power escalation when all users have a given target SINR constraint.

We will start from an ordered network structure, in particular an equally spaced line (or square) of N transmitters, each with a receiver located in its neighborhood at a fixed distance. We will then perturb the ordered network by allowing two different types of connections. Specifically, a finite fraction of the users Np where $0 < p < 1$ may be at distance δ_1 from their base, while having a target SINR γ_1, while the rest may be at distance δ_2 from their corresponding bases, with a target SINR of γ_2. Picking the values p, δ_1, δ_2 appropriately, we may use this model to describe, rather coarsely, but effectively, any "disordered configuration" of users in each base area.

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Although relatively simple, this model takes into account the disruptive effects of randomness in the effort of transmitters to jointly compensate for the interference they cause to each other, in order to maintain a target SINR. As we shall see, it behaves qualitatively different from ordered networks. Interestingly, these results are also valid for both one-dimensional and two dimensional networks (arrays and square-grids). It should be mentioned that in a recent manuscript [12] we have analyzed a different example of a random network: In that case, a finite fraction of transmitter receiver pairs are erased at random. Although quite different as a networking scenario, the results share certain common features, which will be compared elsewhere.

The paper is organized as follows. In the next section we define the network model in the absence of any randomness. In Section III we introduce randomness in the network and calculate the average minimum power per user using results from random matrix theory. We also discuss the relevance of these results in Section IV for both one-dimensional and two dimensional geometries. Finally, in Section V we draw some conclusions, while at the Appendix we discuss some details of the proofs.

II. Model Description

Consider a network of N base-stations on a lattice. The lattice may be one-dimensional with equal spacing ℓ between the base-stations, or two-dimensional (see Fig. 1). In the latter case, which is obviously the more realistic one, it may have any symmetry, e.g. hexagonal or triangular (as analyzed in [13]) but for concreteness, we will assume that the lattice is square. Each base-station is connected via a link to a single user located at a distance δ from its corresponding base-station. The channel coefficient between base-station i and user j and averaged over fading is given by

$$g_{ij} = \frac{\ell^\alpha}{(|\mathbf{m}_i - \mathbf{m}_j|^2 \ell^2 + \delta^2)^{\alpha/2}} \tag{1}$$

where $\mathbf{m}_i$ is the lattice vector of integers corresponding to base-station i. For simplicity, we have normalized the channel gains to unity for $i = j$. We also define $s = \delta/\ell$. For d-dimensional lattices ($d = 1, 2$), it can be expressed in terms of integer linear combinations of the d basis-vectors of the lattice

$$\mathbf{m}_i = \sum_{k=1}^{d} m_i^k \boldsymbol{\epsilon}^k \tag{2}$$

for integer $0 \leq m_i^k < L$, where $N = L^d$. Since the integers m_i^k fully specify the position $\mathbf{m}_i$, we will drop the index i when possible. For simplicity we assume periodic boundary conditions on the lattice, hence $m_i^k \equiv m_i^k + L$. This means that the distance between any two points is taken as their minimum distance on a toroidal geometry (or a circular geometry for one dimension). It should be emphasized that this circulant property of the matrix g_{ij} is a result of the regular structure of the transmitter locations. For large network sizes its effect on the boundaries of the network is negligible. Also, $\alpha \geq 2$ is the pathloss exponent, which signifies how fast the channel strength decays as a function of distance.

Note that the pathloss function (1) normalizes to $g_{ii} = \ell^\alpha/\delta^\alpha = 1/s^\alpha$ per each link ($i = j$) and hence it has the right dependence on δ in this case. Observe also that, technically, for $i \neq j$ it's dependence on δ is only asymptotically correct when $\delta << \ell|\mathbf{m}_i - \mathbf{m}_j|$, that is, each receiving mobile very close to its serving base station (compared to the distance between base stations. For other regimes the function (1) can serve as an approximation. The reason for using this asymptotically correct functional form (1) for pathloss becomes clear below; it allows for analytic probing of the system and leads to analytic results consistent with experimental observations in simulations. It should also be mentioned however, that this pathloss function is strictly correct when each user is located vertically to the line connecting all bases in a one-dimensional geometry [6].

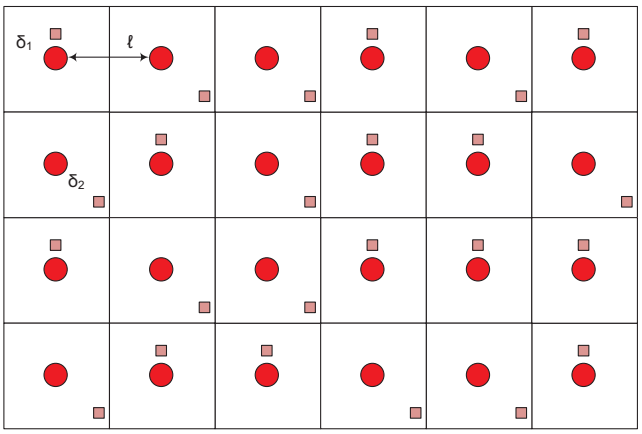

Fig. 1. Schematic figure of wireless network The circles correspond to base-stations, while the squares receiver users. We see that the users are located in two distinct distances from their serving bases.

The users are assumed to try to connect with their own nearby cells. The SINR for each connection is given by

$$\text{SINR}_k = \frac{p_k g_{kk}}{n + \sum_{j \neq k} g_{jk} p_j} \tag{3}$$

where p_k is the transmitted power from the k base to the user k and n is the thermal noise, assumed equal for all for simplicity. For the connection to be possible, a minimum value of the SINR has to be attainable i.e.

$$\text{SINR}_k \geq \gamma \tag{4}$$

Therefore, each base should adjust its power to meet this criterion. As a result, the following set of equations should be simultaneously met

$$n^{-1}\gamma^{-1} g_{kk} p_k - \sum_{j \neq k} n^{-1} g_{jk} p_j \geq 1 \tag{5}$$

The above equations constitute a set of linear (planar) constraints on the powers. The minimum total power is reached

at the apex of the conical section of allowed powers, given by equality of all constraints above. Defining the matrix $\mathbf{M}$ with elements M_{ij} as

$$M_{ij} = \begin{cases} n^{-1}\gamma^{-1}g_{ii} & i = j \\ -n^{-1}g_{ji} & i \neq j \end{cases} \tag{6}$$

we may write the above equation as

$$\mathbf{P} = \mathbf{M}^{-1}\mathbf{J} \tag{7}$$

where $\mathbf{P}$ is the vector of powers that satisfies the equality constraint above and $\mathbf{J} = [1, 1, \ldots, 1]^T$. As a result, the minimum average power per node is given by

$$P_{ave} = \frac{1}{N}\mathbf{J}^T\mathbf{M}^{-1}\mathbf{J} \tag{8}$$

Clearly, for the inverse $\mathbf{M}$ to be well-defined, all its eigenvalues have to be positive.

Now, due to the circulant structure of the matrix, its eigenvectors will be equal to $\mathbf{v_q}$, which at position $\mathbf{m}$ has the value

$$\mathbf{v_{q,m}} = \frac{e^{i\mathbf{q}\cdot\mathbf{m}}}{N^{1/2}} \tag{9}$$

where the vector $\mathbf{q}$ resides in the fundamental cell of the reciprocal lattice of the lattice $\mathbf{m}$

$$\mathbf{q} = \frac{2\pi}{L}\sum_{s=1}^{d} k_s\hat{\boldsymbol{\epsilon}}_s \tag{10}$$

with k_s integer and $N = L^d$, such that $0 \leq k_s < L$ [14]. The eigenvalue corresponding to the vector $\mathbf{q}$ is simply the Fourier transform of any line of the matrix $\mathbf{M}$

$$\begin{aligned} \lambda(\mathbf{q}) &= \sum_{\mathbf{m}} M_{0,\mathbf{m}} e^{i\mathbf{q}\cdot\mathbf{m}} \\ &= n^{-1}\left(\gamma^{-1}s^{-\alpha} - \sum_{\mathbf{m}\neq 0}\frac{e^{i\mathbf{q}\cdot\mathbf{m}}}{(|\mathbf{m}|^2+s^2)^{\alpha/2}}\right) \end{aligned} \tag{11}$$

It is important to note that these eigenvalues are real because the matrix $\mathbf{M}$ is real and symmetric, which is the reason for choosing (1). Finally, since the vector $\mathbf{J}$ is proportional to the $\mathbf{q} = 0$ eigenvector of $\mathbf{M}$ we conclude that

$$P_{ave} = \frac{1}{\lambda(0)} = \frac{n\gamma s^{\alpha}}{1 - \gamma\sum_{\mathbf{m}\neq 0}\frac{s^{\alpha}}{(|\mathbf{m}|^2+s^2)^{\alpha/2}}} \tag{12}$$

where $\lambda(0) = \lambda(\mathbf{q} = 0)$. This solution corresponds to all bases transmitting with the same power. The condition for finite total power in the case of no randomness is simply

$$\gamma\sum_{\mathbf{m}\neq 0}\frac{\ell^{\alpha}}{(|\mathbf{m}|^2\ell^2+\delta^2)^{\alpha/2}} < 1 \tag{13}$$

III. Introduction of Randomness in the Network

The above equations hold in the absence of any disorder in the network. We will now introduce disorder in two simple yet relevant ways. First, we allow users to be located at two distinct distances from their corresponding base δ_1, δ_2, respectively. We also allow for users to have two distinct SINR targets γ_1 and γ_2. Thus, for the $c = 1, 2$ set of users, the equations that need to be met are

$$n^{-1}\gamma_c^{-1}g_{ii}^c p_i - \sum_{j\neq i} n^{-1}g_{ji}^c p_j \geq 1 \tag{14}$$

where

$$\begin{aligned} g_{ij}^c &= \frac{\ell^{\alpha}}{(|\mathbf{m}_i - \mathbf{m}_j|^2\ell^2 + \delta_c^2)^{\alpha/2}} \\ &= \frac{1}{(|\mathbf{m}_i - \mathbf{m}_j|^2 + s_c^2)^{\alpha/2}} \end{aligned} \tag{15}$$

This can be written in matrix form when (14) become equalities, as follows:

$$(\mathbf{M}_1 + \mathbf{E}(\mathbf{M}_2 - \mathbf{M}_1))\,\mathbf{P} = \mathbf{J} \tag{16}$$

where the matrices $\mathbf{M}_{1,2}$ are the analogous to (6) for corresponding $\gamma_{1,2}$ and $s_{1,2}$ in (5). $\mathbf{E}$ is an $N \times N$ diagonal matrix with independent diagonal elements taking the values $e_i = 0, 1$ depending on whether the corresponding user (row) is at distance δ_1 (and with γ_1) or δ_2 (with γ_2), respectively. We may treat $\mathbf{E}$ as a random matrix with diagonal elements taking value

$$\begin{aligned} P[e_i = 0] &= p \\ P[e_i = 1] &= 1 - p \end{aligned} \tag{17}$$

This means that for large N the fraction of users at distance δ_1 will be p, while the fraction at distance δ_2 will be $(1-p)$. Inverting the above matrix we get the power vector $\mathbf{P}$

$$\mathbf{P} = [\mathbf{M}_1 + \mathbf{E}(\mathbf{M}_2 - \mathbf{M}_1)]^{-1}\,\mathbf{J} \tag{18}$$

To obtain the average power as before we multiply on the left with $\mathbf{J}^{\dagger}$ to collect all the components with non-zero power. Thus we have

$$P_{ave} = \frac{1}{N}\mathbf{J}^{\dagger}\,[\mathbf{M}_1 + \mathbf{E}(\mathbf{M}_2 - \mathbf{M}_1)]^{-1}\,\mathbf{J} \tag{19}$$

In the large N limit the average power may be given by the following theorem.

Theorem 1 (Average Minimum Power)**.** *Let $\bar{\beta}^*$ be the solution of the equation*

$$1 - p = \int \frac{d\mathbf{q}}{(2\pi)^d}\frac{\lambda_2(\mathbf{q})}{\lambda_2(\mathbf{q}) + (\bar{\beta} - 1)\lambda_1(\mathbf{q})} \tag{20}$$

where $\lambda_1(\mathbf{q})$, $\lambda_2(\mathbf{q})$ are the $\mathbf{q}$ eigenvalues of the matrices $\mathbf{M}_1$, $\mathbf{M}_2$ in (16), respectively. The integral above is over the volume of the fundamental reciprocal lattice (10). We assume that $\lambda_1(0) \geq \lambda_2(0)$. Then the average power per base-station is given by the equation

$$P_{ave} = \frac{\bar{\beta}^*}{\lambda_1(0)(\bar{\beta}^* - 1) + \lambda_2(0)} \tag{21}$$

Proof: We defer the proof for the Appendix. ■

Remark 1.1. In the appendix the above theorem is proved straightforwardly for one-dimensional lattices using results from [10]. We will argue that this is valid for two-dimensional lattices through simulations and defer the complete proof for a longer version of the work.

It is easy to see that the above result reduces to $P_{ave} = 1/\lambda_1(0)$ when $p \to 0$ and $P_{ave} = 1/\lambda_2(0)$ when $p \to 1$. Indeed in the former case in (20) we see that $\bar{\beta} \to \infty$, while in the latter case, $\bar{\beta} \to 1$.

IV. Discussion

We will start by analyzing the results for the simpler one-dimensional case, where the base-stations are located in a ring seen in Figs. 2-3. First, we see that the average power obviously is a increasing function of γ_0. Second, we see, both analytically and numerically, that the average power is finite for a range of γ above the value for which $\lambda_2(0) = 0$, which is the value of γ for which the $p = 0$ solution diverges. However, this comes at a cost: For such γ for which $\lambda_2(0) < 0$ a second solution to the fixed point equation appears. However, this value of P_{ave} is unstable. This second solution of P_{ave} persists until it merges with the stable value at a critical value of γ (which happens before $\lambda_1(0) + \lambda_2(0) = 0$). Beyond this point no finite average power can be supported for the given value of $p = 1/2$. Thus we see a discontinuous transition at that point, from a finite average minimum power to an infinite value. This is because beyond this point, at least one power has to be infinite. In fact, the finite value of P_{ave} all the way to the critical value suggests that at most a vanishing fraction of bases have unbounded powers. This effect is a direct result of a *domino* effect occurring at a region where a disruptive realization of the randomness forces a small number of users increase their powers in an unbounded fashion in order to compensate for each others' interference and has been suggested at [4]. This behavior is similar to the case where a fraction of the base-user pairs is silent, as discussed in [12].

It is interesting that these results also hold for two-dimensional grids, even though we have not provided a proof in this short paper version. This can be seen in Fig. 4 where the average optimum power for a square grid 50×50 of base-user pairs is plotted as a function of γ. We plot the analytic curve as well as the numerical average over approximately 500 realizations (solid line) and the curve for the realization that has lasted to the largest γ before becoming unfeasible (dashed). We once again observe the same behavior. The "average" curve stops at some point after the value of γ for which $\lambda_2(0) = 0$ while the dashed lines go up to close to the analytic curve.

In Figs. 2-4 we also plot an approximate solution

$$p_{ave,approx} = \frac{\mathrm{e}}{\lambda_1(0)} + \frac{1-\mathrm{e}}{\lambda_2(0)} \tag{22}$$

which only works for small γ.

Another interesting result from the numerical analysis is that beyond the value of γ where $\lambda_2(0) = 0$ the behavior becomes sample dependent. Thus the value at which the power becomes infinite varies somewhat from sample to sample (as well as the whole curve beyond the value where $\lambda_2(0) = 0$). How can we explain this behavior? For example, if a region happens to have a large concentration of bases it may create a situation where the power of a particular base diverges. Such a happenstance may be less probable if each base is affected by a "mean-field" of all bases, including far away ones.

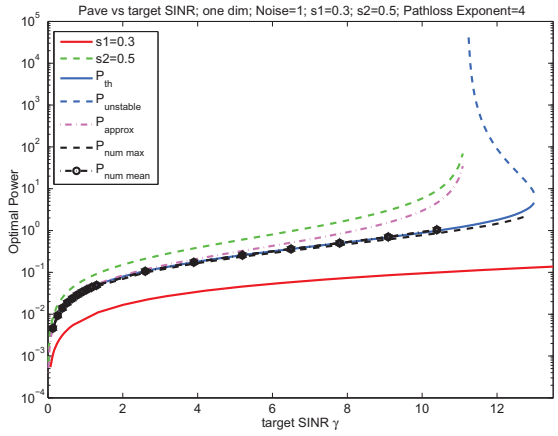

Fig. 2. Plot of average power as a function of the target SINR for various values $s_1 = 0.3$, $s_2 = 0.5$. The path-loss exponent is $\alpha = 4$. The red and green curves give the power per user if all users had s_1 and s_2 distance from their bases. The blue curve is the one resulting from the calculations from Theorem 1. The dashed blue curve corresponds to the unstable solution of the equations which appears when the value of γ exceeds the value at which the green curve diverges. The black curves are the numerical results for linear networks with $N = 300$ nodes. The magenta curve is simply the mean between the green and dashed red curves. We see that it agrees with the correct answer only for small values of γ.

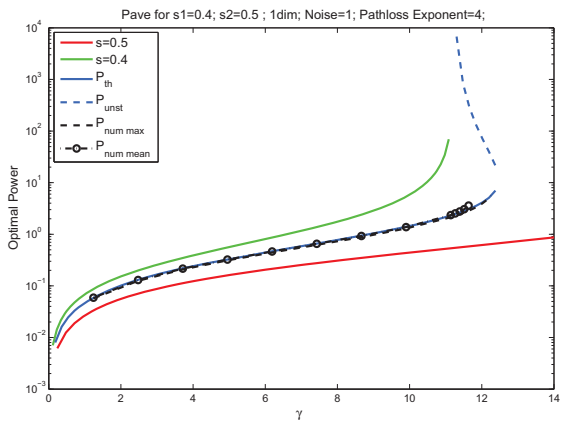

Fig. 3. Plot of average power as a function of the target SINR for various values $s_1 = 0.4$, $s_2 = 0.5$. The path-loss exponent is $\alpha = 4$. The curves have the same meaning as in the previous figure.

V. Conclusion

In this paper we have studied the optimal power vector that achieves an SINR target criterion in a wireless network

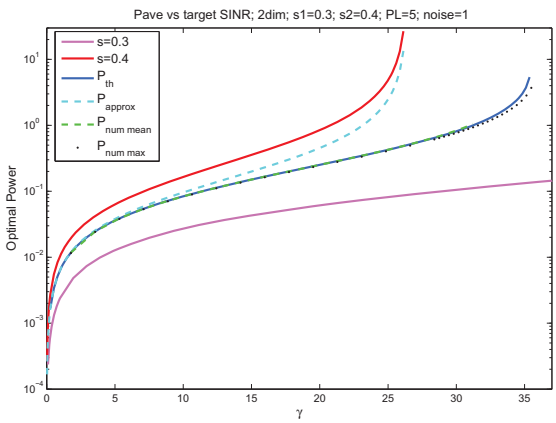

Fig. 4. Plot of average power as a function of the target SINR for various values $s_1 = 0.3$, $s_2 = 0.4$ for a two-dimensional square grid of base-stations. The curves have the same meaning as in the previous figure. The path-loss exponent is $\alpha = 5$. We picked this exponent for computational simplicity (the numerical evaluation of Bessel functions is slower than exponentials). Also, $\alpha = 4$ for one-dimensional lattices is has the same aggregate interference tails as $\alpha = 5$ for two-dimensional lattices. Finally, in the modified Hata pathloss model [15] the pathloss exponent is $\alpha \approx 4.5$, making $\alpha = 5$ not a unreasonable value.

setting where both randomness and interference are relevant. To our knowledge this is the first work that directly takes into account the effects of competing power control of neighboring nodes which may result to domino effects in a local scale due to the randomness. We have applied methods from Random Matrix Theory to obtain the average optimum power. Our numerical results show excellent agreement with predicted analytical calculations, both for one- and two-dimensional network geometries. We observe that in the presence of randomness the optimal power does not go to infinity in a continuous fashion. Rather, beyond a certain point, no finite power solution exists. We also numerically observe that beyond the point where the pure system has finite optimal power, real random systems become unstable, making finite power unfeasible for increasing SINR targets at different points. The ones that remain finite reach our analytical limit. This suggests that randomness creates sample to sample fluctuations, which make local clusters of cells and hence the whole network unfeasible to operate at that target SINR. Therefore, it does not make sense to look for globally optimal power vectors beyond a certain network size. Instead sacrificing the connection of a few "bad" bases by adding some outage may increase the overall behavior dramatically.

Appendix

In the proof we will assume one-dimensional networks. The case of two dimensional networks is deferred to a longer publication. We now move to Theorem 1, which is proved using the basic steps of [10].

Proof of Theorem 1: We start by noting that $\mathbf{J}$ is an eigenvector of both $\mathbf{M}_1$ and $\mathbf{M}_2$ with eigenvalues $\lambda_1(0)$, $\lambda_2(0)$ respectively. We thus may rewrite (19) as the above equation as

$$P_{ave} = \frac{1}{N\lambda_1(0)} \mathbf{J}^\dagger \left[\mathbf{I} + \mathbf{EN}\right]^{-1} \mathbf{J} \tag{23}$$

where $\mathbf{N} = (\mathbf{M}_2 - \mathbf{M}_1)\mathbf{M}_1^{-1}$. After some manipulation we get

$$\begin{aligned} P_{ave} &= \frac{1}{\lambda_1(0)} \left(1 - \frac{1}{N} \mathbf{J}^\dagger \left[\mathbf{I} + \mathbf{EN}\right]^{-1} \mathbf{ENJ}\right) \\ &= \frac{1}{\lambda_1(0)} \left(1 - \frac{\lambda_2(0) - \lambda_1(0)}{\lambda_1(0)} \zeta\right) \end{aligned} \tag{24}$$

where the second line results from the fact that $\mathbf{J}$ is an eigenvector of $\mathbf{N}$. ζ is defined as

$$\begin{aligned} \zeta &= \frac{1}{N} \mathbf{J}^\dagger \left[\mathbf{I} + \mathbf{EN}\right]^{-1} \mathbf{EJ} \\ &= \frac{1}{N} \mathbf{J}^\dagger \mathbf{E} \left[\mathbf{I} + \mathbf{ENE}\right]^{-1} \mathbf{EJ} \\ &= \mathbf{a}_0^\dagger \left[\mathbf{I} + \mathbf{A\Lambda A}^\dagger\right]^{-1} \mathbf{a}_0 \end{aligned} \tag{25}$$

where we use the fact that $\mathbf{E}^2 = \mathbf{E}$ to go from the first to the second equality. The third equality results from expressing $\mathbf{N}$, which is a circulant matrix, in terms of a diagonal eigenvalue matrix $\mathbf{\Lambda}$ and its eigenvector matrix, which is the Fourier matrix $\mathbf{F}$, i.e.

$$\mathbf{N} = \mathbf{F\Lambda F}^\dagger \tag{26}$$

In addition, we have defined the matrix

$$\mathbf{A} = \mathbf{EF} \tag{27}$$

and denote its column vectors by $\mathbf{a}(\mathbf{q})$ and, in particular, $\mathbf{a}(0) = N^{-1/2}\mathbf{EJ} = \mathbf{a}_0$. Using the matrix inversion lemma we get:

$$\zeta = \frac{1}{\beta + (\lambda_2(0) - \lambda_1(0))/\lambda_1(0)} \tag{28}$$

where

$$\beta^{-1} = \mathbf{a}_0^\dagger \left[\mathbf{A\Lambda}_0\mathbf{A}^\dagger + \mathbf{I}\right]^{-1} \mathbf{a}_0 \tag{29}$$

and where $\mathbf{\Lambda}_0$ is the diagonal matrix of eigenvalues of $\mathbf{N}$ with the one corresponding to $\mathbf{q} = 0$ set to zero. In [11] it is shown that β takes (a.s.) a deterministic value in the large N limit, which is the solution of the following equation

$$\frac{p}{\beta - 1} = \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\lambda_1(\mathbf{q})}{(\beta - 1)\lambda_1(\mathbf{q}) + \lambda_2(\mathbf{q})} \tag{30}$$

This equation results from taking the limit $N \to \infty$ in which case the sum over the eigenvalues $\mathbf{q}$ (10) becomes an integral. After some manipulation this equation becomes (20). In addition, after plugging in (28) into (24) we get (21) for P_{ave}. ■

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