



HAL
open science

Boundary conditions control in ORCA2

Eugene Kazantsev

► **To cite this version:**

Eugene Kazantsev. Boundary conditions control in ORCA2. 4ème Colloque National sur l'Assimilation des données (2012), Dec 2012, Nice, France. hal-00764588

HAL Id: hal-00764588

<https://inria.hal.science/hal-00764588>

Submitted on 13 Dec 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Boundary conditions control in ORCA2

Eugene Kazantsev

INRIA, Moise

December, 17, 2012

ORCA2 Configuration : $182 \times 149 \times 31$ nodes in curvilinear (x, y) coordinates with z levels.

$$\begin{aligned}
 \frac{\partial u}{\partial t} &= \underbrace{v(\omega + f) - \frac{\partial(u^2 + v^2)/2}{\partial x} - w \frac{\partial u}{\partial z}}_{\text{Advection}} - \underbrace{\frac{\partial A_u^h \xi}{\partial x} + \frac{\partial A_u^h \omega}{\partial y}}_{\text{Hor. Dissipation}} + \\
 &+ \underbrace{\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}}_{\text{Vert. Dissipation}} + \underbrace{g \int_0^z \frac{\partial \rho(x, y, \zeta)}{\partial x} d\zeta}_{\text{Pressure gradient}} + \underbrace{g \frac{\partial(\eta + T_c \phi)}{\partial x}}_{\text{SSH}} \\
 \frac{\partial v}{\partial t} &= -u(\omega + f) - \frac{\partial(u^2 + v^2)/2}{\partial y} - w \frac{\partial v}{\partial z} - \frac{\partial A_u^h \xi}{\partial y} - \frac{\partial A_u^h \omega}{\partial x} + \\
 &+ \frac{\partial}{\partial z} A_u^z \frac{\partial v}{\partial z} + g \int_0^z \frac{\partial \rho}{\partial y} dz + \frac{\partial(\eta + T_c \phi)}{\partial y} \\
 \xi &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad \omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \text{Divergence, Vorticity} \\
 w &= \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = 0 \quad \text{Vertical velocity} \\
 \phi &= \frac{\partial \eta}{\partial t} \quad \text{Grav. Waves filter} \quad A_u^h = \text{const}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial T}{\partial t} &= \underbrace{-\frac{\partial uT}{\partial x} - \frac{\partial vT}{\partial y} - \frac{\partial wT}{\partial z}}_{\text{Advection}} + \underbrace{A_T^h \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)}_{\text{Hor.diffusion}} + \\
 &+ \underbrace{\frac{\partial}{\partial z} A_T^z \frac{\partial T}{\partial z}}_{\text{Vert.diffusion}} + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} + \text{Surface} \\
 \frac{\partial S}{\partial t} &= -\frac{\partial uS}{\partial x} - \frac{\partial vS}{\partial y} - \frac{\partial wS}{\partial z} + A_T^h \left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) + \frac{\partial}{\partial z} A_S^z \frac{\partial S}{\partial z} + \text{BBL} + \text{Surface} \\
 A_T^z &= \text{Turbulent closure: } \sim \max(A_0^z, C_k l_k \sqrt{\bar{e}}) \tag{1} \\
 \frac{\partial \bar{e}}{\partial t} &= A_v^z \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] - A_T^z N^2 + \frac{\partial}{\partial z} \left[A_u^z \frac{\partial \bar{e}}{\partial z} \right] - c_\epsilon \frac{\bar{e}^{3/2}}{l_\epsilon} \\
 \rho &= \rho(T, S), \quad N^2 = N^2(T, S), \quad R = R(T, S)
 \end{aligned}$$

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \left(S_x S_y v \right) S_y (\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \left(S_x w D_z u \right) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ g \int_0^z D_x S_z \rho(x, y, \zeta) d\zeta + D_{zz} (A_u^z u) + g D_x (\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x (u S_x T) - D_y (v S_y T) - D_z (w S_z T) + A_T^h \left(D_x D_x T + D_y D_y T \right) + \\
&+ D_{zz} (A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = 0
\end{aligned}$$

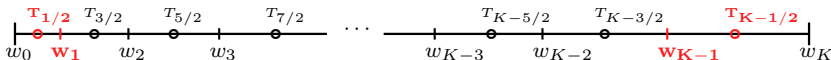
Interpolations and Derivatives

$$\begin{aligned}
(Sw)_{k+1/2} &= \frac{w_{k+1} + w_k}{2} \quad k = 1, \dots, K-1 \\
(DT)_k &= \frac{T_{k+1/2} - T_{k-1/2}}{h} \quad k = 1, \dots, K-1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \left(S_x S_y v \right) S_y (\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \left(S_x w D_z u \right) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ g \int_0^z D_x S_z \rho(x, y, \zeta) d\zeta + D_{zz} (A_u^z u) + g D_x (\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x (u S_x T) - D_y (v S_y T) - D_z (w S_z T) + A_T^h \left(D_x D_x T + D_y D_y T \right) + \\
&+ D_{zz} (A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x, y, \zeta) d\zeta; w(x, y, H) = 0
\end{aligned}$$

Interpolations and Derivatives **Modified Near the boundary**

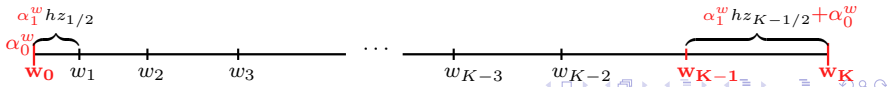
$$\begin{aligned}
(Sw)_{k+1/2} &= \frac{w_{k+1} + w_k}{2}, \quad k = 1, \dots, K-2, \quad (Sw)_{1/2} = \alpha_0^S + \alpha_1^S w_0 + \alpha_2^S w_1 \\
(DT)_k &= \frac{T_{k+1/2} - T_{k-1/2}}{h}, \quad i = 2, \dots, K-2, \quad (DT)_1 = \alpha_0^D + \frac{\alpha_1^D T_{1/2} + \alpha_2^D T_{3/2}}{h}
\end{aligned}$$



$$\begin{aligned} \frac{\partial u}{\partial t} &= \left(S_x S_y v \right) S_y (\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \left(S_x w D_z u \right) + D_x A_u^h \xi + D_y A_u^h \omega + \\ &+ g \int_0^z D_x S_z \rho(x, y, \zeta) d\zeta + D_{zz} (A_u^z u) + g D_x (\eta + T_c \phi) \\ \frac{\partial T}{\partial t} &= -D_x (u S_x T) - D_y (v S_y T) - D_z (w S_z T) + A_T^h \left(D_x D_x T + D_y D_y T \right) + \\ &+ D_{zz} (A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\ \xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = \alpha_0(x, y) \end{aligned}$$

Vertical velocity

$$\begin{aligned} w_{i,j,K-1} &= \alpha_0^{w^b} - \alpha_1^{w^b} h z_{i,j,K-1/2} \xi_{i,j,K-1/2} \\ w_{i,j,k-1} &= w_{i,j,k} - h z_{i,j,k-1/2} \xi_{i,j,k-1/2} \quad \forall k : 2 \leq k \leq K-1 \\ w_{i,j,0} &= w_{i,j,1} + \alpha_0^{w^s} - \alpha_1^{w^s} h z_{i,j,1/2} \xi_{i,j,1/2} \end{aligned}$$



Vertical diffusion

$\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}$ is replaced by

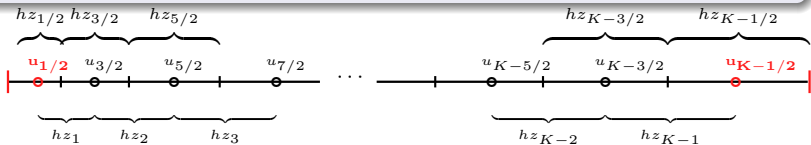
$$\left(D_{zz} u \right)_{i,j,1/2} = \frac{(A_u^z)_1}{h z_1 h z_{1/2}} (\alpha_2^{D_{zz} U^S} u_{3/2} - \alpha_1^{D_{zz} U^S} u_{1/2})$$

$$\left(D_{zz} u \right)_{i,j,k-1/2} = \frac{1}{h z_{k-1/2}} \left(\frac{(A_u^z)_k}{h z_k} (u_{k+1/2} - u_{k-1/2}) - \frac{(A_u^z)_{k-1}}{h z_{k-1}} (u_{k-1/2} - u_{k-3/2}) \right) \quad \forall k : 2$$

$$\left(D_{zz} u \right)_{i,j,K-1/2} = \frac{1}{h z_{K-1/2}} \left[\alpha_2^{D_{zz} U^b} \frac{(A_u^z)_{K-1}}{h z_{K-1}} u_{K-1/2} - \alpha_1^{D_{zz} U^b} \left(\frac{(A_u^z)_K}{h z_K} + \frac{(A_u^z)_{K-1}}{h z_{K-1}} \right) u_{K-3/2} \right] \quad u_{K-3/2}$$

$$\frac{\partial u}{\partial z} \Big|_{w_0} = \alpha_0^{D_{zz} U^S} + \frac{\tau_x}{h z_1 \rho_0}, \quad \frac{\partial v}{\partial z} \Big|_{w_0} = \alpha_0^{D_{zz} U^S} + \frac{\tau_y}{h z_1 \rho_0}, \quad \frac{\partial T}{\partial z} \Big|_{w_0} = \frac{\partial S}{\partial z} \Big|_{w_0} = \alpha_0^{D_{zz} T^S}$$

$$u|_{\text{bottom}} = v|_{\text{bottom}} = \alpha_0^{D_{zz} U^b} \quad T|_{\text{bottom}} = S|_{\text{bottom}} = \alpha_0^{D_{zz} T^b} \quad (2)$$



$$\begin{aligned}
 \frac{u^{n+1} - u^{n-1}}{2dt} &= \underbrace{\gamma(u^{n+1} - 2u^n + u^{n-1})}_{\text{Asselin filter}} - \left(S_x S_y v^n \right) S_y (\omega^n + f) - \\
 &- D_x \frac{S_x (u^n)^2 + S_y (v^n)^2}{2} - S_z \left(S_x w^n D_z u^n \right) + \\
 &+ \underbrace{D_x A_u^h \xi^{n-1} + D_y A_u^h \omega^{n-1}}_{\text{Explicit Euler}} + \underbrace{D_{zz} A_u^z u^{n+1}}_{\text{Impl.Euler}} + \\
 &+ g \int_0^z D_x S_z \rho^n(x, y, \zeta) d\zeta + \underbrace{g D_x (\eta^{n+1} + T_c \phi^{n+1})}_{\text{Implicit}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{T^{n+1} - T^{n-1}}{2dt} &= \underbrace{\gamma(u^{n+1} - 2u^n + u^{n-1})}_{\text{Asselin filter}} - D_x (u^n S_x T^n) - D_y (v^n S_y T^n) - \\
 &- \underbrace{D_z (w^n S_z T^n) + A_T^h \left(D_{xx} T^{n-1} + D_{yy} T^{n-1} \right)}_{\text{Explicit Euler}} + \\
 &+ \underbrace{D_{zz} (A_T^z) T^{n+1}}_{\text{Implicit}} + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL}
 \end{aligned}$$

The models solution depend on initial conditions and a number of parameters:

$$\frac{\partial T}{\partial t} = -D_x(uS_xT) - D_y(vS_yT) - D_z^{(\alpha)}(wS_z^{(\alpha)}T) + A_T^h \left(D_{xx}T + D_{yy}T \right) + D_{zz}^{(\alpha)}(A_T^z)T$$

- The model $x(t) = \mathcal{M}_{0,t}(x_0, \alpha)$

We calculate the derivatives and their adjoints with respect to

$$x_0, \alpha$$

by **TAPENADE 3.6** (Tropics team, INRIA). that allows us

- to avoid a HUGE development/coding (a double of the classical one, at least)
- to obtain immediately the derivative with respect to any parameter we want.

TAPENADE 3.6 (Tropics team, INRIA) with the Memory usage optimization:

search for push/pop

```
CALL PUSHREAL8ARRAY(sold, nx*ny*nz)
CALL PUSHREAL8ARRAY(told, nx*ny*nz)
CALL PUSHREAL8ARRAY(vold, nx*ny*nz)
CALL PUSHREAL8ARRAY(uold, nx*ny*nz)
CALL PUSHREAL8ARRAY(ssh, nx*ny)
CALL PUSHREAL8ARRAY(s, nx*ny*nz)
CALL PUSHREAL8ARRAY(t, nx*ny*nz)
CALL PUSHREAL8ARRAY(v, nx*ny*nz)
CALL PUSHREAL8ARRAY(u, nx*ny*nz)
```

replace by

```
call push_uvts(u,v,t,s,ssh)
```

Procedure push/pop_uvts(u,v,t,s,ssh):

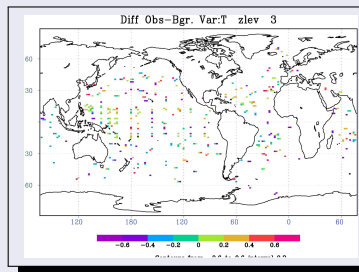
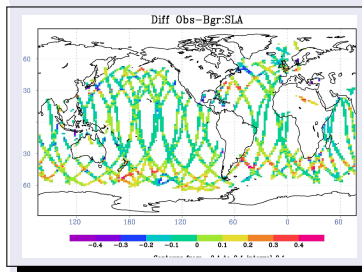
- does not push $n - 1$ step and pops appropriate values (divides the required memory by 2)
- does not push u, v, t, s in lower level routines
- does not push values on continents (divides by 2)
- pushes values in Real*4 format (divides by 2)
- eventually pushes only odd timesteps and interpolate when popping (divides by 2)

Total reduction of required memory is up to 25 times.

10 hours window \implies 10 days window.

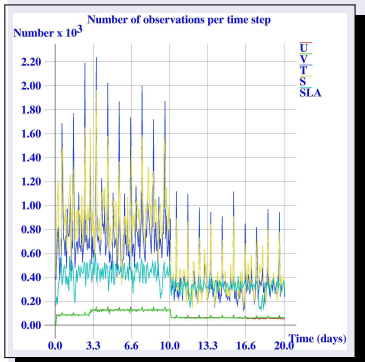
ECMWF data issued from Jason-1 and Envisat altimetric missions and ENACT/ENSEMBLES data banque.

January, 1, 2006.

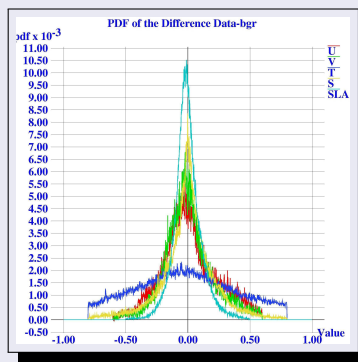


Difference between observations and background during the 1st of January.

January, 1-20 2006.



Number of observation per time step during the 1 – 20 Jan.2006



Probability density function for the difference (observation-background).

The model: $x_N = \mathcal{M}_{0,N}(x_0, \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

$$J = \|x_0 - x_{bgr}\|_{B^{-1}}^2 + \|\alpha - \alpha_{bgr}\|_{B^{-1}}^2 + \sum_{n=0}^N t_n \|\mathcal{H}\mathcal{M}_{0,n}(x_0, \alpha) - y_n\|_{R^{-1}}^2$$

Matrices: $B^{-1} = \text{diag}(10^{-4})$,

$R^{-1} = \text{diag}(1/\sigma_u, 1/\sigma_v, 1/\sigma_T, 1/\sigma_S, 1/\sigma_{ssh})$ where $\sigma_u^2 = \frac{1}{N_{obs}} \sum (u_{obs} - u_{bgr})^2$

Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal)

Data Assimilation – Forecast

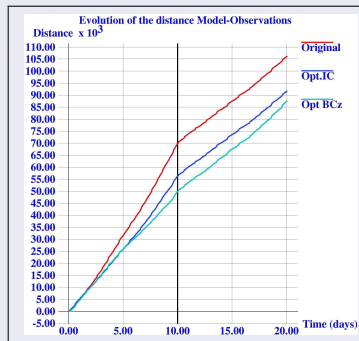
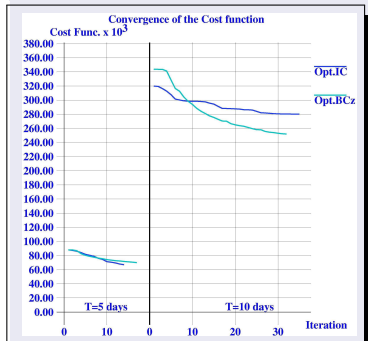
Assimilation window — 10 days,

Test time — 20 days.

The model: $x(t) = \mathcal{M}_{0,t}(x_0, \alpha)$ with $x = (u, v, T, S, ssh)^T$

$$\text{Distance: } \xi(t) = \sum_{n=0}^t \|\mathcal{H}\mathcal{M}_{0,n}(x_0, \alpha) - y_n\|_{R^{-1}}$$

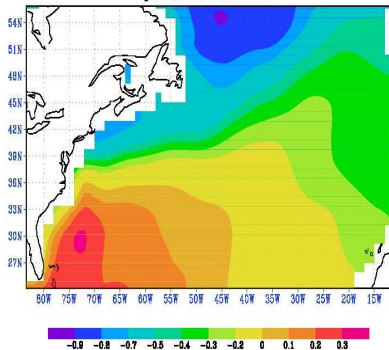
Convergence of J and evolution of ξ



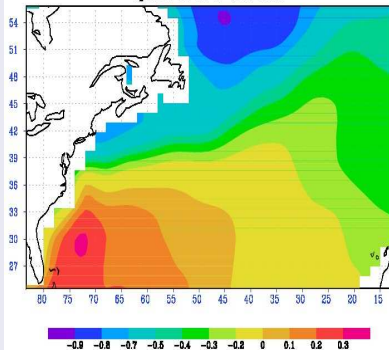
20 Cost function calls with $T = 5$ days and 40 calls with $T = 10$ days.

SSH, North Atlantic, January, 1-30 2006.

Opt.IC:SSH Jan,01 00hr



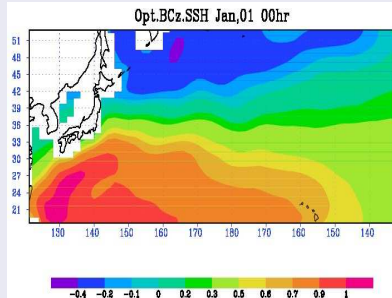
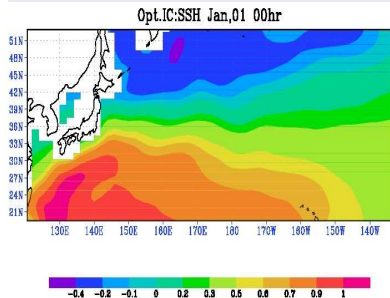
Opt.BCz.SSH Jan,01 00hr



Optimal IC

Optimal BCz

SSH, North Pacific, January, 1-30 2006.



Optimal IC

Optimal BCz

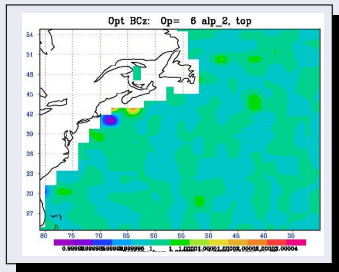
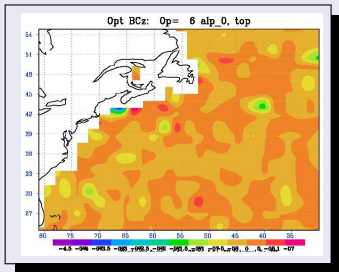
Modified formula

$$w_{i,j,K-1} = \alpha_0^{w^b} - \alpha_2^{w^b} h z_{i,j,K-1/2} \xi_{i,j,K-1/2}$$

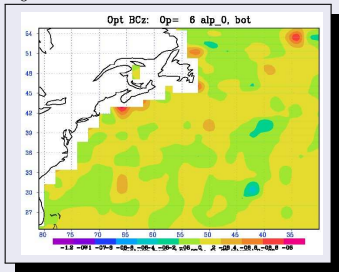
$$w_{i,j,k-1} = w_{i,j,k} - h z_{i,j,k-1/2} \xi_{i,j,k-1/2} \quad \forall k : 1 \leq k \leq K-2$$

$$w_{i,j,0} = w_{i,j,1} + \alpha_0^{w^s} - \alpha_2^{w^s} h z_{i,j,1/2} \xi_{i,j,1/2}$$

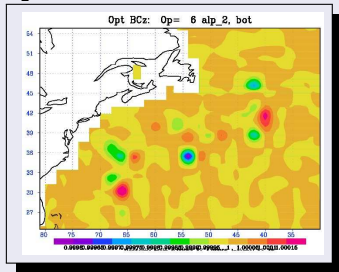
α for the vertical velocity w . North Atlantic.



α_0 on the surface



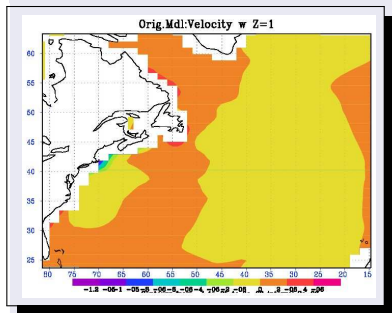
α_2 on the surface



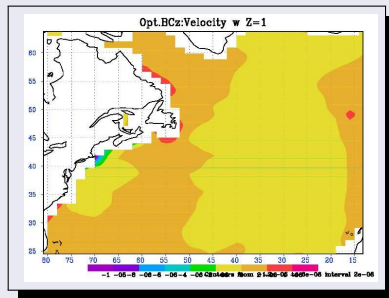
α_0 on the bottom

α_2 on the bottom

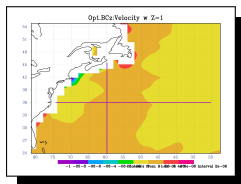
North Atlantic, January, 30, 2006, surface



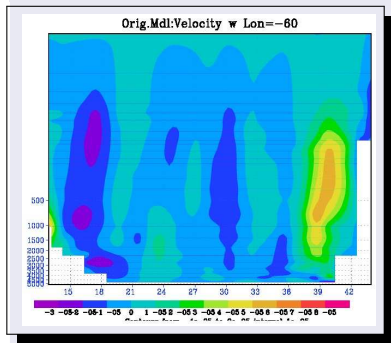
Original model



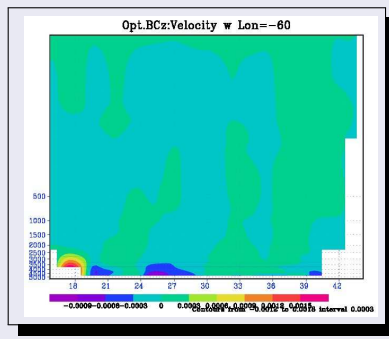
Optimal BCz



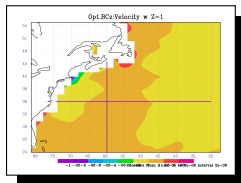
North Atlantic, January, 30, 2006, $y-z$ section



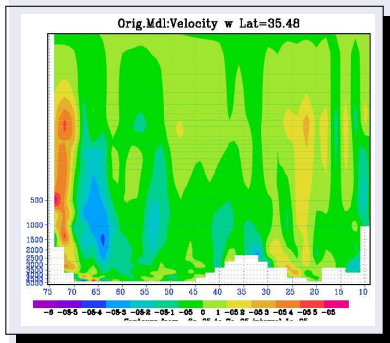
Original model



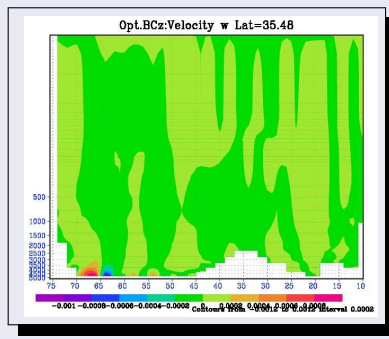
Optimal BCz



North Atlantic, January, 30, 2006, $x - z$ section

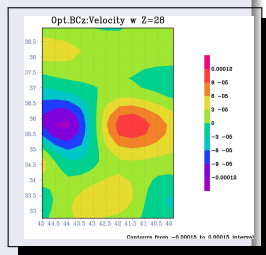
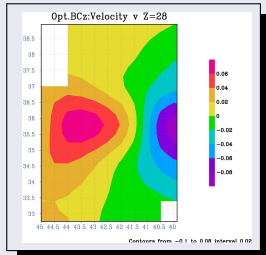
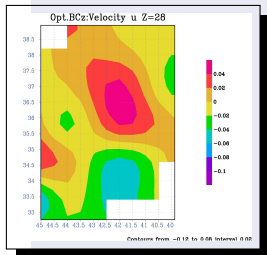


Original model



Optimal BCz

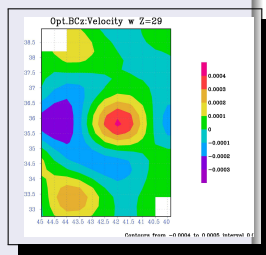
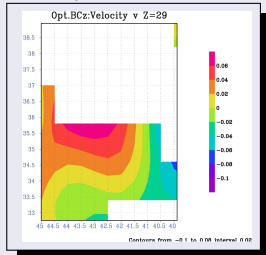
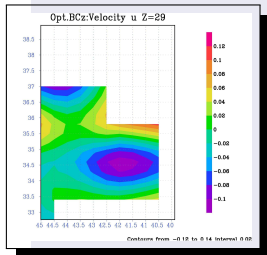
Levels $z = 28$ and $z = 29$



Velocity u

Velocity v

Velocity w



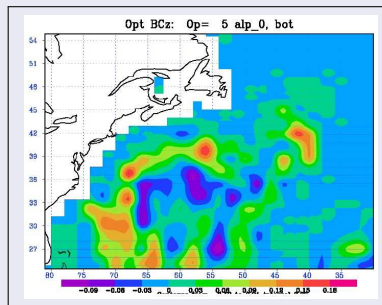
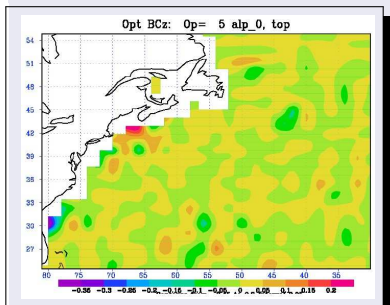
Velocity u

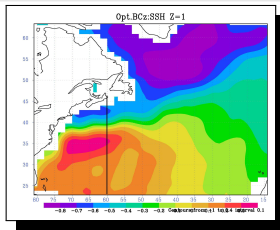
Velocity v

Velocity w

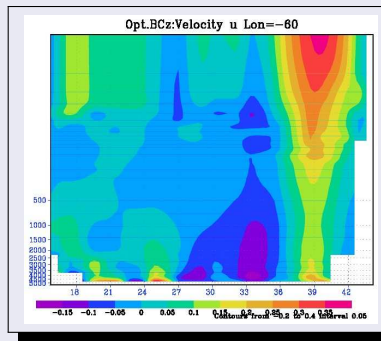
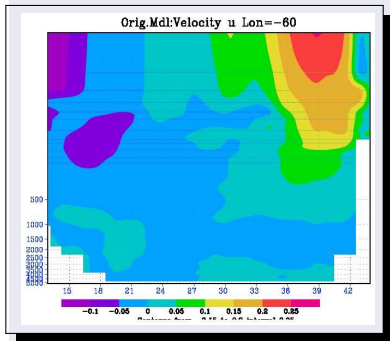
$$\left. \frac{\partial u}{\partial z} \right|_{w_0} = \alpha_0^{D_{zz}U^s} + \frac{\tau_x}{hz_1\rho_0}, \quad \left. \frac{\partial v}{\partial z} \right|_{w_0} = \alpha_0^{D_{zz}U^s} + \frac{\tau_y}{hz_1\rho_0},$$
$$u|_{bottom} = v|_{bottom} = \alpha_0^{D_{zz}U^b}$$

North Atlantic





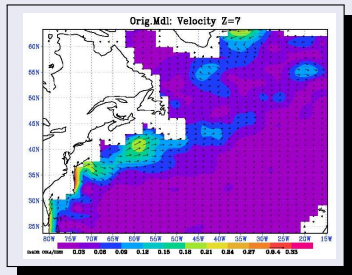
North Atlantic, January, 30, 2006, Velocity u , $y-z$ section



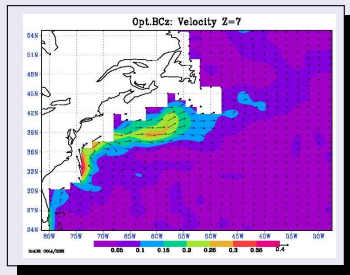
Original model

Optimal BCz

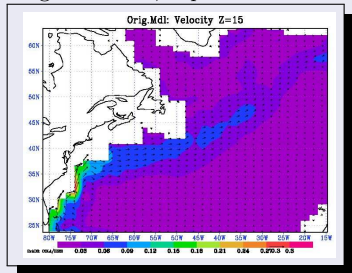
North Atlantic on the Jan.,30,2006



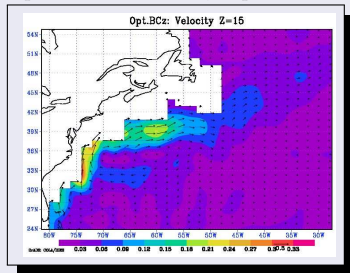
Original model, depth 65 m



Optimal BCz model, depth 65 m



Original model, depth 160 m



Optimal BCz model, depth 160 m

North Atlantic

Modification of the SSH in the North Atlantic is strongly related to the **boundary conditions of u and v especially on the bottom.**

Boundary Conditions influence is **important**

- The **cost function decreases** more under BCz control than under IC control in the assimilation window
- The models **forecast is closer** to observations with optimal BCz than with optimal IC
- Stream **jets are refined** under BCz control

Tapenade allows us

- to generate TLM/AM almost immediately,
- to avoid a HUGE development/coding,
- to obtain immediately the derivative with respect to any parameter we want.

But, it requires more memory

	Resolution	TimeStep per day	Model Size	AM Size	Traj. size per 10 days
ORCA 2	181 × 149 × 31	15	650M	1.6G	660M
ORCA05	722 × 511 × 46	40	13G	33G	35G

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The **physical meaning** of α is difficult to understand;
- The set of α is **not unique**;
- The problem of **identifiability** is not addressed yet;
- The problem of **stability** is not even posed.

Consequently:

It is not a parameter estimation study, but

- a way to **compensate model errors**
- showing the **most influent parameter** (vertical BC for ORCA-2, lateral BC for Shallow-Water).

j'apprécierai beaucoup vos commentaires et critiques



<http://www-ljk.imag.fr/membres/Kazantsev/orca2/index.html>