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# Boundary conditions control in ORCA2

Eugene Kazantsev

INRIA, Moise

December, 17, 2012

ORCA2 Configuration :  $182 \times 149 \times 31$  nodes in curvilinear  $(x, y)$  coordinates with  $z$  levels.

$$\begin{aligned}
 \frac{\partial u}{\partial t} &= \underbrace{v(\omega + f) - \frac{\partial(u^2 + v^2)/2}{\partial x} - w \frac{\partial u}{\partial z}}_{\text{Advection}} - \underbrace{\frac{\partial A_u^h \xi}{\partial x} + \frac{\partial A_u^h \omega}{\partial y}}_{\text{Hor.Dissipation}} + \\
 &+ \underbrace{\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}}_{\text{Vert.Dissipation}} + \underbrace{g \int_0^z \frac{\partial \rho(x, y, \zeta)}{\partial x} d\zeta}_{\text{Pressure gradient}} + \underbrace{g \frac{\partial(\eta + T_c \phi)}{\partial x}}_{\text{SSH}} \\
 \frac{\partial v}{\partial t} &= -u(\omega + f) - \frac{\partial(u^2 + v^2)/2}{\partial y} - w \frac{\partial v}{\partial z} - \frac{\partial A_u^h \xi}{\partial y} - \frac{\partial A_u^h \omega}{\partial x} + \\
 &+ \frac{\partial}{\partial z} A_u^z \frac{\partial v}{\partial z} + g \int_0^z \frac{\partial \rho}{\partial y} dz + \frac{\partial(\eta + T_c \phi)}{\partial y} \\
 \xi &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad \omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \text{Divergence, Vorticity} \\
 w &= \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = 0 \quad \text{Vertical velocity} \\
 \phi &= \frac{\partial \eta}{\partial t} \quad \text{Grav.Waves filter} \quad A_u^h = \text{const}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial T}{\partial t} &= \underbrace{-\frac{\partial uT}{\partial x} - \frac{\partial vT}{\partial y} - \frac{\partial wT}{\partial z}}_{\text{Advection}} + \underbrace{A_T^h \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)}_{\text{Hor.diffusion}} + \\
 &+ \underbrace{\frac{\partial}{\partial z} A_T^z \frac{\partial T}{\partial z}}_{\text{Vert.diffusion}} + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} + \text{Surface} \\
 \frac{\partial S}{\partial t} &= -\frac{\partial uS}{\partial x} - \frac{\partial vS}{\partial y} - \frac{\partial wS}{\partial z} + A_T^h \left( \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right) + \frac{\partial}{\partial z} A_S^z \frac{\partial S}{\partial z} + \text{BBL} + \text{Surface} \\
 A_T^z &= \text{Turbulent closure: } \sim \max(A_0^z, C_k l_k \sqrt{\bar{e}}) \tag{1} \\
 \frac{\partial \bar{e}}{\partial t} &= A_v^z \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right] - A_T^z N^2 + \frac{\partial}{\partial z} \left[ A_u^z \frac{\partial \bar{e}}{\partial z} \right] - c_\epsilon \frac{\bar{e}^{3/2}}{l_\epsilon} \\
 \rho &= \rho(T, S), \quad N^2 = N^2(T, S), \quad R = R(T, S)
 \end{aligned}$$

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \left( S_x S_y v \right) S_y (\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \left( S_x w D_z u \right) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ g \int_0^z D_x S_z \rho(x, y, \zeta) d\zeta + D_{zz} (A_u^z u) + g D_x (\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x (u S_x T) - D_y (v S_y T) - D_z (w S_z T) + A_T^h \left( D_x D_x T + D_y D_y T \right) + \\
&+ D_{zz} (A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = 0
\end{aligned}$$

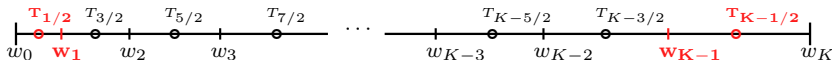
## Interpolations and Derivatives

$$\begin{aligned}
(Sw)_{k+1/2} &= \frac{w_{k+1} + w_k}{2} \quad k = 1, \dots, K-1 \\
(DT)_k &= \frac{T_{k+1/2} - T_{k-1/2}}{h} \quad k = 1, \dots, K-1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \left( S_x S_y v \right) S_y (\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \left( S_x w D_z u \right) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ g \int_0^z D_x S_z \rho(x, y, \zeta) d\zeta + D_{zz} (A_u^z u) + g D_x (\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x (u S_x T) - D_y (v S_y T) - D_z (w S_z T) + A_T^h \left( D_x D_x T + D_y D_y T \right) + \\
&+ D_{zz} (A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x, y, \zeta) d\zeta; w(x, y, H) = 0
\end{aligned}$$

Interpolations and Derivatives **Modified Near the boundary**

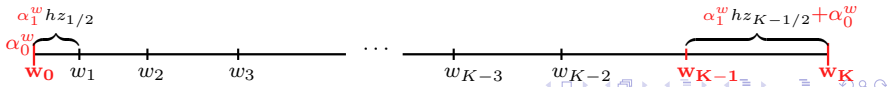
$$\begin{aligned}
(Sw)_{k+1/2} &= \frac{w_{k+1} + w_k}{2}, \quad k = 1, \dots, K-2, \quad (Sw)_{1/2} = \alpha_0^S + \alpha_1^S w_0 + \alpha_2^S w_1 \\
(DT)_k &= \frac{T_{k+1/2} - T_{k-1/2}}{h}, \quad i = 2, \dots, K-2, \quad (DT)_1 = \alpha_0^D + \frac{\alpha_1^D T_{1/2} + \alpha_2^D T_{3/2}}{h}
\end{aligned}$$



$$\begin{aligned}
 \frac{\partial u}{\partial t} &= \left( S_x S_y v \right) S_y (\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \left( S_x w D_z u \right) + D_x A_u^h \xi + D_y A_u^h \omega + \\
 &+ g \int_0^z D_x S_z \rho(x, y, \zeta) d\zeta + D_{zz} (A_u^z u) + g D_x (\eta + T_c \phi) \\
 \frac{\partial T}{\partial t} &= -D_x (u S_x T) - D_y (v S_y T) - D_z (w S_z T) + A_T^h \left( D_x D_x T + D_y D_y T \right) + \\
 &+ D_{zz} (A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
 \xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = \alpha_0(x, y)
 \end{aligned}$$

## Vertical velocity

$$\begin{aligned}
 w_{i,j,K-1} &= \alpha_0^{w^b} - \alpha_1^{w^b} h z_{i,j,K-1/2} \xi_{i,j,K-1/2} \\
 w_{i,j,k-1} &= w_{i,j,k} - h z_{i,j,k-1/2} \xi_{i,j,k-1/2} \quad \forall k : 2 \leq k \leq K-1 \\
 w_{i,j,0} &= w_{i,j,1} + \alpha_0^{w^s} - \alpha_1^{w^s} h z_{i,j,1/2} \xi_{i,j,1/2}
 \end{aligned}$$



## Vertical diffusion

$\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}$  is replaced by

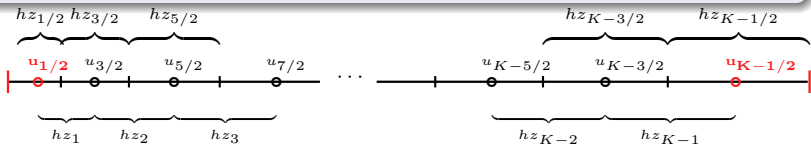
$$\left( D_{zz} u \right)_{i,j,1/2} = \frac{(A_u^z)_1}{h z_1 h z_{1/2}} (\alpha_2^{D_{zz} U^S} u_{3/2} - \alpha_1^{D_{zz} U^S} u_{1/2})$$

$$\left( D_{zz} u \right)_{i,j,k-1/2} = \frac{1}{h z_{k-1/2}} \left( \frac{(A_u^z)_k}{h z_k} (u_{k+1/2} - u_{k-1/2}) - \frac{(A_u^z)_{k-1}}{h z_{k-1}} (u_{k-1/2} - u_{k-3/2}) \right) \quad \forall k : 2$$

$$\left( D_{zz} u \right)_{i,j,K-1/2} = \frac{1}{h z_{K-1/2}} \left[ \alpha_2^{D_{zz} U^b} \frac{(A_u^z)_{K-1}}{h z_{K-1}} u_{K-1/2} - \alpha_1^{D_{zz} U^b} \left( \frac{(A_u^z)_K}{h z_K} + \frac{(A_u^z)_{K-1}}{h z_{K-1}} \right) u_{K-3/2} \right] \quad u_{K-3/2}$$

$$\frac{\partial u}{\partial z} \Big|_{w_0} = \alpha_0^{D_{zz} U^S} + \frac{\tau_x}{h z_1 \rho_0}, \quad \frac{\partial v}{\partial z} \Big|_{w_0} = \alpha_0^{D_{zz} U^S} + \frac{\tau_y}{h z_1 \rho_0}, \quad \frac{\partial T}{\partial z} \Big|_{w_0} = \frac{\partial S}{\partial z} \Big|_{w_0} = \alpha_0^{D_{zz} T^S}$$

$$u|_{\text{bottom}} = v|_{\text{bottom}} = \alpha_0^{D_{zz} U^b} \quad T|_{\text{bottom}} = S|_{\text{bottom}} = \alpha_0^{D_{zz} T^b} \quad (2)$$





$$\begin{aligned}
 \frac{u^{n+1} - u^{n-1}}{2dt} &= \underbrace{\gamma(u^{n+1} - 2u^n + u^{n-1})}_{\text{Asselin filter}} - \left( S_x S_y v^n \right) S_y (\omega^n + f) - \\
 &- D_x \frac{S_x (u^n)^2 + S_y (v^n)^2}{2} - S_z \left( S_x w^n D_z u^n \right) + \\
 &+ \underbrace{D_x A_u^h \xi^{n-1} + D_y A_u^h \omega^{n-1}}_{\text{Explicit Euler}} + \underbrace{D_{zz} A_u^z u^{n+1}}_{\text{Impl.Euler}} + \\
 &+ g \int_0^z D_x S_z \rho^n(x, y, \zeta) d\zeta + \underbrace{g D_x (\eta^{n+1} + T_c \phi^{n+1})}_{\text{Implicit}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{T^{n+1} - T^{n-1}}{2dt} &= \underbrace{\gamma(u^{n+1} - 2u^n + u^{n-1})}_{\text{Asselin filter}} - D_x (u^n S_x T^n) - D_y (v^n S_y T^n) - \\
 &- \underbrace{D_z (w^n S_z T^n) + A_T^h \left( D_{xx} T^{n-1} + D_{yy} T^{n-1} \right)}_{\text{Explicit Euler}} + \\
 &+ \underbrace{D_{zz} (A_T^z) T^{n+1}}_{\text{Implicit}} + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL}
 \end{aligned}$$

The models solution depend on initial conditions and a number of parameters:

$$\frac{\partial T}{\partial t} = -D_x(uS_xT) - D_y(vS_yT) - D_z^{(\alpha)}(wS_z^{(\alpha)}T) + A_T^h \left( D_{xx}T + D_{yy}T \right) + D_{zz}^{(\alpha)}(A_T^z)T$$

- The model  $x(t) = \mathcal{M}_{0,t}(x_0, \alpha)$

We calculate the derivatives and their adjoints with respect to

$$x_0, \alpha$$

by **TAPENADE 3.6 (Tropics team, INRIA)**. that allows us

- to avoid a HUGE development/coding (a double of the classical one, at least)
- to obtain immediately the derivative with respect to any parameter we want.

## TAPENADE 3.6 (Tropics team, INRIA) with the Memory usage optimization:

search for push/pop

```
CALL PUSHREAL8ARRAY(sold, nx*ny*nz)
CALL PUSHREAL8ARRAY(told, nx*ny*nz)
CALL PUSHREAL8ARRAY(vold, nx*ny*nz)
CALL PUSHREAL8ARRAY(uold, nx*ny*nz)
CALL PUSHREAL8ARRAY(ssh, nx*ny)
CALL PUSHREAL8ARRAY(s, nx*ny*nz)
CALL PUSHREAL8ARRAY(t, nx*ny*nz)
CALL PUSHREAL8ARRAY(v, nx*ny*nz)
CALL PUSHREAL8ARRAY(u, nx*ny*nz)
```

replace by

```
call push_uvts(u,v,t,s,ssh)
```

Procedure push/pop\_uvts(u,v,t,s,ssh):

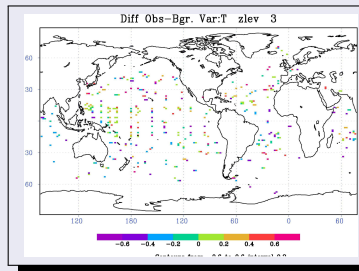
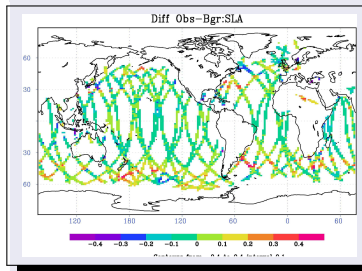
- does not push  $n - 1$  step and pops appropriate values (divides the required memory by 2)
- does not push  $u, v, t, s$  in **lower level** routines
- does not push values **on continents** (divides by 2)
- pushes values in **Real\*4** format (divides by 2)
- eventually pushes only odd timesteps and interpolate when popping (divides by 2)

Total reduction of required memory is **up to 25 times**.

10 hours window  $\implies$  10 days window.

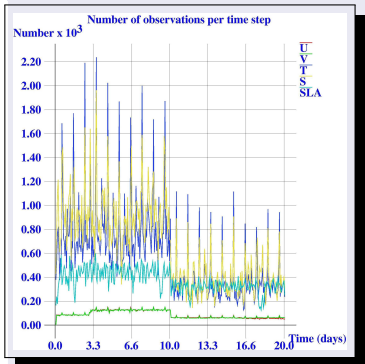
ECMWF data issued from Jason-1 and Envisat altimetric missions and ENACT/ENSEMBLES data banque.

January, 1, 2006.

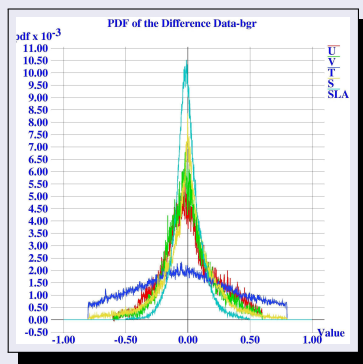


Difference between observations and background during the 1st of January.

January, 1-20 2006.



Number of observation per time step during the 1 – 20 Jan.2006



Probability density function for the difference (observation-background).

The model:  $x_N = \mathcal{M}_{0,N}(x_0, \alpha)$  with  $x = (u, v, T, S, ssh)^T$

## Cost function $J$

$$J = \|x_0 - x_{bgr}\|_{B^{-1}}^2 + \|\alpha - \alpha_{bgr}\|_{B^{-1}}^2 + \sum_{n=0}^N t_n \|\mathcal{H}\mathcal{M}_{0,n}(x_0, \alpha) - y_n\|_{R^{-1}}^2$$

Matrices:  $B^{-1} = \text{diag}(10^{-4})$ ,

$R^{-1} = \text{diag}(1/\sigma_u, 1/\sigma_v, 1/\sigma_T, 1/\sigma_S, 1/\sigma_{ssh})$  where  $\sigma_u^2 = \frac{1}{N_{obs}} \sum (u_{obs} - u_{bgr})^2$

Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal)

## Data Assimilation – Forecast

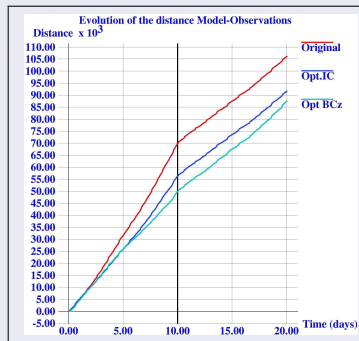
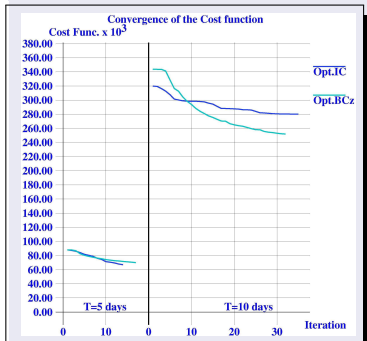
Assimilation window — 10 days,

Test time — 20 days.

The model:  $x(t) = \mathcal{M}_{0,t}(x_0, \alpha)$  with  $x = (u, v, T, S, ssh)^T$

$$\text{Distance: } \xi(t) = \sum_{n=0}^t \|\mathcal{H}\mathcal{M}_{0,n}(x_0, \alpha) - y_n\|_{R^{-1}}$$

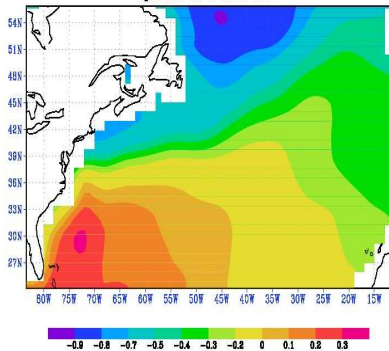
## Convergence of $J$ and evolution of $\xi$



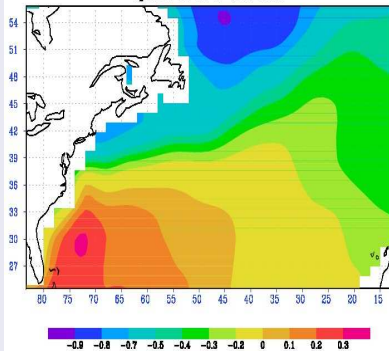
20 Cost function calls with  $T = 5$  days and 40 calls with  $T = 10$  days.

SSH, North Atlantic, January, 1-30 2006.

Opt.IC:SSH Jan,01 00hr



Opt.BCz.SSH Jan,01 00hr

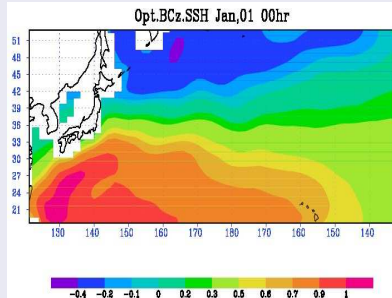
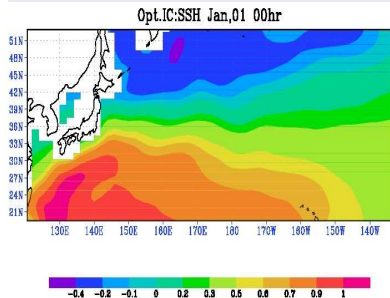


Optimal IC

Optimal BCz



SSH, North Pacific, January, 1-30 2006.



Optimal IC

Optimal BCz

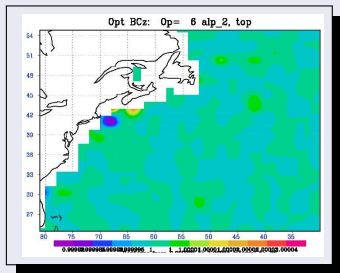
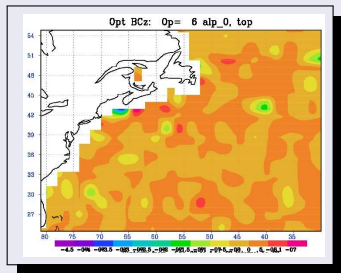
## Modified formula

$$w_{i,j,K-1} = \alpha_0^{w^b} - \alpha_2^{w^b} h z_{i,j,K-1/2} \xi_{i,j,K-1/2}$$

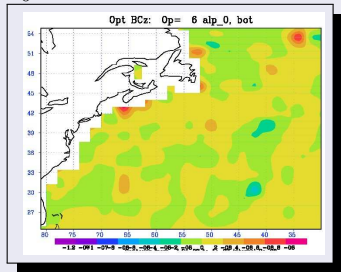
$$w_{i,j,k-1} = w_{i,j,k} - h z_{i,j,k-1/2} \xi_{i,j,k-1/2} \quad \forall k : 1 \leq k \leq K-2$$

$$w_{i,j,0} = w_{i,j,1} + \alpha_0^{w^s} - \alpha_2^{w^s} h z_{i,j,1/2} \xi_{i,j,1/2}$$

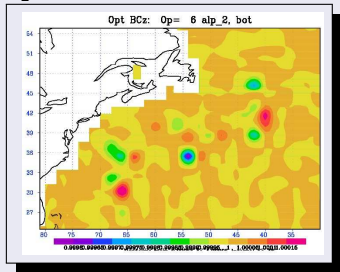
$\alpha$  for the vertical velocity  $w$ . North Atlantic.



$\alpha_0$  on the surface



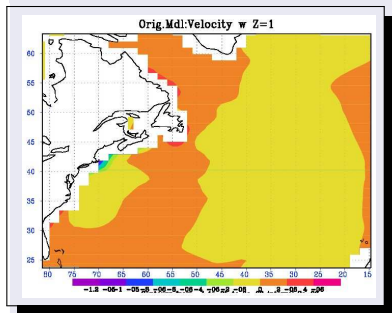
$\alpha_2$  on the surface



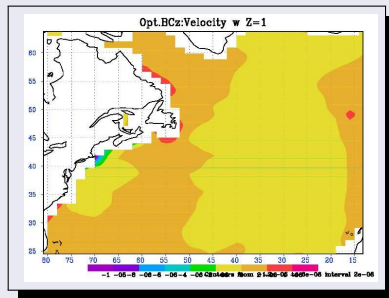
$\alpha_0$  on the bottom

$\alpha_2$  on the bottom

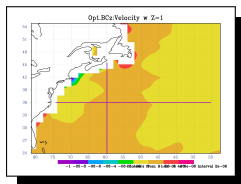
North Atlantic, January, 30, 2006, surface



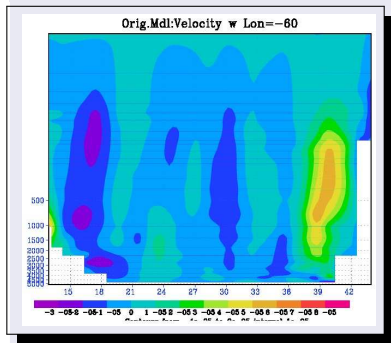
Original model



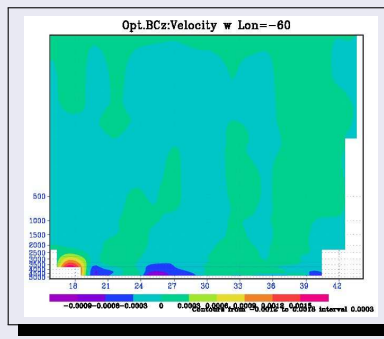
Optimal BCz



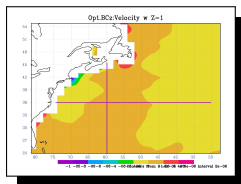
North Atlantic, January, 30, 2006,  $y-z$  section



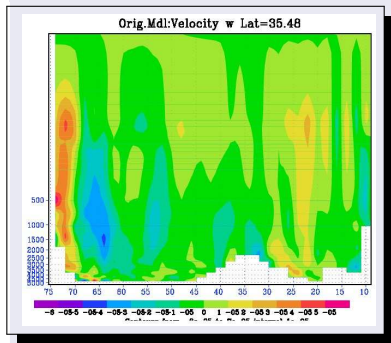
Original model



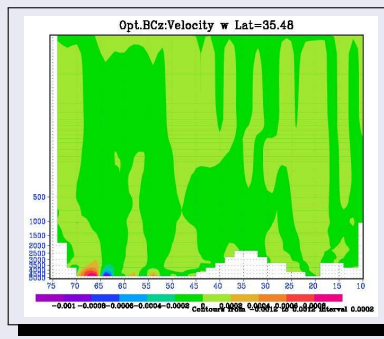
Optimal BCz



North Atlantic, January, 30, 2006,  $x - z$  section

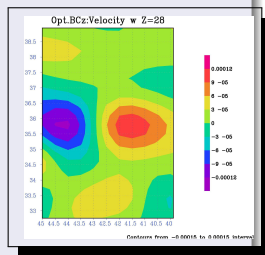
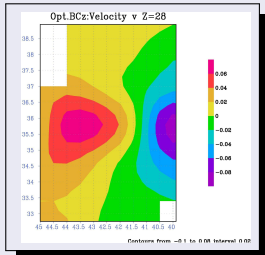
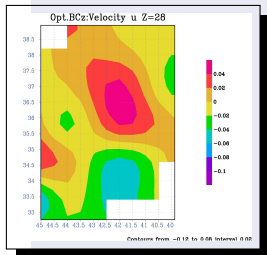


Original model



Optimal BCz

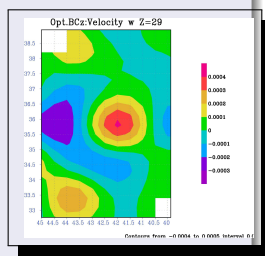
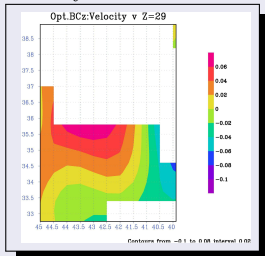
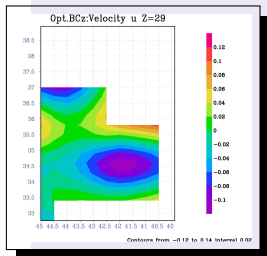
Levels  $z = 28$  and  $z = 29$



Velocity  $u$

Velocity  $v$

Velocity  $w$



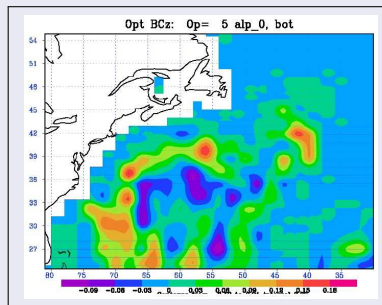
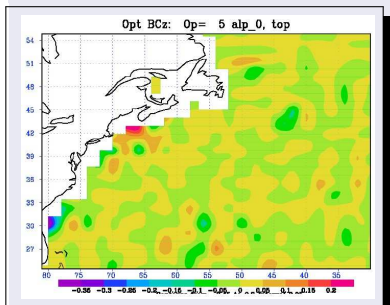
Velocity  $u$

Velocity  $v$

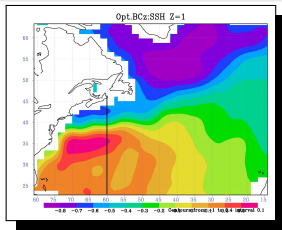
Velocity  $w$

$$\left. \frac{\partial u}{\partial z} \right|_{w_0} = \alpha_0^{D_{zz}U^s} + \frac{\tau_x}{hz_1\rho_0}, \quad \left. \frac{\partial v}{\partial z} \right|_{w_0} = \alpha_0^{D_{zz}U^s} + \frac{\tau_y}{hz_1\rho_0},$$
$$u|_{bottom} = v|_{bottom} = \alpha_0^{D_{zz}U^b}$$

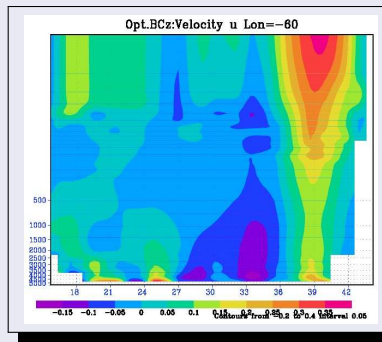
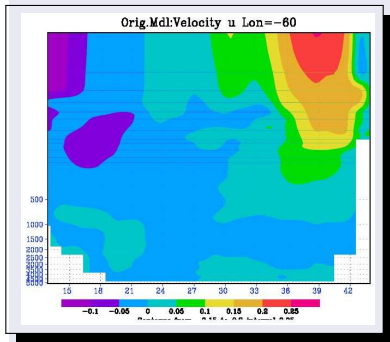
## North Atlantic







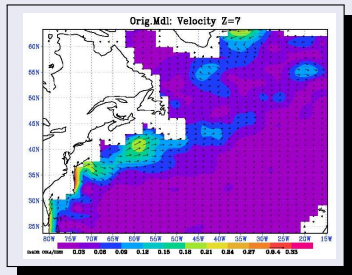
North Atlantic, January, 30, 2006, Velocity  $u$ ,  $y - z$  section



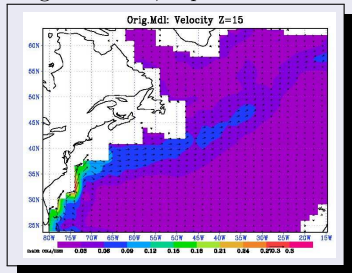
Original model

Optimal BCz

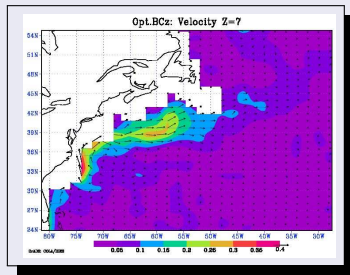
North Atlantic on the Jan.,30,2006



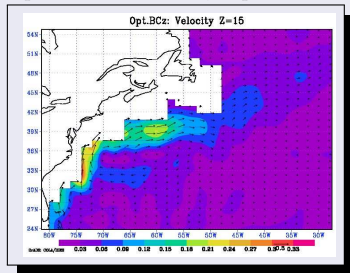
Original model, depth 65 m



Original model, depth 160 m



Optimal BCz model, depth 65 m



Optimal BCz model, depth 160 m

## North Atlantic

Modification of the SSH in the North Atlantic is strongly related to the **boundary conditions of  $u$  and  $v$  especially on the bottom.**

## Boundary Conditions influence is **important**

- The **cost function decreases** more under BCz control than under IC control in the assimilation window
- The models **forecast is closer** to observations with optimal BCz than with optimal IC
- Stream **jets are refined** under BCz control

## Tapenade allows us

- to generate TLM/AM almost immediately,
- to avoid a HUGE development/coding,
- to obtain immediately the derivative with respect to any parameter we want.

But, it requires more memory

	Resolution	TimeStep per day	Model Size	AM Size	Traj. size per 10 days
ORCA 2	$181 \times 149 \times 31$	15	650M	1.6G	660M
ORCA05	$722 \times 511 \times 46$	40	13G	33G	35G

## BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The **physical meaning** of  $\alpha$  is difficult to understand;
- The set of  $\alpha$  is **not unique**;
- The problem of **identifiability** is not addressed yet;
- The problem of **stability** is not even posed.

## Consequently:

It is not a parameter estimation study, but

- a way to **compensate model errors**
- showing the **most influent parameter** (vertical BC for ORCA-2, lateral BC for Shallow-Water).

j'apprécierai beaucoup vos commentaires et critiques



<http://www-ljk.imag.fr/membres/Kazantsev/orca2/index.html>