Power control under best response dynamics for interference mitigation in a two-tier femtocell network

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***Abstract*—Femtocell networks are about to be extensively deployed with the aim of providing substantial improvements to cellular coverage and capacity. However, their deployment is nontrivial because of the extra interference that the femtocell nodes cause to the macrocell nodes with which they share the same portion of the spectrum. This paper proposes a non-cooperative power control approach for interference mitigation in a two-tier femtocell network where the first tier is a conventional cellular network and the second tier is a set of (short-range) femtocells. We define objective functions that are different for Femtocell Mobile Nodes (FMNs) and macrocell Mobile Nodes (MNs). We then define a power control game, we prove the existence of a Nash Equilibrium (NE) for that game and we analyze the necessary and sufficient conditions so that the NE is unique. Based on the best response dynamics method, we propose a distributed iterative power control scheme that, starting from any initial power vector, converges to that NE. We simulate various scenarios that are based on realistic assumptions and topologies. Results show that, in many cases, in the NE, smooth coexistence of all entities of the topology is feasible.**

I. Introduction and Motivation

Due to the broadcast nature of the wireless spectrum, wireless entities such as Access Points (APs), Base Stations (BSs), Mobile Nodes (MNs), etc., that are collocated and share the same portion of the spectrum are inherently competing with each other for access to that spectrum. The Quality-of-Service (QoS) realized by a link typically depends on the Signal-to-Interference-plus-Noise-Ratio (SINR) at the link's receiver, and an entity's target SINR may not be always achieved if another entity is trying to achieve its own and interferes.

At the same time, the demand for mobile data is increasing tremendously. Owners of 3G smartphones increasingly use multimedia services such as streaming video and audio, web-surfing and e-mail. In 2008, the yearly global mobile data traffic was 1.3 Exabytes [1]. The prediction is that by 2014, the mobile data traffic per month will be 1.6 Exabytes. A study in 2011 [2] claims that by 2015 mobile data traffic per month will be 6.3 Exabytes. It is also worth mentioning that more than 70% of this data traffic is generated indoors (mostly at home or at the office) [1]. Consequently, a major challenge for mobile operators is to continue to provide an excellent data experience indoors given the tremendous growth of data traffic. However, a prerequisite for excellent indoor data traffic is excellent signal strength. New wireless cellular standards such as 3GPP's High Speed Packet Access (HSPA) and Long Term Evolution (LTE) achieve considerable improvements in system capacity and throughput, but at the cost of high operational expenses and capital expenditures. A way to bring costs down and improve performance is to deploy, in addition to standard cells, termed *macrocells* in our context, a large number of smaller and cheaper cells which are called *femtocells* and connect to the mobile operator network using residential DSL or cable broadband connections [3]. Femtocells belong to a broader class of radio access technology called *small cells*. Small cells are expected to be a key feature for future LTE networks, where all cells will be self-organizing [4].

Indoor users that are connected to femtocells experience superior indoor reception and achieve better data rates than macrocell users. Often, this is achieved with low user transmission power, so that battery life prolongation is also achieved. Such networks, comprised of a conventional macrocell network overlaid with a number of femtocell base stations (FBSs) are referred to as *two-tier femtocell networks*.

One of the biggest challenges for the successful deployment of these networks is mitigating the interference that the femtocell users cause to the macrocell users when they share the same frequency bands (which is the typical case). If the level of interference is not controlled, the deployment of two-tier femtocell networks is problematic. Observe that cellular networks have been dimensioned without taking into account the existence of femtocells, and therefore it is imperative that MNs be protected. Consequently, the adoption of radio resource management techniques is of crucial importance to alleviate the problems of this femtocell-macrocell interference.

In this work, we formulate a non-cooperative power control game with a view to alleviating the consequences of the interference in a two-tier femtocell network. Game theory is a natural framework to model these interactions. We assume that each node (either femtocell or macrocell) aims at maximizing its own objective function, by modifying its transmission power. We prove the existence and uniqueness of a Nash

Equilibrium for the resulting game and propose a distributed power control algorithm that, based on the best response dynamics method, converges to this unique set of transmission powers.

II. Related Work

Power control, *i.e.*, selecting transmitter powers to achieve a specific target, has been extensively studied since the early 90's. A review of some fundamental approaches can be found in [5]. The key feature of any power control algorithm is whether it is designed for use in a voice or data network. Power control algorithms were firstly applied in voice networks. The idea was to find a set of transmitter powers so that the SINR targets of all the links could be satisfied. Distributed iterative schemes were presented that can always find a solution, in case there is one [6]. In parallel, various power control algorithms were developed that are suitable for data networks. The idea is that each user aims at maximizing its own utility function $U_i(\cdot)$. In the case of power control games, the general form of a utility function is $U_i(P_i, \mathbf{P}_{-i}) = V_i(P_i, \mathbf{P}_{-i}) - C_i(P_i)$, where $V_i(\cdot)$ is a value function that expresses the value that the link perceives and $C_i(\cdot)$ is a cost function that expresses the resources that it has to spend to achieve this value. P_i is the transmission power of user i, whereas $\mathbf{P}_{-i}$ is the vector of the transmission powers of all users except user i.

Considering these approaches, we note the following: Power control in voice networks is simple(r); it is SINR-based and incorporates only this metric; moreover, the SINR targets are "hard" in the sense that if the user cannot satisfy its target, then its value is zero. On the other hand, power control in data networks is (more) complex; it is utility-based and may incorporate various metrics; moreover, SINR targets are "soft", as a user may obtain a nonzero utility value even if the SINR that it perceives is lower than the ideal value.

However, 3G and 4G networks (will) consist of nodes with heterogeneous targets and needs. It is challenging to (try to) unify these approaches with a view to providing algorithms that, depending on the entity, will focus either on voice or on data services. A two-tier femtocell network definitely corresponds to that case. Macrocell (traditional) users are mostly interested in making voice calls. On the other hand, femtocell users focus on data services. As explained in the introduction, femtocell networks are not designed aiming (simply) at providing better coverage of indoor voice calls but are considered an important vehicle towards the unlimited broadband data era and this is their primary focus. A good power control algorithm should be simple, fast, efficient, and flexible. Adopting a hybrid approach that combines the simplicity of the SINR-based approaches with the powerful utility-based approaches is a nontrivial task that may lead to significant contributions.

III. Non-cooperative Power Control in Two-tier Femtocell Networks

A. Game Theory Preliminaries and System Model

A *strategic (or normal form) non-cooperative game G* with a finite number of players consists of the following triplet: A set of players $\mathbf{N} = \{1, 2, ..., N\}$ and, for each player i, a set of strategies (actions) S_i, and a utility (payoff) function $U_i(\cdot)$. A key concept in non-cooperative games is the pure Nash Equilibrium (NE) which is defined as follows:

Definition 1: $\mathbf{s}^\star = [s_1^\star, s_2^\star, \ldots, s_n^\star]^T$ is a pure NE for a game G if $\forall i \in N$ and $\forall s'_i \in S_i$ $U_i(s_i^\star, \mathbf{s}_{-i}^\star) \geq U_i(s'_i, \mathbf{s}_{-i}^\star)$.

Consequently, a pure NE corresponds to a steady state of a game in the sense that no player has an incentive to change unilaterally its own strategy. In the following, we shall deal with pure NEs only (and we will not deal with other types), so we shall omit the term "pure". Given a game G in strategic form, the standard roadmap, which we follow here, is to search for answers to the following questions:

- (Existence of NE): Has the game G at least one NE?
- (Uniqueness of NE): Are there conditions that guarantee the existence of a *unique* NE for the game G?
- (Algorithm for finding a NE): Can we find an algorithm that converges to a NE of the game G?

In our case, we study a CDMA network that consists of N_1 macrocell Mobile Nodes (MNs) and N_2 Femtocell Mobile Nodes (FMNs) that coexist in the same area (e.g., home, office). Following the standard abstraction model [5], a wireless network is considered as a collection of directly interfering links, where each link consists of a transmitter and a receiver. We focus on the uplink and we assume a closed access model [3]. This means that each Femtocell Base Station (FBS) may associate only with predefined FMNs and no MNs can connect to it. The strategy of each node is to update its transmission power P_i that belongs either to $[0, P_{\max}]$ if i is a MN or to $[0, FP_{\max}]$ if i is a FMN ($FP_{\max}$ is the maximum transmit power available to the FMNs).

Let $G_{ij} > 0$ express the link gain from transmitter i to receiver j and $n(>0)$ be the noise of the channel, assumed, for the sake of simplicity, to be the same for all nodes. Let L be the spread factor of the CDMA network. Let R_i be the total interference plus noise that a node receives (note that it is always positive). SINR_i is defined as:

$$\mathrm{SINR}_i = L\frac{G_{ii}P_i}{\sum\limits_{j\neq i} G_{ji}P_j + n} = L\frac{G_{ii}P_i}{R_i}.$$

MN_i utility function: $U_i(P_i, \mathbf{P}_{-i}) = B_i \ln(1 + \mathrm{SINR}_i)$, where B_i is a positive constant, $0 \leq P_i \leq P_{\max}$ and $\mathrm{SINR}_i \leq \mathrm{targetSINR}_i \triangleq \gamma_i$.

FMN_i utility function: $U_i(P_i, \mathbf{P}_{-i}) = B_i \ln(1 + \mathrm{SINR}_i) - c_i P_i$, where B_i, c_i are positive constants, $0 \leq P_i \leq FP_{\max}$.

Each MN_i uses a utility function which is a logarithmic function of the user's SINR. This utility function can be interpreted as being proportional to the Shannon capacity and is weighted by a positive user-specific parameter B_i that corresponds to the user's desire for SINR. Moreover, there are 2 constraints: The transmission power should not exceed $P_{\max}$ and the SINR of user i should belong to the interval $[0, \gamma_i]$.

On the other hand, each FMN_i uses a different utility function. Apart from the value part (which is the same with the one of a macrocell transmitter), the cost part is a linear pricing function of P_i that defines the price that user i has to pay for using a specific amount of power. As previously, the transmission power should not exceed $FP_{\max}$. This utility function is inspired by the one proposed in [7].

The reasons that we choose different objective functions for each category of users are the following: Macrocell users have a higher priority to be served by the mobile operators, as they will be mostly used for inelastic, voice traffic. They can use any transmission power up to $P_{\max}$ (without paying for their choice) to overcome the extra interference that is caused by the femtocell users. On the other hand, femtocells are deployed by indoor users for their own interest. Consequently, a pricing policy is applied to discourage them from creating high interference to the macrocell users. However, as femtocells have generally higher demands for QoS, there is no maximum SINR constraint for them. This means that depending on the conditions (e.g., when the outdoor users are very distant or have achieved their SINR targets), they can increase their SINR (and consequently, their throughput and data rate) as much as possible. Even if we use different values of the parameters for femtocells and macrocells, the above heterogeneous characteristics could not be expressed successfully by a sole utility function.

As a final comment, we point out that the idea of using different objective functions for femtocell and macrocell users has already been proposed in [8]. However, the approach there is highly related to SINR. The authors demand that the SINR of each user i belongs to an interval $[\mathrm{minSINR}_i, \mathrm{maxSINR}_i]$. In case that the SINR targets of macrocell users cannot be achieved, femtocell users are obliged to adjust their targets to the interval $[k \cdot \mathrm{minSINR}_i, k \cdot \mathrm{maxSINR}_i], 0 < k < 1$.

B. Existence of a NE in the two-tier femtocell network game

To prove that the game G has at least one NE, we use the following theorem by Debreu-Fan-Glicksberg (1952) [9]:

Theorem 1: Let G be a strategic non-cooperative game. Suppose that $\forall i \in \mathbf{N} = \{1, 2, ..., N\}$ (where $\mathbf{N}$ is the set of players):

- *The strategy set S_i is a compact and convex set.*
- *The utility $U_i(\mathbf{s})$, where $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T$ is continuous in $\mathbf{s}$ and quasi-concave in s_i.*

Then the game G has at least one NE.

Theorem 2 (existence of a NE): The two-tier femtocell network game G has at least one NE.

Proof: It is straightforward to check that all conditions of *Theorem 1* are fulfilled. ■

C. Best response dynamics scheme

Given the fact that we know that a game has a NE, how can we devise an algorithm that converges to a NE? We shall present the fundamentals of best response dynamics schemes, which may lead to a NE.

TABLE I: An example of a 2-player game. Numbers in cells correspond to the utility of each player.

Player_1 \ Player_2	Bach	Stravinsky
Bach	(2,2)	(0,0)
Stravinsky	(0,0)	(1,1)

Definition 2: Let G a strategic non-cooperative game. The best response strategy of player i is the one that maximizes his utility, taking other players' strategies as given.

An equivalent definition of the NE incorporates the notion of best response:

Definition 3: $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T$ is a NE of a strategic game G with N players IFF every player's strategy is a best response to the other players' strategies.

The idea of best response is useful when we are trying to find an approach to reach a steady state of a game, *i.e.*, a NE of a game. A best response dynamics scheme consists of a sequence of rounds, where in each round after the first, each player i chooses the best response to the other players' strategies in the previous round. In the first round, the choice of each player is the best response based on its arbitrary belief about what the other players will choose. In some games, the sequence of strategies generated by best response dynamics converges to a NE, regardless of the players' initial strategies. However, this does not hold in general. A nice counterexample is presented in TABLE I [10]:

Let us suppose that, at round 1, Player_1 believes that Player_2 will choose Bach, whereas Player_2 believes that Player_1 will choose Stravinsky. So, Player_1 will choose Bach as best response to that belief and Player_2 will choose Stravinsky correspondingly. So, at round 1, they will play (Bach, Stravinsky) and the utilities will be $(0, 0)$. At round 2, the best responses to round 1 will lead to (Stravinsky, Bach) and the utilities will be $(0, 0)$. Consequently, the choices will infinitely switch from (Bach, Stravinsky) to (Stravinsky, Bach) and vice versa. Players will never reach one of the two NE of the game, *i.e.*, (Bach, Bach), (Stravinsky, Stravinsky).

Although the adoption of a best response dynamics scheme is neither a necessary nor a sufficient condition for the reachability of a NE, we shall use this technique as the basis for a distributed power control algorithm for the game that we study and we shall come up with conditions that guarantee the convergence of this scheme.

D. Power control under best response dynamics

We can see the two-tier power femtocell network game G as a collection of N parallel optimization problems, where each (F)MN aims at maximizing its own utility function U_i with no interest for the others. We shall pose these optimization problems and solve them with the use of the Karush-Kuhn-Tucker (KKT) conditions that generalize the method of Lagrange multipliers [11].

- Maximization problem of MN_i:

$$\max_{P_i} U_i(P_i, \mathbf{P}_{-i}) = \max\{B_i \ln(1 + \mathrm{SINR}_i)\},$$
$$\text{subject to: } 0 \leq P_i \leq P_{\max} \text{ and } \mathrm{SINR}_i \leq \gamma_i.$$

The optimal power $P_i^\star$ has the form:

$$P_i^\star = \min\left\{P_{\max}, \gamma_i \frac{R_i}{LG_{ii}}\right\}. \quad (1)$$

Therefore, we arrive at the well-known Simplified Foschini-Miljanic formula with $P_{\max}$ constraint [12]. However, the key difference is that, contrary to [12], where each node's utility value is either 0 (when the target is not achieved) or 1 (when the target is achieved), each user gets a nonzero value even if it has not achieved its SINR target.

- Maximization problem of FMN_i:

$$\max_{P_i} U_i(P_i, \mathbf{P}_{-i}) = \max\{B_i \ln(1 + \mathrm{SINR}_i) - c_i P_i\},$$
$$\text{subject to: } 0 \leq P_i \leq FP_{\max}.$$

The objective function and the inequality constraints functions are differentiable convex functions. Therefore, the KKT conditions are necessary and sufficient conditions for having primal and dual optimality [11]. By solving the system of the KKT conditions, we get the optimal power $P_i^\star$:

$$P_i^\star = \max\left\{0, \min\left\{\frac{B_i}{c_i} - \frac{R_i}{LG_{ii}}, FP_{\max}\right\}\right\}. \quad (2)$$

By definition, $P_i^\star$ is a best response of player i to the other players' strategies. We then present the pseudocode of Algorithm 1 which is a power control scheme under best response dynamics for our game.

Algorithm 1 Power control under best response dynamics for a two-tier femtocell network

for $k = 1 \rightarrow$ MAX_NUMBER_OF_ITERATIONS **do**
  each receiver i passes to its associated transmitter i the level of the total received power $\sum_j G_{ji} P_j(k) + n$.
  each transmitter i computes $\mathrm{SINR}_i = L \frac{G_{ii} P_i(k)}{R_i(k)}$.
  if i is a macrocell transmitter, it updates its power adjusting (1) to:

$$P_i(k+1) = \min\left\{P_{\max}, \gamma_i \frac{R_i(k)}{LG_{ii}}\right\}. \quad (3)$$

  if i is a femtocell transmitter, it updates its power adjusting (2) to:

$$P_i(k+1) = \max\left\{0, \min\left\{\frac{B_i}{c_i} - \frac{R_i(k)}{LG_{ii}}, FP_{\max}\right\}\right\}. \quad (4)$$

  if $|P_i(k+1) - P_i(k)| \leq e$, where e is a small positive quantity, **break**;
end for

It is worth mentioning that Algorithm 1 is fully distributed in the sense that each link does not need to exchange information with other links to decide upon the level of its transmission power at the transmission round $k+1$. More specifically, each $\mathrm{(F)MN}_i$ needs to know the following information: a) its transmission power at the previous transmission round k, b) the values of the parameters L, G_{ii}, c) the total interference that it has received at the previous transmission round d) (if it is a MN) the values of the target SINR γ_i and e) (if it is a FMN) the values of the parameters B_i and c_i. Elements a), b), d) and e) are already known to each (F)MN, whereas element c) can be easily computed through the downlink.

We also mention that Algorithm 1 is a synchronous scheme, in the sense that (F)MNs should update their transmission powers concurrently. This is the reason that we consider slotted time. However, Algorithm 1 works even with asynchronous updates, provided that each (F)MN has updated its power in the semi-open time interval $[k, k+1)$.

E. Uniqueness of the NE for two-tier femtocell network

In this section, we sketch the main ideas of the proof that Algorithm 1 converges to a NE and this is the unique NE of the game. Mathematically, the uniqueness of a NE is equivalent to proving the existence of a unique *fixed point*, which is a point that is mapped to itself by a function. We restate the following notions from distributed optimization [13], which will be useful in the rest of this section.

Definition 4: Let $M(\cdot) : X \rightarrow X$ be a mapping. Let $\mathbf{x}^\star \in X$ be a fixed-point. M is a pseudo-contraction mapping with respect to some norm $\|\cdot\|$ if there exists $q \in [0, 1)$ so that:

$$\|M(\mathbf{x}) - \mathbf{x}^\star\| \leq q\|\mathbf{x} - \mathbf{x}^\star\|, \forall \mathbf{x} \in X.$$

Theorem 3: Suppose that $X \subset \Re^n$ and that the mapping $M(\cdot) : X \rightarrow X$ is a pseudo-contraction with a fixed point $\mathbf{x}^\star \in X$. Then M has no other fixed points and the sequence $\{\mathbf{x}(k)\}$ generated by $\mathbf{x}(k+1) = M(\mathbf{x}(k))$ converges to $\mathbf{x}^\star$.

Let $T_i(k) = G_{ii} P_i(k)$ be the received power from the transmission of $\mathrm{(F)MN}_i$ at time k. (3) and (4) can be rewritten as:

$$T_i(k+1) = \min\left\{T_{\max}, \gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j \neq i} T_j(k)}{L}\right\}. \quad (5)$$

$$T_i(k+1) = \max\{0, \min\{\frac{G_{ii} B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j \neq i} T_j(k)}{L}, FT_{\max}\}\}. \quad (6)$$

Similarly, the received power level at the NE $T_i^\star$ can be rewritten as:

$$T_i^\star = \min\left\{T_{\max}, \gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j \neq i} T_j^\star}{L}\right\}. \quad (7)$$

$$T_i^\star = \max\left\{0, \min\left\{\frac{G_{ii} B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j \neq i} T_j^\star}{L}, FT_{\max}\right\}\right\}. \quad (8)$$

Let also

$$\Delta T_i(k) = T_i(k) - T_i^\star \quad (9)$$

be the distance between the received power from the transmission of $\mathrm{(F)MN}_i$ at time k and the received power level at the NE. We state the following theorem:

Theorem 4: Let $\mathbf{N} = [MN_1, MN_2, \cdots, MN_{N1}, FMN_1, FMN_2, \cdots, FMN_{N2}]$ *be the set of size N that corresponds to the players in the two-tier femtocell network game. The following inequalities hold* $\forall i \in \mathbf{N} = \{1, 2, ..., N\}$:

$$\text{If } i \text{ is a MN, then:} |\Delta T_i(k+1)| \leq \left|\gamma_i \frac{1}{L}\sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

$$\text{If } i \text{ is a MN, then:} |\Delta T_i(k+1)| \leq \left|\frac{1}{L}\sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

Proof: The proof is based on the examination of all possible combinations for the form of the pair $(T_i(k+1), T_i^{\star})$. For each combination, we use properties of the absolute value. Let i be a MN. There are 4 cases. We shall present two of them. All cases are available in [14]. Case #1:

$$T_i(k+1) = \gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j(k)}{L},$$
$$T_i^{\star} = \gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^{\star}| = \left|\gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j(k)}{L} - \gamma_i \frac{n}{L} - \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}\right| = \left|\gamma_i \frac{\sum_{j\neq i}(T_j(k) - T_j^{\star})}{L}\right| = \gamma_i \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case #2:

$$T_i(k+1) = T_{\max}, \quad T_i^{\star} = \gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}.$$

From (5) and (7) we get:

$$\gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j(k)}{L} > T_{\max},$$
$$0 < \gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L} \leq T_{\max}.$$

So we have:

$$\gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}\right) \geq T_{\max} - \left(\gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}\right).$$

We then get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^{\star}| = \left|T_{\max} - \gamma_i \frac{n}{L} - \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}\right| \leq \left|\gamma_i \frac{n}{L} + \gamma_i \frac{\sum_{j\neq i} T_j(k)}{L} - \gamma_i \frac{n}{L} - \gamma_i \frac{\sum_{j\neq i} T_j^{\star}}{L}\right| = ^{\text{Case\#1}} \gamma_i \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

After examining the remaining two cases [14], we find that, for each MN i, the following inequality holds:

$$|\Delta T_i(k+1)| \leq \left|\gamma_i \frac{1}{L}\sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

Let i be a FMN. There are 9 cases. We shall present three of them. All cases are available in [14]. Case #1:

$$T_i(k+1) = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L},$$
$$T_i^{\star} = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L}.$$

Then:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^{\star}| = \left|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \frac{G_{ii}B_i}{c_i} + \frac{n}{L} + \frac{\sum_{j\neq i} T_j^{\star}}{L}\right| = \left|\frac{\sum_{j\neq i}(T_j(k) - T_j^{\star})}{L}\right| = \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case #2:

$$T_i(k+1) = 0, \quad T_i^{\star} = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L}.$$

From (6) and (8) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} < 0,$$
$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L} \geq 0.$$

So we have:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right) > \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L}.$$

We then get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^{\star}| = \left|0 - \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L}\right| \leq |\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right)| = ^{\text{Case\#1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

Case #3:

$$T_i(k+1) = FT_{\max},$$
$$T_i^{\star} = \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^{\star}}{L}.$$

From (6) and (8) we get:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} > FT_{\max},$$
$$0 \leq \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} \leq FT_{\max}.$$

So:

$$\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right) \geq FT_{\max} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right).$$

We then get:

$$|\Delta T_i(k+1)| = |T_i(k+1) - T_i^\star| = \left|FT_{\max} - \frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L}\right| \leq$$
$$|\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j^\star}{L} - \left(\frac{G_{ii}B_i}{c_i} - \frac{n}{L} - \frac{\sum_{j\neq i} T_j(k)}{L}\right)| =$$
$$=^{\text{Case\#1}} \frac{1}{L}\left|\sum_{j\neq i}(\Delta T_j(k))\right|.$$

After examining the remaining six cases [14], we find that, for each FMN i, the following inequality holds:

$$|\Delta T_i(k+1)| \leq \left|\frac{1}{L}\sum_{j\neq i}^{N} \Delta T_j(k)\right|.$$

■

Uniqueness of the NE for the two-tier femtocell network game

Theorem 5: Let L be the spread factor of the system and $\gamma_{\max}$ the maximum SINR target of a FMN.
If $N < \max\left\{\frac{L}{\gamma_{\max}} + 1, L+1\right\}$, then:

- *The two-tier femtocell network game has a unique NE.*
- *The power control scheme under best response dynamics of Algorithm 1 for FMNs and MNs converges to this NE.*

Proof: We introduce the N-size vector that contains all the parameters ΔT_i and we take the maximum norm of that vector. By using *Theorem* 3 and *Theorem* 4, we can prove the existence of a pseudo-contraction under the above condition and the convergence of Algorithm 1 to a unique fixed point (*i.e.*, a NE). As the best response dynamics scheme is a pseudo-contraction in the entire strategy space, this is the unique NE of the two-tier femtocell network game [15]. Let

$$\mathbf{\Delta T} = [\Delta T_{\text{MN\_1}}, \Delta T_{\text{MN\_2}}, \cdots, \Delta T_{\text{MN\_N1}}, \Delta T_{\text{FMN\_1}}, \Delta T_{\text{FMN\_2}}, \cdots, \Delta T_{\text{FMN\_N2}}]^T$$

be a N-size vector. Its maximum norm $\|\mathbf{\Delta T}\|_\infty$ is defined as:

$$\|\mathbf{\Delta T}\|_\infty = \max\{|\Delta T_{\text{MN\_1}}|, |\Delta T_{\text{MN\_2}}|, \cdots, |\Delta T_{\text{MN\_N1}}|, |\Delta T_{\text{FMN\_1}}|, |\Delta T_{\text{FMN\_2}}|, \cdots, |\Delta T_{\text{FMN\_N2}}|\}.$$

Then, by using *Theorem 4*, we get:

$$\|\mathbf{\Delta T}(k+1)\|_\infty = \max\{|\Delta T_{\text{MN\_1}}(k+1)|, |\Delta T_{\text{MN\_2}}(k+1)|, \cdots, |\Delta T_{\text{MN\_N1}}(k+1)|, |\Delta T_{\text{FMN\_1}}(k+1)|, |\Delta T_{\text{FMN\_2}}(k+1)|, \cdots, |\Delta T_{\text{FMN\_N2}}(k+1)|\}^T.$$

We distinguish two cases:

Case #1: The maximum norm $\|\mathbf{\Delta T}(k+1)\|_\infty$ belongs to a FMN. Then:

$$\|\mathbf{\Delta T}(k+1)\|_\infty = \max_i\{|\Delta T_i(k+1)|\} \leq \max_i\left\{\left|\frac{1}{L}\sum_{j\neq i}^{N}\Delta T_j(k)\right|\right\} = \frac{1}{L}\max_i\left\{\left|\sum_{j\neq i}^{N}\Delta T_j(k)\right|\right\} \leq \frac{1}{L}\max_i\left\{\sum_{j\neq i}^{N}|\Delta T_j(k)|\right\} \leq \frac{1}{L}(N-1)\max_j\{|\Delta T_j(k)|\}.$$

So:

$$\|\mathbf{\Delta T}(k+1)\|_\infty \leq \frac{N-1}{L}\|\mathbf{\Delta T}(k)\|_\infty. \tag{10}$$

Case #2: The maximum norm $\|\mathbf{\Delta T}(k+1)\|_\infty$ belongs to a MN. Then:

$$\|\mathbf{\Delta T}(k+1)\|_\infty = \max_i\{|\Delta T_i(k+1)|\} \leq \max_i\left\{\left|\frac{1}{L}\gamma_i\sum_{j\neq i}^{N}\Delta T_j(k)\right|\right\} = \frac{1}{L}\max_i\{\gamma_i\}\max_i\left\{\left|\sum_{j\neq i}^{N}\Delta T_j(k)\right|\right\} \leq \frac{1}{L}\gamma_{\max}\max_i\left\{\sum_{j\neq i}^{N}|\Delta T_j(k)|\right\} \leq \frac{1}{L}\gamma_{\max}(N-1)\max_j\{|\Delta T_j(k)|\}.$$

So:

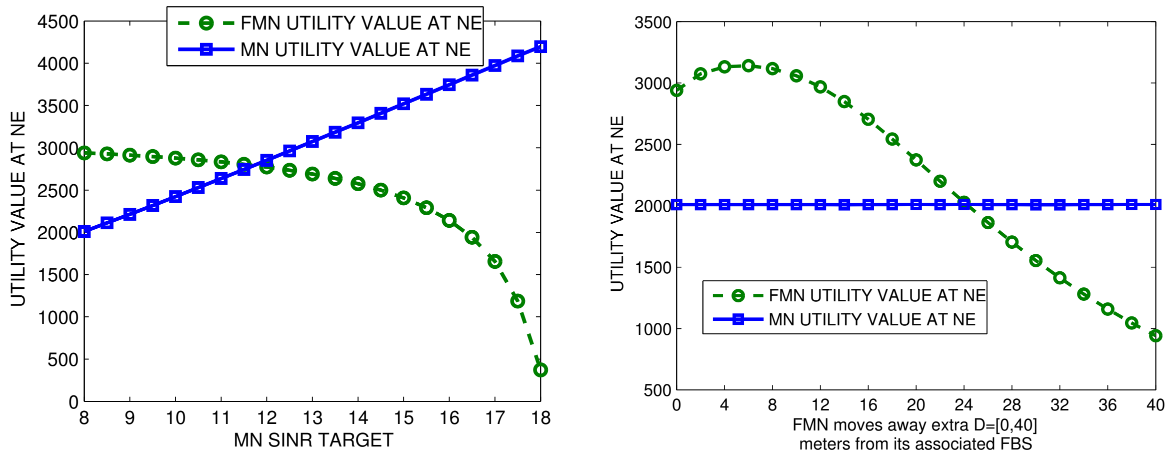
$$\|\mathbf{\Delta T}(k+1)\|_\infty \leq \frac{N-1}{L}\gamma_{\max}\|\mathbf{\Delta T}(k)\|_\infty. \tag{11}$$

From (10) and (11) we get:

$$\|\mathbf{\Delta T}(k+1)\|_\infty \leq \max\left\{\frac{N-1}{L}\gamma_{\max}, \frac{N-1}{L}\right\}\|\mathbf{\Delta T}(k)\|_\infty \Leftrightarrow \|\mathbf{T}(k+1) - \mathbf{T}^\star\|_\infty \leq$$
$$\leq \max\left\{\frac{N-1}{L}\gamma_{\max}, \frac{N-1}{L}\right\}\|\mathbf{T}(k) - \mathbf{T}^\star\|_\infty.$$

From *Definition* 4, this is a pseudo-contraction mapping IFF:

$$\max\left\{\frac{N-1}{L}\gamma_{\max}, \frac{N-1}{L}\right\} < 1 \Leftrightarrow N < \max\left\{\frac{L}{\gamma_{\max}} + 1, L+1\right\}.$$



(a) F)MN evolution of the NE utility under Scenario #1.

(b) F)MN evolution of the NE utility under Scenario #2.

Fig. 1: (F)MN evolution of the NE utility under Scenario #1 and Scenario #2.

Consequently, from *Theorem* 3, the power control game under best response dynamics for FMNs and MNs converges to a unique NE.

Moreover, as the best response dynamics scheme is a pseudo-contraction in the entire strategy space, this is the unique NE of the two-tier femtocell network game [15]. ■

Theorem 5 provides an upper bound on the number of players (MNs and FMNs) that should share the same portion of the spectrum. This bound is proportional to the spread factor of the system L and inversely proportional to the maximum SINR target among all the MNs of the topology. This result is natural and can be interpreted as following: The higher the target of the FMNs, the less the number of them that can achieve it. On the other hand, a high(er) value of the parameter L is desirable as it offers the opportunity to a (F)MN to reinforce its SINR without needing to increase its transmission power. So, if we apply a initial admission control scheme so that the condition of *Theorem* 5 is satisfied, players will converge to a unique NE.

IV. Performance Evaluation

We have simulated our scheme for topologies that consist of one BS placed at the origin $(0,0)$ and associated with two MNs $(\mathrm{MN}_1, \mathrm{MN}_2)$, as well as two FBS $(\mathrm{FBS}_1, \mathrm{FBS}_2)$, each one having two FMNs $(\mathrm{FMN}_1, \mathrm{FMN}_2)$ and $(\mathrm{FMN}_3, \mathrm{FMN}_4)$ respectively.

It is worth mentioning that we distinguish two cases for the path loss model. For indoor-to-indoor communication, where indoor (F)MNs communicate with FBS, we use the ITU P.1238 model for the path loss [16], *i.e.*:

$$PL(\mathrm{dB}) = 20\log_{10}(f) + V\log_{10} d + L_f(z) - 28.$$

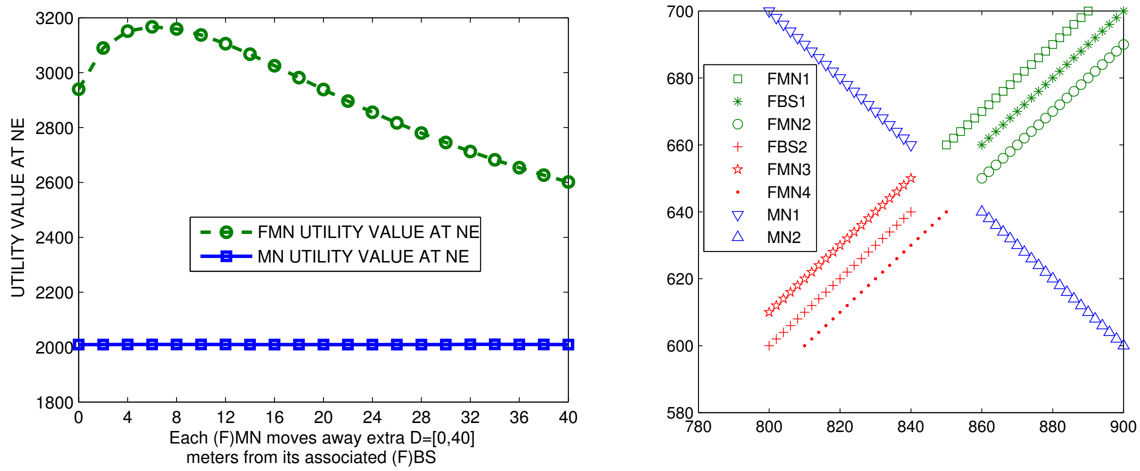
For indoor-to-outdoor communication, where indoor (F)MNs communicate with the BS, we use the Okumura-Hata model for large cities [16]. The Path Loss formula as a function of the distance d (in km) between the (F)MN and the BS is calculated as:

$$PL(\mathrm{dB}) = 125.76 + 35.22\log_{10} d, \quad d > 1 \quad \mathrm{km}.$$

We examine the utility values and the SINR for each (F)MN at the NE. All system parameters are available in [14] and are based on an extensive study conducted by the Small Cell Forum (former Femtoforum) [16]. We have studied various scenarios where we gradually update some of the following parameters of the topology: positions of the MNs, positions of the FMNs, positions of the FBSs, MN SINR targets. We just present three of them and depict the utility value at the NE for each simulation round for each (F)MN. As we have studied symmetric topologies, each (F)MN has the same utility value.

Scenario #1 (Fig.1a) corresponds to the case where the positions of all entities are fixed. In each new simulation round, the target SINR of each MN increases with a step equal to $\Delta_{\mathrm{SINR}} = 0.5$ dB. As expected, the utility value of the MN is increasing as the SINR target increases. In addition, to achieve a higher utility value, each MN uses higher power. As the positions of all entities are fixed, the interference that each FMN receives is increasing. So, the utility value at the NE is decreasing. However, apart from the last simulation round, the SINR achieved per FMN is over 8 dB, which is sufficient for smooth voice communication [16].

Scenario #2 (Fig.1b) corresponds to the case that, in each new simulation round, each FMN gradually moves away from its associated FBS. All other parameters are fixed. Up to round 4, the utility value at the NE of each FMN is increasing as it is able to increase its transmission power to augment its utility. From round 5 and on, the utility value at the NE is decreasing. This happens as each FBS gradually receives less power from each FMN (which transmits at $FP_{\max}$ but the FMN-FBS distance increases). However, apart from the last two rounds, the SINR achieved per FMN is sufficient for voice services. Concerning the MNs, they keep the same level of utility value at the NE.



(a) F)MN evolution of the NE utility under Scenario #3

(b) Evolution of the positions of the nodes under Scenario #3. E.g., FMN_1, FMN_2 and FBS_1 are moving northeast.

Fig. 2: (F)MN evolution of the NE utility under Scenario #3.

Scenario #3 (Fig.2a) corresponds to the case that, in each new simulation round, both the FMNs and the MNs gradually move away from its associated FBS/BS respectively. Fig.2b shows the evolution of the topology. All other parameters are fixed. These changes have no influence on the MN utility value at the NE. Concerning the FMNs, up to round 4, the utility value at the NE follows the same trend with Scenario #2. From round 5 and on, we notice a rather small decrease in the utility value. Although, as in Scenario #2, each FBS gradually receives less power from each FMN, it also receives less interference (as the MNs are moving away both from the BS and the FBSs). This limits the loss of the utility value at the NE.

V. Conclusions

We present a power control scheme based on best response dynamics that promotes the smooth coexistence of users that share the spectrum in a two-tier femtocell network. We argue that in this type of network MNs focus mostly on voice communications, whereas FMNs focus on data communications. Based on that, we define a non-cooperative power control game where each (F)MN aims at maximizing its own objective function. We determine the corresponding transmission powers at the NE and we provide a distributed algorithm that converges to them. Our results clearly indicate that the application of power control by distinguishing the utility functions based on the users' QoS requirements leads in many cases of interest to a smooth coexistence in a two-tier femtocell network.

As future work, we plan to investigate the impact of the cost function (pricing policy) of the FMNs to the achieved NE and examine which pricing policy should be adopted so as to increase the revenue of the wireless operator. Finally, open access femtocells will be studied focusing on the incentives that a MN has to be served by a FBS.

VI. Acknowledgment

This research was funded by the Research Centre of the Athens University of Economics and Business.

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