



HAL
open science

Optimal Sampling Strategies for Minimum Latency Routing with Imperfect Link State

Saikat Guha, Prithwish Basu, Howard Tripp, Timothy Freeman, Dmitriy
Katz, Robert E Hancock, Jim Kurose

► **To cite this version:**

Saikat Guha, Prithwish Basu, Howard Tripp, Timothy Freeman, Dmitriy Katz, et al.. Optimal Sampling Strategies for Minimum Latency Routing with Imperfect Link State. WiOpt'12: Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, May 2012, Paderborn, Germany. pp.81-88. hal-00763366

HAL Id: hal-00763366

<https://inria.hal.science/hal-00763366>

Submitted on 10 Dec 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Optimal Sampling Strategies for Minimum Latency Routing with Imperfect Link State

Saikat Guha, Don Towsley*, Prithwish Basu, Howard Tripp†, Timothy Freeman†,
Dmitriy Katz-Rogozhnikov‡, Robert Hancock†, Jim Kurose*
Raytheon BBN, *UMASS Amherst, †Roke Manor Research UK, ‡IBM Research US

Abstract—Since dynamic wireless networks evolve over time, optimal routing computations need to be performed frequently on time-varying network topologies. However, it is often infeasible or expensive to gather the current state of links for the entire network all the time. We provide a thorough analytical characterization of the effect of various link-state sampling strategies operating under a limited sampling budget on the performance of the minimum-latency routing policy in a special class of dynamic networks. We show that for a two-state Markov link-dynamics model parameterized by probabilities p, q , if links are more likely to turn on than off at each time instant ($p > q$), a “depth-first” sampling strategy is optimal, whereas a “breadth-first” sampling strategy is optimal if links are more likely to turn off than on ($p < q$)—under the Cut Through (CuT) latency model, i.e., when the packet-forwarding latency is negligible compared to the time scale of the link dynamics. We precisely characterize the optimal-latency spatial-sampling schedules for one-shot interrogation. We also present numerical simulation results on comparing various spatio-temporal sampling schedules under an overall sampling rate constraint, and initial results on comparisons of optimal schedules under a Store-and-Advance (SoA) packet-forwarding latency model.

I. INTRODUCTION

Wireless networks are inherently dynamic in nature since wireless links are regularly subjected to outages caused by channel fading, shadowing, mobility or sleep-wake scheduling. Such dynamics result in network topological changes, which affect metrics such as latency, reliability, available capacity, etc. In this paper, we concentrate on the problem of minimum latency routing in a simple class of dynamic networks. The problem of computing minimum latency routes through dynamic networks has been studied before. The simplest model of link dynamics is the one in which each link is up independently with a certain probability p (also referred to as the dynamic Erdős-Rényi (ER) random graph model). p is assumed to be known in advance and the link state is observable (sampled) when the packet (or probe) arrives at an endpoint of that link; and the problem is to dynamically route through the graph so as to minimize *expected routing time (ERT)* between a given source and a destination. This

Corresponding author: sguha@bbn.com. This research was sponsored by the U.S. Army Research Laboratory and the U.K. Ministry of Defense and was accomplished under Agreement Number W911NF-06-3-0002.1. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the U.S. Army Research Laboratory, the U.S. Government, the U.K. Ministry of Defence or the U.K. Government. The U.S. and U.K. Governments are authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

problem is #P-complete even for ER dynamic graphs where the transmission delay on active links (also referred to as edge length) is zero [1]; however some settings with *resampling* up/down state of a link in the local neighborhood of a node are known to be optimally solvable in polynomial time by Dijkstra-style dynamic programming [2]–[4].

While the ER random graph is a basic model for studying dynamic graphs, it does not capture temporal correlation in link dynamics. Since node mobility imposes temporal correlations, a more powerful model that admits temporal correlation is the (p, q) dynamic discrete-time Markovian model [5] (Fig. 1(e)). If $q = 1 - p$, this reduces to the ER model. Nikolova and Karger [4] gave optimal polynomial-time routing algorithms for resampling settings in directed acyclic graphs (DAGs) (the edges outside the one-hop neighborhood are not sampled and are assumed to be in steady state) and in undirected disjoint-path graphs (with 0/1 edge lengths). Similar Dijkstra-based algorithms were given for the resampling setting and for DAGs (in the more general setting of (q, p) Markovian edges) by Ogier and Rutenburg [1] in 1992.

In this paper, we study a variant of the above problem where the selection of the optimal route is constrained by a “budget” for sampling the state of various links in the network. We focus on analytically characterizing optimal sampling strategies (i.e., which link states to sample) that yield a minimum-latency path in a disjoint multi-path topology, under a *sampling budget constraint*. While heuristics such as hazy-sighted link state routing [6] have long been known, this, to our knowledge, is the first attempt at *analytically characterizing* the tradeoff between link state sampling strategies (such as *breadth-first* and *depth-first*) and end-to-end routing performance computed using the available (incomplete) link state information, in a dynamic network.

II. PROBLEM FORMULATION

We consider the sampling problem in a canonical dynamic network whose underlying topology comprises multiple disjoint paths between a source and a destination node. Each path is composed of links whose dynamics follow an i.i.d. Markov(p, q) process, which transitions in every time slot (see Fig. 1(a), (e)). With passage of time, the link probabilities eventually converge to the following steady state probability distribution: $(\pi_0, \pi_1) = (\frac{q}{p+q}, \frac{p}{p+q})$. We assume an idealized model of link forwarding latency, where once a link is up, it takes no time to traverse the link. This is referred to as

the cut-through (CuT) model. This is reasonable when the end-to-end latency is dominated by the latency experienced during the process of waiting for an off link to turn on, and not propagation or queueing latency. We do not expect that the basic nature of findings in this paper will be different for other link latency models, e.g., Store-and-Advance, where it takes one unit of time to traverse an up link; hence, we omit that analysis due for brevity. Last, let

$$P_{x_1|x_0}(s) = \pi_{x_1} + (-1)^{x_1}(\bar{x}_2 p - x_2 q)\beta^s / (p + q) \quad (1)$$

denote the probability that a link is currently in state x_1 given it was in state x_0 , s time steps ago, where $\beta \equiv 1 - (p + q)$.

We consider two broad categories of sampling strategies in this paper: (1) *one-shot* – in each time slot, a specified fraction of links in the network are always sampled; and (2) *multi-shot* – a specified fraction of links are sampled per unit time, following a particular spatio-temporal schedule. We consider two cost models for our analysis and simulations. In the *uniform* cost model for our theory results in Section III, the cost of sampling any link is fixed regardless of its location. In the *linear* cost model (explored in simulation results in Section IV), the cost of sampling a link is linear in the distance from the source.

III. THE SINGLE-SHOT OPTIMAL SPATIAL SAMPLING PROBLEM

a) *Expected traversal time of a single path for a given sampling pattern:* We first consider a single path consisting of L links. We introduce the concept of a *sampling set* $A = \{a_1, \dots, a_m\} \subseteq \{1, \dots, L\}$. A is the set of links to be sampled. Let $T(x_1, \dots, x_{m_k}; A)$ denote the ETT given that the sampled links are in states $X_{a_i} = x_i$, $i = 1, \dots, m$. We do not have a closed form expression for this except when $L = 2$. However, we have developed an efficient recursive algorithm of complexity $O(L^2)$ for computing $T(x_1, \dots, x_{m_k}; A)$ for any $0 < p, q < 1$, $A \subseteq \{1, \dots, L\}$, [7].

b) *Optimal allocation of samples on a single path:*

Definition 1: Let Y, Z be nonnegative random variables. Y is smaller than Z in the sense of convex order ($Y \leq_{cx} Z$) iff $\mathbb{E}[f(Y)] \leq \mathbb{E}[f(Z)]$ for all convex functions f .

The following derives from the above definition.

Lemma 1: Let Y, Z be nonnegative random variables with $Y \leq_{cx} Z$. Then $\mathbb{E}[\min\{Z, d\}] \leq \mathbb{E}[\min\{Y, d\}]$, for any constant $d \geq 0$.

Let A_k denote the sampling set associated with path P_k , $m_k = |A_k|$, and $T_k(x_1, \dots, x_{m_k}; A_k)$ the ETT for P_k given that links in A_k are in states $X_{k,a_{k,i}} = x_i$, $x_i \in \{0, 1\}$, $i = 1, \dots, m_k$. A special sampling set $A_k = \{1, 2, \dots, m_k\}$ corresponds to sampling the first m_k links of path k .

We have the following result.

Theorem 1: If $p + q < 1$, then

$$\begin{aligned} \mathbb{E}\left[\min_{1 \leq k \leq n} T_k(X_{k,1}, \dots, X_{k,m_k}; \{1, \dots, m_k\})\right] &\leq \\ \mathbb{E}\left[\min_{1 \leq k \leq n} T_k(X_{k,1}, \dots, X_{k,m_k}; A_k)\right] &\quad (2) \end{aligned}$$

for any collection of sampling sets $A_k \in \{1, \dots, L_k\}$, $k = 1, \dots, n$ such that $|A_k| = m_k$.

Proof: We prove by contradiction. Assume that ERT is minimized by some collection of sampling templates such that $A_k \neq \{1, \dots, m_k\}$ for some $k \in \{1, \dots, n\}$. Condition on the states of the sampled links in the remaining paths over all instances of time and let d denote the minimum ETT over these paths. Focus on path k ; $\exists h$ s.t. $1 < h \leq L$ and $a_{h-1} < a_h - 1$, i.e., the link upstream from h is not in A_k . Consider a new sampling set $A'_k = \{a'_1, \dots, a'_{m_k}\}$ such that $a'_j = a_j$, $j \neq h$, and $a'_h = a_h - 1$.

Let W_k and W'_k denote the end to end delays on path k given the link states in A_k and A'_k ; $T_k(X_1, \dots, T_{m_k}; A_k)$ $T_k(X_1, \dots, T_{m_k}; A'_k)$ are their averages. We couple the link states of the first $a_h - 2$ links under A_k and A'_k so that the delays incurred through the first $a_h - 2$ links are the same. Using the fact that $P_{x_1|x_0}(s)$ is monotonic in s when $p + q < 1$ in conjunction with a coupling argument, we can establish a stochastic ordering of the delays through the remaining links and thus conclude that $W'_k \leq_{st} W_k$ when $X_h = 1$ and $W_k \leq_{st} W'_k$ when $X_h = 0$. Now, define two independent rvs Y and Z such that $Y = \mathbb{E}[W_k | X_h = 1]$, $Z = \mathbb{E}[W'_k | X_h = 1]$ with probability π_1 and $Y = \mathbb{E}[W_k | X_h = 0]$, $Z = \mathbb{E}[W'_k | X_h = 0]$ with probability π_0 . It follows from the definition that $Y \leq_{cx} Z$. Application of Lemma 1 implies that A'_k provides a lower ERT than A_k for all possible link states at all time instances for the remaining paths producing the contradiction. ■

c) *Expected routing time (ERT) on N two-link paths with imperfect link-state information:* Consider a path with two links. The link states are denoted x and y , where $x, y \in \{0, 1\}$. Integers s_x and s_y are the ‘‘stale-ness’’ of the link states, meaning, link state x was collected s_x time slots in the past. $s_x = 0$ means x is the true link state, and $s_x \rightarrow \infty$ is the steady-state limit. The ETT for a two-link path with stale-state on the two links, assuming the CuT model, can be computed as (derivation omitted):

$$\begin{aligned} T(x, y, s_x, s_y) &= \frac{P_{1|x}(s_x)P_{0|y}(s_y) + P_{0|x}(s_x)}{p} \\ &+ P_{0|x}(s_x) \left[\frac{\pi_0}{p} + \frac{(p\bar{y} - qy)\beta^{s_y+1}}{(p+q)(1-\beta(1-p))} \right], \end{aligned}$$

where $P_{x_1|x_0}(s)$ is defined in (1). Two useful special cases are $P_{1|x}(0) = x$ and $P_{0|x}(0) = \bar{x}$, where $\bar{x} \equiv 1 - x$. Specializing to $s_x = s_y = 0$:

$$\begin{aligned} T_{xy} &\equiv T(x, y, 0, 0) = \frac{x\bar{y} + \bar{x}}{p} + \bar{x} \left[\frac{\pi_0}{p} + \dots \right. \\ &\left. \dots \left(\frac{\bar{y}p - yq}{p+q} \right) \left(\frac{\beta}{1-\beta(1-p)} \right) \right]. \quad (3) \end{aligned}$$

In light of Theorem 1, we disregard cases where only the second link is sampled. The ETTs for various sampling patterns are given by, $T_{11} = 0$, $T_{1*} = \pi_0/p$, $T_{0*} = (1 + \pi_0)/p$, $T_{**} = 2\pi_0/p$, $T_{10} = 1/p$, $T_{0x} = T_{0*} + (-1)^x \pi_{\bar{x}} \beta / [1 - \beta(1 - p)]$,

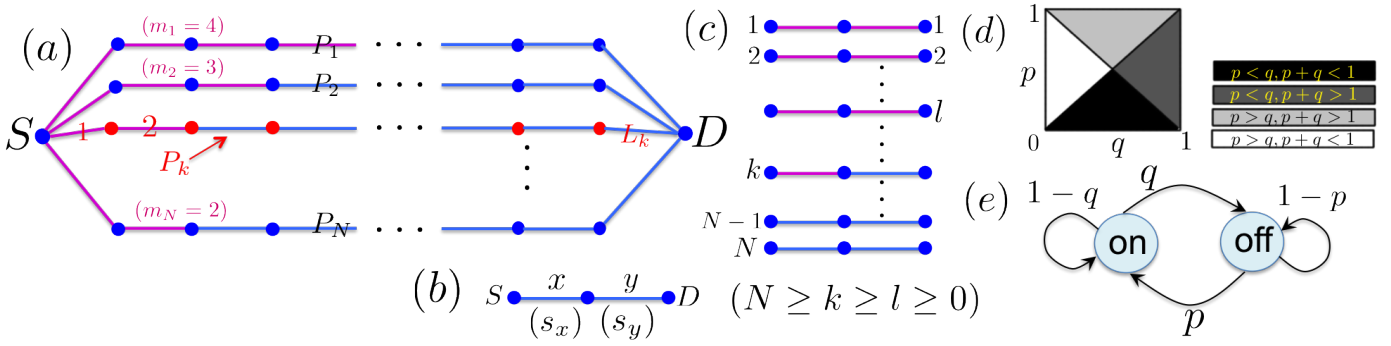


Fig. 1. The problem setup with N disjoint paths. Magenta-colored links show the m_k links that are sampled in path P_k . For the 2-link case, total number of links sampled, $m = k + l$; l paths are fully sampled. An additional $k - l$ paths' 1st links alone are sampled. The four regions of the (p, q) space that govern the relative ordering of the ETTs on a two-link path, are shown.

where a subscript $(*)$ indicates an un-sampled link (assumed to be in steady state). The relative magnitudes of these ETT values are as follows:

$$T_{11} < T_{1*} < T_{10} < T_{**} < T_{01} < T_{0*} < T_{00}, \quad (4)$$

for $p < q$ and $p + q < 1$. For $p > q$, $T_{**} < T_{10}$, and when $p + q > 1$, $T_{00} < T_{0*} < T_{01}$. The ERT is, by definition of our path-selection rule, the minimum ETT conditioned on the link-state samples. Let k path's first links, and l of those k path's second links be sampled (Fig. 1(c)), $N \geq k \geq l \geq 0$. We get:

$$T(k, l) = (1 - \pi_0^{k-l}) \left[(1 - \pi_1^2)^l T_{1*} \right] + V(k, l), \quad (5)$$

where $V(k, l)$ is given by Eq.(5).

d) Optimal sampling strategy: Given m links to sample in the network, $0 < m \leq \sum_{k=1}^N L_k$, which m links should one sample to obtain the minimum expected routing time (ERT)? Assume a decision rule where we commit to the minimum-ETT path after obtaining the link-state samples. Note that it is not clear whether this is always an optimal strategy. For example, if the minimum-ETT path has the first link down, and therefore the transmission fails, one could recompute the ETTs in the second link (using $(s_x = 1, s_y = 1)$ stale-state ETT formulas), and then attempt to transmit on the now-ETT-minimum path, which may be different from the path chosen earlier as the minimum-ETT path. The latter strategy cannot perform worse than the first, and could actually perform better.

Optimal sampling strategy for the two-link case. For $p > q$, for m odd, $k_{\text{opt}} = (m + 1)/2$, whereas for m even, $k_{\text{opt}} = (m/2) + 1$ for $2q > p > q$ and $k_{\text{opt}} = m/2$ for $p > 2q$. $p > q$ means the links are more often up than down, making sampling full paths better (i.e., $k \approx m/2$), because it is likely that one of those paths will have all links up, producing a zero ETT. On the other hand, for $p \leq q$, k_{opt} transitions from $\sim m/2$ (at $p = q$) to $\min(m, N)$ (at $q/p \gg 1$), at a discrete set of threshold values of p/q (see Fig. 2 and Fig. 3). $q/p \gg 1$ is when the links are mostly down, and it is thus better to sample the first links on as many paths possible (to maximize the chances of finding a path with the first link up, and thereby

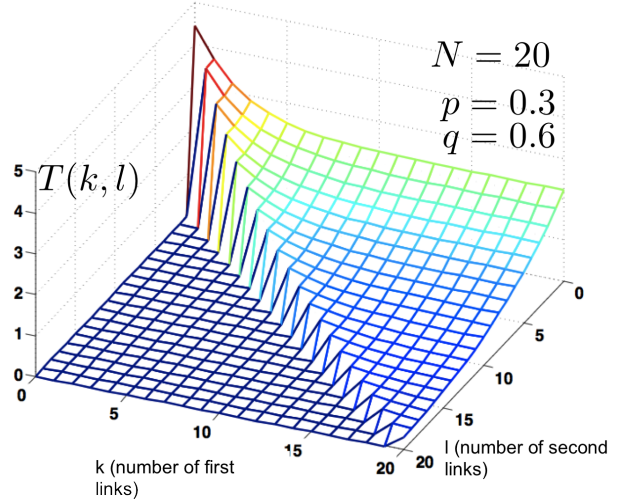


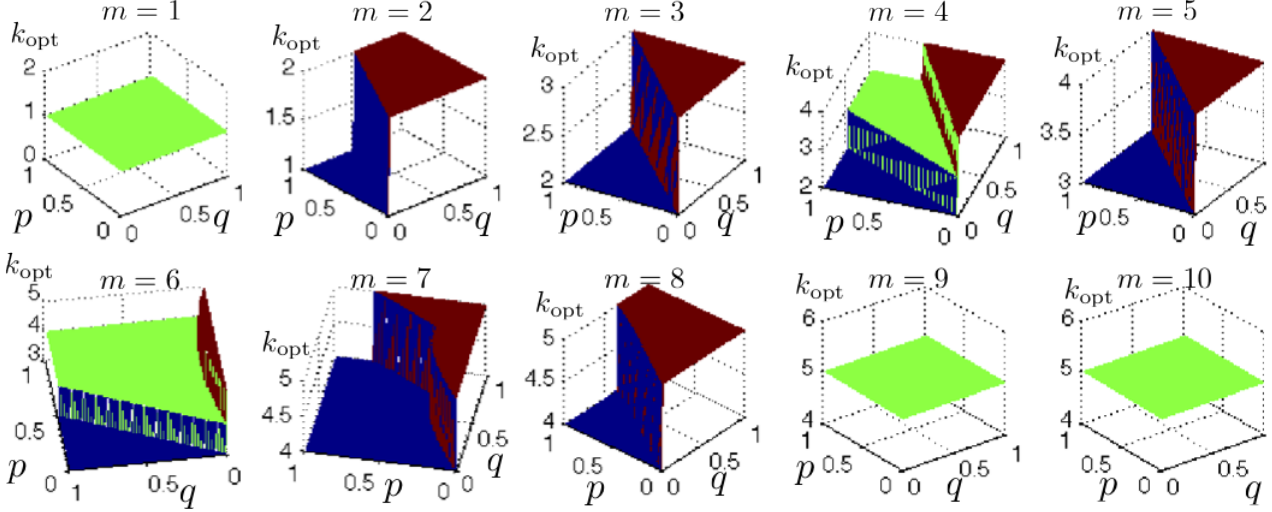
Fig. 4. Expected routing time as a function of number of first links (k) and number of second links (l) sampled. The ERT-steepest-descent trajectory from $(0, 0)$ gives the global optimum sampling when the total number of links sampled $m = k + l$ is the constraint.

moving ahead). The transition to a breadth-first probing as p/q decreases, happens quite rapidly at low p/q . Fig. 2 shows this. An interesting observation is that $T(N - 1, 0) = T(N, 0) = (\pi_0^N + \pi_0)/p$. So, if $m = N$, its better to leave a path un-sampled and sample the 2nd link of a path, than it is to sample the first links of each path. However, this is probably specific to the CuT and the uniform cost models.

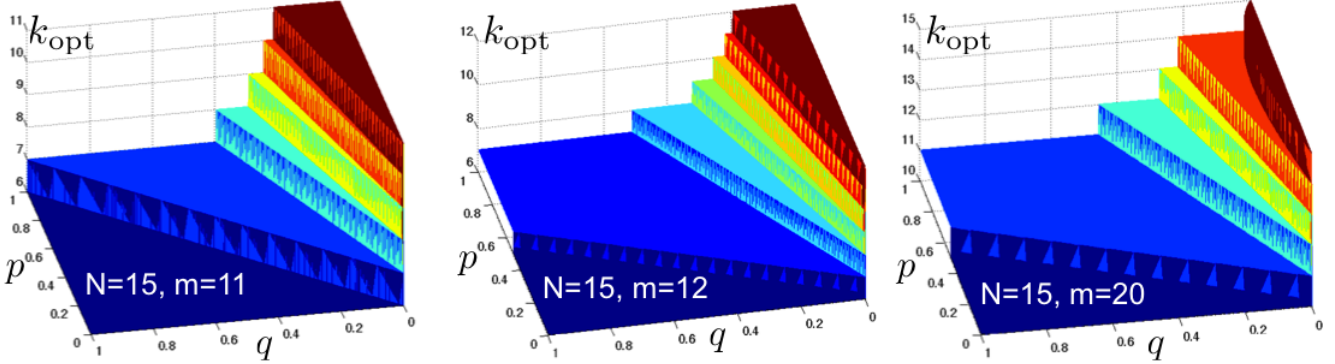
Greedy optimization of ERT achieves the global optimum. We have verified numerically that adding links one at a time to the set of links sampled, while at each step allocating a new link in a way that the incremental decrease in ERT is maximized, is also the global optimum of ERT for whatever is the total number of links in the set (see Fig. 4).

Asymptotic results for the general case for $p/q \ll 1$ and $p/q \gg 1$. For 2-link paths, we observed regions in (p, q) space where depth first sampling is best ($p/q \gg 1$) and where breadth first sampling is best ($p/q \ll 1$). It is interesting to hypothesize that this is the case when path lengths are greater than two, $L > 2$, and indeed this is the case as given by the

$$V(k, l) = \begin{cases} \pi_0^{k-l} \left[(1 - \pi_1^2)^l \right] & p > q, N > k \geq l \geq 0 \\ \pi_0^{k-l} \left[\pi_0^l T_{**} + \left((1 - \pi_1^2)^l - \pi_0^l \right) T_{10} \right] & p \leq q, N > k \geq l \geq 0 \\ \pi_0^{k-l} \left[\pi_0^{2l} T_{0*} + \pi_0^l (1 - \pi_0^l) T_{01} + \left((1 - \pi_1^2)^l - \pi_0^l \right) T_{10} \right] & p + q < 1, N = k \geq l \geq 0 \\ \pi_0^{k-l} \left[(\pi_0 \pi_1)^l T_{0*} + \pi_0^l (1 - \pi_1^l) T_{00} + \left((1 - \pi_1^2)^l - \pi_0^l \right) T_{10} \right] & p + q \geq 1, N = k \geq l \geq 0 \end{cases} \quad (5)$$



(a) $N = 5, L = 2$ (k_{opt} = Number of paths whose first links should be sampled)



(b) Optimal sampling is “depth-first” when $p \gg q$, and progressively swaying toward “breadth-first” as q increases beyond $q > p$, and completely so for $q \gg p$.

Fig. 2. Optimal sampling is “depth-first” when $p > q$, which progressively sways toward “breadth-first” as q gets larger than p , and completely so for $q \gg p$. (k_{opt} = Number of paths whose first links should be sampled.)

following theorem (proof omitted for brevity).

Theorem 2: Given a budget of $m < NL$ samples and $p + q < 1$, the sampling policy that results in the minimum ERT when $p/q > m^2$ is the depth first policy and when $p/q < 1/m^2$ is breadth first policy.

A physically-motivated constraint on the probing model. Until now we have focused on the class of sampling policies that allow the flexibility to sample any link regardless of the states of other links. However, the situation can arise where the sampling operation is physically implemented by a query that traverses the path from the source to the link in question.

However, such a query fails if any intermediate link on that path is down. Thus a link can only be sampled if all links between it and the source are up. More precisely link i on a path consisting of L links, $i \leq L$ can be sampled only if $X_j = 1, j = 1, \dots, i - 1$. In addition, because whether a link can be sampled depends on previous samples, in this case we consider a model where links are sampled sequentially, that is when sampling m th link, we know the state of the previous $m - 1$ links sampled. Given such a constraint on what links can be sampled, we have the following result.

Theorem 3: Given n paths each containing L links and a

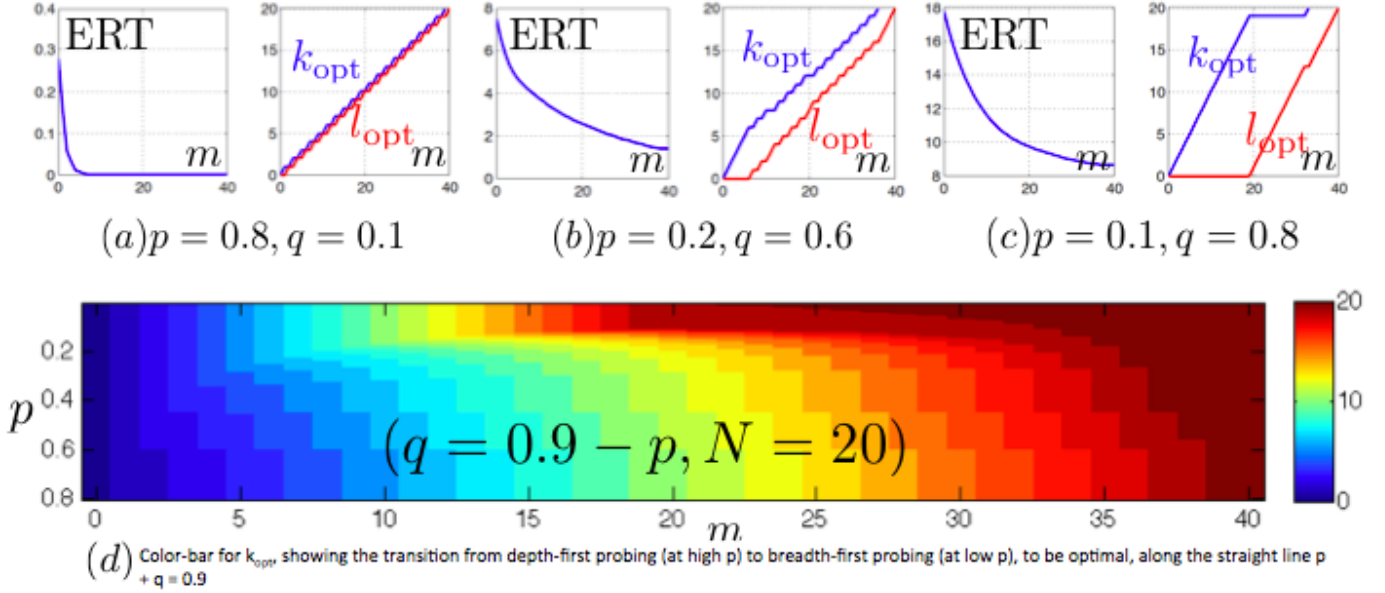


Fig. 3. Optimal choices of number of first-links sampled, k and number of second-links sampled, l are shown as more samples m become available. When $p > q$, $k_{\text{opt}} \approx m/2$ (depth-first), whereas for $q \gg p$, $k_{\text{opt}} = \min(m, N)$.

sampling budget $m \leq nL$, the depth first sampling policy minimizes the ERT for any set of parameters $0 < p, q < 1$.

Proof: We will analyze the depth-first algorithm, *DEPTH*, and compare that with any other algorithm, *ALG*, and show that *DEPTH* always does at least as well as *ALG*.

Let x_{ij} , $i = 1, \dots, L$, $j = 1, \dots, N$ be i th link on the j th path. Let Y_j , $j = 1, \dots, N$, be the number of links that are sampled on path j . We will call Y_j the “up-length” of path j . We will say that j is *open* if all links sampled on path j are up, and *closed* if the last link sampled on path j is down.

Observe that if two paths are both *open* or both *closed*, the one with higher up-length (higher Y) has lower ERT.

Without loss of generality, both *ALG* and *DEPTH* will use the following tie-breaker rules:

- 1). When sampling a link on a path of up-length a , the path with the lowest index of up-length a is sampled.
- 2). At the end, if one of several identical lowest ERT paths is chosen (to send the packet along), it is always the path with the lowest index.

Lemma 2: At any point in execution of *ALG*, if path j is *open* and $j < k$, then $Y_j \geq Y_k$. In other words, if a path is *open*, it is at least as long as any higher index paths.

Proof: We provide a simple proof by recursion, using tie-breaker rules:

This Lemma holds when we have 0 samples. For it to be violated at the m th sample, we would need to sample a higher index path when a lower index *open* path of same up-length exists, and this is impossible due to our tie-breaker rules. ■

Lemma 3: If at the end path k is chosen as the one with lowest ERT, all paths above it are *closed*.

Proof: Here we will separately consider the case of path k being open and case of path k being closed.

If k is *open*, this lemma directly follows from Lemma 2 and the tie-breaking rule for choosing paths.

If k is *closed*, by Lemma 2 if there is an *open* path j with index $j < k$, then $Y_j \geq Y_k$. But then, j would have lower ERT than k , thus would be chosen instead. Hence, contradiction. ■

Lemma 4: Suppose *ALG* chooses path j as lowest ERT. When *DEPTH* has spent as many samples total as *ALG* spent on paths with index $\leq j$, *DEPTH* has sampled path j to the same depth as *ALG*.

Proof: This follows directly from Lemma 3. ■

From this, Theorem 3 directly follows. ■

IV. NUMERICAL RESULTS: ONE-SHOT AND MULTI-SHOT SPATIO-TEMPORAL LINK SAMPLING

We now report initial simulation results from comparison of various spatio-temporal sampling strategies. All the results reported in this section were carried out for the store and advance (SoA) model, unless specifically noted. Unlike the CutThrough (CuT) model used in all of our theory analysis in Section III where a packet crosses an up link instantly, in the SoA Model a packet takes one time slot to traverse an up link. During this one time slot, the network evolves by one time step of the Markov process.

A. Comparison of spatio-temporal probing strategies under uniform sampling costs

In order to gain intuition on the various temporal strategies, we first evaluate the ERT under the uniform cost model for $N = 2$, $L_1 = L_2 = 2$. The ERTs for each possible path state were approximated numerically, averaged over 10^6 runs. The simulation was carried out over 10^6 time slots, with the average fractional ERT increase over the true minimum ERT

with all links sampled at all time slots (ERT_{\min}), was plotted for each of the following strategies (see Fig. 6(a)):

When the network is more stable, probing the links uniformly seems to give the best performance. As the network becomes more unstable, it becomes more effective to probe just the first links of each path, concurrent with our theory results for spatial sampling for the single-shot case.

B. Comparison of spatio-temporal probing strategies under the linear sampling cost model

We next consider the $N = 3$, $L_k = 3, \forall k$ (9-link) network as depicted in Fig. 1(a). Cost equals the hop count of a link from the source node S (under the linear cost model). Note that the analyses in Section III assume the uniform cost model. To vary cost, probing is done at 8 rates in the range from every 2 time slots to every 20 time slots. Let us also assume that 3 links are probed each period (i.e., 33% of the state is gathered). The strategies are considered for selecting the subset of 3 links to probe in each slot, are listed in the table in Fig. 7.

The results are summarized in Fig. 6(b). PrefHop seems to perform particularly well at high costs. It has two advantages over other strategies, (i) for a given cost, it can get more samples per unit time (because links further away are costlier to probe), and (ii) probing closer links more often is an overall better strategy. In fact, PrefHop outperforms other strategies even for the uniform cost model. This observation begs for a systematic study of an optimal temporal probing schedule as a function of hop count, thereby grounding the heuristic hazy-sighted link state routing (HSLs) protocol [6] in theory, which is similar in spirit to PrefHop, but obeys a logarithmic decay of sampling rate with distance from the source.

We also simulated probing more (than 3) links. Fig. 6(c) shows results for the random strategy, probing all links from 2 to 8 links per period (i.e. 22 – 89% of the links). Linear cost and $p = q = 0.25$ is assumed. It seems that if we are prepared to accept high costs, it is more “cost-effective” to probe a more links less frequently, than it is to probe fewer links more frequently (for the same total cost).

C. Numerical results on one-shot and multi-shot spatio-temporal probing strategies for a large network

So far, we have not extended the exact analytic results to path lengths greater than 2. So, we carried out simulations to see how the transition from ‘depth first’ to ‘breadth first’ occurs as p/q is decreased from ∞ to 0, for larger networks. Consider a $N = 5$, $L_k = 5, \forall k$ (5 paths with 5 links in each) network. We take $q = 0.2$, and vary $p \in [0.05, 0.8]$. The source can probe 10 links, before choosing the shortest path based on the knowledge it has obtained. The CuT model is now used for the transversal times on each path. The performance measure used is fractional difference in ERT between the path selected and the actual ERT-minimum path. A selection of sets of links are probed, roughly ranging from ‘breadth first’ to ‘depth first’, with some which are somewhere between the two (see Fig. 8).

All links cost the same to probe, but for each path, the closest links are probed first, as this has already been shown

Strategy	Links probed for each path
1	[5, 5, 0, 0, 0]
2	[5, 3, 2, 0, 0]
3	[4, 4, 2, 0, 0]
4	[4, 3, 2, 1, 0]
5	[4, 2, 2, 2, 0]
6	[3, 3, 2, 2, 0]
7	[2, 2, 2, 2, 2]

Fig. 8. Link probing patterns chosen for numerical study on a 5-by-5 instantiation of the problem set up shown in Fig. 1, to transition from “depth-first” to “breadth first” sampling. For example, [5, 5, 0, 0, 0] implies sample (all) 5 links of path P_1 , all 5 links of path P_2 , and no links of paths P_3 , P_4 , and P_5 .

to be optimal. Fig. fig:fivelinknet(a) plots the fractional ERT difference versus p . It shows the same general trends as the simulations and analytic results for the two-link case. At low p , where $p \ll q$, the ‘breadth first’ strategy is the most effective, and vice versa.

The one-shot sampling techniques above are not directly useful in the multi-shot case, as it is not (or at least very unlikely to be) desirable to sample exactly the same links in every time slot. In the one shot case, we showed analytically that for a given path, the closest links should always be sampled. This may not always be the case in the multi-shot situation, but we know that the closer links should at least be probed more often, so we keep this restraint for now. Each path is probed to a randomly determined depth, chosen by a uniform roulette selection. The sampling strategy moves from ‘breadth first’ to ‘depth first’ as a parameter, λ , is increased from 0 to 1. The results for the simulation are shown in Fig. 9(b). The results look very similar to those of the one shot case. Fig. 9(b) shows the same general trend, of the ‘breadth-first’ strategies performing increasingly well as p is increased, and the ‘depth-first’ strategies getting increasingly worse.

V. CONCLUSIONS AND ONGOING WORK

We considered the problem of finding an optimal link-probing schedule (i.e., which link states to sample and when), under a total sampling budget constraint, for a dynamic network with (p, q) Markovian on-off link dynamics. The objective is to pick the minimum-latency path between a source destination pair in a disjoint multi-path topology.

We obtained analytic results for the optimal solution for multiple disjoint two-link paths, as well as asymptotically-optimal solutions for the $p \gg q$ and $p \ll q$ cases, both for the CuT (Cut Through) link-forwarding latency model. The results suggest that when the links are almost always up ($p \gg q$), a ‘depth first’ probing approach is the most effective. In other words, probing full paths is more likely to get the packet further by finding a long contiguous set of links on a path that are open, which the packet can traverse (instantly, in the CuT model). On the other hand, when the links are almost always down ($p \ll q$), a ‘breadth first’ sampling approach is more effective, since probing the first few links of all paths is

singleRandom: Randomly samples one link per slot

singleCycle: Cycles through all the links, one per slot

singlePref: Cycles through links, one per slot, while probing the first links twice as often as the second links

singleFirst: Cycles through the two first links, one per slot

doubleFirst: Probes the two closest links, every other time slot

doubleRandom: Probes two random links, every other slot

doublePathCycle: Probes entire path every other slot

doubleHopPref: Cycles through hops, one every other slot, in the order 1, 2, 1

doubleHopCycle: Probes a whole hop every other slot

all: Probes all four links every fourth slot

Fig. 5. Description of strategies used for temporal sampling on the “diamond network” ($L_1 = L_2 = 2, N = 2$). Results shown in Fig. 6(a).

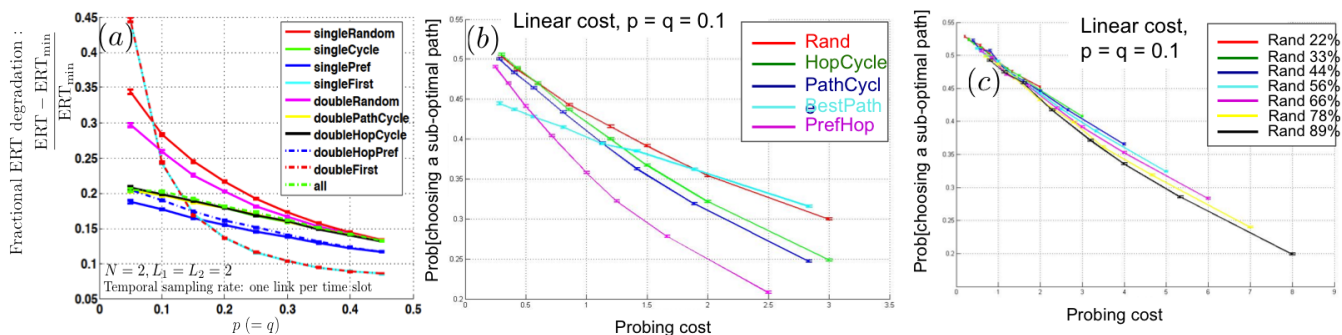


Fig. 6. Simulation results: (a) Comparing temporal sampling schedules on the 2-path case, each path having 2 links. (b) Comparison of the performance of temporal sampling strategies for the 3-by-3 network, when constrained to probe 3 links in a time slot, as a function of probing cost under a linear cost model, and the SoA packet-forwarding model. (c) For the 3-by-3 network, comparison of (randomly) probing more links less frequently to probing fewer links more frequently (2 to 8 links, of the 9, are sampled randomly in each time slot).

Random: Each probing period, randomly choose 3 of the 9 links

HopCycle: Group the links into three sets grouped by hop count and probe each of the sets cyclically

PathCycle: Group the links into three sets group by path and probe each of the sets cyclically

Best Path: Each time period probe 3 links on the path that S calculates as having the lowest ETT

PrefHop: Probe via groups based on hop count (as in HopCycle), however sample nearer links more frequently. Sample the 3 one-hop links in the 1st, 3rd and 5th periods, the 3 two-hop links in the 2nd and 4th periods, and the 3 three-hop links, in the 6th period, and then cycle back to the beginning

Fig. 7. Description of strategies for spatio-temporal sampling with the linear cost model shown in Figs. 6 (b) and (c).

likely to find at least one path whose first link (or perhaps the first few links) is (are) up.

Most of our simulations (such as those in Fig. 6) used the SoA (Store and Advance) link-forwarding latency model. The general behavior of these results indicate that it is best to probe nearby links more often, and far away less often, if ever. The analytic work used the CuT model, and found that for $p \gg q$, it is best to probe whole paths. These conclusions appear to contradict. The main difference between the two packet-forwarding models is that in the CuT model, a packet crosses an up link instantly, and in the SoA model, it takes one time slot to traverse a link, during which the network evolves by one step. This could make obtaining depth-first information less and less *valuable* as the length of each path N grows, even if p is high. Hence, with increasing path lengths, for the SoA latency model, the ‘breadth first’ strategy remains

(more or less) always the best even as p is increased. Fig. 10 summarizes our preliminary numerical results on this. We are working on understanding these differences in the behavior of link forwarding latency models, more systematically.

In further ongoing work, we are pursuing extensions of both our analytical and numerical analyses to more general networks with non-disjoint paths. We are investigating link-probing mechanisms as well as cost constraints that are informed more accurately by physically-motivated probing mechanisms. Furthermore, we are working on extensions to a continuous-time Markovian link model. Finally, we are working on characterizing the ultimate limits of the tradeoff between imperfect link-state versus end-to-end routing latency, via a rigorous information-theoretic rate-distortion calculation, where we characterize the latency of the lossy compression of the link-state information using ideas from recent work on

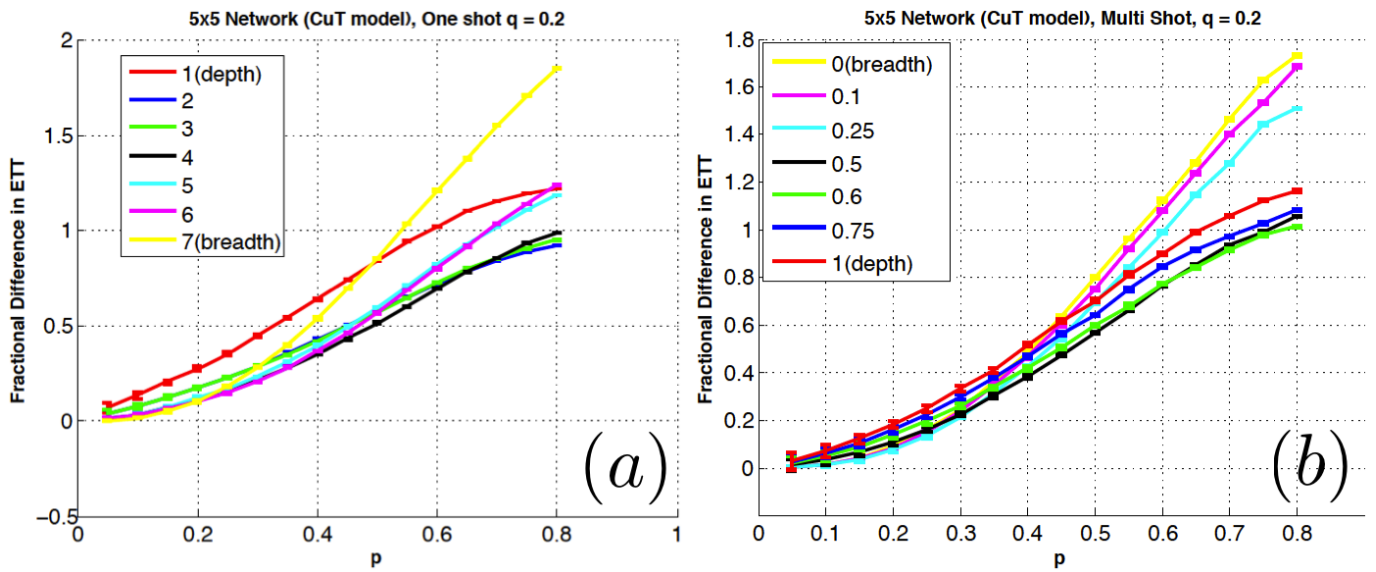


Fig. 9. Numerical results for the five-link five-path network, with single-shot and multi-shot sampling strategies, ranging from ‘breadth’ first to ‘depth first’. The figures show the fractional ERT difference, for the CuT (Cut Through) packet-forwarding-latency model, as a function of p when $q = 0.2$ is held fixed.

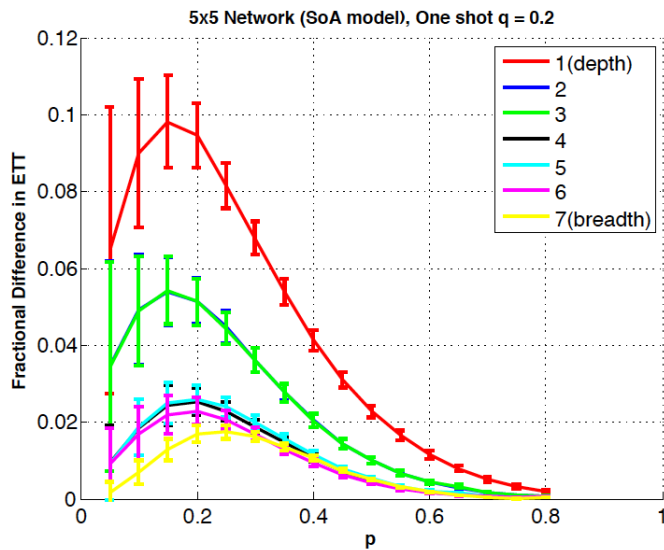


Fig. 10. Numerical results for the five-link five-path network. The plot shows the fractional ERT difference, for the SoA (Store and Advance) packet-forwarding-latency model, as a function of p when $q = 0.2$ is held fixed.

fine-block-length lossy source coding [8].

REFERENCES

- [1] R. G. Ogier and V. Rutenburg, “Minimum-expected-delay alternate routing,” in *INFOCOM*, 1992, pp. 617–625.
- [2] A. Bar-Noy and B. Schieber, “The canadian traveller problem,” in *SODA*, 1991, pp. 261–270.
- [3] G. H. Polychronopoulos and J. N. Tsitsiklis, “Stochastic shortest path problems with recourse,” *Networks*, vol. 27, no. 2, pp. 133–143, 1996.
- [4] E. Nikolova and D. R. Karger, “Route planning under uncertainty: The canadian traveller problem,” in *AAAI*, 2008.
- [5] A. Seetharam, J. Kurose, D. Goeckel, and G. Bhanage, “A Markov Chain Model for Coarse Timescale Channel Variation in an 802.16e Wireless Network,” in *IEEE INFOCOM*, 2012.

- [6] C. A. Santiviáñez, R. Ramanathan, and I. Stavrakakis, “Making link-state routing scale for ad hoc networks,” in *Proc. ACM Mobihoc*, 2001, pp. 22–32. [Online]. Available: <http://doi.acm.org/10.1145/501417.501420>
- [7] P. Nain, D. Towsley, M. Johnson, P. Basu, A. Bar-Noy, and F. Yu, “Traversal Times in Markovian Paths,” Raytheon BBN Technologies, Tech. Rep., Jan. 2012. [Online]. Available: <http://www.ir.bbn.com/~pbasu/pubs/markovpath.pdf>
- [8] V. Kostina and S. Verdú, “Fixed-length lossy compression in the finite blocklength regime: Gaussian source,” in *IEEE Information Theory Workshop (ITW)*, 2011.