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**Abstract:** A high gain nonlinear observer is implemented to estimate the dynamics evolution of a schistosomiasis infection described by a continuous non linear model [1]. A reduction schistosomiasis infection model is proposed and used to build the observer. Numerical simulations are included in order to test the behavior and the performance of the proposed observers.

**Key-words:** Nonlinear dynamical systems, Observer, Schistosomiasis.

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# Synthèse d'observateur d'un modéle non linéaire de la Bilharziose

**Résumé :** Un observateur 'grand gain' non-linéaire est mis en Ïuvre pour évaluer l'évolution de dynamique d'une infection de la Bilharziose décrite par un modèle continu non linéaire [1]. On propose un modèle réduit du modèle [1] de la Bilharziose pour construire l'observateur. Des simulations numériques ont été faites pour tester le comportement et la performance de l'observateur proposé.

**Mots-clés :** Systèmes dynamiques non-linéaires, Observateur, Bilharziose

# 1 Introduction

Human schistosomiasis is a behavioural and occupational disease associated with poor human hygiene, insanitary animal husbandry and economic activities. Among human parasitic diseases, schistosomiasis ranks second behind malaria in terms of socio- economic and public health importance in tropical and subtropical areas. Urinary schistosomiasis, caused by the species Schistosoma haematobium, is common in Africa and the Middle East. The main clinical sign of schistosomiasis infection is haematuria due to deposition of eggs by the adult female worms in the wall of the bladder and urinary tract [1]. The most effective form of treatment for the infected individual is the use of the drug praziquantel which kills the worms with high efficacy. Control programs at community level often consist of mass chemotherapy possibly supplemented by snail (intermediate host) control. Since school-age children are the most heavily infected age group, suffer the most morbidity and are the major source of infection for the community, school targeted chemotherapy can be a cost effective approach to control the morbidity [2, 3].

The spread and persistence of schistosomiasis are one of the more complex host-parasite processes to model mathematically because of the different steeps growth of larval assumed by the parasite and the requirement of two hosts (definitive human hosts and intermediate snail hosts) during their life cycle.

An efficiency method to control the schistosomiasis infection which may require relatively little funding is biological control. Particularly, trematode parasites or competitive snails of the intermediate snail hosts have been proved to be effective in controlling schistosomiasis in the Caribbean area (Pointier and Jourdane, 2000). In fact, Allen and al. proposed a model incorporating this kind of control strategies and it is assumed that a snail species can be found that is resistant and is able to outcompete the intermediate host snail species.

Ordinary differential Equations (ODEs) are used to describe population evolution over a continuous time period. Deterministic ODEs are one of the major modeling tools and are used in our case. Symbolically one writes

$$\begin{cases} \dot{X}(t) &= F(X(t), U(t)) \\ Y(t) &= h(X(t)) \end{cases} \tag{1}$$

with $X(t) \in \mathbb{R}^n$, $U(t) \in \mathbb{R}^m$ $m \leq n$, $Y(t) \in \mathbb{R}^p$ $p < n$.

If it is possible to have the value of the state at some time $t_0$ then it is possible to compute $X(t)$ for all $t \geq t_0$ by integrating the differential equation with the initial condition $X(t_0)$. Unfortunately, it is often not possible to measure the whole state at a given time and therefore it is not possible to integrate the differential equation because one does not know an initial condition. One can only have a partial information of the state and this partial information is precisely given by $Y(t)$ the output of the system. Therefore we shall show how to use this partial information $Y(t)$ together with the given model in order to have a reliable estimation of the unmeasurable state variables. A state observer is usually employed, in order to accurately reconstruct the state variables of the dynamical system. In the case of linear systems, the observer design theory developed by Luenberg [12], offers a complete and comprehensive answer to the problem. In the field of nonlinear systems, the nonlinear observer design problem is much more challenging and has received a considerable amount of attention in the literature.
An observer for (1) is a dynamical system

$$\dot{\hat{X}}(t) = \hat{F}(\hat{X}(t), U(t), Y(t))$$

whose task is state estimation. It is expected to provide an estimate state $\hat{X}(t)$ of the state $X(t)$ of the original system. The output is in general a function of the state variable and the input, that is, $Y(t) = h(X(t))$. This estimate will be produced by an auxiliary dynamical system which uses the output information $Y(t)$ provided by the system (1).
One usually requires at least that $\left|\hat{X}(t) - X(t)\right|$ goes to zero as $t \to \infty$. When the convergence of $\hat{X}(t)$ towards $X(t)$ is exponential, the system (1) is an "exponential observer". More precisely, system (1) is an exponential observer for system (1) if there exists $\lambda > 0$ such that, for all $t \geq 0$ and for all initial conditions $\left((X(0), \hat{X}(0)\right)$, the corresponding solutions of (1) and satisfy $\left|\hat{X}(t) - X(t)\right| \leq \exp(-\lambda\, t) \left|\hat{X}(0) - X(0)\right|$.
In this situation a good estimate of the real unmeasured state is rapidly obtained. One must notice that we need not care about the choice of the initial condition of the observer since the convergence of $\hat{X}(t)$ towards the real state $X(t)$ does not depend on this choice.
It happens here that the structure of the system (1) ( i.e the functions $F$ and $Y$) are known from the physical laws or from the user's experience, i.e from a priori knowledge.

We will consider a schistosomiasis model with biological control, [24], described by:

$$\left\{\begin{aligned}
\frac{du_1}{dt} &= -t_{15}\, u_5\, u_1 + r_{12}\, u_2,\\
\frac{du_2}{dt} &= t_{15}\, u_5\, u_1 - r_{12}\, u_2,\\
\frac{du_3}{dt} &= b_3\,(u_3+u_4+u_5) - t_{32}\, u_2\, u_3 - d_3\, u_3 - c_{33}\, u_3(u_3+u_4+u_5)\\
&\quad -c_{36}\, u_3\, u_6 - t_{38}\, u_3\, u_8,\\
\frac{du_4}{dt} &= t_{32}\, u_2\, u_3 + t_{38}\, u_3\, u_8 - d_4\, u_4 - c_{44}\, u_4(u_3+u_4+u_5)\\
&\quad -c_{46}\, u_4\, u_6 - r_{54}\, u_4,\\
\frac{du_5}{dt} &= r_{54}\, u_4 - d_5\, u_5 - c_{55}\, u_5(u_3+u_4+u_5) - c_{56}\, u_5\, u_6,\\
\frac{du_6}{dt} &= b_6\, u_6 - c_{64}\, u_6(u_3+u_4+u_5) - c_{66}\, u_6\, u_6 - d_6\, u_6,\\
\frac{du_7}{dt} &= b_7(u_7+u_8) - t_{75}\, u_5\, u_7 - c_{77}\, u_7(u_7+u_8) - d_7\, u_7,\\
\frac{du_8}{dt} &= t_{75}\, u_5\, u_7 - d_8 u_8 - c_{88}\, u_8(u_7+u_8).
\end{aligned}\right. \tag{2}$$

where

$u_1(t)$ = the susceptible (uninfected) human population size,
$u_2(t)$ = the infected human population size,
$u_3(t)$ = the susceptible snail host population size,
$u_4(t)$ = the population size of the infected snails which are not yet shedding cercariae,
$u_5(t)$ = the infected and shedding snail population size (shedding population size),
$u_6(t)$ = the competitor snail population size (resistant to infection),
$u_7(t)$ = the susceptible mammal population size,
$u_8(t)$ = the infected mammal population size.

In addition, the population of snails as well as mammals are assumed to be competitive. Birth and death rates for the various sub populations will be denoted by $b_i$ et $d_i$, respectively, for $i = 1, 2, ...8$. For simplicity the birth rate of different sub-populations of mammals and intermediate host snails will be assumed to be equal, i.e $b_3 = b_4 = b_5$ and $b_8 = b_7$. The transmission parameters for the model are:

$t_{15}$ = transmission rate from infected snails to uninfected humans,
$t_{32}$ = transmission rate from infected humans to uninfected snails,
$t_{38}$ = transmission rate from infected mammals to susceptible snail,
$t_{75}$ = transmission rate from infected snails to susceptible mammals.

Competition parameters are defined for the populations:
$c_{33}$ is the competition parameter between $u_3$ and $u_3$, $u_4$, $u_5$,
$c_{44}$ and $c_{55}$ are the competition parameters between $u_4$ and $u_5$, respectively, and $u_3$, $u_4$, and $u_5$,
$c_{36}$ is the competition parameter for snails $u_6$ with snails $u_3$,
$c_{46}$ and $c_{56}$ are defined analogously,
$c_{64}$ is the competition parameter for snails $u_3$, $u_4$ and $u_5$ with $u_6$,
$c_{66}$ is the competition parameter for $u_6$ with $u_6$,
$c_{77}$ and $c_{88}$ are the competition parameter for the mammals populations.

Also, $r_{12}$ is the rate that infected humans recover and $r_{54}$ denotes the rate that the latent snail population $u_4$ becomes shedding $u_5$.

It is assumed for simplicity that $d_3 = d_4 = d_5$, $c_{33} = c_{44} = c_{55}$, $c_{77} = c_{88}$ and $c_{46} = c_{56} = c_{36}$.

In the previous work, [1], we proposed a reduced system (3) derived to the one for convenience stability analysis.

$$\left\{\begin{array}{lcl} \dfrac{d\,u_2}{dt} &=& t_{15}\,(N_H^* - u_2)\,x_5\,N_{si}^* - r_{12}\,u_2 \\ \dfrac{d\,x_3}{dt} &=& -\,(t_{32}\,u_2 + t_{38}\,N_M^*\,x_8 + b_3)\;x_3 + b_3 \\ \dfrac{d\,x_5}{dt} &=& r_{54}\;(1 - x_3 - x_5) - b_3\,x_5 \\ \dfrac{d\,x_8}{dt} &=& t_{75}\,N_{si}^*\,x_5\;(1 - x_8) - b_7\,x_8 \end{array}\right. \tag{3}$$

Where we define $x_i = \dfrac{u_i}{N_{Si}}$ for $i = 3, 4, 5$, $x_i = \dfrac{u_i}{N_M}$ for $i = 7, 8$,
$N_H = u_1 + u_2$, $N_M = u_7 + u_8$ and $N_{si} = u_3 + u_4 + u_5$.
The stars at these quantities means that we take it at the equilibrium point.

In this model, the states of snails et mammalians are not available for measurement. The only available information at time $t$ is the value of the infected human populations : this mean that one can detect by clinical signs the number of infected populations at each time $t$. The value of the infected human populations can be seen as the measurable output of the reduced model. If it is possible to have the value of the state at some time $t_0$ then it is possible to compute states for all $t \geq t_0$ by integrating the differential equation with the initial condition. Unfortunately, it is often not possible to measure the whole state at a given time and therefore it is not possible to integrate the differential equation because one does not know an initial condition. One can only have a partial information of the state and this partial information is precisely given by $u_2(t)$ the output of the system. Therefore we shall show how to use this partial information together with the given model in order to have a dynamical estimate of the real unknown state variable. This estimate will be produced by an auxiliary dynamical system called an observer which uses the information $u_2(t)$ provided by the system (3).

This paper exhibits a high gain observer for a reduced non linear model of schistosomiasis proposed by ([24]). There are several ways to deal with the synthesis of nonlinear observers. The most general method that we know, to deal with the construction of nonlinear observers, is the "high-gain observer method". (It is much more general than the " output injection method" developed in [17], [18], [19], [20], which applies to a very special class of systems only). This "high-gain observer" method, has been initiated in [22], [21], [23] and we will use it in this paper.

The construction of an observer requires some properties of observability (detectability precisely) and requires essentially the existence of globally defined and globally lipschitzian change of coordinates. If the system is uniformly observable for any input, then a globally convergent observer is found for a non linear system with inputs. The paper is organized as follows : Section 2 gives the problem formulation and the non linear design, Simulation and conclusions are given in Sections 3 and 4.

## 2 An observer for a schistosomiasis model

The following mathematical model of schistosomiasis was described in :

$$\left\{\begin{array}{lcl}\dfrac{d\,u_2}{dt} & = & t_{15}\,(N_H^* - u_2)\,x_5\,N_{si}^* \; - r_{12}\,u_2 \\ \dfrac{d\,x_3}{dt} & = & -\,(t_{32}\,u_2 + t_{38}\,N_M^*\,x_8 + b_3)\;x_3 + b_3 \\ \dfrac{d\,x_5}{dt} & = & r_{54}\;(1 - x_3 - x_5) - b_3\,x_5 \\ \dfrac{d\,x_8}{dt} & = & t_{75}\,N_{si}^*\,x_5\;(1 - x_8) - b_7\,x_8 \end{array}\right. \tag{4}$$

$u_2$, $x_3$, $x_5$, $x_8$ denote the numbers human hosts, uninfected snails, infected snails and infected mammals respectively. $t_{15}$, $t_{32}$, $t_{38}$, $t_{75}$ are the transmission rates from infected snails to uninfected humans, from infected humans to uninfected snails, from infected mammals to susceptible snails, from infected snails to susceptible mammals. Also, $r_{12}$ is the rate that infected humans recover and $r_{54}$ denotes the rate that the latent snail population becomes shedding $x_5(t)$.

Here we assume that the total population size $u_2(t)$ is measured on-line. This total human populations can be considered as a measurable output of the system (4) and it is given by $y(t) = u_2(t)$.

We then obtain the following coupled system:

$$\left\{\begin{array}{lcl}\dfrac{d\,u_2}{dt} & = & t_{15}\,(N_H^* - u_2)\,x_5\,N_{si}^* \; - r_{12}\,u_2 \\ \dfrac{d\,x_3}{dt} & = & -\,(t_{32}\,u_2 + t_{38}\,N_M^*\,x_8 + b_3)\;x_3 + b_3 \\ \dfrac{d\,x_5}{dt} & = & r_{54}\;(1 - x_3 - x_5) - b_3\,x_5 \\ \dfrac{d\,x_8}{dt} & = & t_{75}\,N_{si}^*\,x_5\;(1 - x_8) - b_7\,x_8 \\ y(t) & = & u_2(t) \end{array}\right. \tag{5}$$

We consider system (5) which is a non linear system.Our claim is to construct an observer (estimator) i.e an auxiliary system which will give a dynamical estimate $(z_1(t), z_2(t), z_3(t))$ of the state $x_3(t), x_5(t), x_8(t))$ of the system (4). For the construction of such auxiliary system, we will use a method called High Gain construction (see for instance [kupka]). This method provide an exponential observer; the estimation error will converges top zeros with exponential speed, i.e.,

$$\|z(t) - x(t)\| \leq \exp\left(-\lambda\,t\right)\|\hat{z}(0) - \hat{x}(0)\|$$

it has been proved in [1] that there is a positively invariant compact set for system (4). This set is in the form $\mathbb{D} = [0, N_H] \times [0,1] \times [0,1] \times [0,1]$, where $N_H$ is the total population size of humans.

Let us denote by $\mathbb{F}$ the vector field defining the dynamics of the system (4), and $h$ the output function, that is $y(t) = h(x(t)) = u_2(t)$ and $\mathbb{F} = \begin{pmatrix} t_{15}\,(N_H - u_2)\,x_5\,N_{si}\; - r_{12}\,u_2 \\ -\,(t_{32}\,u_2 + t_{38}\,N_M\,x_8 + b_3)\;x_3 + b_3 \\ r_{54}\;(1 - x_3 - x_5) - b_3\,x_5 \\ t_{75}\,N_{si}\,x_5\;(1 - x_8) - b_7\,x_8 \end{pmatrix}$.

Let $\Phi$ be the function $\Phi : \mathring{\mathbb{D}} \to \mathbb{R}^3$ ($\mathring{\mathbb{D}}$ is the interior of $\mathbb{D}$) define as follows:

$$\Phi(x) = \begin{pmatrix} h(x) \\ L_F h(x) \\ L_F^2 h(x) \\ L_F^3 h(x) \end{pmatrix},$$

where $L$ denote the Lie derivative operator with respect to vector field $\mathbb{F}$. Thus

$$\Phi(x) = \begin{pmatrix} u_2 \\ \\ -r_{12}u_2 + N_{Si}t_{15}\,(N_H - u_2)\,x_5 \\ \\ N_{Si}t_{15}\,(N_H - u_2)\,(r_{54}\,(1 - x_3 - x_5) - b_3 x_5) \\ +\,(-r_{12} - N_{Si}t_{15}x_5)\,(-r_{12}u_2 + N_{Si}t_{15}\,(N_H - u_2)\,x_5) \\ \\ (-r_{54} + r_{54}x_3 + b_3 x_5 + r_{54}x_5) \\ (N_{Si}t_{15}\,(b_3\,(N_H - u_2) + N_H\,(r_{12} + r_{54} + 2N_{Si}t_{15}x_5) - u_2\,(2r_{12} + r_{54} + 2N_{Si}t_{15}x_5))) \\ -\,(r_{12}u_2 - N_H N_{Si}t_{15}x_5 + N_{Si}t_{15}u_2x_5) \\ \left(r_{12}^2 + 2N_{Si}r_{12}t_{15}x_5 + N_{Si}t_{15}\,(r_{54}\,(-1 + x_3 + x_5) + x_5\,(b_3 + N_{Si}t_{15}x_5))\right) \\ +N_{Si}r_{54}t_{15}\,(N_H - u_2)\,(-b_3 + b_3x_3 + t_{32}u_2x_3 + N_M t_{38}x_3x_8) \end{pmatrix},$$

The Jacobian of $\Phi$ can be written:

$$\frac{d\,\Phi}{d\,x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -r_{12} - N_{Si}\,t_{15}\,x_5 & 0 & N_{Si}\,t_{15}\,(N_H - u_2) & 0 \\ \alpha_0 & -N_{Si}\,r_{54}\,t_{15}\,(N_H - u_2) & \alpha_1 & 0 \\ \alpha_2 & \alpha_3 & \alpha_4 & N_M\,N_{Si}\,r_{54}\,t_{15}\,t_{38}\,(N_H - u_2)\;x_3 \end{pmatrix},$$

where :

$$
\begin{aligned}
\alpha_0 \quad &= (r_{12} + N_{Si}\, t_{15}\, x_5)^{\,2} + N_{Si}\, t_{15}\, (b_3 x_5 + r_{54}\, (-1 + x_3 + x_5)) \\
\alpha_1 \quad &= -N_{Si}\, t_{15}\, \left(b_3\, (N_H - u_2) + N_H\, (r_{12} + r_{54} + 2\, N_{Si}\, t_{15}\, x_5) - u_2\, (2\, r_{12} + r_{54} + 2\, N_{Si} t_{15}\, x_5)\right) \\
\alpha_2 \quad &= N_{Si}\, r_{54}\, t_{15}\, t_{32}\, (N_H - u_2)\, x_3 + N_{Si}\, t_{15}\, (b_3 + 2\, r_{12} + r_{54} + 2\, N_{Si}\, t_{15}\, x_5) + (-r_{12} - N_{Si} t_{15} x_5) \\
&\quad (-b_3 x_5 - r_{54}\, (-1 + x_3 + x_5)) \left((r_{12} + N_{Si}\, t_{15} x_5)^{\,2} + N_{Si}\, t_{15}\, (b_3 x_5 + r_{54}\, (-1 + x_3 + x_5))\right) \\
&\quad + N_{Si}\, r_{54}\, t_{15}\, (b_3 - x_3\, (b_3 + t_{32}\, u_2 + N_M\, t_{38}\, x_8)) \\
\alpha_3 \quad &= N_{Si}\, r_{54}\, t_{15}\, \left(2\, b_3\, (N_H - u_2) + N_H\, (r_{12} + r_{54} + t_{32}\, u_2 + 3 N_{Si}\, t_{15}\, x_5 + N_M\, t_{38}\, x_8)\right) \\
&\quad - N_{Si}\, r_{54}\, t_{15}\, (u_2\, (3 r_{12} + r_{54} + t_{32} u_2 + 3\, N_{Si}\, t_{15}\, x_5 + N_M\, t_{38}\, x_8)) \\
\alpha_4 \quad &= b_3^2\, N_H\, N_{Si}\, t_{15} + b_3\, N_H\, N_{Si}\, r_{12}\, t_{15} + N_H\, N_{Si}\, r_{12}^2\, t_{15} + 2\, b_3\, N_H N_{Si}\, r_{54}\, t_{15} + N_H\, N_{Si}\, r_{12}\, r_{54}\, t_{15} \\
&\quad + N_H\, N_{Si}\, r_{54}^2\, t_{15} - 3\, N_H\, N_{Si}^2\, r_{54}\, t_{15}^2 - b_3^2\, N_{Si}\, t_{15}\, u_2 - 3\, b_3\, N_{Si}\, r_{12}\, t_{15}\, u_2 - 3\, N_{Si}\, r_{12}^2\, t_{15}\, u_2 \\
&\quad - 2\, b_3\, N_{Si}\, r_{54}\, t_{15}\, u_2 - 3\, N_{Si}\, r_{12}\, r_{54}\, t_{15}\, u_2 - N_{Si}\, r_{54}^2\, t_{15}\, u_2 + 3\, N_{Si}^2\, r_{54}\, t_{15}^2{}_2 + 3\, N_H\, N_{Si}^2\, r_{54}\, t_{15}^2\, x_3 \\
&\quad - 3\, N_{Si}^2\, r_{54}\, t_{15}^2\, u_2\, x_3 + 6\, b_3\, N_H\, N_{Si}^2\, t_{15}^2\, x_5 + 4\, {}_H\, N_{Si}^2\, r_{12}\, t_{15}^2\, x_5 + 6\, N_H\, N_{Si}^2\, r_{54}\, t_{15}^2\, x_5 - 6\, b_3\, N_{Si}^2\, t_{15}^2\, u_2\, x_5 \\
&\quad - 6\, N_{Si}^2\, r_{12}\, t_{15}^2\, u_2\, x_5 - 6\, N_{Si}^2\, r_{54}\, t_{15}^2\, u_2\, x_5 + 3\, N_H\, N_{Si}^3\, t_{15}^3\, {}_5^2 - 3\, N_{Si}^3\, t_{15}^3\, u_2\, x_5^2
\end{aligned}
$$

The determinant of $\frac{d\,\Phi}{d\,x}$ can be expressed by :

$$\Gamma(u_1, x_3, x_5) = N_M\, N_{Si}^3\, r_{54}^2\, t_{15}^3\, t_{38}\, (N_H - u_2)^{\,3}\, x_3$$

So the map $\Phi$ is a diffeomorphism from $\dot{\mathbb{D}}$ to $\Phi(\dot{\mathbb{D}})$. This implies that the system (5) is observable. In the news coordinates defined by $(z_1, z_2, z_3)^T = z = \Phi(x) = (h(x), L_F h(x), L_F^2 h(x))^T$, our system can be written in the canonical form as follow:

$$
\left\{
\begin{array}{rcl}
\dot{z}(t) & = & \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{A} z(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \Psi(z(t)) \end{pmatrix}, \\
y(t) & = & z_1(t) = \underbrace{(1, 0, 0, 0)}_{C}\, z(t)
\end{array}
\right. \tag{6}
$$

where : $\Psi(z) = L_F^4 h(\Phi^{-1}(z)) = L_F^4 h(x) = \phi(x)$

The function $\phi$ is smooth(it is a polynomial function of $x = (u_2, x_3, x_5, x_8)$) on the compact set $\mathbb{D}$. Hence, it is globally Lipschitz on $\mathbb{D}$. Therefore it can be extended by $\tilde{\phi}$, a Lipschitz function on $\mathbb{R}^3$ which satisfies $\tilde{\phi}(x) = \phi(x)$, for all $x \in \mathbb{D}$. In the same way we define $\tilde{\Psi}$ the Lipschitz prolongation of the function $\Phi$. So we have the following system defined on the whole space $\mathbb{R}^3$. The restriction of (7) to the domain $\mathbb{D}$ is the system 6:

$$\begin{cases} \dot{z} &= A\,z + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(z) \end{pmatrix}, \\ y &= C\,z \end{cases} \tag{7}$$

Hence, we have shown that the system (5) satisfies the conditions of the following result which provides the observer construction.

**Proposition 2.1** *Under the assumption that*
***H1:*** *$\Phi$ is diffeomorphism from $\mathring{\mathbb{D}}$ to $\Phi(\mathring{\mathbb{D}})$. ($\mathring{\mathbb{D}}$ is the interior of $\mathbb{D}$).*
***H2:*** *$\phi$ can be extended from $\mathbb{D}$ to $\mathbb{R}^3$ by a $\mathbb{C}^\infty$ function, globally Lipschitz on $\mathbb{R}^3$. Then an exponential observer for system (7) is given by the following system :*

$$\dot{\tilde{z}} = A\,\tilde{z} + \tilde{\Phi}(\tilde{z}) + S^{-1}(\theta)\,C^T\,(y - C\;\tilde{z}).$$

*where $\theta$ is the solution of*

$$0 = -\theta\,S(\theta) - A^T\,S(\theta) - S(\theta)\,A^T + C^T\,C,$$

*and $\theta$ is large enough.*

$$\text{Here, } S(\theta) = \begin{pmatrix} \frac{1}{\Theta} & -\frac{1}{\Theta^2} & \frac{1}{\Theta^3} & -\frac{1}{\Theta^4} \\ -\frac{1}{\Theta^2} & \frac{2}{\Theta^3} & -\frac{3}{\Theta^4} & \frac{4}{\Theta^5} \\ \frac{1}{\Theta^3} & -\frac{3}{\Theta^4} & \frac{6}{\Theta^5} & -\frac{10}{\Theta^6} \\ -\frac{1}{\Theta^4} & \frac{4}{\Theta^5} & -\frac{10}{\Theta^6} & \frac{20}{\Theta^7} \end{pmatrix}.$$

*Precisely $\theta \geq 2\,n\,c\,K\,\sqrt{S}$, where $K$ is the Lipschitz coefficient of the function $\Phi$, $n$ is the dimension of the space, and $S = \sup_{i,j} \|S(1)_{i,j}\|$*

For the proof one can see Going back to the original system (4) via the transformation $\Phi^{-1}$, we have:

$$\dot{\hat{x}} = \tilde{\mathbb{F}}(\hat{x}) + \left[\frac{d\,\Phi}{d\,x}\right]^{-1}_{x=\hat{x}} \times S(\theta)^{-1}\,C^T(y - h(\hat{x}))$$

This observer is particularly simple since it is only a copy of (4), together with a corrective term depending on $\theta$.

# 3 Simulations

We present here some simulation results that show the efficiency of the observer of system (4). For the simulation we extend the function $\phi$ by continuity in order to make it globally Lipschitz on $\mathbb{R}^3$ in the following way : We denote $\tilde{\phi}$ the prolongation pf $\phi$ to $\mathbb{R}^3$ and the function $\pi$ the projection on the domain $\mathbb{D}$ and we construct $\tilde{\phi} = \phi \circ \pi$. The extend function $\tilde{\phi}$ has the same Lipschitz coefficient as $\phi$.

Using the same parameters values, when we do not use the Lipschitz prolongation of the function $\phi$ to the whole $\mathbb{R}^3$, the state estimation $\hat{x}$ computed by the observer tends to infinity in finite time. This actually happens in the beginning of the integration process . When the Lipschitz prolongation of the function $\phi$ to the whole $\mathbb{R}^3$ is done, the convergence of the estimates delivered by the observer is quite fast.

The parameters are: $t_{15} = 2,23.10^{-7}$, $t_{38} = 2,0.10^{-7}$, $t_{32} = 1,05.10^{-7}$, $t_{75} = 1,02.10^{-7}$, $r_{12} = 4,47.10^{-3}$, $r_{54} = 2,50.10^{-3}$, $b_3 = 11,00.10^{-3}$. $d_6 = 1,00.10^{-3}$, $d_3 = 8,86.10^{-3}$, $d_7 = 1,00.10^{-6}$, $c_{33} = 5,11.10^{-7}$, $c_{36} = 5,11.10^{-8}$, $c_{77} = 7,00.10^{-11}$, $c_{66} = 1,50.10^{-8}$, $c_{64} = 25,11.10^{-9}$, $b_6 = 6,60.10^{-3}$, $b_7 = 5,20.10^{-6}$,$a_6 = b_6 - d_6$, $a_3 = b_3 - d_3$, $\theta = 10$

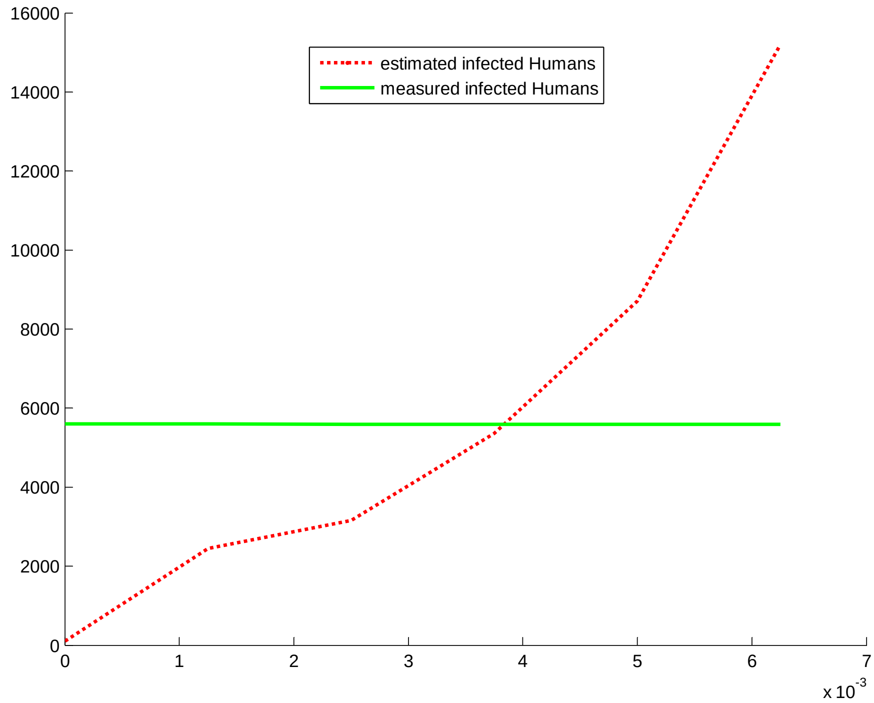


Figure 1: Simulation of system with its observer :$u_2$ (solid line) and its estimate $\hat{u}_2$(dash line) when $\phi$ is not extended

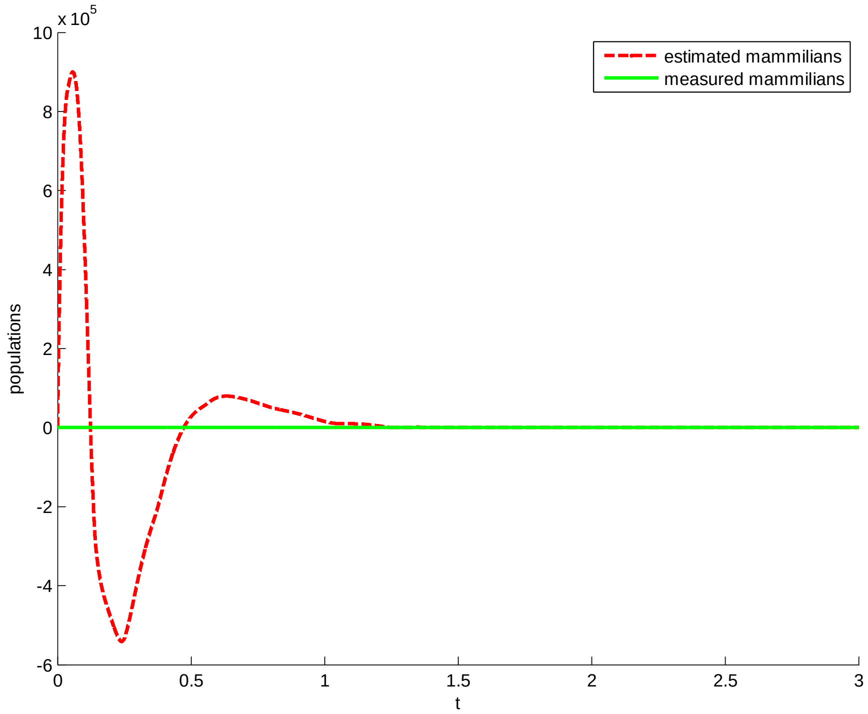


Figure 2: Simulation of system with its observer : $u_2$(solid line) and its estimate $\hat{u}_2$ (dash line) when $\phi$ is extended

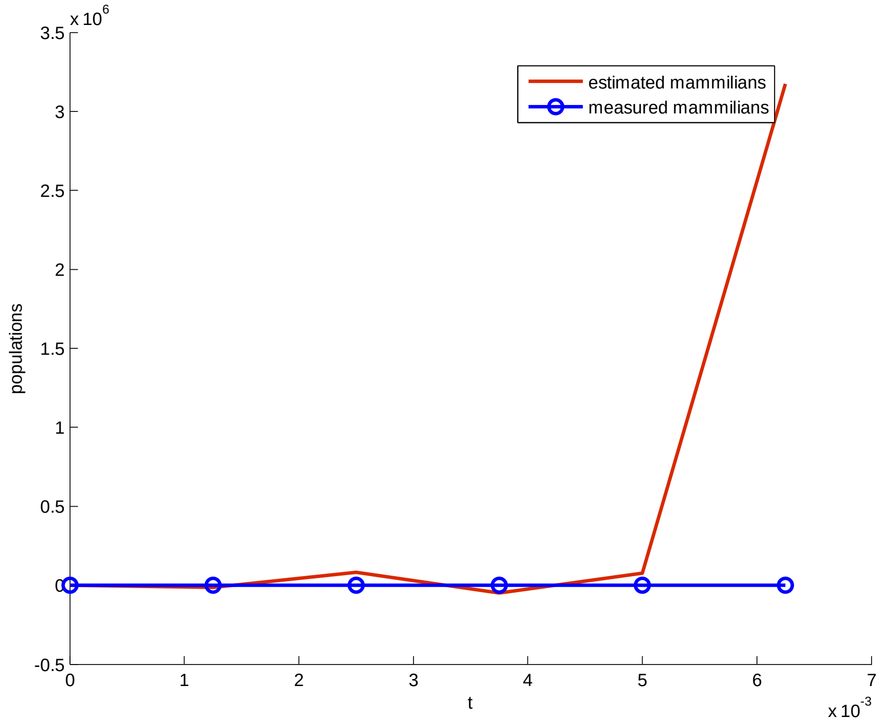


Figure 3: Simulation of system with its observer : $u_2$(solid line) and its estimate $\hat{u}_2$ (dash line) when $\phi$ is not extended

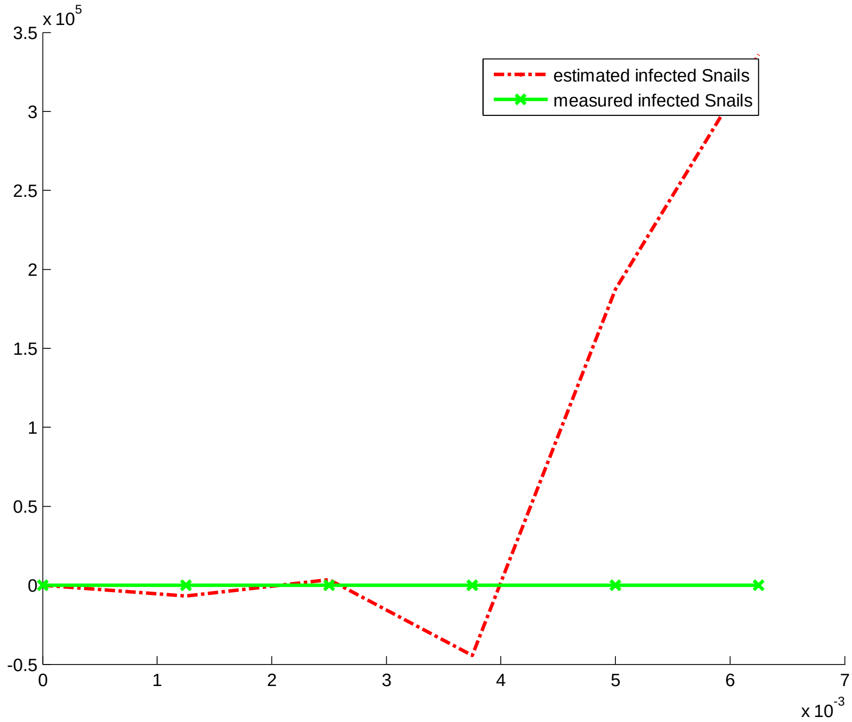


Figure 4: Simulation of system with its observer : $u_2$(solid line) and its estimate $\hat{u}_2$ (dash line) when $\phi$ is not extended

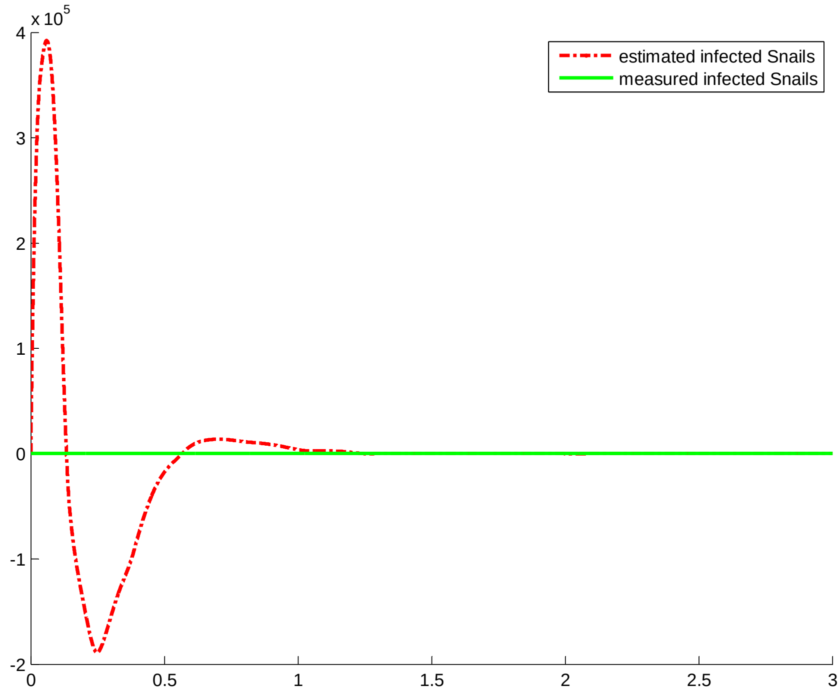


Figure 5: Simulation of system with its observer : $u_2$(solid line) and its estimate $\hat{u}_2$ (dash line) when $\phi$ is extended

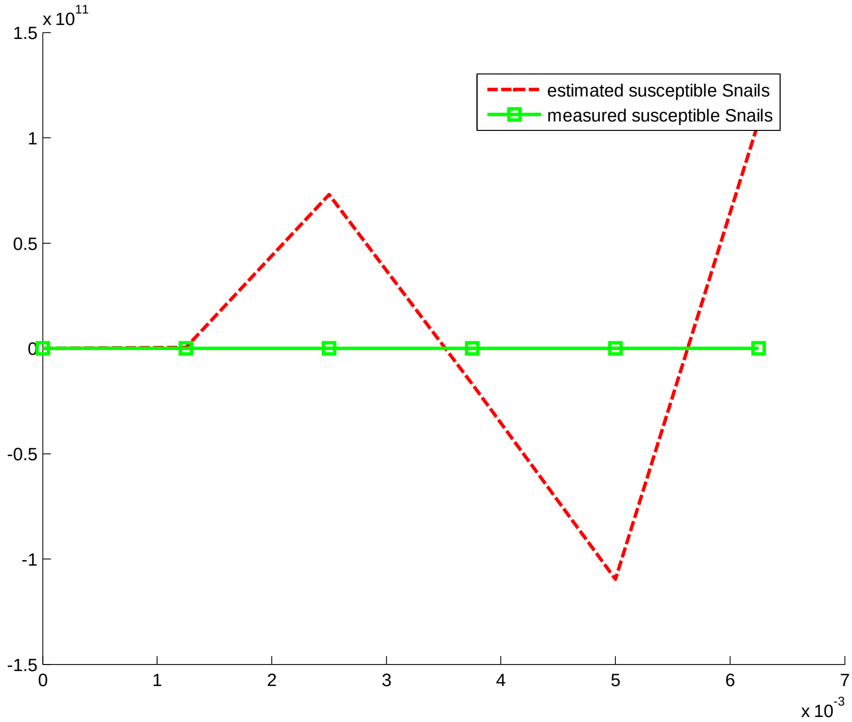


Figure 6: Simulation of system with its observer : $u_2$(solid line) and its estimate $\hat{u}_2$ (dash line) when $\phi$ is not extended

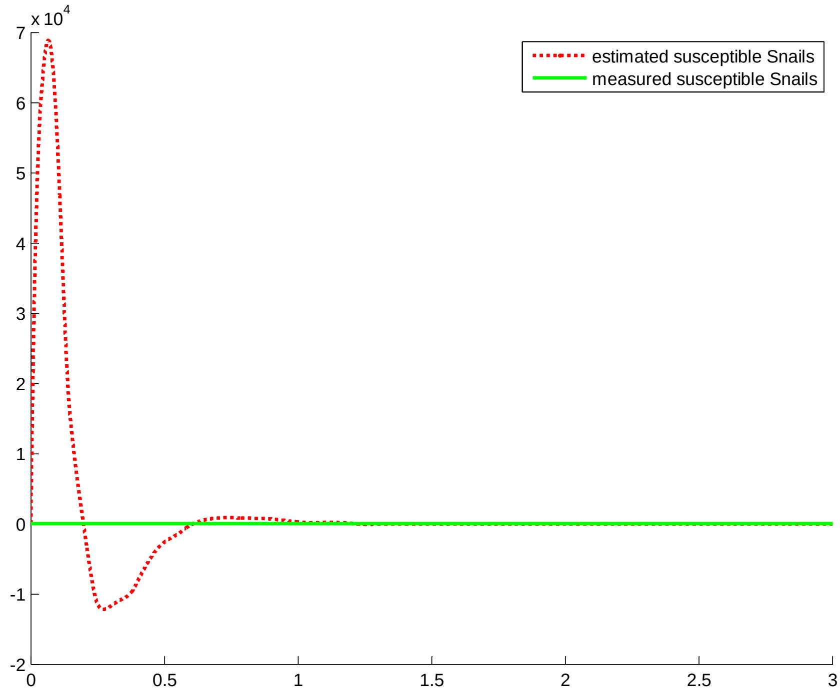


Figure 7: Simulation of system with its observer : $u_2$(solid line) and its estimate $\hat{u}_2$ (dash line) when $\phi$ is extended

# 4 Concluding remarks

In this paper a non linear observer was designed and tested in simulation to estimate the total population size of snails and mammalians. With the use of judicious value of the gain parameters $\theta$ we obtain satisfactory estimation of the real state. The observer's convergence is quite fast and does not depend on the initial condition choice. Therefore one can get a "good" estimate of the unmeasurable real state quickly. However the observers given in this paper assume that the model is good enough and the parameters values are available. Unfortunely that is not the case in enough mathematical models of infections disease. In the future work, we present a way of coping with uncertainties in the model when estimating the variables, given the outputs. Our basic idea is simple: given deterministic bounds on the uncertainties, we obtain deterministic guaranteed dynamic intervals containing the variables to estimate.

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