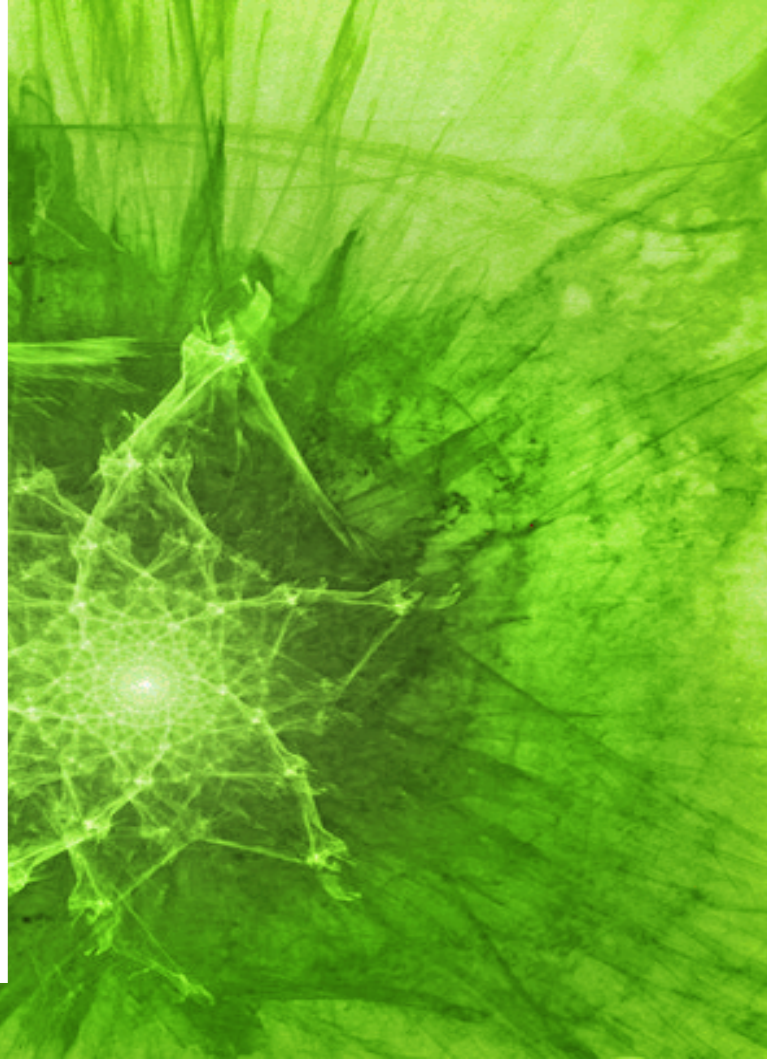


R. Dubois
H. Yahia
Oriol PONT

GeoStat Team, INRIA Bordeaux – IHU LIRYC

Singularity analysis of Potential Maps

FisEs 2012 – Palma



Approach: Heart Dynamics as a Complex System

Singularity analysis:

Preserves **physical principles** of the signal

Takes advantage of known **symmetries**

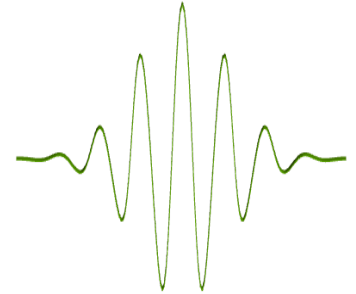
Model-agnostic signal **processing**

Singularity exponent of a signal

Singularity exponent $h(\mathbf{x})$ degree of singularity / regularity of signal s around point \mathbf{x}

$$\mu_S [\Psi_r(\mathbf{x})] \sim r^{\dim(\mathbf{x}) + h(\mathbf{x})} \quad d\mu_S \sim \| \nabla s \| dx$$

μ is a measure defined from gradient modulus and $\Psi_r = \Psi (\mathbf{x} / r)$ is a wavelet kernel

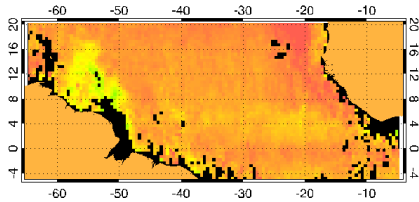


Ocean Circulation

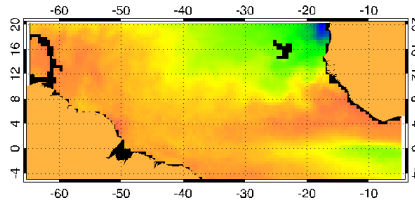
Pont *et al.* JGR 2012

Singularity and local predictability:

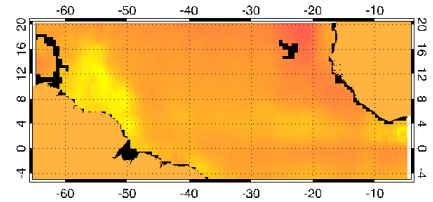
$$\mu_s [\Psi(\mathbf{x})] = A(\mathbf{x}) \| \Psi \|^{h(\mathbf{x})} + o(\| \Psi \|^{h(\mathbf{x})}) \quad d\mu_s = |\nabla s| dx$$



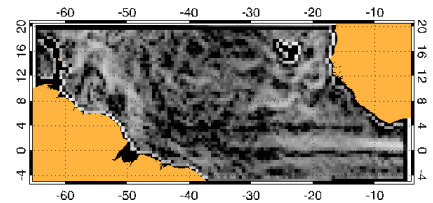
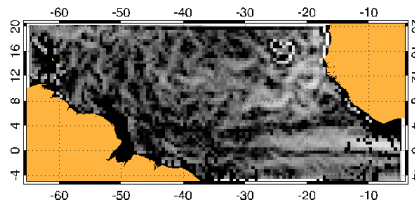
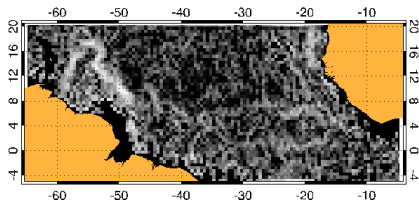
salinity



temperature



salinity fused



Derived fields, fusion:

$$d\mu_s = S(\mathbf{x}) d\mu_r$$

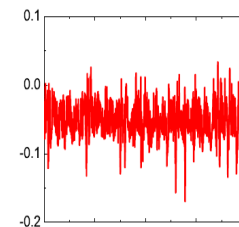
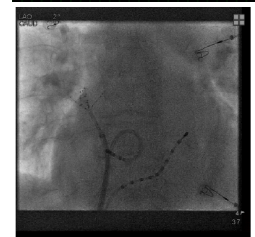
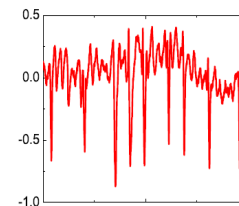
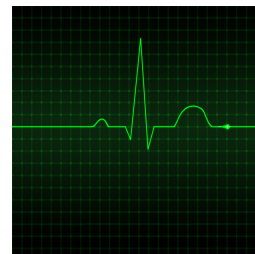
Singularity exponents – validation

Validated for :

ECGs

Endocardial catheter (potential time series)

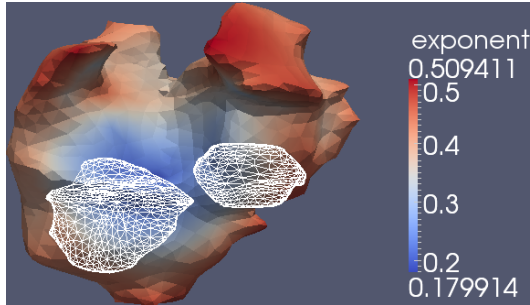
Electrocardiographic potential maps



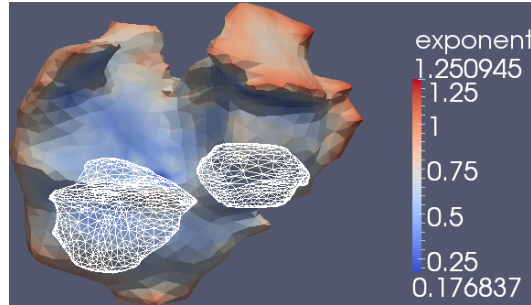
$$\mu_S [\Psi_r(x)] \sim r^{\dim(x) + h(x)}$$

Singularity exponent epicardial maps

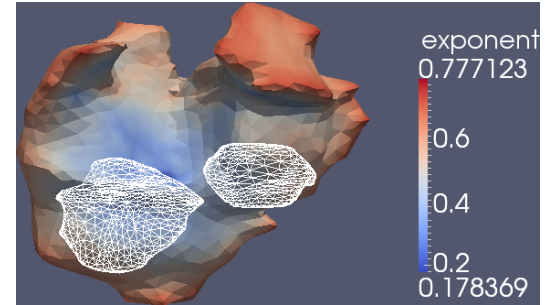
space



time



space-time

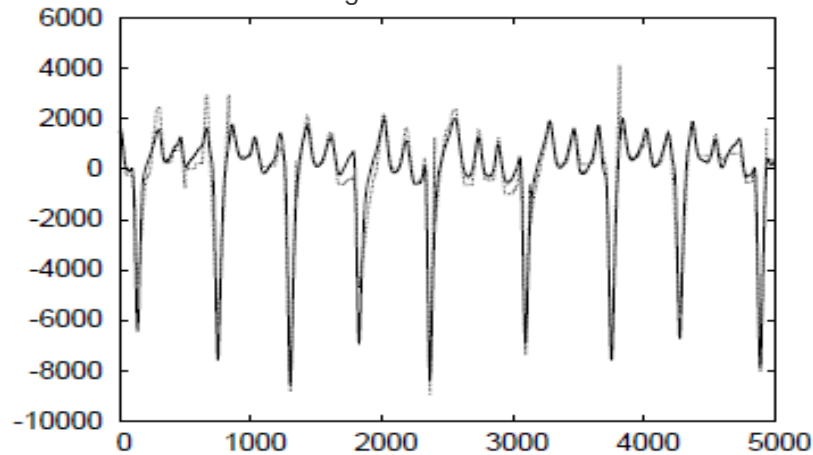


versatility in the domain considered

$$\mu_S [\Psi_r(x)] \sim r^{\dim(x) + h(x)}$$

Signal well reconstructed from most singular manifold

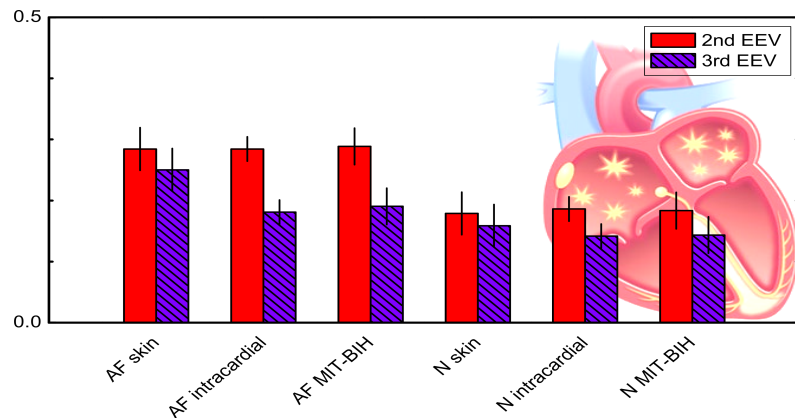
$$\hat{s}(f) = (i f / f^2) \cdot F(\nabla s \delta_{\text{most sing.}})$$



signal s , frequency f , most singular manifold $\delta_{\text{most sing.}}$

Signal well reconstructed from most singular manifold

Fast dynamics: MSM orientation, 3-states no memory (Markov)



$$P(\sigma_0, \sigma_\tau) = \langle P(\delta_\infty(t) = \sigma_0, \delta_\infty(t + \tau) = \sigma_\tau) \rangle$$

$$P(\sigma_0) = \langle P(\delta_\infty(t) = \sigma_0) \rangle_t = \langle P(\sigma_0, \sigma_\tau) \rangle_{\sigma_\tau}$$

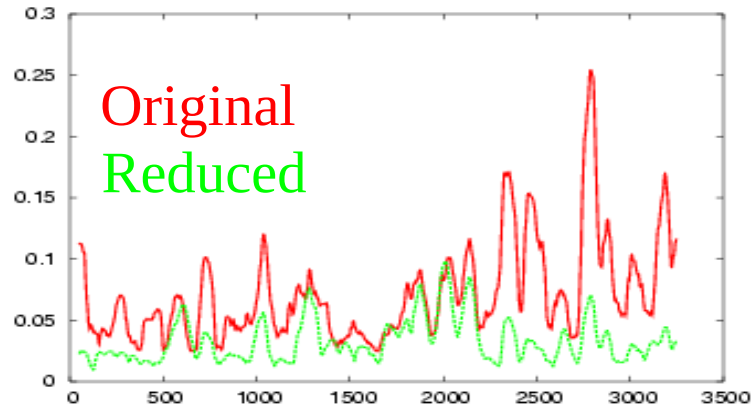
$$P(\sigma_2 | \sigma_0) = \sum_{\sigma} P(\sigma_2 | \sigma) P(\sigma | \sigma_0)$$

eigenvalues of transition matrix discriminate AF in both ECG and endocardial (Pont et al. CinC 2011)

Signal well reconstructed from most singular manifold

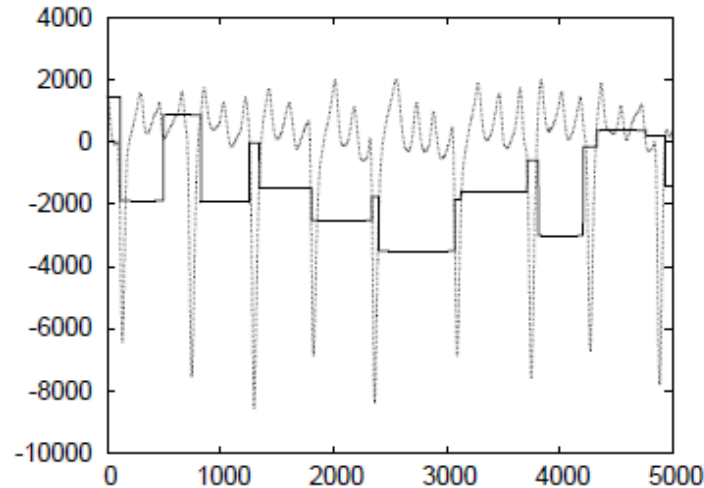
Fast dynamics: MSM orientation, 3-states no memory (Markov)

Slow dynamics: modulation over the reduced MSM-orientation reconstruction



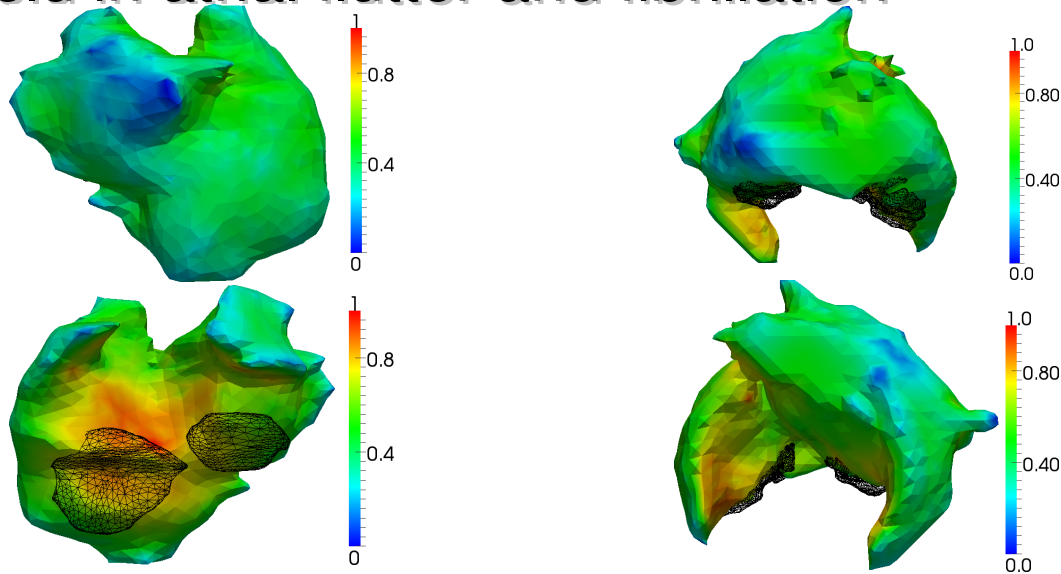
Source field, slow dynamics

$$SF(\mathbf{x}) = d\mu_s / d\mu_r$$

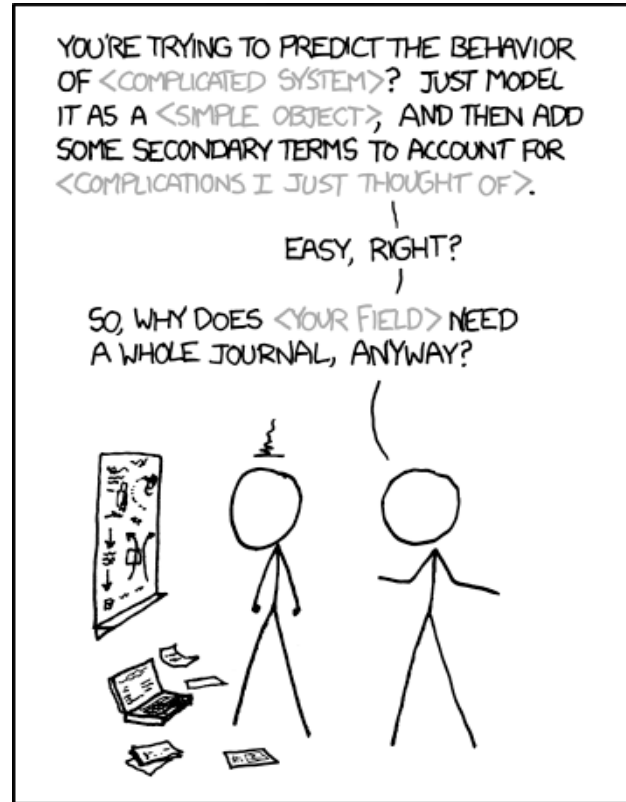


Radon-Nikodym derivative between original signal and reduced signal diff measures

Source field in atrial flutter and fibrillation



robust numerical estimate – physical non-heuristic, sharp descriptor



Thanks for your attention!