Modeling the Dynamic Effects of Discourse: Principles and Frameworks

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1 Introduction

In the study of the meaning of natural language expressions, the sentence level provides a natural entry point. Its relevance depends, of course, on the focus we want to put on meaning: as related to thought, to communication, to truth, etc. In this paper, we concentrate on the model theoretic view of meaning, in particular *via* first-order logic representation. This view is commonly referred to as Montague semantics because of Richard Montague's influential work, but is not limited there to [1]. It naturally brings in inference capabilities that, for instance, allows us to discuss the consequences that are true of a world a sentence describes.

In relating natural language utterances to logical representations, a key feature associated with this view is the compositionality principle. This principle basically states that the meaning of a sentence derives from the meaning of its parts and how they combine syntactically. However, some of these parts can only take on meaning with respect to previously uttered sentences. Typical examples of such parts are pronouns. But they are not the only ones.

In Sect. 2 we will present phenomena that illustrate the challenges posed by discourse to truth-conditional semantics and compositionality. We will show in Sect. 2.1 that proposals to address these challenges rely on the additional device of *contexts* and on the way sentences can access and modify these contexts. This capability is usually referred to as the *context change potential* of a sentence. Depending on the phenomenon, contexts need to represent different kinds of information: propositions, discourse referents, and

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[1] For an historical and epistemological perspective, see [56].

variations on these elements. We will also show in Sect. 2.2 that taking into account the rhetorical structure of discourse leads to even richer structuring of the context.

We will then devote Sect. 3 to the presentation of frameworks that have been designed to model these phenomena. We will also concentrate on formalisms that give an account of the dynamics of discourse in Sect. 3.1. We will introduce the well-established formalisms of Discourse Representation Theory (DRT) [39, 41] in Sect. 3.1.2, Dynamic Predicate Logic (DPL) [30] in Sect. 3.1.3, and the more recently developed approach based on continuation semantics [31] in Sect. 3.1.4. Finally, we will introduce Segmented Discourse Representation Theory (SDRT) [5], which combines the effects of dynamics and discourse structure.

2 Dynamics and Coherence in Discourse: Principles

2.1 Dynamics

We have been discussing, at a rather general level, some phenomena that stress a desirable distinction between the semantic content of a single sentence and the content of that sentence when it is uttered in a larger text or discourse. While this provides a general idea of the notion of dynamics which underlies the content of a discourse, we can be more precise. The dynamic feature of a discourse representation appears in the requirement of a notion of *context* in the discourse modeling.

This notion of context is a key feature in the various approaches to discourse modeling. A context stores the elements that have been used so far and are used in sentences to assert things about the world. But sentences can, in turn, access and modify the context and make it ready for the next sentence. Actually, much more than simple texts can modify the context as it is used in a discourse. For instance, finger-pointing at an object can make it salient in a discourse and referable to just as if it had been introduced using a linguistic expression.

The folowing sections will be devoted to the presentation of a range of phenomena that have been considered in the literature. We defer the formalization of context and its use to Sect. 3.1. Using various examples, we will describe what kind of information is relevant to describing context.

2.1.1 Presupposition

Presupposition corresponds to the fact that when some expressions are uttered, even if no other clue appears in the preceding discourse (for instance

when it is the first sentence in a discourse), the listener may infer certain information that is not explicitly stated. It is even the case that if this information was previously denied, the whole discourse becomes infelicitous. Example (1a) is such a sentence, and (1b) states the implicit information, the so-called *presupposition*. This presupposition corresponds to the hypothesis the listener will assume, even if he or she has no further evidence for it. The presupposition is said to be *accommodated* and can be used to infer (2). Otherwise, if it were false, as in discourse (3) where it is linguistically and explicitly denied, this part of the discourse would become infelicitous.

(1) a. John stopped smoking.

  b. *(Presupposed: John used to smoke)*

(2) Someone used to smoke.

(3) a. John never smoked.

  b. *John stopped smoking.

This intuitively describes a property of the context: it can be updated with non-explicitly-uttered content and it has an effect on the semantic value of the explicitly uttered content.

Expressions enabling this kind of behavior are called *presupposition triggers*. There is a wide range of them, including, for instance (taken from [9, 10]):

- change of state verbs (*stop*, *begin*, etc.);
- definite description (*the man*, proper nouns, possessives, etc.);
- factive verbs (*know*, *regret*, etc.);
- iterative adverbs (*again*, *too*, *in return*, etc.);
- counterfactual conditionals (*If I had known, then I would not have come*) that presuppose the falsity of the *if* clause.

One way to characterize presupposition is to rely on the robustness of its effects on embedding in complex structures. For instance, both (4a) and (5a) entail (6). However, while (4b), which negates (4a), still entails (6), this is not the case for (5b).

(4) a. John regrets that Mary left.

  b. John does not regret that Mary left.

(5) a. Mary left.

  b. Mary did not leave.

(6)   Someone left.

This means that presupposed content embedded under negation can escape this embedding and become a presupposition for the whole sentence. To test whether a clause has presupposed content, it is thus possible to embed it under a negation and check whether this presupposed content is still available. This is called the *embedding under negation test*. More generally, such differences in behavior between asserted and presupposed content can be used to test and identify the presupposed content of an utterance. The way the presupposed content can escape the complex clause it is embedded in is called *projection*. The issue then arises of predicting the presuppositions of a complex clause from the presuppositions of its subclauses. This is the *projection problem*. As Beaver [9] states,

> (...) the projection problem fits quite naturally into a larger Fregean picture of how language should be analyzed. The projection problem for presupposition is the task of stating and explaining the presuppositions of complex sentences in terms of the presuppositions of their parts.

This makes the projection problem fall within the scope of *compositionality*.

In addition to the embedding under negation test, other constructions, for instance the ones exemplified in (7), still imply (2) (and even (1b)) and may be used to study what is projected, when, and where.

(7)   a.   If John stopped smoking, then he feels healthier

       b.   Did John stop smoking?

       c.   Maybe John stopped smoking

       d.   Peter knows that John stopped smoking

Without discussing the details of the different formalizations of this phenomenon, we would like to stress that the actual definition of the context and meaning of a sentence are at stake here. Each phenomenon is studied with respect to the minimal structure and minimal content of the context that permits its modeling. For Karttunen [42], the context C of a sentence is the set of sentences that are presupposed. The (local) context of each subclause is computed from the syntactic structure in which it occurs and from the context of the clause.

For instance, if we assume a context C for (7a), the antecedent of the condition, the subclause S *John stopped smoking*, also has C as local context. The consequent subclause *he feels healthier* has C *and* S in its local context.

For a sentence to be uttered felicitously, its context and the local context of its subclauses must all entail the presupposition they trigger. So, the context of (7a) should at least entail that *John used to smoke*. To see why the antecedent is added to the local context of the consequent, we can contrast (8a) and (8b), where the presupposition *John stopped smoking* is triggered in the consequent by the factive verb *regrets*. In (8a), because the

antecedent is added to the local context of the consequent, it trivially entails the presupposition. This is true whatever the context of the whole sentence may be the *if . . . then* construction can filter presuppositions. They are *locally* accommodated.

On the other hand, (8b) cannot provide such an entailment because, whatever the context of the sentence, the addition of the antecedent to the local context of the consequent raises a contradiction. Hence it is considered infelicitous.

(8) a. If John stopped smoking, then he regrets he stopped smoking

  b. *If John didn't stop smoking, then he regrets he stopped smoking

This gives us a first example of what the context can contain, and how it can be updated. Here, the context basically records a set of propositions possibly extended with the asserted content of subclauses. Karttunen [42] uses such a context only to predict the felicity of a assertion. The truth conditions of each sentence do not interact with their context. But examples such as (3) show that the asserted content of a sentence somehow restricts the possible contexts that are available to assess the felicity of a subsequent sentence. If, at the beginning of the discourse, any model is available, as soon as (3a) is uttered, only models that can satisfy its asserted content will be considered. Since such a model cannot entail the presupposed content of (3b), this sentence becomes infelicitous.

According to Gazdar [27], the lack of interaction between truth-conditional content, presupposed context, and the way some lexical items may have presuppositions accommodated by Karttunen [42], prevents the latter from providing explanatory content to a presupposition. Heim [37] proposes another account of presupposition that more closely combines those different aspects. Interestingly enough, this approach introduces the *context change potential* of a sentence, in terms of which the truth of a sentence is defined: "the truth-condionnal aspect of the meaning of any expression is predictable on the basis of its context change potential". This compositional treatment makes it explicit how the evaluation of a complex clause in context relies on modification of the context by the subclauses.

2.1.2 Context Update

In order to take into account these interactions between the context against which presuppositions are evaluated and the asserted content of a sentence as proposed by Heim [37], Muskens et al [54] introduce the following notations: $\lceil S \rceil$ denotes the possibilities (represented by a set of valuations, for instance) that are compatible with the asserted content of S. Then, when two sentences combine, we have $\lceil S_1.S_2 \rceil = \lceil S_1 \rceil \cap \lceil S_2 \rceil$. It is easy to see that, for a sequence of sentences $S_1, S_2, \ldots, S_n$, $\lceil S_1.S_2.\ldots.S_n \rceil = \lceil S_1 \rceil \cap \lceil S_2 \rceil \cap \cdots \cap \lceil S_n \rceil$.

Muskens et al [54] also define the *context change potential* $\|S\|$ of a sentence S as a function from context to context: $\|S\| = \lambda C.C \cap \lceil S \rceil$. This operator specifies how the possibilities compatible with a sentence S combine with the context against which the presuppositions are tested.

Then, if a sentence S_1 is processed with context C, the context in which a subsequent sentence S_2 has to be processed is not the same C, but rather C restricted by $\lceil S_1 \rceil$, that is $C \cap \lceil S_1 \rceil = \|S_1\|$. This leads to a typical feature of discourse dynamics, where the effects of combining sentences in a discourse are described by function composition as shown in (9).

$$\begin{aligned}(9) \qquad \|S_1.S_2\| &= \lambda C.\|S_2\|(\|S_1\|(C)) \\ &= S_2 \circ S_1\end{aligned}$$

Following Heim [37], this operator allows Muskens et al [54] to propose a dynamic version of the logical connectives ($\overline{\wedge}$, $\overline{\neg}$, $\Rrightarrow$) and a connective / such that ϕ/ψ means that ϕ is the presupposition of ψ. These connectives are defined in (10). The definition of / in (10c) means that when a sentence ψ that triggers presupposition ϕ is uttered in a context C, if ϕ is implied by C (that is does not restrict C), then $\|\psi\|(C)$ can be evaluated. Otherwise, the result is undefined. Of course, when applied to an undefined result, $\|\psi\|$ is also undefined. (10a) stipulates that when ϕ is negated, whatever satisfies ϕ should be removed from the context. (10b) stipulates the same function composition as (9).

$$(10a) \qquad \|\overline{\neg}\phi\| = \lambda C.C \setminus \|\phi\|(C)$$

$$(10b) \qquad \|\phi \overline{\wedge} \psi\| = \lambda C.\|\psi\|(\|\phi\|(C))$$

$$(10c) \qquad \|\phi/\psi\| = \lambda C.\text{if } \|\phi\|(C) = C \text{ then } \|\psi\|(C) \text{ else undefined}$$

In this approach, the context is modeled by a set of valuations rather than by a set of propositions. Each of the formalizations is then evaluated with respect to these valuations. This gives us another modeling of context.

While function composition here explicitly marks the dynamic nature of the connectives, Muskens et al [54] point out that the connectives of (10) are not intrinsically dynamic. They provide an equivalent interpretation where the context change potential of a clause in a context does not require evaluation of the context change potential of subclauses in any other context. This gives rise to the characterization of an operator F as static: there exists a P such that $F(C) = C \cap P$ for all contexts C.

An example of an actual dynamic operator is given with the epistemic modal *might* of Update Semantics [66]. This operator accounts for example (11). (11a) is felicitous because, intuitively, the modal leaves open whether or not it is sunny in the set of possibilities. As a result, all possibilities are available in evaluating the second part which, in turn, reduces the set of possibilities to those where it is not sunny.

On the other hand, the first sentence in (11b) restricts the possibilities to those where it is not sunny. There is no possibility left where it might be sunny.

(11)   a.   It might be sunny. It is not sunny.

       b.   It is not sunny. *It might be sunny.

(12) gives the interpretation of a sentence of the form $\Diamond\phi$.

$$(12) \qquad \|\Diamond\phi\| = \lambda C. \text{ if } \|\phi\|(C) \cap C \neq \emptyset \text{ then } C \text{ else } \emptyset$$

We can show that $\|\Diamond\textbf{sunny}\|$ is not static. Let us assume it is static. Then there is a P such that $\|\Diamond\textbf{sunny}\|(C) = C \cap P$ for any C. Let us choose C such that it is true of all its possibilities that $\neg\textbf{sunny}$ holds, then $\|\textbf{sunny}\|(C) \cap C = \emptyset = C \cap P$. Hence in none of the possibilities of P, $\neg\textbf{sunny}$ holds. This means that, in all possibilities P, $\textbf{sunny}$ holds. So, for any C that contains both possibilities, $\|\Diamond\phi\|(C) \subsetneq C$. This contradicts with $\|\Diamond\phi\|(C) = C$ according to (12). So $\|\Diamond\phi\|$ is not static.

2.1.3 Anaphora

An anaphora is a specific linguistic expression whose interpretation is a reference. For example, in (13a), *him* is an anaphora because it is coreferential with the subject, *Carlotta's dog*. The most common anaphoras are pronouns, which refer to their antecedents, but anaphoras can also be nominal phrases or adverbial phrases. They play a crucial role in maintaining the coherence of a discourse. The study of these phenomena is relevant to various fields, at least including linguistics, as in Binding Theory of Generative Theory; Computational Linguistics with the question of how to pick up the right referent; Cognitive Sciences as indicators of how humans process natural language.

(13)   a.   Carlotta's dog thinks that John loves him.

       b.   John parks his car.

       c.   Every man thinks of his mother.

In a simple anaphora as in (13b), *his* picks up its interpretation in the local context, which co-refers to John. Anaphoras can also deal with quantification, as in (13c). The semantics of such anaphoras consists in the semantic interpretation of the referent element or the variable bound by the quantifier. Note that when the referential element come first, it is anaphora. Otherwise, when it is after, this is called cataphora.

The use of anaphora can be more complex than in the previous examples, where the reference is intra-sentential. In a discourse, the anaphora must be

resolved extra-sententially in a set of discourse referents. This increases ambiguity because many discourse referents are introduced. Morpho-syntactic features are not sufficient to distinguish the referent, but syntactic and/or rhetorical relations should help to resolve this problem.

One way to resolve an anaphora is to deal with the quantified antecedent. Examples proposed by Evans [23] in defining e-pronouns may help us discuss the relations between anaphoras and quantified expressions.

(14)   a.   Few professors came to the party. They had a good time.

     b.   Every professor came to the party. * He had a good time.

The interpretation of discourse (14a) relies on the conjunction of the two sentences, entailing that *they* refer to a subset of professors, albeit *few* of them. But, in a more realistic interpretation, *they* should refer to all the *few* professors who attended the party. Anaphoras can refer to more than the quantified expressions which trigger references to more general sets of entities. But the reverse is not true, as shown in (14b), where reference to a specific entity in the set defined by the quantified expression is not acceptable. Another classic problem in resolving anaphoras is that of donkey sentences. We will precisely define this in Sect. 3.1.1, where we address the limits of Montague's approach.

Anaphoras can also be of another type, as in definite noun phrase anaphoras where the antecedent is referred to a definite noun phrase representing either the same concept or a semantically close one or one-anaphora, where the anaphoric expression is provided by a *one*-noun phrase.

2.1.4 Modal Subordination

Although maintaining a list of discourse referents in context seems adequate in the cases in the previous section, there are other cases where the context needs to be somewhat extended. Modal subordination is such a case. It has been studied in particular with respect to its interaction with anaphora resolution and accessibility. While presupposition requires the context to store a set of propositions, and anaphora a set of discourse referents, modal subordination requires both.

Classical examples of anaphoric links between pronouns and their antecedents across modalities are given in (15) from Sells [64] and in (16) from Roberts [61]. In these two examples, the second clause contains a linguistic expression (quantifier, mood operator, adverb, *etc.*) that makes the sentence dependent on the previous one. Here, the anaphoric pronouns would refer to a discourse referent that is under the scope of a modal. This implies that a subpart of the discourse is potentially defined in a possible world. The use of the present tense in the last sentence induces the interpretation outside the potential described world. We see in (15) that indefinites introduced in

the antecedent can be retrieved in the modally subordinated sentence as well. However, this fails in the other case (15b).

(15)   If John bought a book$_i$, he'll be reading it$_i$ by now.

  a.   It$_i$'ll be a murder mystery.

  b.   * It$_i$ is a murder mystery.

(16)   If Edna forgets to fill the birdfeeder, she will feel very bad.

  a.   They will get hungry.

In the interpretation of (15a), the modal force in the consequent and the modally subordinated sentence are the same. This is not the case in (17): (17a) introduces a *modal base*, i.e. a description of the possibility that is involved; then (17b) is evaluated relative to this modal base. The context should therefore be updated.

(17)   a.   A thief might break into the house.

  b.   He would take the silver.

Similarly, (18a) shows that discourse referents introduced in the factual world are accessible to pronouns introduced in a modal clause. The reverse is not true, as (18b) shows. This contrast suggests that, in addition to keeping track of the modal base, the context should distinguish between two sets of discourse referents: one for discourse referents introduced in factual clauses and available for any reference; one for discourse referents introduced in modal clauses that are only available to reference under modalities.

(18)   a.   A thief has broken into the house. He might take the silver.

  b.   A thief might break into the house. *He will take the silver.

Modal subordination also interacts with negation. Generally, negation blocks the accessibility of entities under its scope from parts of the discourse that are outside its scope, as (19a) shows. But it becomes possible to refer to them through the modal, as in (19b).

(19)   John didn't buy a mystery novel.

  a.   *It is *War and Peace*.

  b.   He would be reading it by now.

In (19a), *It* could not refer to the novel which is under the scope of the negation and therefore does not exist. In (19b), *would* corresponds to the consequent of a counterfactual conditional. It could be interpreted as *If John had bought a mystery novel, then he would be reading it by now*. The second possible interpretation is simply that there is no mystery novel, as expressed in the first part of (20).

(20) $\neg(\exists x \, \mathbf{novel}(x) \wedge \mathbf{buy}(\mathbf{John}, x)) \wedge (\exists y \, \mathbf{novel}(y) \wedge \mathbf{buy}(\mathbf{John}, y)$
$$\implies \mathbf{read}(\mathbf{John}, y))$$

If modal subordination is related to conjunction, it is also related to disjunction, as in example (21), attributed by Roberts [61] to Barbara Partee.

(21)   Either there is no bathroom in this house, or it is/must be in a strange place.

The standard interpretation of (21) fails to capture the semantics because the bathroom is introduced in the scope of the negation, and then is not accessible. The use of the modal *must* allows the sentence to be interpreted as if the two disjuncts belonged together. The negation is not copied, as it is not part of a condition applied to a referent. Consequently, the disjunction is felicitous.

Roberts [61] also introduced generalized subordinations in discourse: see example (22a). Here, the interpretation of (22b) and (22c) is possible only with (22a) and the restriction of the interpretation of adverbs (*always* and *usually*).

(22)   a.   Harvey courts a girl$_i$ at very convention.

b.   She$_i$ always comes to the banquet with him.

c.   The girl$_i$ is usually very pretty.

2.2 Coherence and Discourse Structure

We have illustrated the phenomena discussed so far by providing a very linear structure for the discourse. Equation (9) stresses a single composition mode for sentences. However, it is well known by linguists as by school teachers that texts need to be structured in order to be coherent and understandable. Keeping in mind the objective of understanding the meaning of a complex discourse, we must conclude that this structure is to be taken into account.

As when building a semantic representation of a sentence out of its syntactic structure, we need to be able to find out the underlying structure of a discourse in order to give it meaning. While syntactic theorists now more or less agree on the possible syntactic structures (mainly constituency trees or dependency graphs), there is no such consensus for discourse structure. Marcu [46] lists the questions that an adequate account of text structure should answer. They include:

- What is the abstract structure of texts? What are the constraints that characterize this structure?

- What are the elementary units of texts?
- What are the relations that could hold between two textual units?
- Is there any correlation between these relations and the concrete realization of texts?

In most theories, the abstract structure is not linear, but hierarchical. This hierarchy arises from a distinction between two kinds of discourse relations: *coordinating* relations and *subordinating* relations. These notions reflect the different roles of a discourse unit: either to expand upon the discourse, or to make it more precise by providing examples, explanations, etc. In Rhetorical Structure Theory (RST) [44], *rhetorical relations* hold between two non-overlapping elementary units. One member of a given relation is called the *nucleus* and the other the *satellite*. An example would be the **Elaboration** relation that holds between (23a) and (23b), while a **Narration** relation holds between (23b) and (23c). Figure 1 shows the associated hierarchy structure.

(23)   a.   John went to the hospital.

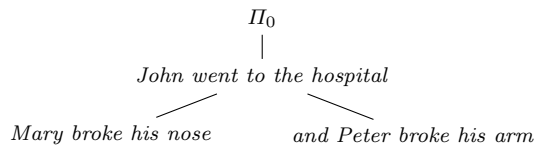   b.   Mary broke his nose,

      c.   and Peter broke his arm.

Fig. 1 Discourse structure for (23)
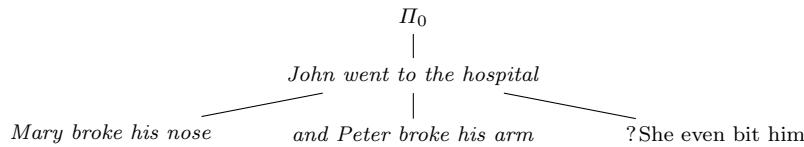
Fig. 2 Discourse structure for (23–25)

Characterizing rhetorical relations and discourse units is a difficult task. Some theories favor intention-based approaches [34, 44] taking into account communication goals, while others [57, 5] favor semantics-based approaches using state or event description.

An important question for discourse relations is how to infer them: what they are and what they link. RST and Segmented Discourse Representation Theory (SDRT) [5] provide different solutions. An adequate instantiation of the context should contain the relevant data to help pick the right relation.

In giving a precise description of how to build a Segmented DRS (SDRS), Asher and Lascarides [5] also suggest the elements that should be put into context.

An important element is probably the structure built so far, or at least the *accessible* attachment points. It has been observed, for instance, that a new discourse relation cannot attach just anywhere in the hierarchy, but rather only on the *right frontier* (if the structure is a tree, this corresponds to the nodes on the path from the root to the rightmost leaf). For instance, if (23) is followed by one of (24), which elaborate on John's injuries, (24) can only attach either to (23a) (as whith (24b) for instance) or to (23c) (as for (24a)). It cannot attach to (23b). In any case, the *it* of (24a) cannot refer to John's nose.

(24)   a.   It was even bleeding.

       b.   He was bleeding.

This *right frontier constraint* seems to be quite strong in attachment points for discourse relations. It also seems to apply to a certain extent to anaphoras. This would explain why extending (23) with (25) to get the structure of Fig. 2 seems wrong [14]. Different anaphoras however behave differently with respect to this constraint. For instance, pronouns seem to follow it rather strictly, while definite descriptions do not [3]. This suggests a model of saliency that is related to discourse structure. [18] also shows how anaphora resolution is improved by taking into account the hierarchical structure of texts.

(25)   She even bit him.

Other inputs for inferring discourse relations of course include lexicalization. Words such as *then*, *because*, etc. strongly suggest what relation is involved. But relations are not necessarily lexicalized as in (26). Much has to be considered in order to infer the correct (**Consequence**) relation, including the preceding topic, temporal relations between events, inferences based on background knowledge, etc.

(26)   John fell. Mary pushed him.

This shows that the context can contain a lot of heterogeneous information. Models and theories of context should be able to provide a way to capture this diversity.

3 Frameworks

In this section, we will introduce the formal devices that have been designed to model the phenomena described in the previous sections. With

regards to expressing discourse dynamics, we limit ourselves to three frameworks: Discourse Representation Theory (Sect. 3.1.2), Dynamic Predicate Logic (Sect. 3.1.3), and continuation semantics for discourse (Sect. 3.1.4). We will then introduce Segmented Discourse Representation Theory, which adds an account of discourse structures to the dynamic semantics (Sect. 3.2).

This first section is devoted to illustrating the limits of standard (static) Montague's semantics in discourse phenomena.

3.1 Dynamic Effects

This section aims to describe formal accounts of the phenomena characterized in Sect. 2.1. We will rely on well-established formalisms and on associated models linking natural language expressions and their representations. As emphasized above, much effort is dedicated to populating the context and describing how expressions contribute to it compositionally.

Let us first recall some of the shortcomings of Montague's sentence semantics [49, 50] as regards intrasentential and intersentential anaphora.

3.1.1 Limits of Montague Semantics

The most frequent examples of problems with anaphoric links are so-called *donkey sentences*, as illustrated, familiarly, by Geach [28]. Let us first look at (27), presented with its expected semantic representation.

(27)   If John owns a donkey, he is rich.

$$(\exists x.\mathbf{donkey}(x) \wedge \mathbf{owns}(\mathbf{John}, x)) \implies \mathbf{rich}(\mathbf{John})$$

(28)   If John owns a donkey, he beats it.

According to the compositionality principle, the expected meaning of (28), because its syntactic structure is similar to thath of (27), is:

$$(\exists x.\mathbf{donkey}(x) \wedge \mathbf{owns}(\mathbf{John}, x)) \implies \mathbf{beats}(\mathbf{John}, x)$$

In the second formula, however the second occurrence of x is *free*. It is *outside* the scope of the existential quantifier. Moreover, instead of an existential quantification, typically introduced by the indefinite article, we expect to have a *universal* quantification that claims something about all the donkeys John owns:

$$(\forall x.(\mathbf{donkey}(x) \wedge \mathbf{owns}(\mathbf{John}, x)) \implies \mathbf{beats}(\mathbf{John}, x))$$

Such examples outline issues both with the composition of the meaning of the clauses (the variable is not bound) and with the lexical semantics (since the indefinite seems to be associated on the one hand with an existential quantifier and on the other hand with a universal quantifier).

Another kind of problem related to pronoun interpretation is exemplified in (29) and (30). The discourse in (29) is felicitous since an antecedent is available to interpret the pronoun in (29b). On the other hand, (30b) is infelicitous when uttered in the context of (30a). The question here is how the negation compositionally affects the contribution of the indefinite such that there is no further possible reference to the variable it introduces. Such observations have given rise to accessibility constraints on discourse antecedents.

(29)   a.   John owns a donkey.

    b.   It is grey.

(30)   a.   John doesn't own a donkey.

    b. *It is grey.

To deal with these phenomena, contexts must now keep track of *discourse referents*. Basically, indefinite noun phrases such as *a donkey* are considered as putting a new item into the context. If correctly recorded, this item can later be accessed by pronouns. The following sections describe different approaches to implementing this intuition. We will then introduce the interpretation given in [52, 54] as an execution of programs that change machine states. The control on this execution can be described with continuations, as in functional programming. This view was first expressed by de Groote [31].

3.1.2 Discourse Representation Semantics

Discourse Representation Theory (DRT) is a formalism introduced and developed by Kamp [39], Kamp and Reyle [41]. As exemplified above, the key idea is to provide a context where discourse referents can be stored and accessed. A sentence is interpreted in this context and, in turn, can also *update* it by adding new discourse referents. This formalism shares many features with the independent formalism of File Semantics proposed by Heim [35, 36]. It is worth noting that, according to Kamp [40], though DRT has been proposed to overcome the limits of semantic modeling when moving from single sentences to longer texts, the first phenomena under consideration were related to time and ways of expressing the difference between the French imperfect and preterit. Only afterwards was it found to be useful for dealing with donkey sentences.

(31)   a.   A man entered.

    b.   He smiled.

(32) $\exists x.\mathbf{man}(x) \wedge \mathbf{entered}(x) \wedge \mathbf{smiled}(x)$

(32) shows the expected semantics for this discourse (31). This results from a representation of (31a) in an empty context. Because of the existential, (31a) contains, it updates the context with a new discourse referent x. In addition, the formula keeps track of the properties this discourse referent satisfies: $\mathbf{man}(x)$ and $\mathbf{entered}(x)$. In DRT, this representation is called a *Discourse Representation Structure (DRS)*. It consists of an *universe* that contains the discourse referents and a list of *conditions*. It is often represented with boxes, as in (33).

(33)

$$
\boxed{\begin{array}{l} x \\ \hline \\ \mathbf{man}(x) \\ \mathbf{entered}(x) \end{array}}
$$

The contribution of (31b) in (34) looks quite similar. An additional condition, called *link*, states that the new entity should refer to some (yet to be determined) other discourse referent.

(34)

$$
\boxed{\begin{array}{l} y \\ \hline \\ \mathbf{smiled}(y) \\ y =? \end{array}}
$$

The two DRSs then merge into a new one. The way two DRSs merge depends much on the syntactic rule that combines the two expressions they correspond to. In the case of adding a new sentence to a discourse, the operation is quite simple and consists in joining the universes and conditions. The '?' in the link is instantiated with a discourse referent that is *accessible* from the position that the pronoun occupies. We will say more about accessibility later. For the moment, it is enough to state that the discourse referents in the universe of a DRS are all accessible to the conditions the DRS contains. This finally gives us the DRS of (35).

(35)

$$
\boxed{\begin{array}{l} x\,y \\ \hline \\ \mathbf{man}(x) \\ \mathbf{entered}(x) \\ \mathbf{smiled}(y) \\ y = x \end{array}}
$$

Remark 1. Note that the combination of the two DRSs is safe as long as their universes do not intersect. Because the variables are technically not bound, without α-conversion[2], defining the merge operation becomes quite complex. The semantics of DRSs and of the merge operation need to be carefully adapted in order to avoid the so-called *destructive assignment problem*. van Eijck and Kamp [21] provide a detailed discussion of this topic.

Remark 2. Linking a pronoun to its antecedent is allowed only when the latter belongs to the accessible discourse referents of the former. We will make this notion explicit later on, but it is important to note that *it does not resolve the anaphora*. In a sentence like (36), the two discourse referents introduced by the first sentence for *John* and *Mary* are both equally accessible to the two pronouns in the second sentence. A resolution algorithm must choose which of all the accessible discourse referents is the most suitable. Such an algorithm typically relies on morphosyntactic information (gender, case, etc. depending on the language), or on background knowledge, as in (37). Since there is no distinction in French between pronouns referring to human and non-human entities, both *Jean* and *l'âne* are accessible to *il* and *le*.

(36) John met Mary. He smiled at her.

(37) Jean possède un âne.   Il   le   bat.
      John owns   a   donkey. PRO-nom PRO-acc beats.

      John owns a donkey. He/It beats it/him.

Definition 1 (DRSs in   van Eijck and Kamp [21]). Let V be a set of variables, C a set of constants and P a set of predicates. The *terms* T, the *conditions* K, and the *DRSs* D are defined by:

Terms $T ::= V \mid C$
Conditions $K ::= \top \mid P(T, T, \ldots, T) \mid V = T \mid V \neq T \mid \neg D$

$$
\text{DRSs} \quad D ::= \quad
\begin{array}{|l|}
\hline
V\,V\,\ldots\,V \\
\hline
K \\
K \\
\vdots \\
K \\
\hline
\end{array}
$$

To save space, we sometime write a DRS D with universe $\{x_1, \ldots, x_n\}$ and conditions $\{K_1, \ldots, K_m\})$ as $D = (\{x_1, \ldots, x_n\}, \{K_1, \ldots, K_m\})$. For two DRSs $D_1 = (\{x_1, \ldots, x_n\}, \{K_1, \ldots, K_m\})$ and D_2, we also define

$$D_1 \implies D_2 \overset{\Delta}{=} \neg(\{x_1, \ldots, x_n\}, \{K_1, \ldots, K_m \, \neg D_2\})$$

---

[2] The operation that allows bound variable renaming in λ-terms and logical formulas.

The condition $V \neq T$ corresponds to the modeling of sentences like (38a) to get (38b) (from van Eijck and Kamp [21]).

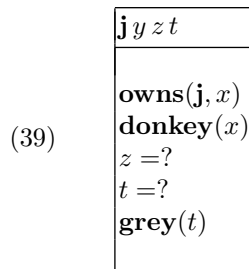(38) a. A man walked in. Another man followed him.

b.

$$
\boxed{\begin{array}{l} x \\ \hline \mathbf{man}(x) \\ \mathbf{walked\_in}(x) \end{array}}
\cdot
\boxed{\begin{array}{l} y\,z \\ \hline \mathbf{man}(y) \\ y\neg = ? \\ z = ? \\ \mathbf{followed}(y,z) \end{array}}
=
\boxed{\begin{array}{l} x\,y\,z \\ \hline \mathbf{man}(x) \\ \mathbf{walked\_in}(x) \\ \mathbf{man}(y) \\ y\neg = x \\ z = x \\ \mathbf{followed}(y,z) \end{array}}
$$
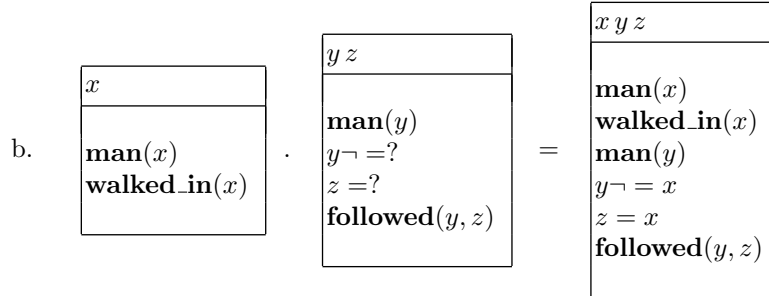
Definition 2 (Subordination and Accessibility). Let K_1 and K_2 be DRSs. K_1 *subordinates* K_2 if:

- $\neg K_2$ is a condition of K_1
- or there exists K_3 such that K_1 subordinates K_3 and K_3 subordinates K_2.
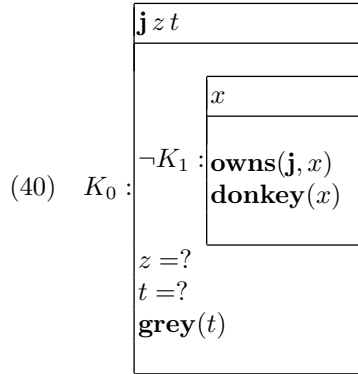
The discourse referents of K_1 are *accessible* from K_2 if:

- $K_1 = K_2$
- or K_1 subordinates K_2

This definition of accessibility explains the contrast between (29b) and (30b). The former builds the DRS of (39), while the latter builds the DRS of (40). In (39), all the discourse referents in the universe are accessible for linking; therefore the pronoun can find an antecedent. But in (40), K_1 does not subordinate K_0 (while K_0 subordinates K_1) ; hence the discourse referents of K_1 cannot be accessed from K_0. The pronoun *it* therefore cannot find an antecedent.[3]

(39)

$$
\boxed{\begin{array}{l} \mathbf{j}\,y\,z\,t \\ \hline \mathbf{owns}(\mathbf{j},x) \\ \mathbf{donkey}(x) \\ z = ? \\ t = ? \\ \mathbf{grey}(t) \end{array}}
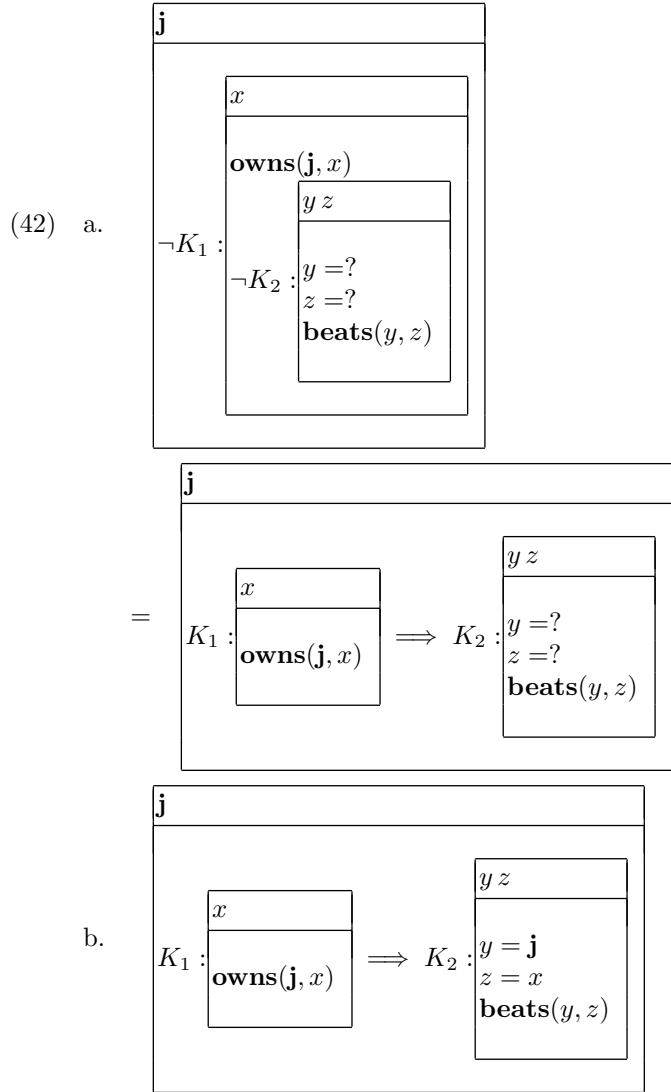$$

---

[3] We do not discuss here the status of discourse referents for proper nouns. They usually are considered as belonging to the universe of the topmost DRS and are therefore always accessible.

(40) K_0 :

$$
\boxed{\begin{array}{l}
\mathbf{j}\, z\, t \\[4pt]
\hline \\[-6pt]
\neg K_1 : \boxed{\begin{array}{l} x \\ \hline \mathbf{owns}(\mathbf{j}, x) \\ \mathbf{donkey}(x) \end{array}} \\[20pt]
z = ? \\
t = ? \\
\mathbf{grey}(t)
\end{array}}
$$

The last example we will deal with in this section is (28), repeated below. For the syntactic structure *if s_1, s_2* we associate the DRS $(p, K_1 \implies K_2)$, where K_i is the DRS associated with s_i, $i \in \{1, 2\}$, and from which p, the set of discourse referents introduced by proper names, has been removed. So the DRS associated with (28) is described in (42a). Because K_2 is subordinated both by K_1 and K_0, both the discourse referents $\mathbf{j}$ in the universe of K_0 and x in the universe of K_1 are accessible to K_2. Thus, the links can be instantiated so as to result in the DRS of (42b). This also shows that any continuation of the discourse will be subordinated neither by K_1 nor by K_2, wherefore none of the discourse referents they introduce will remain accessible (except for the proper names, as already mentioned). This explains why (41) is infelicitous.

(28)   If John owns a donkey, he beats it.

(41)   a.   If John owns a donkey, he beats it.

       b. *It suffers.

(42)　a.

$$\neg K_1 : \boxed{\begin{array}{l} \mathbf{j} \\ \hline \boxed{\begin{array}{l} x \\ \hline \mathbf{owns}(\mathbf{j}, x) \\ \neg K_2 : \boxed{\begin{array}{l} y\ z \\ \hline y =? \\ z =? \\ \mathbf{beats}(y, z) \end{array}} \end{array}} \end{array}}$$

$$= \boxed{\begin{array}{l} \mathbf{j} \\ \hline K_1 : \boxed{\begin{array}{l} x \\ \hline \mathbf{owns}(\mathbf{j}, x) \end{array}} \Longrightarrow K_2 : \boxed{\begin{array}{l} y\ z \\ \hline y =? \\ z =? \\ \mathbf{beats}(y, z) \end{array}} \end{array}}$$

b.

$$\boxed{\begin{array}{l} \mathbf{j} \\ \hline K_1 : \boxed{\begin{array}{l} x \\ \hline \mathbf{owns}(\mathbf{j}, x) \end{array}} \Longrightarrow K_2 : \boxed{\begin{array}{l} y\ z \\ \hline y = \mathbf{j} \\ z = x \\ \mathbf{beats}(y, z) \end{array}} \end{array}}$$

So far we have only described what can be considered as the *formulas* of DRT. We also need to explain how they are interpreted. In particular, DRSs can be provided a truth definition. Such a definition may place more or less emphasis on its *relational* nature. In all cases, it relies on *assignments*.

Definition 3 (Models and assignments). A *first order model* $\mathcal{M} = \langle M, I \rangle$ has a non-empty domain M and an interpretation function that maps n-ary predicate names (the relation symbols used in DRS conditions) to n-ary relations on M[4].

---

[4] 0-ary relations are constants.

An *assignment s* for $\mathcal{M} = \langle M, I \rangle$ is a mapping from a set of variables to elements of M. G is the set of all assignment functions.

Let h and g be assignments and x a variable. Let us say that $h[x]g$ if and only if for all $y \neq x$, $h(y) = g(y)$ (h and g differ at most in the value they assign to x).

An assignment basically describes a *state* when stipulating what actual values should be used in performing a computation. For instance, computing y = x + 1 does not yield the same result for y when the current state is such that x is assigned 1 (usually noted in programming language x := 1) as if x is assigned 2 (x := 2).

Although several interpretation of DRSs have been given (see [21] for instance), they are equivalent in some sense. In particular, as Groenendijk and Stokhof [30] show, DRSs can be translated into Dynamic Predicate Logic (DPL) formulas such that DRS interpretations can be derived directly from the semantics of DPL formulas[5]. We will present DPL and its semantics in the next section.

Much work based on DRT has been proposed to account for various phenomena. In addition to the aforementioned references, a good introduction in French is given by Corblin [17]. For an account of presupposition within this framework, the reader can refer to [63, 29]. For modal subordination, we can mention [61, 25, 24, 29].

3.1.3 Dynamic Predicate Logic

Given the semantics of (28), repeated below, and its possible interpretations:

- strictly following the compositionality principle in (43a)
- actually expected, respecting first order logic syntax and semantics in (43b),

it is argued by Groenendijk and Stokhof [30] that DRSs actually mimic what is observed in (43b), in particular with respect to the different parts from which this formula is built and to the scope of the quantifier. This has been considered a weakness regarding adherence to the compositionality principle[6]. The alternative that is provided is *to propose another semantics* so that the formula (43a) can represent the meaning of (28) and come out with the same truth conditions as expressed in FOL formulas (43b).

(28)   If John owns a donkey, he beats it.

---

[5] For an epistemological view of the evolution of DRS interpretation, see [40].

[6] Subsequent work addressed this criticism and showed that DRT could be expressed compositionally, for instance in Muskens [53], Amsili and Bras [1].

(43)   a. $(\exists x.\mathbf{donkey}(x) \wedge \mathbf{owns}(\mathbf{John}, x)) \implies \mathbf{beats}(\mathbf{John}, x)$

     b. $(\forall x.(\mathbf{donkey}(x) \wedge \mathbf{owns}(\mathbf{John}, x)) \implies \mathbf{beats}(\mathbf{John}, x))$

Groenendijk and Stokhof [30] stress that the approach it provided here is inspired by programming languages:

> In this paper we give an alternative account of the phenomena (. . .) by replacing the standard semantics of the language of first-order predicate logic by a dynamic semantics, which is inspired by systems of dynamic logic as they are used in the denotational semantics of programming languages.

We will elaborate on this comparison at the end of this section. But it is worth noting that the interpretation of a program can be regarded as a relation between an assignment (the input state) and another assignment (the output state) that assign possibly different values to a variable in the input and the output.

Definition

The intuition behind this relational semantics is as follows: starting with an arbitrary assignment g that assigns variables to constants in the model, the meaning of a sentence S specifies the conditions on h, another assignment, such that h can be viewed as one of the possible outputs of $[\![S]\!](g)$. Typically, a sentence that introduces a condition $P(x)$ in DRT will require that $h = g$ and that the interpretation of P holds for the constant that g interprets x as (item 1 of Definition 4). If the sentence also introduces the discourse referent x, it is interpreted as: whatever the input assignment g was, the new assignment may differ from g only on the value it assigns to x (item 7 of Definition 4), in particular because this value must now satisfy certain conditions introduced by the sentence or by the remainder of the discourse.

Definition 4 (DPL syntax and semantics).

The syntax of DPL is standard first order logic syntax with equality. In order to differentiate between "dynamic" logical connectives and "static" ones, we use the following notation:

- $\equiv$ for dynamic equality
- $\overline{\wedge}$ for dynamic conjunction
- $\underline{\vee}$ for dynamic disjunction
- $\Rightarrow$ for dynamic implication
- $\overline{\exists}$ for dynamic existential quantification
- $\overline{\forall}$ for dynamic universal quantification

Let $\mathcal{M} = \langle M, I \rangle$ be a model and g be an assignment. We define $[\![t]\!]_g^{\mathcal{M}} = g(t)$ if t is a variable and $[\![t]\!]_g^{\mathcal{M}} = I(t)$ if t is a constant.

The interpretation function $[\![\cdot]\!]^{\mathcal{M}} \subset G \times G$ (namely $[\![\cdot]\!]^{\mathcal{M}}$ is a relation between assignment functions) is then defined as:

1. $[\![Rt_1 \dots t_n]\!]^{\mathcal{M}} = \{\langle g, h\rangle | h = g \ \wedge \ I(R)\langle [\![t_1]\!]_h, \dots, [\![t_n]\!]^{\mathcal{M}}_h\rangle\}$
2. $[\![t_1 \equiv t_2]\!]^{\mathcal{M}} = \{\langle g, h\rangle | h = g \ \wedge \ [\![t_1]\!]^{\mathcal{M}} = [\![t_2]\!]^{\mathcal{M}}\}$
3. $[\![\bar{\neg}\phi]\!]^{\mathcal{M}} = \{\langle g, h\rangle | h = g \ \wedge \ \neg\exists k.\langle h, k\rangle \in [\![\phi]\!]^{\mathcal{M}}\}$
4. $[\![\phi \bar{\wedge} \psi]\!]^{\mathcal{M}} = \{\langle g, h\rangle | \exists k.\langle g, k\rangle \in [\![\phi]\!]^{\mathcal{M}} \ \wedge \ \langle k, h\rangle \in [\![\psi]\!]^{\mathcal{M}}\}$
5. $[\![\phi \Rightarrow \psi]\!]^{\mathcal{M}} = \{\langle g, h\rangle | h = g \wedge \forall k.\langle h, k\rangle \in [\![\phi]\!]^{\mathcal{M}} \ \Longrightarrow \ \exists j.\langle k, j\rangle \in [\![\psi]\!]^{\mathcal{M}}\}$
6. $[\![\phi \bar{\vee} \psi]\!]^{\mathcal{M}} = \{\langle g, h\rangle | h = g \wedge \exists k.\langle h, k\rangle \in [\![\phi]\!]^{\mathcal{M}} \vee \langle h, k\rangle \in [\![\psi]\!]^{\mathcal{M}}\}$
7. $[\![\bar{\exists} x.\phi]\!]^{\mathcal{M}} = \{\langle g, h\rangle | \exists k.k[x]g \ \wedge \ \langle k, h\rangle \in [\![\phi]\!]^{\mathcal{M}}\}$
8. $[\![\bar{\forall} x.\phi]\!]^{\mathcal{M}} = \{\langle g, h\rangle | h = g \wedge \forall k.k[x]h \ \Longrightarrow \ \exists j.\langle k, j\rangle \in [\![\phi]\!]^{\mathcal{M}}\}$

Examples

We can now check the effect of this semantics on the previous examples. Let us start with example (31), repeated below, together with the first order logic formula representing its meaning in a strictly compositional way, as in (44).

(31)   A man entered. He smiled.

(44) $(\exists x.\mathbf{man}(x) \bar{\wedge} \mathbf{entered}(x)) \bar{\wedge} \mathbf{smiled}(x)$

Table 1 Example model $\mathcal{M}$

	John	Bill	Mary
man	⊤	⊤	
entered	⊤	⊤	⊤
smiled	⊤		⊤

Let us assume a very simple model with three entities and their properties, as described in Table 1. By definition,

$$
\begin{aligned}
[\![\mathbf{man}(x)]\!]^{\mathcal{M}} &= \{\langle g, h\rangle | g = h \ \wedge \ \mathbf{man}(g(x))\} \\
&= \{\langle g, g\rangle | \mathbf{man}(g(x))\} \\
[\![\mathbf{entered}(x)]\!]^{\mathcal{M}} &= \{\langle h, k\rangle | h = k \ \wedge \ \mathbf{entered}(h(x))\} \\
&= \{\langle h, h\rangle | \mathbf{entered}(h(x))\}
\end{aligned}
$$

So,[7]

$[\![\mathbf{man}(x) \bar{\wedge} \mathbf{entered}(x)]\!]$

$\quad = \{\langle g, h\rangle | \exists k.\langle g, k\rangle \in [\![\mathbf{man}(x)]\!] \ \wedge \ \langle k, h\rangle \in [\![\mathbf{entered}(x)]\!]\}$

$\quad = \{\langle g, h\rangle | \exists k.k = g \ \wedge \ \mathbf{man}(g(x)) \ \wedge \ h = k \ \wedge \ \mathbf{entered}(k(x))\}$

$\quad = \{\langle g, h\rangle | h = g \ \wedge \ \mathbf{man}(g(x)) \ \wedge \ \mathbf{entered}(g(x))\}$

$\quad = \{\langle g, g\rangle | \mathbf{man}(g(x)) \ \wedge \ \mathbf{entered}(g(x))\}$

---

[7] From now on, we omit the $\mathcal{M}$ superscript since the model is implicitly known. We thus note $[\![\cdot]\!]$ instead of $[\![\cdot]\!]^{\mathcal{M}}$.

And

$$\llbracket \overline{\overline{\exists}} x.\mathbf{man}(x) \,\overline{\wedge}\, \mathbf{entered}(x) \rrbracket$$
$$= \{\langle g, h\rangle | \exists k.k[x]g \,\wedge\, \langle k, h\rangle \in \llbracket \mathbf{man}(x) \wedge \mathbf{entered}(x)\rrbracket\}$$
$$= \{\langle g, h\rangle | \exists k.k[x]g \,\wedge\, k = h \,\wedge\, \mathbf{man}(k(x)) \,\wedge\, \mathbf{entered}(k(x))\}$$
$$= \{\langle g, h\rangle | h[x]g \,\wedge\, \mathbf{man}(h(x)) \,\wedge\, \mathbf{entered}(h(x))\}$$

So, $\langle g, h\rangle \in \llbracket \overline{\overline{\exists}} x.\mathbf{man}(x) \,\overline{\wedge}\, \mathbf{entered}(x) \rrbracket$ requires:

- g and h can only differ in the value they assign to x;
- $\mathbf{man}(h(x))$ and $\mathbf{entered}(h(x))$ must hold.

There is no other requirement on g. With respect to the toy model in Table 1, all assignments h must then assign x either to **John** or to **Bill**. This is the condition on the possible output state after processing the first sentence in (31).

The second sentence provides for the following interpretation:

$$\llbracket \mathbf{smiled}(x) \rrbracket = \{\langle g, h\rangle | h = g \,\wedge\, \mathbf{smiled}(h(x))\}$$

For $\langle g, h\rangle \in \llbracket \mathbf{smiled}(x) \rrbracket$ in the model described by Table 1, this then requires that $g = h$ and either $g(x) = \mathbf{John}$ or $g(x) = \mathbf{Mary}$.

Remark 3. Since we are considering assignment functions, it make sense to talk about $h(X)$. But the only requirement so far is that **smiled** is true of x. The important thing is that the representation is ready to combine with sentences that put additional conditions on x.

Putting the two sentences together with the conjunction yields:

(45) $\llbracket (\overline{\overline{\exists}} x.\mathbf{man}(x) \,\overline{\wedge}\, \mathbf{entered}(x)) \,\overline{\wedge}\, \mathbf{smiled}(x) \rrbracket$

$$= \{\langle g, h\rangle | \exists k.\langle g, k\rangle \in \llbracket \overline{\exists} x.\mathbf{man}(x) \wedge \mathbf{entered}(x)\rrbracket \,\wedge\, \langle k, h\rangle \in \llbracket \mathbf{smiled}(x)\rrbracket\}$$
$$= \{\langle g, h\rangle | \exists k.k[x]g \wedge \mathbf{man}(k(x)) \wedge \mathbf{entered}(k(x)) \wedge h = k \wedge \mathbf{smiled}(h(x))\}$$
$$= \{\langle g, h\rangle | h[x]g \wedge \mathbf{man}(h(x)) \wedge \mathbf{entered}(h(x)) \wedge \mathbf{smiled}(h(x))\}$$

This means that whatever the input state, the output state can only differ in the value it assigns to x, but the output state must make true of x the conditions **man**, **entered** and **smiled**. Specifically, the assignment h such that $h(x) = \mathbf{Mary}$ and which is a possible input and output state for *He smiles* is ruled out in the conjunction (which is a composition of relations) because it cannot be an output state of *A man entered*.

Comments

The following so-called donkey equivalences [20] hold:

(46) $$(\bar{\exists}x.\phi) \bar{\wedge} \psi \cong (\bar{\exists}x.\phi \wedge \psi)$$

(47) $$(\bar{\exists}x.\phi) \Rrightarrow \psi \cong (\bar{\forall}x.\phi \Rrightarrow \psi)$$

We used (46) above to show that *he smiled* gets the correct interpretation, with the existential "dynamically extending its scope" over the **smiled** predicate.

Similarly, (47) explains why sentence (28), repeated below, correctly gets a universal quantification over the individuals that are donkeys.

(28)   If John owns a donkey, he beats it.

Dynamic logic has been used to account for anaphora [30], presupposition [9, 10], update semantics [66], modal subordination [65, 62, 55, 6], etc.

The scope theorem stated in (46) makes the logic at hard quite different from the usual first order logic. Moreover, it also suffers from the destructive assignment problem. This problem can be viewed as equivalent to that seer in imperative programming languages. Basically, it involves the fact that an assignment x := 2 in a program hides previous assignments (for instance x:= 1). Suggestions using states to remedy this, such as Dekker's Predicate Logic with Anaphora (PLA) [19], have been made.

Groenendijk and Stokhof [30] have already mentioned the parallel between computer programs and the way such programs modify machine states to design DPL. This parallelism has been further explored, as in [52, 54] or van Eijck and Visser [22].

The following sections present another approach to dynamics that was also inspired by computer science. Interestingly, it moves us into the paradigm of functional programming languages and the way control is modeled in this setting by means of so-called *continuation*. This provides a way to escape the drawbacks inherited from imperative programming.

3.1.4 Continuation Semantics

In mathematics, a function accepts parameters and returns a value. In imperative programming, using states allows for the implementation of *side effects*. These are effects or changes of states that are not rendered in the return value of a function. For instance, an assignment such as x:=2 can occur in any function, no matter the actual output, and change the states. It is thus possible to add a statement changing the assignment in any function. Assume, for instance, a function that adds 1 to its input. Translating the standard mathematical definition into a programming language would produce the definition on the left in Fig. 3. But nothing prevents mixing the intended meaning of this function with some other "hidden" change. In the program on the right in Fig. 3, the function f has the side effect of assigning 3 to z.

```
Function  f ( x ) ;
begin
return ( x+1)
end ;
```

```
Function  f ( x ) ;
begin
z :=3;
return ( x+1)
end ;
```

Fig. 3 A function with no side effect   A function with side effects

Functional programming involves function evaluation, just as in mathematics. It is a programming paradigm that avoids states and side effects. It also makes functions first-class citizens, i.e. functions are considered just like any other values and can be parameters as well. A very important notion that comes with this paradigm is that of type systems and type theory. Functional programming as elaboration on λ-calculus and type theory has existed in formal semantics at least since Montague [49, 50].[8] In extensional Montague semantics, we usually consider the set of atomic types to be $\{e, t\}$, respectively denoting entities and truth values. In intensional Montague semantics, we usually consider the set of atomic types to be $\{e, t, s\}$, following Gallin [26], where s denotes possible worlds. In the continuation semantics approach, we use additional atomic types. But let us first illustrate what a continuation is. We assume the type $\mathbb{N}$ of integers. We are considering functions of type $\mathbb{N} \to \mathbb{N}$. $f = \lambda x.x + 1$ is such a function: it takes an integer as parameter and returns an integer.

It is not possible to describe all the computations in which the result of f will be used. However, we can abstract over them because we know they will take an integer (the result of some $f(x)$) as parameter. And, if we consider only computations that in turn produce integers, the type of these abstractions over computations is then $(\mathbb{N} \to \mathbb{N})$. We can thus systematically change f into $\overline{f}$ of type $\mathbb{N} \to (\mathbb{N} \to \mathbb{N}) \to \mathbb{N}$ with an additional parameter of type $(\mathbb{N} \to \mathbb{N})$. This parameter is the *continuation* of the computation in which the result $f(x)$ is involved.

Let us now assume that we have two functions, f and g, of type $\mathbb{N} \to \mathbb{N}$. Composing them with the function composition $g \circ f = \lambda x.g(f(x))$ is a standard operation. Can we relate that to some operation on $\overline{f}$ and $\overline{g}$? First, according to the definition of $\overline{\cdot}$, $\overline{g \circ f} = \lambda x\, k.k(g(f\, x))$. Then, if we consider $g \circ f$ applied to x in some continuation k, we can also say that g and k are in the continuation of f[9]. So $\overline{f}$ is applied to x and to some continuation k'. k' is such that when applied to some value x', the result of $g\, x'$ is given to the continuation k. This means that what is evaluated is $\overline{g}\, x'\, k$.

---

[8] We are talking about the standard notions of simply-typed λ-calculus with β-conversion. For an introduction to these concepts, see Carpenter [15].

[9] The *application* of functions to parameters is left associative. We use the following notations: $f(x) = f\, x$ and $(\ldots((f\, x_1)\, x_2)\ldots x_n) = f\, x_1\, x_2 \ldots x_n$ when f takes n parameters and is of type $\alpha_1 \to \alpha_2 \to \ldots \to \alpha_n \to \alpha$ and every x_i is of type α_i.

We now have:

$$\overline{g} \,\overline{\circ}\, \overline{f} = \lambda x \, k.\overline{f} \, x \, (\lambda x'.\overline{g} \, x' \, k)$$
$$= \lambda x \, k.\overline{f} \, x \, (\lambda x'.(\lambda x'' \, k''.k''(g \, x'')) \, x' \, k)$$
$$\rightarrow_\beta \lambda x \, k.\overline{f} \, x \, (\lambda x'.(k(g \, x')))$$
$$= \lambda x \, k.(\lambda x'' \, k'.k'(f \, x'')) \, x \, (\lambda x'.(k(g \, x')))$$
$$\rightarrow_\beta \lambda x \, k.(\lambda x'.(k(g \, x')))(f \, x)$$
$$\rightarrow_\beta \lambda x \, k.k(g(f \, x))$$
$$= \overline{g \circ f}$$

Continuation semantics for discourse, introduced by de Groote [31], uses a similar approach, except that we have the usual semantic types e and t, while on the other hand the sentences (the f functions) will have as parameters an additional type γ for the *environment*. In a static approach, the type associated with sentences would take an environment and return a truth value (type $\gamma \rightarrow t$). Since we want to have the dynamic counterpart with continuations, they will be interpreted with type $\gamma \rightarrow (\gamma \rightarrow t) \rightarrow t$. de Groote [31] calls the first parameter of type γ of a sentence the *left context*. This corresponds to the context made from the sentences preceding the current sentence. The second parameter, the continuation of type $(\gamma \rightarrow t)$, is called the *right context*, that is the context made from the sentences following the current one: the remaining discourse. Let us have a look at an example with discourse (31), repeated below. Sentences are enriched with their continuation semantics.

(31)   a.   A man entered.

$\lambda e \, k.\exists x.(\mathbf{man}(x)) \wedge (\mathbf{entered}(x)) \wedge (k(x :: e))$

       b.   He smiled.

$\lambda e \, k.(\mathbf{smiled}(\mathtt{sel} \, e)) \wedge (k \, e)$

These semantic recipes make use of two additional operators:

- the :: (update) operator, of type $e \rightarrow \gamma \rightarrow \gamma$ that inserts entities into the context;
- the $\mathtt{sel}$ operator, of type $\gamma \rightarrow e$, which selects and retrieves an entity from a context.

Just as in DRT, the $\mathtt{sel}$ operator is meant to implement an anaphora resolution algorithm. It should thus be fed with additional data such as morphosyntactic information. But we need not go into further details here.

Remark 4. In the semantics of (31a), it should be noted that the variable x over which it is quantified is added to the context which is given to the continuation. Similarly, this continuation *is in the scope* of the existential quantifier.

This is how indefinites extend their scope to the remaining part of the discourse.

We can also provide a way to combine sentences using $\bar{\circ}$, the dynamic version of (9) from Sect. 2.1.2:

(48)
$$[\![S_1.S_2]\!] = [\![S_2]\!]\bar{\circ}[\![S_1]\!]$$
$$= \lambda e\,k.[\![S_1]\!]\,e\,(\lambda e'.[\![S_2]\!]\,e'\,k)$$

So, the semantics of (31) is:

$$[\![(31a).(31b)]\!] = \lambda e\,k.[\![(31a)]\!]\,e\,(\lambda e'.[\![(31b)]\!])$$
$$= \lambda e\,k.[\![(31a)]\!]\,e\,(\lambda e'.(\lambda e\,k.(\mathbf{smiled}(\mathtt{sel}\,e))\wedge(k\,e))\,e'\,k)$$
$$= \lambda e\,k.[\![(31a)]\!]\,e\,(\lambda e'.(\mathbf{smiled}(\mathtt{sel}\,e'))\wedge(k\,e'))$$
$$= \lambda e\,k.(\lambda e\,k.\exists x.(\mathbf{man}(x))\wedge(\mathbf{entered}(x))\wedge(k(x::e)))$$
$$e\,(\lambda e'.\mathbf{smiled}(\mathtt{sel}\,e')\wedge(k\,e'))$$
$$= \lambda e\,k.\exists x.(\mathbf{man}(x))\wedge(\mathbf{entered}(x))$$
$$\wedge(\lambda e'.(\mathbf{smiled}(\mathtt{sel}\,e'))\wedge(k\,e'))(x::e))$$
$$= \lambda e\,k.\exists x.(\mathbf{man}(x))\wedge(\mathbf{entered}(x))$$
$$\wedge((\mathbf{smiled}(\mathtt{sel}\,(x::e)))\wedge(k\,(x::e)))$$

We now see that the $\mathtt{sel}$ operator has to select an entity from the environment $x::e$. So x is indeed available, and the formulas can be given the standard semantics.

This approach combines very well with Montague's semantics principle and type homomorphism. In (extensional) Montague semantics, the interpretation of the syntactic type of sentence S is interpreted by t. All other interpretations for noun phrases (NP) or nouns (N) follow:

$$[\![S]\!] = t$$
$$[\![NP]\!] = (e \to [\![S]\!]) \to [\![S]\!]$$
$$[\![N]\!] = e \to [\![S]\!]$$

These interpretation still hold, except that $[\![S]\!]$ is now $\Omega = \gamma \to (\gamma \to t) \to t$.

$$[\![S]\!] = \Omega$$
$$[\![NP]\!] = (e \to [\![S]\!]) \to [\![S]\!]$$
$$[\![N]\!] = e \to [\![S]\!]$$

Moreover, by means of a definition of dynamic connectives, standard lexical semantics derives a dynamic version:

$$P \mathbin{\overline{\wedge}} Q = \lambda e\,k.P\,e\,(\lambda e'.Q\,e'\,k)$$
$$\overline{\neg}P = \lambda e\,k.(\neg P\,e(\lambda e'.\top)) \wedge (k\,e)$$
$$\overline{\exists}x.P = \lambda e\,k.\exists x.P\,x\,(x::e)\,k$$

The other connectives result from the application of the de Morgan laws. Furthermore, by translating a simple proposition such as $\mathbf{man}(x)$ into a dynamic one $\lambda ek.(\mathbf{man}(x)) \wedge (k\,e)$, we can give the dynamic lexicon that was used to analyze (31):

$$\llbracket man \rrbracket = \overline{\lambda x.\mathbf{man}(x)}$$
$$= \lambda x.\lambda e\,k.(\mathbf{man}(x)) \wedge (k\,e)$$
$$\llbracket a \rrbracket = \lambda P\,Q.\overline{\exists}x.(P\,x) \mathbin{\overline{\wedge}} (Q\,x)$$
$$= \lambda P\,Q.\lambda e\,k.\exists x.(P\,x\,(x::e)\,k) \wedge (Q\,x\,(x::e)\,k)$$
$$\llbracket entered \rrbracket = \overline{\lambda s.s(\lambda x.\mathbf{entered}(x))}$$
$$= \lambda s.\lambda e\,k.s(\lambda x.(\mathbf{entered}(x)) \wedge (k\,e))$$
$$\llbracket smiled \rrbracket = \overline{\lambda s.s(\lambda x.\mathbf{smiled}(x))}$$
$$= \lambda s.\lambda e\,k.s(\lambda x.(\mathbf{smiled}(x)) \wedge (k\,e))$$
$$\llbracket he \rrbracket = \lambda P.\lambda e\,k.P\,(\mathtt{sel}\,e)\,e\,k$$

For further explanations on how to automatically derive a dynamic lexicon from a static one, we refer the reader to [32] and [43].

Remark 5. There are several points to stress:

- $\llbracket he \rrbracket$ is not derived from a static semantics. This simply means that it has no counterpart in a static semantics and is only made available when moving to the dynamic interpretation;
- looking at $\overline{\neg}P$, note that $\neg P$ is fed with the *trivial* continuation. This means that $\neg P$ is completely evaluated within that context. Then, the remainder of the discourse, represented by k, is *not* in the scope of the negation. Moreover, it is fed with the same context as P. This means that whatever discourse referent P introduces, it will not be passed to k. This corresponds to the accessibility constraint as expressed in DRT.

de Groote [31] presents the basics on continuation semantics for discourse and anaphora. Martin and Pollard [48, 47] present an elaboration on this basis and also deal with presupposition, as do [33, 43]. Asher and Pogodalla [7] give an account of modal subordination using continuation semantics. They also provide in [8] a continuation semantics for SDRT. All these accounts stress that the continuation semantics for discourse is quite flexible with respect to what should be put into the context. This may be entities as well as properties, order relations, etc.

3.2 Discourse Structure

In the following, we focus on Segmented Discourse Representation Theory (SDRT), which is an extension of DRT introduced by Asher and Lascarides [5]. This is a dynamic representational theory of discourse that proposes to model the links between the semantic content of a sentence and the general structure of the discourse. Although in this short presentation we have linked SDRT to DRT, SDRT has either DRT, DPL or Continuation Semantics as its model theoretic. Thus the interpretation of rhetorical structures occurs at different levels (depending of the model). Left contexts postulated by SDRT are quite different from those needed to reproduce DRT or DPL in Continuation Semantics.

Rhetorical relations in discourse are needed for discourse semantics. Asher and Lascarides [5] propose two examples to justify this assertion.

(49) a. π_1: John had a great evening last night.

b. π_2: He had a fantastic meal.

c. π_3: He ate salmon.

d. π_4: He devoured lots of cheese.

e. π_5: He won a dancing competition.

f.   * It was a beautiful pink.

From a semantic perspective, discourse (49) does not contain any expressions which block accessibility. Therefore the pronominal anaphora in the last sentence should be resolved in the discourse. DRT over-generates by accepting the last sentence. Only an analysis relying on discourse structure allows us to explain the non-accessibility of the referent, here *salmon*.

The rhetorical relation between the first two sentences is a kind of ***Elaboration***, which means that the second sentence gives details about the first one: ***Elaboration***(π_1, π_2). On the other hand, the relation between π_3 and π_4 is a kind of ***Narration***. π_4 is a temporal progression of π_3. According to Asher [2], ***Elaboration*** induces a subordination, whereas ***Narration*** induces coordination. Fig. 4 shows the corresponding hierarchical structure.

A second argument for rhetorical relations given by Asher and Lascarides [5] is about temporal structure. In (50a), the sentence order reflects the temporal one, whereas in (50b) it does not. But both have the same tense and aspectual classes. Only the rhetorical relations differ: (50a) is a ***Narration*** whereas (50b) is an ***Elaboration***.

(50) a.   John fell. Mary helped him up.

b.   John fell. Mary pushed him.
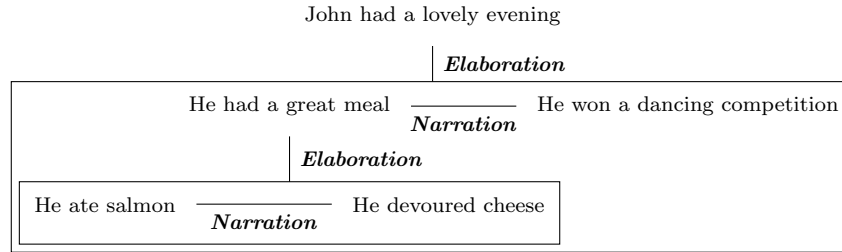
John had a lovely evening

Fig. 4 Rhetorical representation of discourse (49)

One interesting feature of SDRT is the computational perspective of its definitions, which allows one to propose algorithms that produce representations. The task when using SDRT is to define rules (and then semantic targets) to trigger the use of rhetorical relations. An SDRS is a formal representation of a discourse structure, which can be a DRS, a rhetorical relation, or a boolean combination of the two.

This process can be divided into three steps: first, associate a DRS with the assertion; next, determine the open attachment sites (following the right frontier constraint, defined in the following); then, perform the update of the structure with the new information.

Note that we do not define the argument of the rhetorical relation. In [2] the relation is proposed over a proposition, whereas in [5] they are over labels which contain propositions. The difference between the two versions is that, in the second, rhetorical relations occur over coherent subparts of the discourse and are included in a label.

Although we will not explain all the details of the building steps, let us briefly explain the SDRS of (49). The discourse starts with a sentence π_1; then it is elaborated with π_2. The meal needs to be elaborated on with π_3 and π_4, with is a narration relation. Then the process introduces an abstract view of **Narration**(π_3, π_4) and reifies it with π_7. Finally, π_5 rises up in the structure to the π_2 label as a **Narration**. Then the process introduces an abstract view of **Narration**(π_2, π_5) and reifies it with π_6. This is represented using a set of labels: $A = \{\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7\}$. The last sentence that occurs in the input representation is π_5. For each label, we give the representation following the SDRT definition. We introduce a function which associates either a DRS or rhetorical relations (or a logical combination of the two) with the full structure F, which is such that:

$$
\begin{aligned}
F(\pi_1) &= K_{\pi_1} & F(\pi_0) &= \boldsymbol{Elaboration}(\pi_1, \pi_6) \\
F(\pi_2) &= K_{\pi_2} & F(\pi_6) &= \boldsymbol{Narration}(\pi_2, \pi_5) \wedge \boldsymbol{Elaboration}(\pi_2, \pi_7) \\
F(\pi_3) &= K_{\pi_3} & F(\pi_7) &= \boldsymbol{Narration}(\pi_3, \pi_4) \\
F(\pi_4) &= K_{\pi_4} & LAST &= \pi_5 \\
F(\pi_5) &= K_{\pi_5} &&
\end{aligned}
$$

A more readable way to present these relations would be graphically as Fig. 5 shows. We assume that the representation of the discourse is at this step when the last sentence introduces the impossible anaphora.
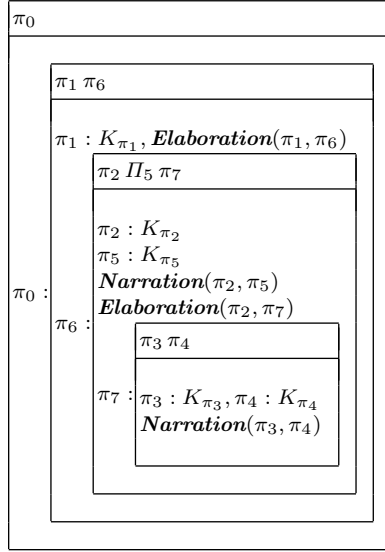

Fig. 5 graphical representation of discourse (49)

If we want to add the next sentence of the discourse, (49f), to the representation, we need to chose where this sentence must be attached. The Right Frontier Constraint (RFC) enables us to restrict the potential options. Intuitively, this constraint assumes that the last sentence is a possible location, as well as any nodes that subordinate it. This follows the right border of the representation. In the example, we could attache (49f) to π_5, π_6, or π_1. The main consequence of this is that the set of accessible discourse referents that the process could use to resolve anaphora is now in this frontier. Thus, *it* cannot refer to the *salmon*. The use of the rhetorical structure limits the over-generation that we discussed previously.

From an SDRS, it is easily possible to derive a logical form based on algorithms developed for DRT. We can then build logical representations of discourse. A major challenge for such frameworks, but also for all those that deal with the semantic-pragmatic interface, lies in defining the process that automatically identifies the rhetorical relations. Even if we find evidence in

syntax and semantics, generally with aspectual informations and adverbs, it is still a problem to define them well. We need to encode knowledge in order to infer rhetorical relations, which contain (at least) compositional and lexical semantics, world knowledge, and cognitive states. The logical design of SDRT leads us to believe that this framework could derive part of such information.

4 Conclusion

We have shown that moving from single sentences to larger texts and discourses leads us to consider specific phenomena. These phenomena share a perspective on sentence behavior within a discourse. In addition to stating facts about the world, sentences need to access and update contexts where enough information is stored in order to correctly interpret the elements of the sentence in particular pronouns. Depending on the phenomenon, the context should minimally consist of:

- a set of propositions or valuations for presupposition;
- a set of discourse referents for declarative discourse;
- two sets of discourse referents and one of propositions for modal subordination;
- a great deal of additional information (discourse unit referents, discourse structure, topic, etc.) for rhetorical structure inference.

We have presented several important frameworks to account for these phenomena, with their specificities. It is worth stressing that these frameworks have been evolving from rather specific tools, such as DRSs, into somewhat more standard (but not completely) logical tools with DPL and PLA, and to even more standard ones with continuation semantics for discourse. There is an interesting parallelism here with the evolution of programming language theory in computer science, our acknowledged inspiration. At the same time, this comes back to Montague's treatment of noun phrases, where type raising is indeed a continuation passing style (CPS) treatment of entities.

The rationality of these frameworks shows through in their ability to model phenomena in natural language. Computational linguistics offers an interesting testbed, and some have been implemented on a rather large scale, for instance by Bos [12, 13], and Marcu [46]. These frameworks also provide ways to analyze natural language usage. Rebuschi et al [59, 60] present SDRT analysis in a pathological context. The claim is that such a specific use of natural language should break down the formal properties expressed by the framework. An interesting point, which is valid at both the cognitive and formal levels, is that schizophrenic interlocutors break at least the right frontier constraint. This suggests that the breaking of right frontier constraint captures a pathological phenomenon. It should thus have cognitive significance.

The identification of a pathological use of formal frameworks also opens new perspectives for such approaches.

The frameworks we have discussed make no special assumptions about the syntactic structures from which meaning is derived. In the terminology of Jackendoff [38], they also present themselves as *generative systems*. This means they have their own rules of well-formedness for building acceptable structures. The fact that not all of them actually correspond to natural language expressions is expressed in the specification of the syntax-semantics interface. The relation this interface defines indeed considers only a subset of all possible semantic forms. An interesting question is how this model can distribute a cognitive model over various elements: syntax, the syntax-semantics interface, and semantics (or pragmatics). Morrill [51] proposes a model of incremental processing and acceptability for type-theoretical syntax. Could we derive a similar model for semantic processing, in particular for generating expressions from semantic representations ?

With respect to representation construction, formalisms provide a large part of systematic process. But they also provide links external to the linguistic process. These links are mainly in the anaphora resolution part, i.e. in the `sel` operator, and the inference of rhetorical relations. For these operations, there is a lot of freedom with respect to the structure of the context and to the processes that operate on it. Their computational complexity may be associated with cognitive capacities or otherwise defined preferences. This is probably reflected in recent work on text summarization and text simplification [45, 16] with a view to deciding, according to the structure of the discourse, which parts are regarded as more or less important than other ones and should be kept. More generally, these computations, possibly inspired by cognitive models, could be the place to go to reduce the gap between the theoretical ambiguity of semantic models and the generally disambiguated readings people make.

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