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Stochastic Analytic Evaluation of End-to-End Performance of Linear Nearest Neighbour Routing in MANETs with Aloha

Bartłomiej Błaszczyszyn[†] and Paul Muhlethaler^{*}

Abstract: Planar Poisson models with the Aloha medium access scheme have already proved to be very useful in studies of mobile ad-hoc networks (MANETs). However, it seems difficult to quantitatively study the performances of end-to-end routing in these models. In order to tackle this problem, in this paper we study a *linear stationary route embedded in an independent planar field of interfering nodes*. We consider this route as an idealization of a “typical” route in a MANET obtained by some routing mechanism. Such a decoupling allows us to obtain many numerically tractable expressions for local and mean end-to-end delays and the speed of packet progression, assuming slotted Aloha MAC and the Signal-to-Interference-and-Noise Ratio (SINR) capture condition, with the usual power-law path loss model and Rayleigh fading. These expressions show how the network performance depends on the tuning of Aloha and routing parameters and on the external noise level. In particular we show a need for a well-tuned lattice structure of fixed relaying nodes, which helps to relay packets on long random routes in the presence of a non-negligible noise. We also consider a *Poisson-line MANET model*, in which *all* nodes are located on roads forming a Poisson-line process. In this case our linear route is rigorously (in the sense of Palm theory) the typical route in this Poisson-line MANET.

Index Terms—MANET, Aloha, SINR, MAC, routing, end-to-end delay, packet speed, transport capacity, Poisson, Poisson-line, lattice.

I. INTRODUCTION

The idea of mobile ad-hoc networks (MANETs) — spontaneous wireless networks, operating without a backbone infrastructure, whose users/nodes forward packets for each other in order to enable multi-hop communications — is continuously inspiring practitioners and poses interesting theoretical questions regarding its performance capabilities. An almost ubiquitous stochastic assumption in the theoretical studies of these problems is that the nodes of the network are distributed (at a given time) as points of a planar (2D) Poisson point process. In conjunction with the Aloha medium access (MAC) scheme — simpler but less efficient than CSMA usually considered in this context by practitioners — 2D Poisson MANET models allow for quite explicit evaluation of many interesting performance metrics, such as the SINR outage probability, the related density of packet progress, mean local delay and many others [1–5]. Most available explicit expressions regard local (one-hop) performance characteristics of MANETs. Poisson models also allow one to discover some intrinsic theoretical limitations of MANETs, regarding e.g. the scaling of the capacity of dense and/or extended networks [6, 7], or the speed of packet progression on long routes [8]. These latter results cannot be

derived from a one-hop analysis of the system because they also involve some assumptions on the routing layer. In fact, introducing routing even to the simplest 2D Poisson-Aloha model is very difficult and, to the best of our knowledge, there are not very many *quantitative* results regarding truly multi-hop performances of MANETs, as e.g. mean end-to-end packet delivery delay. In this paper we propose some stochastic models that allow just such an analysis to be performed.

A simplified 2D Poisson-Aloha MANET model proposed in [1, 3], called the bipolar model, offers a very explicit evaluation of the SINR outage probability in the case of Rayleigh fading. It does not however allow routing issues to be addressed since it assumes that each transmitter has its dedicated receiver outside the actual Poisson MANET. The analysis of the outage probability as well as the mean local delay (the number of time slots required to successfully transmit the packet) at a typical node of the network can be relatively easily extended to the nearest neighbour (inside the MANET) receiver model, as shown in [9]. Practitioners, as well as some theoretical results confirm that it is quite natural to assume that the next-relay node should be chosen from the neighbours of the transmitter. However there is no easy way to go beyond one hop in this analysis and to build an analytically tractable, complete end-to-end routing model, even in the simplest 2D Poisson-Aloha setting with Rayleigh fading. The reason for this is that, while the source-node can be considered as a typical node in the MANET and the powerful Palm theory of point process can be used in the analysis of the first hop, further relay nodes on a given path (traced by the Dijkstra algorithm or any reasonable local routing on 2D Poisson MANET) cannot be seen as typical nodes of the MANET. In fact, the route followed by a packet is a random subset of the MANET point pattern (depending on the routing algorithm) and the typical point “seen” by the packet on a long route is not the typical point of the whole MANET in the sense of Palm theory¹, cf *routing paradox* in [11, § 21.7.2.2].

A. Route-in-MANET model

In order to overcome this impasse and to propose an analytically tractable model, in this paper we propose to *decouple a tagged route from the remaining network, external to the route part of the MANET*; i.e. consider two stochastically independent point patterns models, respectively, some given path on which packets of the same flow are forwarded, and the rest of the MANET, whose nodes are only sources of interference for

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¹unless a high node mobility can be assumed, which results in completely independent re-sampling of the node placement after each hop, which might be reasonable in the context of delay-tolerant networks [10], but which we do not want to assume in this paper

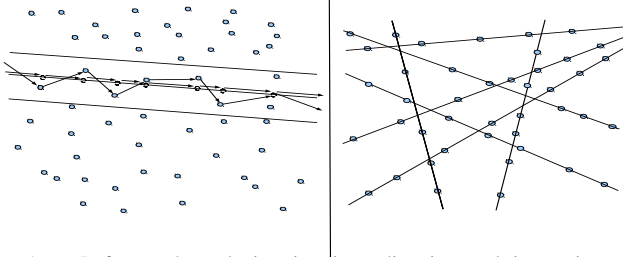


Fig. 1. Left : nodes relaying in given direction and in a given strip approximated by a line of nodes. Right: nodes on roads forming Poisson-line process

the packet transmission on the tagged route. Moreover, we assume that the *tagged route is modeled by a linear (1D) stationary point process*, embedded in a 2D-stationary point process of external, interfering nodes. These assumptions allow us to *distinguish the typical node of the route from the typical point of the MANET*, and appropriately use Palm theory to manipulate the two objects. More precisely, the advantage of the 1D-modeling of the tagged route is that packets forwarded on it through the nearest neighbour transmissions “see”, at any relaying node, the typical (1D Palm) distribution of the whole route. This is a well-known *point-shift invariance property of the Palm distribution in 1D*, which does not have a natural extension in higher dimensions. Another advantage of our decoupling of the two parts of the MANET is that we can go beyond Poisson assumptions in both of them. Specifically, we will also consider a *Poisson route appended with a lattice structure of relaying nodes*, which happens to be crucial to improve the routing performance on long routes in the presence of external interference and/or noise. Regarding external interference, we are able to study the impact of the clustering of nodes, considering *Poisson-cluster point processes*.

Before describing our main results, let us further motivate and justify our route-in-MANET model. We consider the tagged route as an *idealization of a “typical” route traced in a MANET by some routing mechanism*. Its linearity (1D-structure) is clearly a simplification unless an interesting, but specific, scenario of linear Vehicular Ad hoc NETWORKS (VANETs) is considered, where it is quite natural to assume that the nodes are on the line. For the usual 2D MANETs we think of packets as being relayed in a given direction, e.g. in a strip as shown on the left sketch on Figure 1. We can approximate the “real” route by a “virtual” one, on the line of the given direction, by taking the orthogonal projections of the real nodes. Such a situation represents an approximation of a *geographic routing*. Last but not least, in this paper we consider a *Poisson-line MANET model*, in which *all nodes are located on roads forming a Poisson-line (back-ground) process*, cf [12], see the right sketch on Figure 1. In this case our *decoupled, tagged route is rigorously (in the sense of Palm theory) the typical route in this Poisson-line MANET*.

B. Summary of the results

We consider slotted Aloha MAC to be used both by the nodes of the tagged route and of the interfering part of the MANET and the Signal-to-Interference-and-Noise Ratio (SINR) capture/outage condition, with the power-law path loss model and Rayleigh fading. We assume that all the nodes of

the MANET are backlogged, i.e., they always have packets to transmit in their buffers. We are interested in the routing performance of a tagged packet relayed by successive nodes on the route with priority in all queues on the route. The main results of this paper are the following:

- In the absence of external (to the route) noise and interference we evaluate the *mean local delay* on the Poisson route, i.e. the expected number of time slots required for the packet to be transmitted by the typical node of the route to its nearest neighbour in a given direction. The inverse of this delay is intrinsically related to the *speed of the packet progression on asymptotically infinite routes*, which can also be related to the *route transport capacity* (number of bit-meters pumped per unit length of the route).
- The local mean delay is minimized (equivalently, the progression speed is maximized) at a unique value of the Aloha medium access probability p . Moreover, the routing is unstable — the delay is infinite (the speed is null) — for p larger than some critical value. This observation is fully compliant with the phase-transition phenomenon of the mean local delay discovered in the 2D Poisson MANET model in [9].
- We evaluate the *mean end-to-end delay* of the packet routing between two given nodes of the route. It shows a cut-off phenomenon, namely that *routing on distances shorter than some critical value is very inefficient in terms of exploiting the route transport capacity*.
- Next, we study the impact of noise and external interference on our previous observations. Confirming the theoretical finding of [8] in 2D Poisson MANETs, we observe that the routing on a Poisson route over long distances is unfeasible: *the speed of packet progression on such routes is null* due to existence of the large hops in random (Poisson) paths. Evaluating the end-to-end delay on finite distances with noise, we identify another, double cut-off phenomenon: existence of *critical values of end-to-end distance and noise power, beyond which the speed of packet progression is close to 0*, again making the routing inefficient in terms of the exploitation of its transport capacity.
- In order to allow efficient routing over long routes one can complete the Poisson route with a fixed lattice of (equidistant) relaying nodes. For this model we evaluate the mean local delay and show how the route transport-capacity can be maximized by an optimal choice of the inter-relay distance of the lattice structure.

C. Related work

In [13] the authors study the delay and throughput in a wireless multihop network with sources that form a Poisson point process and relays which are placed equidistantly on the source-destination line. The delay includes both the service times and waiting times in the buffers of the typical route. Local delays in Poisson MANETs are studied in [14] and the author finds that (randomized) power control is essential to keep the delay finite (cf. the phase transition discovered in [9]). Moreover, analytical results and bounds on these delays are presented. Different routing strategies in large wireless ad

hoc networks are studied in [15]. In particular, short-hop and long-hop routing schemes are compared. Using techniques from first-passage percolation and epidemic processes, in [16] the authors show that the end-to-end delay scales linearly with the distance in the absence of external noise. This is consistent with the results of [8], where the crucial and destructive impact of noise was discovered. Capacity results for clustered processes are presented in [17].

The remaining part of this paper is organized as follows: In Section II we present our model. In Section III we study the end-to-end delays when the interference is limited to the interfering nodes on the route. In Section IV we study the impact of external noise and interference.

II. LINEAR NEAREST NEIGHBOUR ROUTING MODEL IN A SPATIAL MANET WITH SLOTTED ALOHA

A. Linear route

Denote by $\mathcal{R} = \{X_i, i = \dots, -1, 0, 1, \dots\}$, with $X_i \in \mathbb{R}$, locations of nodes participating in the routing of some *tagged flow of packets*. This route is assumed not to change on the time scale considered in this paper. The notational convention is such that $X_i < X_{i+1}$ and in our *nearest neighbour (NN) routing* model we assume that the packet is routed from each X_i to its nearest-to-the-right neighbour X_{i+1} .

The following two stochastic assumptions regarding \mathcal{R} will be considered in this paper.

1) *Poisson route*: If not specified otherwise, we suppose that $\mathcal{R} = \Phi$ forms a Poisson point process of intensity λ , which means that the 1-hop distances $X_{i+1} - X_i$ are independent (across i) exponential random variables with mean $1/\lambda$.

The default assumption in our numerical examples throughout the paper is $\lambda = 0.01$ (nodes per meter; all length in this paper are tacitly expressed in meters).

2) *Poisson route with fixed relay nodes*: We will also consider a route of the form $\mathcal{R} = \Phi \cup \mathcal{G}$, where the Poisson route Φ is augmented with equidistantly located “fixed” relay nodes $\mathcal{G} = \{n\Delta + U_\Delta, n \in \mathbb{Z}\}$, where $\Delta > 0$ is some fixed parameter and where U_Δ is a uniform random variable on $[0, \Delta)$, independent of Φ , making \mathcal{R} stationary.

B. Route in MANET

We consider \mathcal{R} as an idealization of a “typical” route obtained in a MANET by some routing mechanism. A simple way of extending \mathcal{R} to a 2D MANET model consists in embedding \mathcal{R} in an external field of nodes $\Psi = \{Y_i\}$, on plane \mathbb{R}^2 . When doing so we will always consider that \mathcal{R} and Ψ are independent.

Two particular assumptions regarding Ψ will be considered.

1) *Poisson field of interferers*: In this model, we assume that Ψ is a Poisson point process of some given intensity μ on \mathbb{R}^2 . Poisson linear route $\mathcal{R} = \Phi$ embedded in a Poisson process of interferers will be our *default simplified MANET model*.

2) *Poisson-line interferers*: We will also consider a case where the interferers are located on a *Poisson process of lines* (roads) on \mathbb{R}^2 of rate ν total road-length per unit of surface. Assuming that on each line of this process there is an independent Poisson process of points of intensity λ' nodes per unit of line length (cf Figure 1, right) we obtain a doubly stochastic Poisson point process on \mathbb{R}^2 with intensity $\mu = \lambda'\nu$

nodes per unit of surface; see [12]. In particular, assuming $\lambda = \lambda'$, one can rigorously consider the Poisson linear route $\mathcal{R} = \Phi$ embedded in such a Poisson line process of interferers, as the typical route of the *Poisson-line MANET*.

C. Aloha MAC

We assume that the nodes of \mathcal{R} try to access the channel (transmit) packets according to the Aloha scheme, in perfectly synchronized time slots $n = 1, 2, \dots$. Each node in each time slot independently tosses a coin with some bias for heads, which will be referred to as the *Aloha medium access probability* (Aloha MAP). Nodes whose outcome is heads transmit their packets, the others do not transmit. We denote by p the MAP of nodes in \mathcal{R} , and by p' the MAP of nodes in Ψ . The above situation will be modeled by marking the points $X_i \in \mathcal{R}$ with random, Bernoulli, medium access indicators e_i^n equal to 1 for the nodes which are allowed to emit in the slot n and 0 for the nodes which are not allowed to emit. We have $p = \mathbf{P}(e_i^n = 1) = p$ for all i, n . When there is no ambiguity, we will skip the time index n in the notation. We will sometimes also use the notation $e_{X_i} = e_i$.

At each time slot n , Aloha splits \mathcal{R} into two point processes $\mathcal{R}^1 = \mathcal{R}^{1,n}$ of emitters (having MAC indicator e equal to 1 at time n) and (potential) receivers $\mathcal{R}^0 = \mathcal{R}^{0,n}$. It is known that when $\mathcal{R} = \Phi$ is a Poisson process of intensity λ then \mathcal{R}^1 and \mathcal{R}^0 are independent Poisson processes with intensity λp and $\lambda(1 - p)$, respectively.

D. Signal propagation

Each transmitting node uses the same transmission power S , which for simplicity is assumed to be equal to 1W (all powers, including this of the external noise are tacitly expressed in Watts). The signal-power path-loss is modeled by the power-law function $l(r) = (Ar)^\beta$ where $A, \beta > 0$ are some constants and r is the distance between the transmitter and the receiver. Our default assumption in the numerical examples is $A = 1\text{m}^{-1}$, $\beta = 4$.

Signal-power is also perturbed by random fading F that is independently sampled for each transmitter-receiver pair at each time slot n . Thus, the actual signal-power received at y from x at time n is equal to $SF_{(x,y)}^n/l(|x - y|)$. In this paper we will restrict ourselves to an important special case when F is *exponentially distributed*, which corresponds to the situation of independent *Rayleigh fading*. By eventual renormalization of A we can assume without loss of generality that F has mean 1.

E. Reception

When a node located at x transmits a signal with power S and this signal is being received by a node located at y , then the success of this reception depends on the signal-to-interference-and-noise ratio (SINR)

$$\text{SINR}_{(x,y)} = \text{SINR}_{(x,y)}^n = \frac{SF_{(x,y)}^n/l(|x - y|)}{W^n(y) + I_{\mathcal{R}^{1,n}}(y)}, \quad (2.1)$$

where $I_{\mathcal{R}^{1,n}}$ is the shot-noise process of $\mathcal{R}^{1,n}$: $I_{\mathcal{R}^{1,n}}(y) = \sum_{X_i \in \mathcal{R}^{1,n}} SF_{(y,X_i)}^n/l(|y - X_i|)$ representing the interference created by the nodes of route \mathcal{R} and $W^n(y)$ represents an external (to \mathcal{R}) noise. This external noise can be a constant

ambient noise $W^n(y) = W$ or a random field. In particular, $W^n(y)$ may comprise the interference created by the external field of interferers Ψ , which do not belong to the route \mathcal{R} . In this case:

$$W^n(y) = W + I_{\Psi^{1,n}}(y) = W + \sum_{Y_i \in \Psi^{1,n}} SF_{(y,Y_i)}^n / l(|y - Y_i|), \quad (2.2)$$

where $\Psi^{1,n}$ is the subset of nodes of Ψ transmitting at time n .

The common assumption throughout this paper is that *noise process $W^n(y)$ is independent of the route-interference process $I_{\mathcal{R}^{1,n}}(y)$ and that the process $W^n(y)$ is stationary in n .*

In this paper we assume a fixed bit-rate coding, i.e., y successfully receives the signal from x if

$$\text{SINR}_{(x,y)} \geq T, \quad (2.3)$$

where $\text{SINR}_{(x,y)}$ is given by (2.1) and T is the SINR-threshold related to the bit-rate given some particular coding scheme. Our default assumption in the numerical examples is $T = 10$.

Recall that in our NN routing model each transmitter in \mathcal{R}^1 transmits to its nearest-to-the-right node in \mathcal{R} , without knowing whether it is authorized to transmit at this time. Successful reception requires that this selected node is *not* authorized by Aloha to transmit (i.e., it is in \mathcal{R}^0), and that the SINR condition (2.3) for this transmitter-receiver pair is satisfied. This corresponds to the usual separation of routing and MAC layers.

III. END TO END DELAYS ON A POISSON ROUTE IN THE ROUTE-INTERFERENCE LIMITED CASE

In this section we assume Poisson route $\mathcal{R} = \Phi$ and that the noise process $W^n(y) \equiv 0$ is negligible with respect to the interference created by the nodes participating in the routing. In the following subsection we begin with a simple calculation of the capture probability.

A. Capture probability

For arbitrary $a, b > 0$ denote $C(a, b) = \int_a^\infty 1/(u^b + 1) du$ and

$$C(b) = C(0, b) = \frac{\pi}{b \sin(\pi/b)}. \quad (3.4)$$

Moreover, for given T and β let us denote: $C_1 = C_1(T, \beta) = T^{1/\beta}(C(T^{-1/\beta}, \beta) + C(\beta))$.

Proposition 3.1: *The probability of successful transmission of a typical node of Poisson route $\mathcal{R} = \Phi$ authorised by Aloha to transmit to its relay node in NN routing model without noise is equal to:*

$$P_{NN}(p) = \frac{1-p}{1+pC_1}. \quad (3.5)$$

Proof: Note directly from the form of the SINR in (2.1) that $P_{NN}(p)$ does not depend on A . Hence in the remaining part of the proof we take $A = 1$. Let us assume without loss of generality that the typical transmitter $X_0 = 0$ is located at the origin, which corresponds to Palm distribution \mathbf{P}^0 of the Poisson route $\mathcal{R} = \Phi \cup \{0\}$. By the known property of the Poisson point process, the distance R from X_0 to its nearest neighbour to the right, X_1 has exponential distribution with parameter λ . Moreover given $R = r$, all the other nodes of the Poisson route \mathcal{R} form a Poisson point process of intensity λ on $(-\infty, 0) \cup (r, \infty)$. Conditioning on

$R = r$ the probability of the SINR condition (2.3) is equal to $\mathbf{P}^0\{FS \geq TI_{\Phi^1}l(r) \mid R = r\} = \psi_{I_{\Phi^1}|R=r}(Tl(r))$, where we have assumed $S = 1$ and $\mathcal{L}_{I_{\Phi^1}|R=r}$ denotes the Laplace transform of the shot noise generated at the receiver (located at r) by Poisson point process of intensity $p\lambda$ on $(-\infty, 0) \cup (r, \infty)$. It is known (and it can be derived from the formula for the Laplace functional of the Poisson p.p. (see e.g. [11, (16.4)])) that

$$\mathcal{L}_{I_{\Phi^1}|R=r}(\xi) = \exp\left\{-\lambda p \left(\int_r^\infty + \int_0^\infty\right) \left(1 - \mathbf{E}[e^{-\xi F/l(s)}]\right) ds\right\}.$$

Plugging in the conditional probability of the SINR condition and unconditioning with respect to R we obtain

$$P_{NN}(p) = \lambda(1-p) \int_0^\infty e^{-\lambda r} \exp\left\{-\lambda p \left(\int_r^\infty + \int_0^\infty\right) \left(1 - \mathbf{E}[e^{-l(r)TF/l(s)}]\right) ds\right\} dr,$$

where $(1-p)$ in front of the first integral is due to the fact that successful reception is possible only if the nearest node is not authorized to transmit at the given time slot. Assuming exponential fading F and the power law path-loss $l(\cdot)$ we evaluate $1 - \mathbf{E}[e^{-l(r)TF/l(s)}] = \frac{1}{1+(s/r)^\beta/T}$ and obtain

$$P_{NN}(p) = \lambda(1-p) \int_0^\infty e^{-\lambda r} \left(1 + pT^{1/\beta}(C(T^{-1/\beta}, \beta) + C(\beta))\right) dr, \quad (3.6)$$

which boils down to the right-hand side of (3.5). ■

B. Local delays

$P_{NN}(p)$ expressed in (3.5) is the probability that the typical $X_0 = 0$ node can successfully send a tagged packet to the next relay node in a single transmission. However, a single transmission is not sufficient in Aloha. Hence, after an unsuccessful transmission the transmitter will try to retransmit the packet's respecting Aloha, possibly several times, until the packet reception. Let us denote by L_0 the number of consecutive time slots needed for the tagged node to successfully transmit a given packet to its nearest neighbour in the Poisson route Φ . We call L_0 *local delay* at the tagged node. Since we have assumed that fading (as well as Aloha indicators) are independent across the time slots, given Φ , L_0 is a geometric random variable with mean $1/(p\pi(\Phi))$, where $\pi(\Phi)$ is the probability of successful reception in a given time slot given the location of nodes Φ . (This latter conditional probability regards only uncertainty due to fading conditions and MAC statuses of nodes in the network.)

In what follows we will give the expression for the unconditional expected local delay $\mathbf{E}[L_0(\Phi)]$, i.e.; *average local delay* at a typical route node, where the averaging regards all possible Poisson configurations of other nodes Φ . As we will see, this expectation is finite only if p is sufficiently small (for any given T and β).

Let us denote

$$\begin{aligned} \mathcal{D}_1(p) &= \mathcal{D}_1(p; T, \beta) \\ &= T^{1/\beta} \left(\int_{T^{-1/\beta}}^\infty \frac{1}{u^\beta + 1 - p} du + \int_0^\infty \frac{1}{u^\beta + 1 - p} du \right). \end{aligned}$$

Proposition 3.2: *Under the assumptions of Proposition 3.1,*

the mean local delay in the NN routing model is equal to

$$\mathbf{E}^0[L_0] = \frac{1}{p(1-p)(1-p\mathcal{D}_1(p))}$$

provided

$$p\mathcal{D}_1(p) < 1 \quad (3.7)$$

and $\mathbf{E}[L_0] = \infty$ otherwise.

Proof: We need to evaluate $\pi(\Phi)$, i.e. the probability of successful reception in a given time slot given Φ . As previously, suppose that the receiver (the nearest neighbour) is at distance r from the tagged node. Following the same arguments as in the proof of Proposition 3.1, we find that $\pi(\Phi)$ given the receiver at distance r , which we denote by $\pi(r, \Phi)$ is equal to $(1-p)\mathbf{E}[e^{-l(r)TI(r, \Phi)}|\Phi]$, where $I(r, \Phi)$ is the interference at the receiver. Hence $\pi(\Phi)$ can be expressed as a conditional Laplace transform of the shot-noise node given Φ . An explicit expression for this is known (cf. the proof of [11, Lemma 17.30]) and in our case gives

$$\pi(r, \Phi) = (1-p) \exp \left\{ \sum_{X_i \in \Phi \setminus 0} \log \left(1 - \frac{p}{1/T(|X_i - r|/r)^\beta + 1} \right) \right\}.$$

Taking the expectation of the inverse of the latter expression with respect to the distribution of the homogeneous Poisson process of intensity λp truncated to $(-\infty, 0) \cup (r, \infty)$, again by the known expression for the Laplace transform of a shot-noise, we obtain

$$\begin{aligned} \frac{1}{p} \mathbf{E}^0 \left[\frac{1}{\pi(r, \Phi)} \middle| R = r \right] &= \frac{1}{p(1-p)} \exp \left\{ -\lambda \left(\int_r^\infty + \int_0^\infty \right) \left(1 - h^{-1}(s, r) \right) ds \right\} \\ &= \frac{\exp \{ \lambda p r \mathcal{D}_1(p) \}}{p(1-p)}. \end{aligned}$$

where

$$h(s, r) = 1 - \frac{p}{\frac{1}{T}(s/r)^\beta + 1}. \quad (3.8)$$

Consequently, we have :

$$\mathbf{E}^0[L_0] = \frac{\lambda}{p(1-p)} \int_0^\infty e^{-\lambda r(1-p\mathcal{D}_1(p))} dr. \quad (3.9)$$

It is easy to see that this integral is infinite if $p\mathcal{D}_1(p) \geq 1$ and $1/((1-p)(1-p\mathcal{D}_1(p)))$ otherwise. This completes the proof. ■

Remark 3.3: Similar phase transition (existence of the critical p) for the mean local delay was discovered in 2D MANETs in [9]. Note that $\mathcal{D}_1(p) \geq C_1$ with the equality attained when $p \rightarrow 0$. Hence, for small p we have $\mathcal{D}_1(p) \approx C_1$, and the critical p for which the average expected delay explodes is approximately $1/C_1$. Expression (3.9) can be used to tune p to the value which minimizes the mean local delay. In the next section we will see that this value of p also optimizes the performance of the NN routing on long routes, for almost any realization of the Poisson route.

C. Long-distance speed of packet progression

We call long-distance speed the speed at which a packet progresses over a long distance. Let us denote by D_i $i = 1, \dots$ the local delay of the packet on its hop from X_i to X_{i+1} . Then

$\sum_{i=0}^{k-1} D_i$ is the end-to-end delay on k hops on the route that starts at $X_0 = 0$. We denote by $v = \lim_{k \rightarrow \infty} |X_k| / \sum_{i=0}^k D_i$ the long-distance speed of the packet progression (expressed in the units of distance per slot). Note also that v multiplied by the number of bits carried by one packet corresponds to the route transport capacity defined as the number of bit-meters “pumped” by the route per slot.

Proposition 3.4: Under the assumptions of Proposition 3.1, the mean long-distance speed of the transmission of packets in NN routing model is equal to

$$v = \frac{p(1-p)(1-p\mathcal{D}_1(p))}{\lambda}$$

provided $p\mathcal{D}_1(p) < 1$ and 0 otherwise.

Proof: The mean empirical speed during the first k hops is

$$\frac{X_k}{\sum_{i=0}^{k-1} D_i} = \frac{(\sum_{i=0}^{k-1} (X_{i+1} - X_i))/k}{(\sum_{i=0}^{k-1} D_i)/k}.$$

By the ergodic property when k tends to infinity the numerator tends almost surely to $1/\lambda$ and the denominator tends almost surely to $\mathbf{E}^0[L_0]$. This completes the proof. ■

Remark 3.5: Inspecting (3.6), one can attribute the existence of the critical value of p for the speed of packet progression to hops, traversed by a tagged packet on the infinite route, being statistically too long when $p\mathcal{D}_1(p) \geq 1$.

Example 3.6: Figure 2 shows the mean long-distance speed for $\lambda = 0.01$, $\beta = 4$ and $T = 10$. We observe that there is an optimal value of p which maximizes the long-distance speed of packet progression and that this speed drops to 0 for p larger than some critical value.

D. Density of progress

Some MANET models (in particular the bipolar one) use the mean density of progress d to optimize the performance of the model with respect to p (cf e.g. [3]). Recall that in 2D MANET d is defined as the expected total progress of all the successful transmissions per unit of surface. In our 1D model, by Campbell’s formula d can be expressed as

$$d_{NN} = p(1-p) \mathbf{E}^0 \left[|X_1| \mathbf{1}(\text{SINR}_{(0, X_1)} \geq T) \right].$$

Proposition 3.7: Under the assumptions of Proposition 3.1 the density of progress in the NN routing model is equal to

$$d_{NN} = \frac{p(1-p)}{(1+pC_1)^2} \quad (3.10)$$

and is maximized for p equal to:

$$p_{NN}^* = \frac{C_1 + 1 - \sqrt{C_1^2 - 1}}{2C_1}.$$

Moreover, $0 < p_{NN}^* < 1$.

Proof: Using Campbell’s formula, the density of progress in NN model can be expressed as

$$d_{NN} = \lambda p(1-p) \mathbf{E} \left[\mathbf{RP} \{ \mathbf{1}(FS \geq TI_{\Phi^1}(r)) \mid R = r \} \right]$$

with the notation as in the proof Proposition 3.1. Following the same arguments as in this latter proof we obtain

$$d_{NN} = \lambda^2 p(1-p) \int_0^\infty r e^{-\lambda r(1+pC_1)} dr,$$

which is equal to the right-hand side of (3.10). Taking the derivative of the latter expression in p we find that its sign is equal to that of the polynomial $P(p) = 1 - p(2 + C_1) - 2p^2 C_1^2 + 2p^3 C_1^2$. Note that $P(-\infty) = -\infty$, $P(0) = 1$, $P(1) = -1 - C_1 < 0$ and $P(\infty) = \infty$. Hence in the interval $(0, 1)$, $P(\cdot)$ has a unique root: p_{NN}^* which maximizes d_{NN} . The explicit expression follows from the general formulas for the roots of cubic equations. ■

Remark 3.8: Note that density of progress is a “static” quantity, calculated with respect to one slot. It fails to discover the existence of the critical value of p for the performance of the MANET revealed by the analysis of the local delay and packet progression speed. Also, one may be tempted to approximate this speed by d/λ . Note however in Figure 2, that the optimization of this latter quantity in p cannot be directly related to the maximization of the packet progression speed. For this reason we will not consider the density of progress any further in this paper.

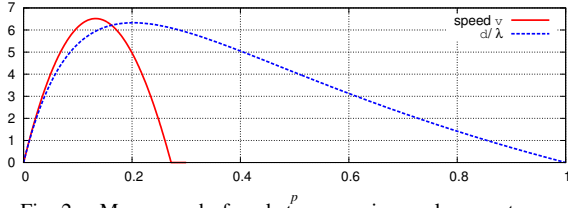


Fig. 2. Mean speed of packet progression on long routes.

E. Mean end-to-end delay and speed of packet progression on finite segments of routes

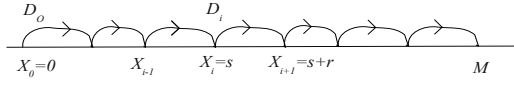


Fig. 3. Transmission delay from node 0 to node M

We come back to the delays and the speed of packet progression. In this section we will study these characteristics on finite segments of routes. In this regard we assume that in addition the node located at the origin $X = 0$ (also called O), a second node, being the final destination of a given packet is located at a fixed position $M > 0$ on the line. Mathematically, this corresponds to the Palm distribution \mathbf{P}^{0M} of the Poisson pp, given these two fixed points. We want to compute the mean end-to-end delay for a given packet to leave the node 0 and to reach node M following NN routing, as depicted on Figure 3. We denote by $L_{0M} = \sum_{X_i \in [0, M)} D_i$ this end-to-end delay.

Proposition 3.9: Under the assumptions of Proposition 3.1 the mean end-to-end delay in the NN routing on the distance M is equal to

$$\mathbf{E}^{0M}[L_{0M}] \quad (3.11)$$

$$= \frac{1}{p(1-p)} \left(e^{-\lambda M} E(M) \right. \quad (a)$$

$$+ \int_0^M \lambda e^{-\lambda r} E(r) G_M(0, r) dr \quad (b)$$

$$+ \lambda \int_0^M \int_0^{M-s} E(r) G_0(s, r) G_M(s, r) \lambda e^{-\lambda r} dr ds \quad (c)$$

$$+ \lambda \int_0^M E(M-s) G_0(s, M-s) e^{-\lambda(M-s)} ds \quad (d)$$

with $E(r) = e^{\lambda p r \mathcal{D}_1(p)}$, $G_0(s, r) = h(s+r, r)^{-1}$ and $G_M(s, r) = h(M-s-r, r)^{-1}$.

Proof: The mean sum of the delays from the source node 0 to the destination node at M under \mathbf{E}^{0M} can be expressed using Campbell’s formula as

$$\mathbf{E}^{0M}[L_{0M}] = \mathbf{E}^{0M}[D(0)] + \lambda \int_0^M \mathbf{E}^{0sM}[D(s)] ds,$$

where the first term corresponds to the average exit time from node 0 and $\mathbf{E}^{0sM}[D(s)]$ is the average (under Palm given three extra points, at 0, s , M) exit time from a “current” node located at $s \in (0, M)$. These two expectations differ from $\mathbf{E}^0[L_0]$, expressed in (3.9), because there is a fixed node at M which acts as an additional interferer but also limits hop lengths. Moreover, for the transmission from the current node $s \in (0, M)$, the node at 0 also acts as an additional interferer. We will show how the terms in (a)–(d) of (3.11) reflect these circumstances. Remark first from (3.9) that the function $E(r)$ is the expected exit time from a typical node, given its receiver is located at the distance r and given no additional (non-Poisson) interfering nodes. Now, it is easy to conclude that the term (a) gives the expected end-to-end delay when it is necessary to make the direct hop from 0 to M (when there is no Poisson relay node between them). The term (b) gives the expected exit time from 0 to its nearest receiver when it is located at some $r \in (0, M)$. Indeed

$$\begin{aligned} \mathbf{E}^{0M}[D(0); r < M] \\ = \frac{1}{p(1-p)} \int_0^M \lambda e^{-\lambda r} e^{\lambda p r \mathcal{D}_1(p)} h(M-r, r)^{-1} dr, \end{aligned}$$

where the factor $h(M-r, r)^{-1} = G_M(0, r)$ (not present in (3.9)) is due to the fact that M acts as an additional interferer for the transmission from 0 to $r < M$. Similarly, the term (d) corresponds to the direct hop from the running node at s to M (when there is no Poisson relay node between them) and term (c) corresponds to the delay of going from s to its nearest receiver $r \in (s, M)$. The node at 0 interferes with both these transmissions, which is reflected by the factors $G_0(s, M-s)$ and $G_0(s, r)$ in the terms (d) and (c), respectively. The node at M also interferes with the transmission from s to $r \in (s, M)$, whence $G_M(s, r)$ in (c). ■

Let us call the ratio $v_{0M} := M/\mathbf{E}^{0M}[L_{0M}]$ (slightly informally) the *average (or mean) speed of the packet progression on distance M* .

Example 3.10: In Figure 4 we present the speed of the packet progression on distance M assuming $T = 10$ and $\beta = 4$, for M varying from 100 to 2000 (meters). The calculation is performed for the value of p that maximizes the asymptotic long-distance speed calculated in Proposition 3.4, presented as the horizontal line on Figure 4. The existence of the destination node in a finite horizon has a double impact on the packet progression. On the one hand it “attracts” the packet reducing the negative impact of long hops (cf Remark 3.5). Indeed, no hop can be longer than the direct hop to M . On the other hand, it “repels” the packet, creating an additional interference, which

is more significant when the packet is close to the destination. Figure 4 shows two phases related to these two “actions”. One can interpret these observations by saying that *routing on distances shorter than some critical values (here $M \approx 500$) is very inefficient in terms of the exploitation of the route transport capacity.*

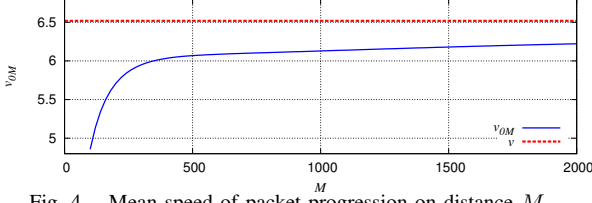


Fig. 4. Mean speed of packet progression on distance M .

IV. IMPACT OF EXTERNAL NOISE AND INTERFERENCE

In the previous section we assumed that the external noise is negligible ($W = 0$). In this section we will study the impact a non-null external noise field $W^n(y)$. Recall that we assume that noise field $\{W^n(y) : y \in \mathbb{R}\}$ is independent of the route process \mathcal{R} and that it is stationary in n ; i.e., $W^n(\cdot)$ is equal in distribution to $W^0(\cdot) = W(\cdot)$. We denote by $\mathcal{L}_{W(r)}$ the Laplace transform of $W(r)$ with $r \in \mathbb{R}_+$

A. Capture probability

It is easy to extend the results of Propositions 3.1 and 3.7 to the case of arbitrarily noise field $W^n(y)$.

Proposition 4.1: *The probability of successful transmission of a typical node of Poisson route $\mathcal{R} = \Phi$ authorised by Aloha to transmit to its relay node in the NN routing model with noise field $W(\cdot)$ is equal to*

$$p_{NN} = \lambda(1-p) \int_0^\infty e^{-r\lambda(1+pC_1)} \mathcal{L}_{W(r)}(TW(r)(Ar)^\beta) dr. \quad (4.12)$$

Proof: The proof is straightforward and follows the same lines as in the proofs of Proposition 3.1. ■

B. Long routes

The following results show a very negative impact of arbitrarily small noise on the performance of NN routing on long routes.

Proposition 4.2: *The mean local delay in Poisson $\mathcal{R} = \Phi$ NN routing model with noise field satisfying $\mathbf{P}(W(r) > w) \geq \epsilon$ for some $w, \epsilon > 0$, is infinite $\mathbf{E}[L_0] = \infty$. Consequently the speed of packet progression on long routes is almost surely null.*

Proof: As in the proof of proposition 3.2 we can show that

$$\begin{aligned} \mathbf{E}[L_0] &= \frac{\lambda}{p(1-p)} \int_0^\infty e^{-\lambda r} e^{\lambda p r \mathcal{D}_1(p)} \mathcal{L}_{W(r)}(-TW(Ar)^\beta) dr \\ &\geq \frac{\epsilon \lambda}{p(1-p)} \int_0^\infty e^{-\lambda r(1-p \mathcal{D}_1(p)) + Tw(Ar)^\beta} dr = \infty. \end{aligned}$$

Remark 4.3: The reason why the long distance speed is null lies in the fact that the hops in a Poisson route can be arbitrary long and that such hops slow down (to zero) the packet progression in the presence of noise, cf Remark 3.5, this time for all $p > 0$. A method to cope with overly long hops consists in adding “fixed” points to the Poisson route \mathcal{R} . In fact, we

will show in Section IV-C that adding only the fixed destination node M is sufficient to make the end-to-end delay finite in the presence of noise. For the infinite routes one needs however a “fixed” regular grid of relaying nodes; cf Section IV-D.

C. Short routes

Our setting is now as in Section III-B, i.e., we consider Poisson route Φ under Palm with fixed origin and destination nodes 0 and M , and we compute end-to-end delay assuming some external noise field of the form (2.2) created by a stationary pattern of interfering nodes Ψ . We assume that this field is stationary and stochastically independent from the Poisson route Φ . Moreover, we assume that the nodes of Ψ use Aloha MAC with probability p' .

Denote $B(r) = e^{TW(Ar)^\beta}$ and $E'(r) = \mathcal{L}_\Psi(\log h_p(\cdot, r))$ that is the Laplace transform of the point process Ψ taken for the function $h_{p'}(\cdot, r)$ given by (3.8) with p replaced by p' .

Proposition 4.4: *Under the assumptions of Proposition 3.1 and in the presence of the external noise of the form (2.2), as explained above, the mean end-to-end delay from 0 to M in Poisson NN routing is given by (3.11) with $E(\cdot)$ replaced by $E(\cdot) \times E'(\cdot) \times B(\cdot)$.*

Proof: Observe first that the conditional capture probability given the receiver in Φ at distance r , and given the interfering pattern Ψ , which we denote by $\pi(r, \Phi, \Psi)$, is equal to

$$\begin{aligned} \pi(r, \Phi, \Psi) &= (1-p) \exp\left\{ \sum_{X_i \in \Phi} \log\left(1 - \frac{p}{1/T(|X_i|/r)^\beta + 1}\right) \right\} \\ &\quad \times \exp\left\{ \sum_{Y_i \in \Psi} \log\left(1 - \frac{p'}{1/T(|Y_i|/r)^\beta + 1}\right) \right\} \\ &\quad \times \exp(TW(Ar)^\beta); \end{aligned}$$

cf the proof of Proposition 3.2. Hence, by independence, the mean local delay given the distance to the receiver $R = r$ is equal to

$$\frac{1}{p} \mathbf{E}^0 \left[\frac{1}{\pi(r, \Phi, \Psi)} \middle| R = r \right] = \frac{E(r)E'(r)B(r)}{p(1-p)}.$$

The remaining part of the proof follows the same lines as the proof of Proposition 3.9. ■

The function E' (corresponding to the Laplace transform of Ψ) can be explicitly given in the case of Poisson and Poisson-cluster point processes. The latter allows us to study the impact of the clustering of interference in a MANET. In particular we have the following expressions.

Fact 4.5: *For Poisson field of interfering nodes of Ψ with intensity μ on the plane we have*

$$E'(r) = E'_P(r) = \exp\left(\frac{2\pi^2 \mu p' T^{2/\beta}}{\beta(1-p')^{1-2/\beta} \sin(\frac{2\pi}{\beta})} r^2\right).$$

For Poisson-line interferers (cf Section II-B2) we have

$$\begin{aligned} E'(r) = E'_{PL}(r) &= \exp\left(2\nu r \int_0^\infty \left(1 - \exp(-2\lambda' r \right. \right. \\ &\quad \left. \left. \times \int_0^\infty \frac{p'}{\frac{1}{T}(s^2 + t^2)^{\beta/2} + 1 - p'} dt) ds\right) \right) \end{aligned}$$

Proof: Both expressions follow from the evaluation of the Laplace transforms of the respective point processes. For a Poisson process this gives

$$E'_P(r) = \exp\left(2\mu\pi r^2 \int_0^\infty \frac{p'}{\frac{1}{T}s^\beta + 1 - p'} ds\right)$$

and can be evaluated to the required expression (which also appeared in [11, Equation 17.43]). The following formula for the Laplace transform of the Poisson-line field of interferers

$$\mathcal{L}_\Psi(f) = \exp\left(-2\nu \int_0^\infty 1 - e^{-2\lambda' \int_0^\infty 1 - \exp(-f(\sqrt{s^2+t^2})) dt} ds\right),$$

is given in [12], from which our expression for $E'_{PL}(r)$ follows. ■

Remark 4.6: Note that

$$E'_{PL}(r) \geq \exp\left(8\pi\lambda'\nu r^2 \int_0^\infty \frac{p'}{\frac{1}{T}s^\beta + 1 - p'} ds\right).$$

Hence, for the same intensities of points $\mu = \lambda'\nu$, $E'_{PL}(r) \geq E'_P(r)$. This implies that the mean end-to-end delay is larger in the Poisson-line field of interferers than in the homogeneous Poisson field. As the former process shows more clustering, we can conclude that *clustering in the interfering field increases end-to-end delays*.

Example 4.7: We present now some numerical illustrations how the average speed of the packet progression on the distance M , $v_{0M} = M/E^{0M}[L_{0M}]$, depends on the model parameters. We assume first only constant noise W . Figure 6 shows the speed of packet progression with respect to W . We have assumed $T = 10$, $\beta = 4$ and $p = 0.15$, the value which maximizes the average long-distance speed without noise. The curves correspond to different routing distances $M = 100, 1000, 10000$ (meters). Figure 5 shows the same speed of packet progression with respect to M for $W = 10^{-11}, 10^{-13}, 10^{-15}$ (Watts). For $M = 100, 1000, 10000$ we keep $p = 0.15$ the value which maximizes the average long-distance speed without noise. Both Figures 5 and 6 show a sharp cut of phenomenon: the speed of packet progression drops rapidly to zero beyond some critical noise level and/or route length. The impact of the noise is stable below its critical value (for a given distance). Comparing Figure 4 to Figure 5 we see that after the “fast acceleration” (cf Example 4.7) the packet reaches a constant speed which can be maintained on routes shorter than some critical length depending on the level of noise. Figure 7 shows the mean end-to-end speed with respect to the spatial density of the field of Poisson interferers λ for two values of the Aloha parameter $p' = 0.15$ and $p' = 0.015$ used by the interferers. As in the case of constant noise, the speed rapidly falls from an almost constant value (corresponding to the case without interferers) to zero.

D. Equipping long Poisson routes with fixed relay stations

As we already mentioned, in order to solve the problem of long hops slowing down the packet progression in the presence of external noise, one can add fixed relay stations thus allowing efficient routing on long Poisson routes. In this section we will study this problem.

In this regard we assume regularly spaced “fixed” relay nodes $\mathcal{G} = \{n\Delta + U_\Delta, n \in \mathbb{Z}\}$, where $\Delta > 0$ is some fixed distance.

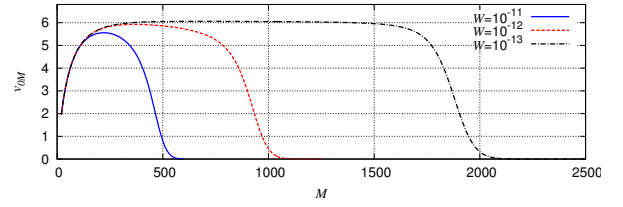


Fig. 5. Mean speed of packet progression on distance M with noise $W = 10^{-11}, 10^{-12}, 10^{-13}$.

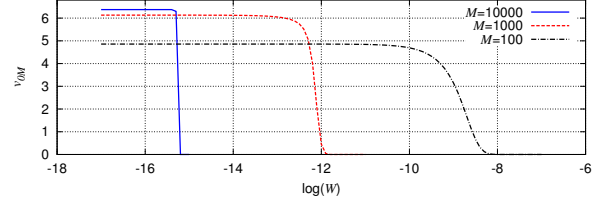


Fig. 6. Mean speed of packet progression on distance $M = 100, 1000, 10000$ as function of the noise.

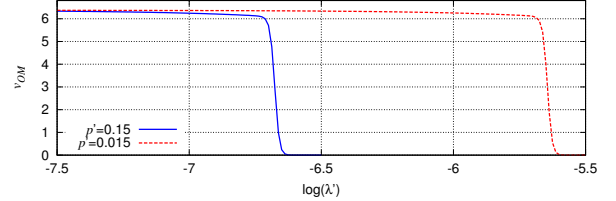


Fig. 7. Mean speed of packet progression on distance $M = 10000$ in the spatial Poisson field of interfering nodes of intensity λ' and two MAC parameters.

Recall, that these nodes are added to the Poisson route Φ , they use the same Aloha parameter p and the NN routing is considered on $\Phi \cup \mathcal{G}$.

We wish to compute the mean local delay on $\Phi \cup \mathcal{G}$ in the presence of noise, and thus characterise the speed of packet progression on long routes equipped with fixed relay stations. This will allow us to optimise the performance of the network in the parameter Δ . Note that a small value of Δ makes the fixed relay nodes too dense. This would create a lot of interference and make the routing hops very small. On the other hand a large value of Δ leads to long hops which slow down the packet progression.

Proposition 4.8: Assume $\mathcal{R} = \Phi \cup \mathcal{G}$ to be a Poisson route equipped with fixed relay stations with the inter-relay distance Δ , as described above. The mean local delay in the NN routing on this model in the presence of the external noise of the form (2.2) as in the Proposition 4.4 is equal to

$$\begin{aligned} E^0[L_0] &= \frac{\lambda}{p(1-p)(\epsilon + \lambda)} \left(\frac{1}{\Delta} \int_0^\Delta E(z)E'(z)e^{-\lambda z}e^{H(0,z)}dz \right. \\ &\quad + \frac{\lambda}{\Delta} \int_0^\Delta \int_0^z E(r)E'(z)e^{-\lambda r}e^{H(z-r,r)+\log(h(z-r,r))}drdz \Big) \\ &\quad + \frac{\epsilon}{p(1-p)(\epsilon + \lambda)} \left(\frac{\lambda}{\Delta} \int_0^\Delta E(z)E'(z)e^{-\lambda z}e^{H(-z,z)}dz \right. \\ &\quad \left. + e^{-\lambda\Delta}B(\Delta)E(\Delta)e^{H(0,\Delta)-\log(h(\Delta,\Delta))} \right) \end{aligned} \quad (4.13)$$

with $\epsilon = \frac{1}{\Delta}$ and $H(z, r) = \sum_{n \in \mathbb{Z}, n \neq 0} \log(h(n\Delta + z, r))$.

Proof: Let us suppose that a typical node 0 of $\Psi \cup \mathcal{G}$ is at $X = 0$. We know that this typical node belongs to Ψ with

probability $\frac{\lambda}{\epsilon+\lambda}$ and to \mathcal{G} with probability $\frac{\epsilon}{\epsilon+\lambda}$. Thus we have

$$\mathbf{E}^0[L] = \frac{\lambda}{\epsilon+\lambda} \mathbf{E}^0[L|0 \in \Psi] + \frac{\epsilon}{\epsilon+\lambda} \mathbf{E}^0[L|0 \in \mathcal{G}].$$

Let us now assume that the typical node 0 is in Ψ and we use the fact that the nodes in Ψ are independent of the nodes in \mathcal{G} . If its closest node (towards the right) is in \mathcal{G} , the average delay to leave node 0 is: $B(z)E(z)e^{H(0,z)}$. Conditioning by the fact there is no node of Ψ in $[0, z]$ (that occurs with probability $e^{-\lambda z}$), we find the first integral in the first line of (4.13). We still assume that the typical node is in Ψ but now we also assume that the closest node (towards the right) is in Ψ . Computing the average delay in this case, we find the second double integral of the first line of (4.13); $\lambda e^{-\lambda r}$ is actually the density of the right-hand neighbour's position. The contribution $e^{\log(h(z-r,r))}$ corresponds to the fact that the node in \mathcal{G} located at z is not taken into account by $e^{H(z-r,r)}$ since the summation in H is for $n \neq 0$.

The second part of the contribution for $\mathbf{E}^0[L]$ corresponds to the case where node 0 is in \mathcal{G} . If we assume that the right-hand neighbour of 0 is in Ψ , the contribution of the delay corresponds to the integral of the proposition in the second line of $\mathbf{E}^0[L]$. There is no correction needed for $H(-z, z)$ since the typical node 0 is in \mathcal{G} and does not contribute to the delay as it is in $H(-z, z)$. The last term of the second line in (4.13) corresponds to the case where node 0 and its right-hand neighbour are in \mathcal{G} . This occurs with probability $e^{-\lambda \Delta}$. The node in \mathcal{G} located at Δ does not contribute to the delay since we analyze the delay from 0 and its right-hand neighbour in \mathcal{G} (thus located at Δ). Thus $H(0, \Delta)$ must be corrected by $e^{-\log(h(\Delta, \Delta))}$. ■

Example 4.9: In Figure 8, we plot the mean long-distance speed $v = 1/(\lambda \mathbf{E}^0[L_0])$ with respect to Δ with fixed noise of levels $W = 10^{-11}$ and $W = 10^{-12}$. Comparing Figure 5 to Figure 8 one can observe that the optimal inter-relay distance Δ of the fixed structure corresponds to the smallest length M of the route on which the mean speed of packet progression attains its maximum without the fixed infrastructure. In other words, the relay stations should “break down” long routes into segments of the length optimal for the end-to-end routing given the noise level.

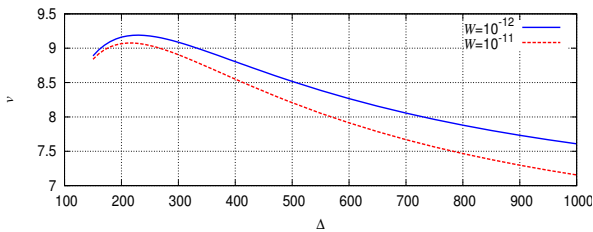


Fig. 8. Mean long-distance speed on the Poisson route equipped with fixed relaying nodes spaced by distance Δ , in the presence of noise $W = 10^{-11}$ and $W = 10^{-12}$.

V. CONCLUDING REMARKS

We have studied performances of end-to-end routing in MANETs, using a linear nearest-neighbour routing model embedded in an independent planar field of interfering nodes using Aloha MAC. We have developed numerically tractable expressions for several performance metrics as the end-to-end delay and speed of packet progression. They show how the

network performance can be optimized by tuning Aloha and routing parameters. In particular we show a need for a well-tuned lattice structure of wireless relaying nodes (not backbone) which help to relay packets on long random routes in the presence of a non-negligible noise. Our analysis does not yet take into account packet queuing at the relay nodes, which we believe can be introduced to our model in future work.

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