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PAPERTONNETZ: MUSIC COMPOSITION WITH INTERACTIVE PAPER

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ABSTRACT

Tonnetze are space-based musical representations that lay out individual pitches in a regular structure. They are primarily used for analysis with visualization tools or on paper and for performance with button-based tablet or tangible interfaces. This paper first investigates how properties of Tonnetze can be applied in the composition process, including the two-dimensional organization of pitches, based on a chord or on a scale. We then describe *PaperTonnetz*, a tool that lets musicians explore and compose music with Tonnetz representations by making gestures on interactive paper. Unlike screen-based interactive Tonnetz systems that treat the notes as playable buttons, *PaperTonnetz* allows composers to interact with gestures, creating replayable patterns that represent pitch sequences and/or chords. We describe the results of an initial test of the system in a public setting, and how we revised *PaperTonnetz* to better support three activities: discovering, improvising and assembling musical sequences in a Tonnetz. We conclude with a discussion of directions for future research with respect to creating novel paper-based interactive music representations to support musical composition.

1. INTRODUCTION

A Tonnetz, or "tone-network" in German, is a two-dimensional representation of the relationships among pitches. Individual pitches are laid out along multiple axes: each intersection is displayed as a cell with a specific pitch. Created by Euler in 1739, Tonnetze were first used to represent *just intonation*, with equal distances between the pitches. In the mid 1800s, Riemann explored the use of these spaces to chart harmonic motion between chords and modulation among keys.

More recently, Tonnetze have been used to analyze classes of pitches, representing neo-Riemannian transformations [1] in which every cell is associated with one of 12 pitches, independently of octave. The vertical axis organizes pitches in cycles of fifths; the two diagonal axes represent minor third and major third cycles. Figure 1 illustrates a region of a neo-Riemannian Tonnetz that extends infinitely in all directions and helps visualize harmonic chord progressions.

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Creating spatial relationships that follow precise mathematical properties, such as the interval cycles that underlie harmonic progressions, offers opportunities for both analysis and composition. Systems such as *Harmony Space* [2] and *Isochords* [3] allow composers to analyze chords, harmony and MIDI sequences using different Tonnetz representations.

We are interested in another role for Tonnetze, *i.e.* to support music composition. We were inspired by Jean-Marc Chouvel's composition process [4], in which he designs his own Tonnetz, prints them on paper, and then generates musical sequences. He draws paths through sets of Tonnetz cells, which provides the inherent order necessary to define musical sequences. After exploring his ideas, he then transforms them into a standard format on a musical score (see Figure 2). Although this works well, we wanted to take his approach one step further to provide a direct link between paths drawn on a paper Tonnetz and on-line composition tools.

This paper presents *PaperTonnetz* which lets composers create their own Tonnetz representations, print them on interactive paper, and 'play' them with a pen. We use Anoto technology [1] to detect the position of a digital pen as it draws ink on paper and produces the corresponding pitch. *PaperTonnetz* can be used as a simple performance tool, similar to the tablet-based systems described above, but with an emphasis on drawing paths rather than pressing buttons. However, *PaperTonnetz* can also be used as a sophisticated composition tool, enabling the composer to capture, replay, transform and compare sequences and chords. Our goal is to combine the flexibility of paper and the power of the computer to help composers explore new creative possibilities.

Section 2 describes related work on the use of Tonnetze to support performance as well as related research on interactive paper to support composition. Next, we discuss how Tonnetze can be designed to form a composition space. We then introduce *PaperTonnetz* and describe our initial study of users, both novices and experienced musicians, as they tried it at a public event. We explain how the results influenced the design of *PaperTonnetz* to explicitly support discovery, improvisation and assembly of complex musical sequences. We conclude with a discussion of the benefits of using interactive paper and Tonnetze to support music composition as well as directions for future research.

[1] http://www.anoto.com

2. RELATED WORK

Tonnetze may be implemented on a computer, enabling musicians to use them for performances. For example, *C-Thru* [2], creates physical Tonnetz keyboard in which each cell is a physical hexagonal button. The user can play individual notes by pressing one button at a time, or play several simultaneously to create a chord.

Tablet-based systems such as *Musix* and *IsoKey* offer users a customizable isomorphic layout on the screen. Like *C-Thru*'s products, users can press the hexagonal cells to produce individual pitches, melodies and chord patterns. They also allow users to switch among different Tonnetz layouts during a performance. Maupin *et al.* [5] examined the harmonic and melodic characteristics of different square and hexagonal pitch layouts and identified which layouts are better suited to different types of improvisation, within particular harmonic contexts.

Although these systems provide interesting performance opportunities, they were not designed to support composition. Computer-based tools that are specifically intended for composition are usually dedicated to the later stages of the creative process [6], such as assigning sequences in time, editing scores or experimenting with advanced forms of sound synthesis. Most of these tools involve learning a specific language, such as MAX/MSP [3] or OpenMusic [7], although a few provide more direct methods, such as Buxton *et al.'s* [8] techniques for interpreting hand-drawn musical notations.

Yet even composers with computer skills and access to advanced composition systems prefer to sketch their early creative ideas on paper [9, 10]. Paper helps creative professionals externalize their ideas [11] and composers often create their own personal languages to express musical ideas [12], which are difficult to translate into computer terms.

One method for bridging the gap between physical paper and composition software is interactive paper. The most common approach uses Anoto technology [13], which uses a digital pen to capture the precise location as the pen moves on the paper. The pen contains an integrated video camera in the tip. The paper is printed with tiny, almost invisible dots that create a unique, identifiable pattern. When the user draws on this paper with the pen, the camera detects the precise location of the pen on the page, as well as the time it was written, the pen that was used and even which page. This information can be processed by the pen, for simple task such as playing a sound, or simultaneously transmitted to a more sophisticated application on a computer.

In previous work, we explored several techniques for incorporating interactive paper into the composition process. For example, *Musink* allows composers to define their own vocabulary of annotations on musical scores, which can then be interpreted as functions in computer-aided music composition software such as OpenMusic. Similarly, *Ink-Splorer* [6] lets composers experiment with different curves drawn on paper to control computer-based algorithms and then use them as a resource for new compositions.

[2] http://www.c-thru-music.com
[3] http://www.cycling74.com

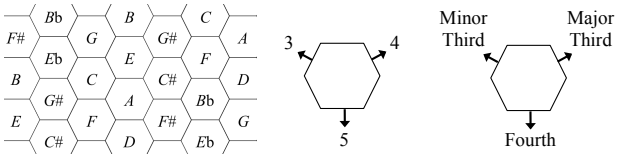

Figure 1. The neo-Riemannian Tonnetz frequently used to visualize chord progressions because of its harmonic properties.

We are interested in allowing composers to use Tonnetz as an aid to the composition process. This requires creating a Tonnetz-based composition space, discussed in section 3, and an interactive paper-based system for letting composers capture and modify musical expressions, described in section 4.

3. TONNETZ AS A COMPOSITION SPACE

3.1 Composition example

Chouvel's work, entitled *Traversée* [4], offers a compelling example of how a Tonnetz can be used as a tool for exploring composition ideas. After creating and printing a Tonnetz on paper, Chouvel drew differently colored paths through the hexagonal cells, each representing a set of pitch sequences. He called the resulting geometrical shapes 'constellations' because they look like the line drawings that connect individual stars in a map of the night sky. Once drawn, he was able to perform geometrical transformations on the paths and examine the results, represented as notes on a standard musical score. Figure 2 illustrates this process: note that the paths that link different pitches may be treated as a sequence, to create a melody, or in parallel, to create a chord.

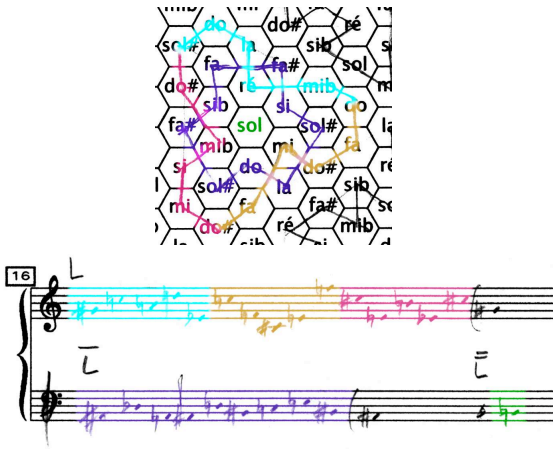

Figure 2. Chouvel's work on paper for his piece *Traversée*. The composer translates drawn shapes in a score and operates geometrical transformations to create variations. Picture from Jean-Marc Chouvel.

Chouvel's approach makes it possible to explore a wide variety of geometrical transformations, including translations, rotations and homothetic transformations. His Ton-

netz can thus support complex musical operations that are rarely formalized in traditional music theory. We build upon Chouvel's approach to support three aspects of composition:

- expressing musical ideas by drawing paths through a Tonnetz space
- performing geometrical transformations on those ideas
- translating those ideas into playable music.

3.2 Two dimensional pitch layouts

3.2.1 Tonnetz layouts

We focus on pitch-class Tonnetze with three interval classes laid out in a hexagonal pattern. Pitch-class Tonnetze have reached to many algebraic considerations. Works presented in this paper are inspired and belong to recent researches on the use of spatial computing to investigate these group-based representations [14]. The three axes (vertical, diagonal left and diagonal right) of hexagonal Tonnetze correspond to the three elementary moves illustrated on Figure 3. Here, every sequence of three steps returns to the starting point. Thus the sum of the interval classes i1, i2 and i3 is always equal to 0 (mod. 12) with respect the hexagonal layout.

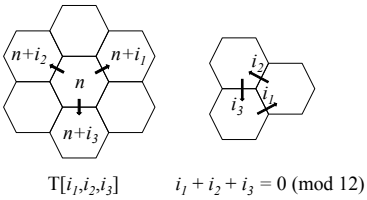

Figure 3. Tonnetz notation and interval relationships.

Figure 3 shows how pitch classes can be represented with a hexagonal cell with six neighbors, each of which can be accessed by adding to the pitch class the three interval classes and their respective inverses. For example, in the Tonnetz Figure 1, one can jump from a fifth by moving up the vertical axis. Moving in the opposite direction adds a fourth to the current pitch class. This offers the composer a one-step method for transposing any melodic sequence or chord.

Catanzaros [15] lists the 12 possible combinations of interval classes that respect this constraint and can be laid out in a hexagonal pattern. Maupin *et al.* [5] discuss the advantages and disadvantages of different Tonnetz layouts for melody and harmony, within the context of a performance. These aspects can be discussed from a compositional point of view. The composer can choose a Tonnetz with neighborhood intervals that are similar to the desired chord intervals to represent it as a compact shape. In particular, every three-note chord can be represented with a triangular shape, in one of the 12 hexagonal Tonnetz. For example, T[3,4,5] is neo- Riemannian Tonnetz in which any three adjacent hexagons form a major or a minor chord.

3.2.2 Heptatonic layouts

We have also been exploring more complex problems, such as how to use a hexagonal layout to represent a seven-pitch (heptatonic) scale. Figure 4 shows how seven pitch classes can be laid out in a chromatic circle such that any move between two pitch classes can be designated by one, two or three steps, up or down from the initial pitch class.

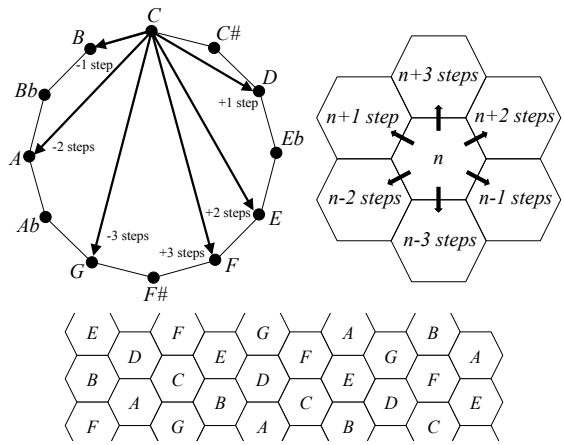

Figure 4. *C* Major heptatonic hexagonal space. Every pitch of the space belongs to this scale. Thus any path inside produces a melody in C Major.

These six translations can be associated with the six possible moves in the hexagonal space. Although the resulting space noticed in [16] is not a perfect Tonnetz since the axes do not represent constant semitone intervals, it still offers a powerful tool for composition.

In summary, Tonnetze can be used to represent interval cycles, chords, and scales, and offer composers a powerful new tool to support composition. The next section describes our work on *PaperTonnetz*, which offers both a novel form of Tonnetz and the use of interactive paper to link the expression of musical ideas on paper to real-world compositions.

4. STUDY OF INTERACTIVE PAPER TONNETZ

We chose the MCM'11 conference in Paris (Mathematic and Computation in Music, June 2011), to learn how users, both non-musicians and musicians, would respond to an interactive paper Tonnetz interface. Figure 5 shows two hexagonal Tonnetze: T[3,4,5] and T[1,2,9] which were printed with the Anoto dot pattern to create interactive paper. These Tonnetze provide different musical properties: the former uses large intervals, the latter uses small intervals.

Participants used the ADP-301 digital pen to hear pitches in real time, as they draw lines from one hexagonal cell to another. The pitch changes as the pen crosses a boundary and continues for as long as the pen stays in that cell. Participants can thus create melodic sequences and control timing through their gestures.

The Tonnetz does not dictate a particular octave, so we chose an arbitrary set, with the 12 pitches from C4 (60) to B4 (71). Users could choose 'parsimonious mode' to obtain more melodic results: We minimized the interval

between subsequent notes by re-calculating each new pitch class. For example, moving from a C to a B in this mode creates a short pitch transition from C4 (60) to B3 (59) instead of from C4 (60) to B4 (71).

4.1 Technical details

The first version of *PaperTonnetz* was written in Java and uses ADP-301 digital pens to communicate via Bluetooth. The Tonnetze are printed with Anoto dot patterns that can distinguish among 20 unique pages. The Open Sound Control Protocol [17] communicates run-time data to a Max/MSP patch, which plays pitches in real-time or routes them to other MIDI- or OSC-compatible applications.

4.2 User study

We set up a table at MCM'11 with copies of both types of Tonnetz. Approximately 30 people participated, including both professional musicians and non-musicians. We began with a quick explanation of Tonnetze and interactive paper technology. We then watched as they tried and compared the two Tonnetz patterns. We answered their questions about interactive paper, computing and music theory and then asked them to tell us their perceptions of the two different layouts and of the interactive paper Tonnetz. We concluded by asking them whether and how such a tool might help them either to perform or to compose music.

Figure 5. A participant discovering the two layouts. When the pen enters a region, the corresponding pitch is played.

4.3 Results

Trying the prototype: Most participants began by tapping a particular cell to produce a sound. They would then scribble with the pen, covering a large number of cells and listening to the result. The real-time sonic feedback encouraged them to explore different patterns and to compare the differences between the two Tonnetze.

Sonification and Representation: Participants realized that drawing particular shapes resulted in identifiable sound patterns, which gave them clues about the differing properties of the two Tonnetze. Non-musicians were able to detect differences, but did not understand them. Musicians, however, performed systematic explorations in order to understand the musical relationships among the cells. They attempted to draw specific melodies or scales in order to understand how they were represented in the network.

Comparing Tonnetze: After becoming familiar with how the system works, most participants tried to compare the two Tonnetze. We suggested that they draw straight lines and geometrical shapes such as triangles or polygons in both networks and compared the sound results. Although both novices and musicians were able to hear clear differences between the two Tonnetze, only the musicians were able to explain what they were hearing. Table 1 compares

	T[3,4,5]	T[1,2,9]
Non-musicians	Small cyclic moves on neighbor regions sound well.	Continuous paths can create melodies that can be sung.
Musicians	Produce easily arpeggios. Difficult to play a scale.	Many scales are easy to perform. Melodies are easy to reproduce.

Table 1. Participants comparison between the two Tonnetz.

the reactions of musicians and non-musicians with respect to each of the two Tonnetze. Their reactions were consistent with the harmonic and melodic properties of hexagonal notes layouts studied in [5].

Limitations: The musicians identified a number of shortcomings of the first prototype. They pointed out the need to express chords and felt that version 1 would not be sufficient to fully explore a musical idea. They wanted to reuse previously drawn curves, preferring to tap instead of redrawing the path in order to listen to it again. Most musicians asked for additional pitch layouts; some even suggested specific alternatives. A few musicians also asked if they could generate personal networks that contained irregular shapes and neighborhood properties.

5. PAPERTONNETZ: A COMPOSITION TOOL

Based on feedback from the first study, we decided to redesign the *PaperTonnetz* prototype into a more general composition tool (see online video [4]). We incorporated a number of ideas from Chouvel, including highlighting the *path*, defined as a trace of ink over several cells, as the central interaction element. We gave composers the ability to listen to chord and note sequences in real time and provided a link between the paths on paper and notes represented on a musical score. Finally, we allow composers to choose among different Tonnetze or design their own. Once the composer has laid out the desired Tonnetz in the workspace, the document can be printed with any good-quality printer. *PaperTonnetz* prints the user's design onto an Anoto dot pattern and the user can interact with the resulting page with a digital pen. Figure 6, provides an example of how a composer can use *PaperTonnetz* to explore ideas for a new composition.

5.1 Creating paper interface

In this scenario, the composer, "Hugo", creates a composition in *C minor* key with three voices: bass part, lead part

[4] `http://vimeo.com/40072179`

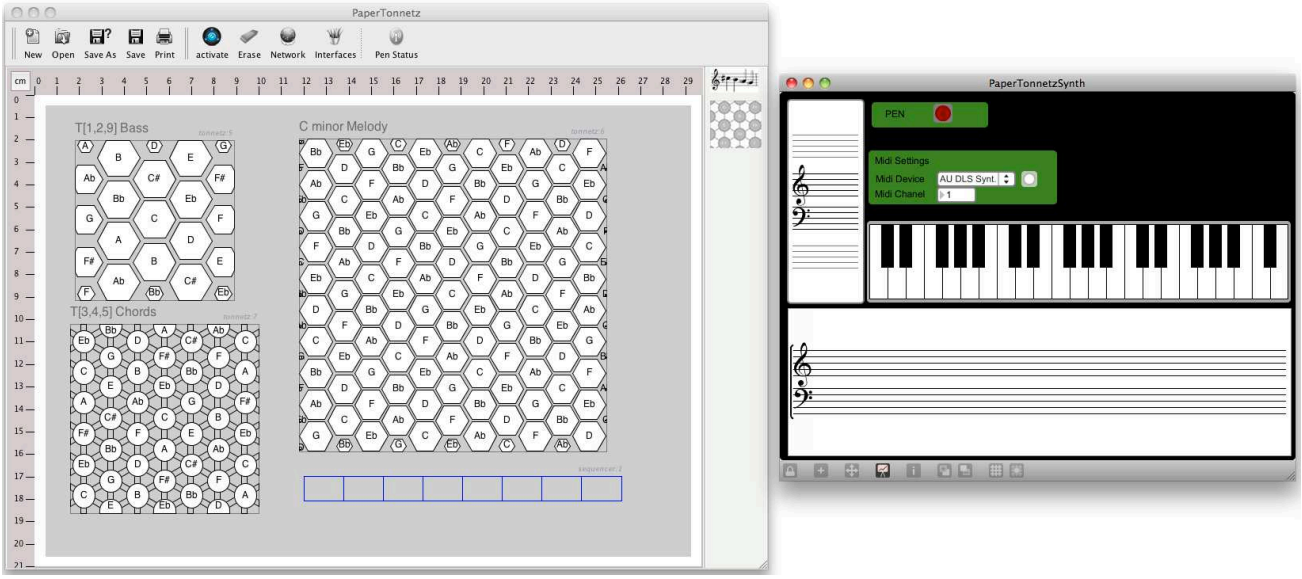

Figure 6. Left: *PaperTonnetz* main interface representing the virtual page with three Tonnetze and one sequencer. Right: The Max/MSP patch to play and visualize created sequences.

and chords part. The interface consists in series of commands on the top, two icons representing sequencers and Tonnetze to the right and a large workspace for assembling the elements of the printed paper page in the center. In this example, Hugo begins by dragging a Tonnetz icon to the workspace. He resizes it and places it to the left of the workspace. Next, he configures it: He chooses a hexagonal pattern and the specific Tonnetz intervals, after which he labels it: ”C minor Melody”. This Tonnetz is in heptatonic *C minor* and requires the most space, since Hugo will use it to create a variety of melodies in that key.

Hugo repeats drags another Tonnetz icon to create the bass part in the upper left corner. He creates a smaller Tonnetz, zoomed in with larger individual cells. He labels it: ”T[1,2,9] Bass” to help him remember the particular intervals he chose. This one is particularly appropriate for the bass part, since the intervals are short, so drawing paths requires only small movements.

Finally, Hugo adds the chord part, in the lower left hand corner. Here, he chooses to represent pitches as circles rather than hexagons, with pairs of lines between them. If the user touches the double lines, the two interconnected pitches are played together as a chord. If they touch the triangular space within three circles, the three corresponding pitches are played together. He labels it ”T[3,4,5] Chords”, a Tonnetz that we have already seen is particularly appropriate for playing major and minor chord patterns, since each can be played by drawing a compact, triangular path.

5.2 Assembling Sequences

Hugo next includes a sequencer by dragging the sequencer icon to the space below the ”C minor Melody” Tonnetz. The sequencer will allow the composer to create and assemble music sequences that are drawn in the other three Tonnetz spaces. The sequencer consists in a band of rectangles, each of which can contain a track (a set of paths within the Tonnetz). Tracks are stored as MIDI files that can be sent to the MAX/MSP synthesizer to be played or for additional processing.

5.3 Supporting Composition

Hugo prints a paper copy of the workspace, which now contains three different Tonnetze (for the melody,bass and chords) as well as a sequencer to capture and assemble individual paths that become standard MIDI tracks.

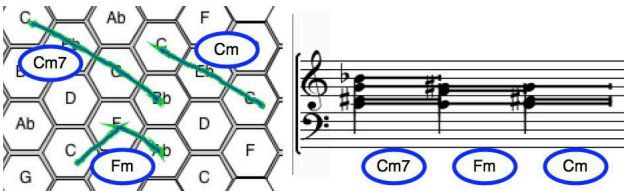

Figure 7. Left: Visualization of drawn paths in the *Cminor* heptatonic Tonnetz. Right: A sequence created with the three paths added as chords.

In Figure 7, Hugo has drawn three paths in the ”C minor Melody” heptatonic Tonnetz, which correspond to three chords, *C minor seventh*, *C minor* and *F minor*. These are translated into notes on the score at the left, based on the length of time spent in each cell. He can replay any path simply by touching it with the pen: this preserves the rhythmic properties of the original path. Hugo can also play the path as a chord by holding the pen on a path: all notes are played simultaneously. Any created path can be reused many times, either as a sequence or as a chord.

Hugo can also assemble sequences of paths, to create musical tracks. He draws a stroke in one of the boxes in the sequencer. If he clicks on a path, it is added as a sequence of notes; if he holds the path, the notes in the path are added as a chord. He can use the same process to add additional sequences or chords. To complete the track, Hugo draws

a second stroke in the sequencer box. This creates a track that is displayed on a score (Figure 7: right). Here, Hugo has created a series of three chords. He clicks on the box to hear the sequence which is rendered from a *bach.roll* [5] object played with a Max/MSP patch). Hugo repeats the process, creating a series of tracks, which can be played in series or at the same time.

5.4 Supporting Improvisation

Although the paper Tonnetz is designed primarily for composition, it can also be used for performance. As in our user study, novices and students can explore the different auditory relationships among pitches by clicking on cells and drawing paths through them. The duration of each pitch is defined by the length of time that the pen stays in each cell. More advanced musicians will be able to capture their improvisations and explore musical ideas in real time.

6. DISCUSSION

6.1 Discovering, Improvising and Assembling

We designed *PaperTonnetz* to accommodate three primary musical activities: discovery, improvisation and assembly, each of which adds requirements to the user interface.

Discovery: *PaperTonnetz* makes each cell and path interactive so they can reveal their musical properties and functionalities. This allows both novices and experts to discover its properties, with real-time auditory feedback.

Improvisation: *PaperTonnetz* allows previously drawn paths to be accessible for future interaction, including playing, transformation, and refinement. The user can create individual chords and melodies and then reuse, recombine and modify them to explore different possibilities.

Assembly: *PaperTonnetz* enables expert users to capture musical sequences based on their improvisations and to export them to other music composition systems. For example, music sequencers can handle the MIDI sequences produced by *PaperTonnetz* for advanced arranging and editing. *PaperTonnetz* supports both MIDI and OSC protocols, so any musical sequence created with it can be imported and edited in any other dedicated application. This could be improved by providing a plug-in version of *PaperTonnetz* that can be hosted by a compatible digital audio workstation.

6.2 Interacting with drawn gestures

We use the notion of *path* as a main interactive element dynamically created by the user. We believe that a trace left on paper by the pen can be a simple and powerful way to get back to previous improvisation. On the other hand, this approach includes some practical limitations, such as the saturation of some regions after many drawings. As we see in Figure 2, the composer reuses the same path with a different orientation and position in space. Then, providing tools and interaction to reuse drawn paths in different networks is a future direction. Using a screen-based tablet can efficiently help such tasks. However, paper affords sketching and writing by using common and personal notation. Then, it supports composers in expressing and exploring their musical ideas [6, 9, 10].

We are also interested in using the gestures properties to control musical parameters. Mapping the pen pressure to the musical velocity is a possible future investigation but the technology is not yet sufficiently reliable to control it properly.

6.3 Spatial Representations

Our first prototype proposed the use of two Tonnetze, T[3,4,5] and T[1,2,9]. The second version of *PaperTonnetz* goes one step further by supporting a range of different Tonnetze. We have also started exploring how to advance some of the graphical properties of Tonnetze in space by taking advantage of the physical properties of paper. Paper can be cut, folded and physically moved without losing its functionalities.

We envision that composers could cut their preferred paths or areas on paper and then recombine them, keeping and reusing their most interesting material. Moreover, paper can be used to create three-dimensional shapes, *e.g.*, a cube, that represent new continuous spaces to explore. For example, in the case of heptatonic hexagonal spaces, switching from one face to another would result in a tonality modulation (Figure 8).

Figure 8. Left: A 3D paper shape. Right: Two planes representing tonalities of *C Major* and *F Major*.

Another possible direction with hexagonal layouts is to represent microtonal pitch spaces. Chouvel [17] has previously discussed this direction, while recent work [18] proposed an application to handle these concepts. This could open new perspectives for music composition.

In this work, we have mostly focused on hexagonal Tonnetz representations but our goal is to enable composers to create their own musical spaces with arbitrary shapes or more complex neighboring relationships between elements. In order to enable continued strokes with the pen, we are interested in the different ways to tessellate the plan. We can imagine both regular (*e.g.*, the hexagonal layout) and irregular tessellations. Such tessellations could include several types of shapes, linked with different neighboring relationships.

[5] http://www.bachproject.net

7. CONCLUSION

In this paper, we investigated the use of Tonnetz representations to support computer aided composition. There is a wide range of pitch layouts based on Tonnetze to represent musical properties or to simplify the performance of scales. We proposed new layouts based on heptatonic scales to extend composition possibilities in two-dimensional pitch spaces. We presented *PaperTonnetz*, a user interface for composing with interactive paper-based Tonnetz representations. A user study helped us redesign *PaperTonnetz* and integrate handwritten gestures into interaction with paper so that users can create chords and musical sequences. We presented a set of interaction techniques to facilitate the discovery, improvisation and assembly of musical sequences on paper. We also extended our tool to enable composers to build their own interactive paper interfaces.

In future work, we plan to collaborate with composers to extend its control capabilities for rhythm and pitch, especially the use of several octaves. We are also interested in integrating PaperTonnetz into existing musical software such as OpenMusic and common digital audio workstation applications.

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