

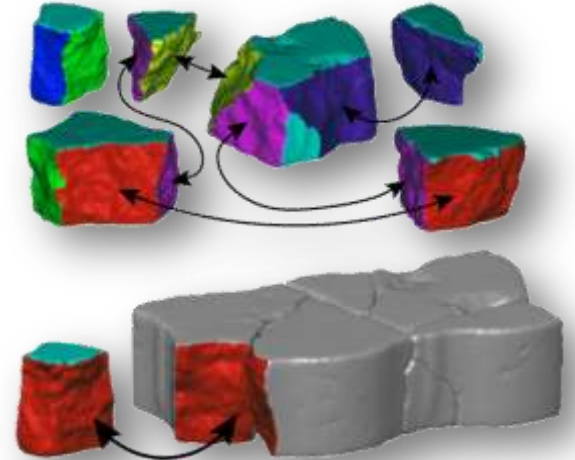
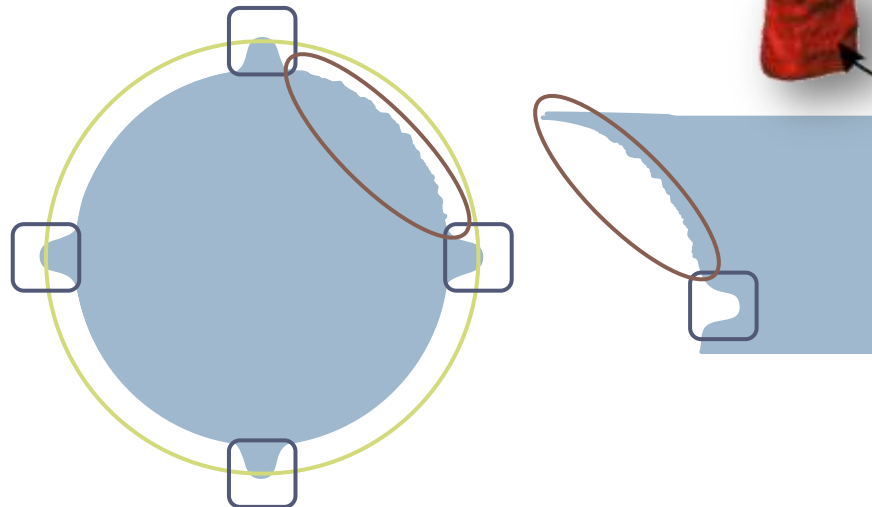
Growing Least Squares for the Analysis of Manifolds in Scale-Space

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Inria - Univ. Bordeaux - IOGS - CNRS

Context

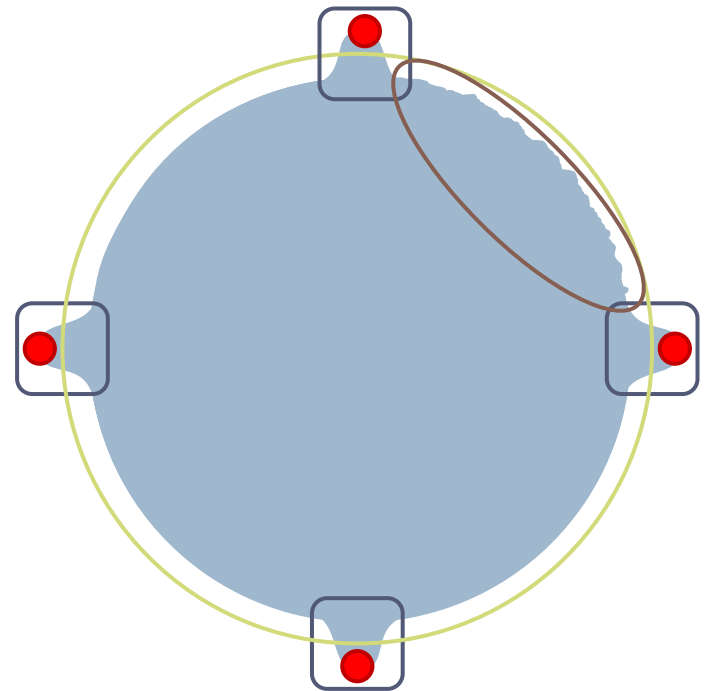
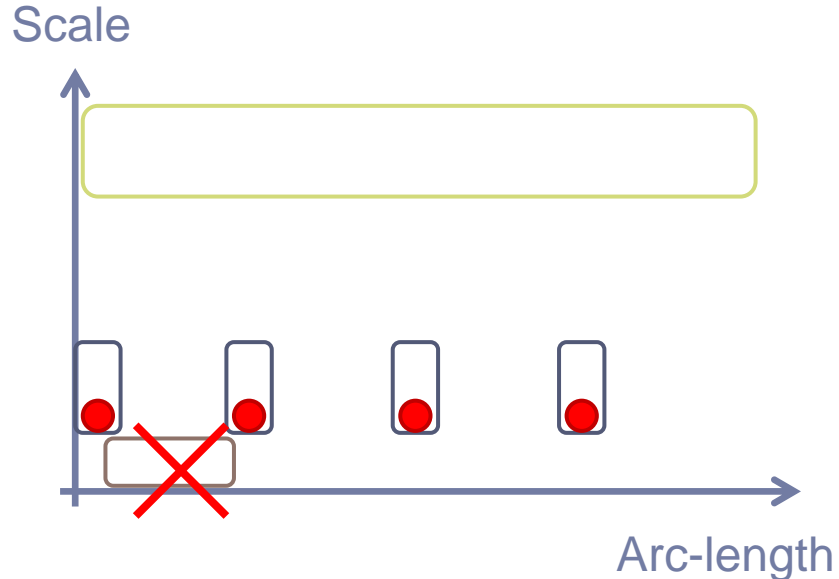
- ▶ Shape matching [HFGHP06,IT11]
- ▶ Focus on geometric properties
 - ▶ Describe shape features
 - ▶ Find their pertinent scales



- ▶ Requires a multi-scale analysis

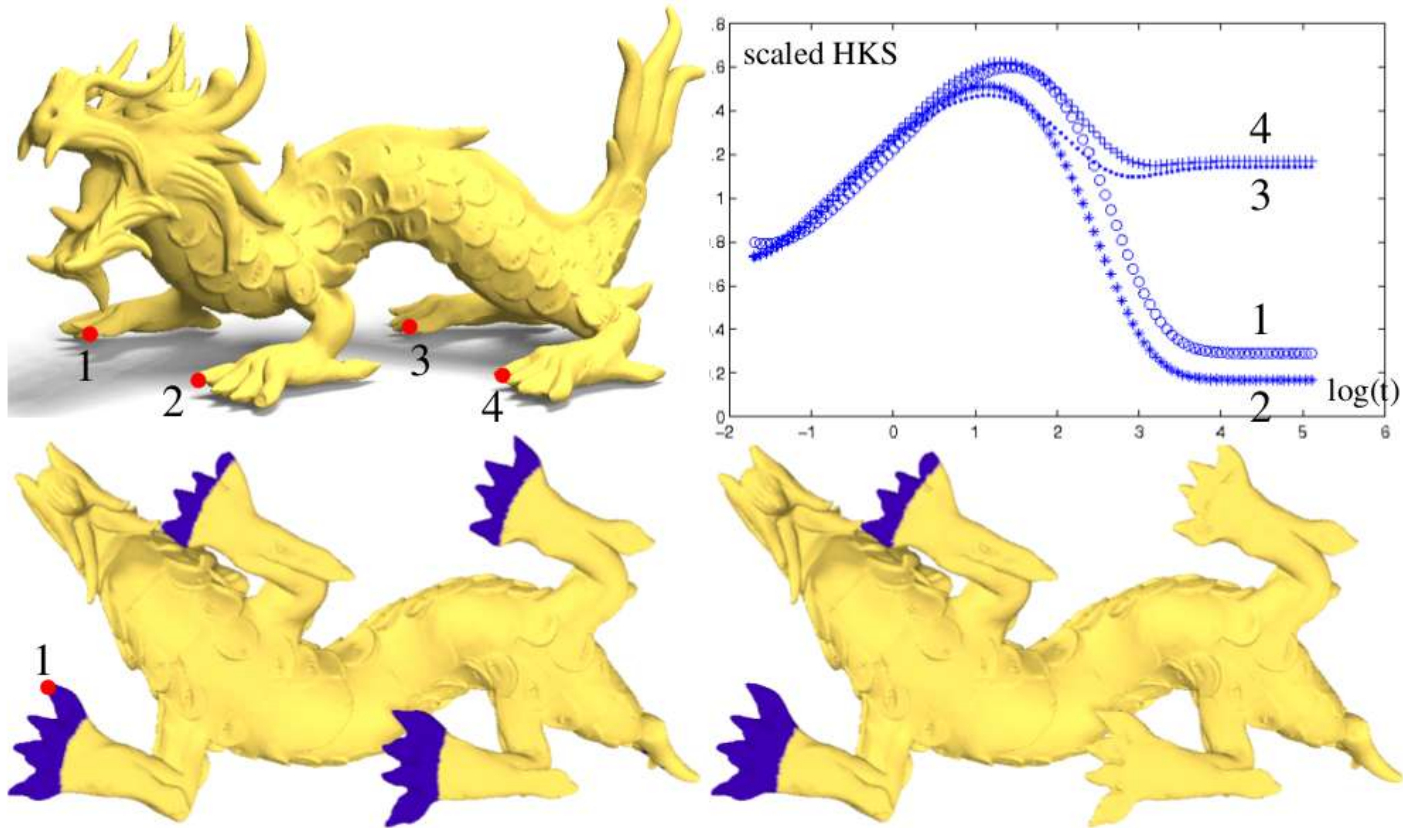
Problem statement

- ▶ Find pertinent structures in scale-space
- ▶ Process
 - ▶ Ignore noise
 - ▶ Extract points
 - ▶ Detect similarity



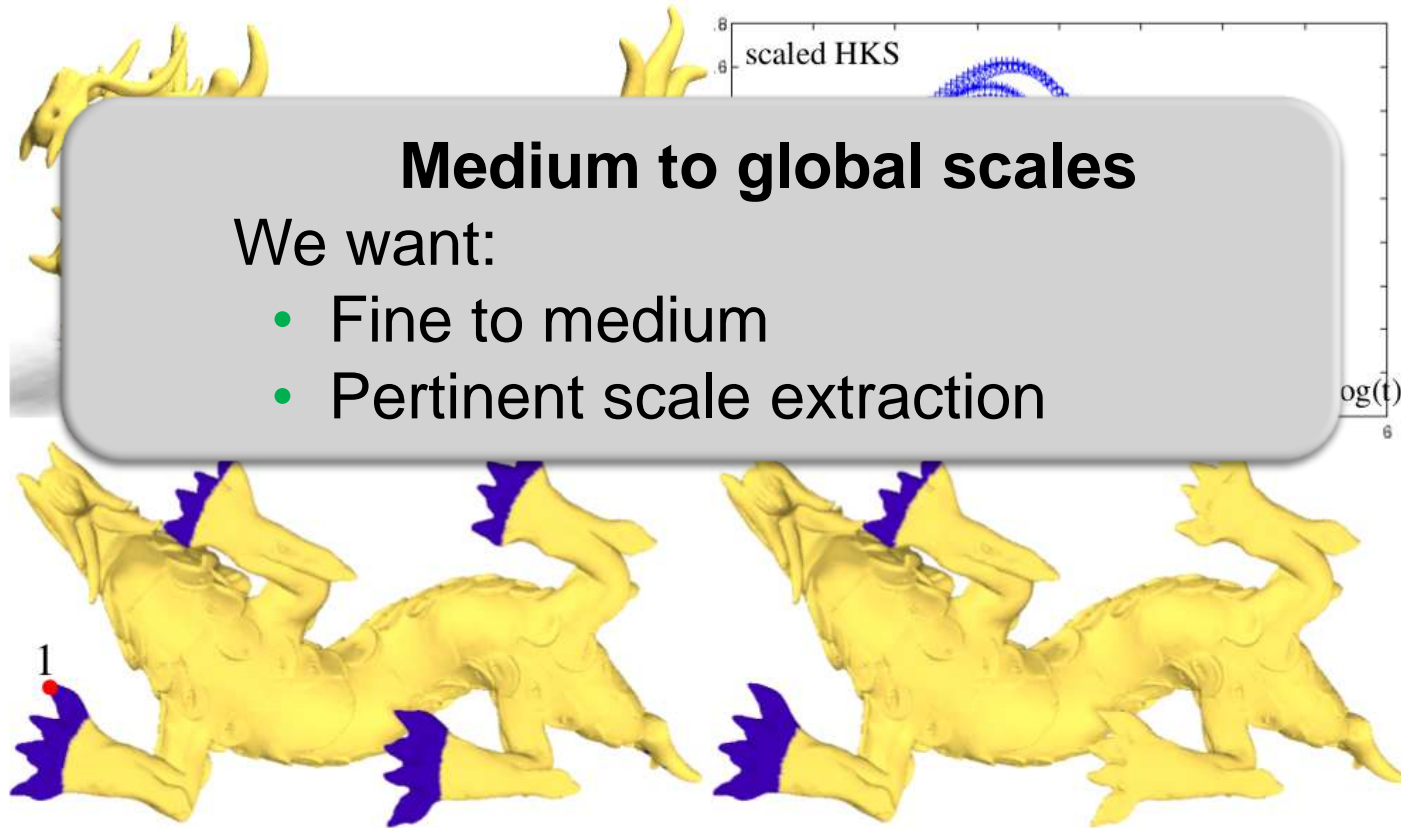
Previous work - 1/3

- ▶ Intrinsic characterization of the shape:
 - ▶ Heat diffusion (HKS) [SOG09,BK10,ZRH11]



Previous work - 1/3

- ▶ Intrinsic characterization of the shape:
 - ▶ Heat diffusion (HKS) [SOG09,BK10, ZRH11]



Previous work - 2/3

- ▶ Multi-scale geometric descriptor: curvature

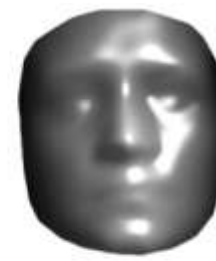
- ▶ Mean Curvatures estimated via smoothing [ZBVH09,MFK* 10]



(a) $\sigma_i = 1.83$ mm



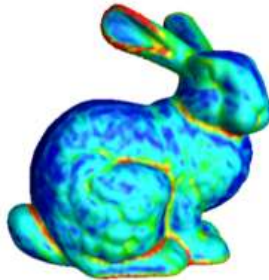
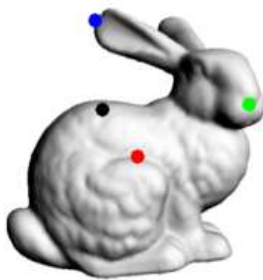
(b) $\sigma_i = 3.66$ mm



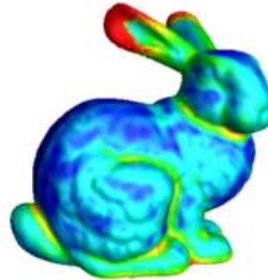
(c) $\sigma_i = 6.18$ mm

- ▶ Covariance Analysis

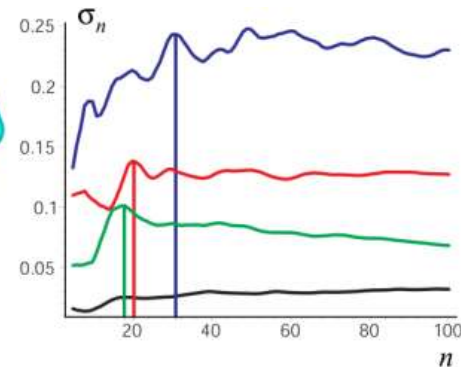
[PKG2003, LG2005, YLHP06]



σ_{20}



σ_{80}

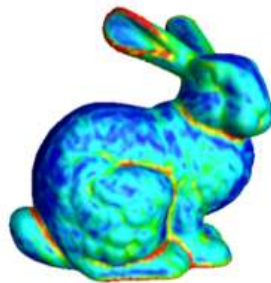
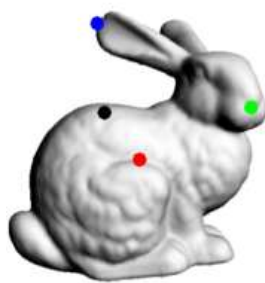


Previous work - 2/3

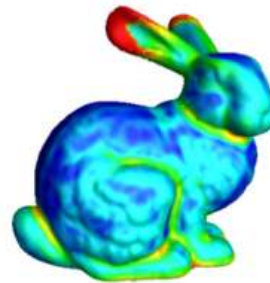
- ▶ Multi-scale geometric descriptor: curvature
 - ▶ Mean Curvatures estimated via smoothing [ZBVH09,MFK* 10]

Curvature is not enough...

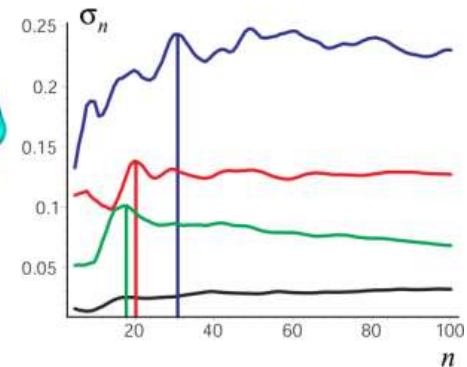
- ▶ Covariance Analysis [PKG2003, LG2005, YLHP06]



σ_{20}



σ_{80}



Previous work - 3/3

- ▶ **Scale-space analysis**
 - ▶ Curvature extrema extraction

[ZBVH09,MFK* 10,DK11]



Previous work - 3/3

- ▶ Scale-space analysis
 - ▶ Curvature extrema extraction

[ZBVH09,MFK* 10,DK11]



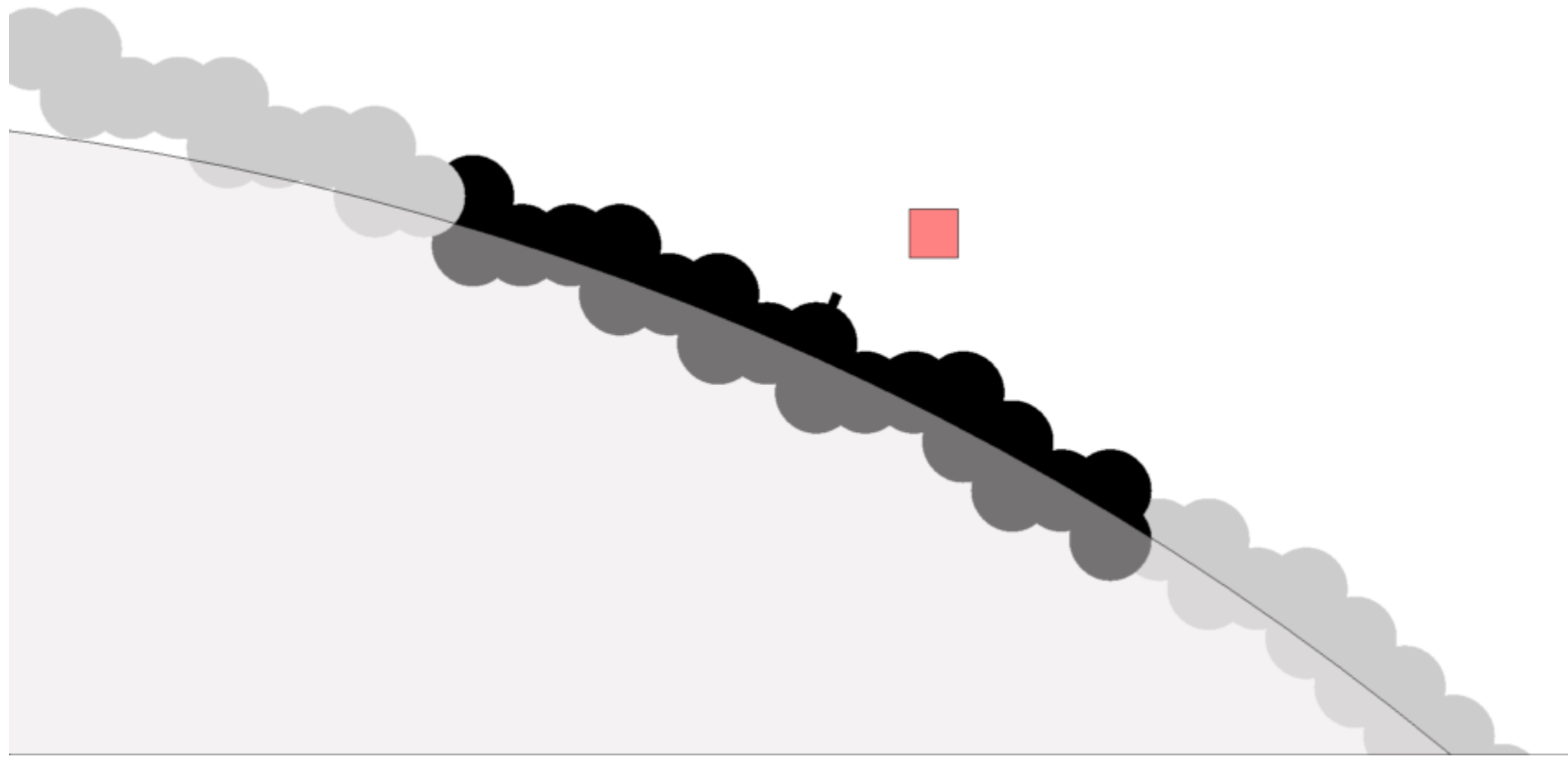
**Not exhaustive pertinence
detection**



Our approach: Growing Least Squares

- ▶ Regression method: fit hyper-sphere
 - ▶ Multi-scale
 - ▶ Support huge point-sets
 - ▶ Characterization by 2nd order proxy
- ▶ Analytic detection of pertinent scales
 - ▶ Dense description
- ▶ Bonus
 - ▶ 2D curves
 - ▶ 3D surfaces

Our approach: Growing Least Squares



Pipeline

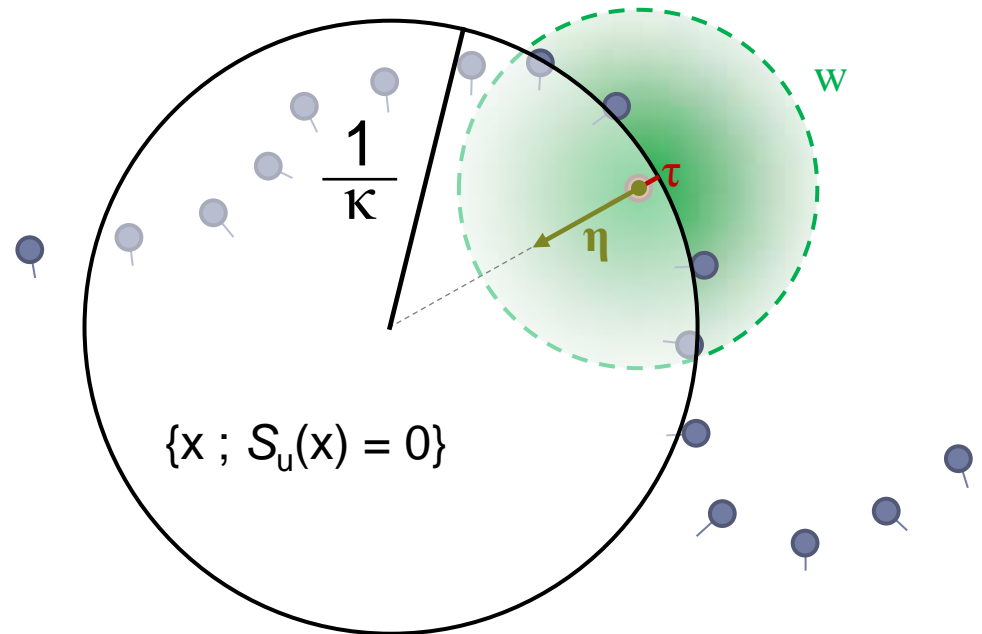
1. **Geometric descriptor** based on 2nd-order regression,
2. Continuous & analytic **geometric variation** in scale.



Geometric Descriptor – 1/3



- ▶ Algebraic hyper-sphere fitting procedure [GG2007]



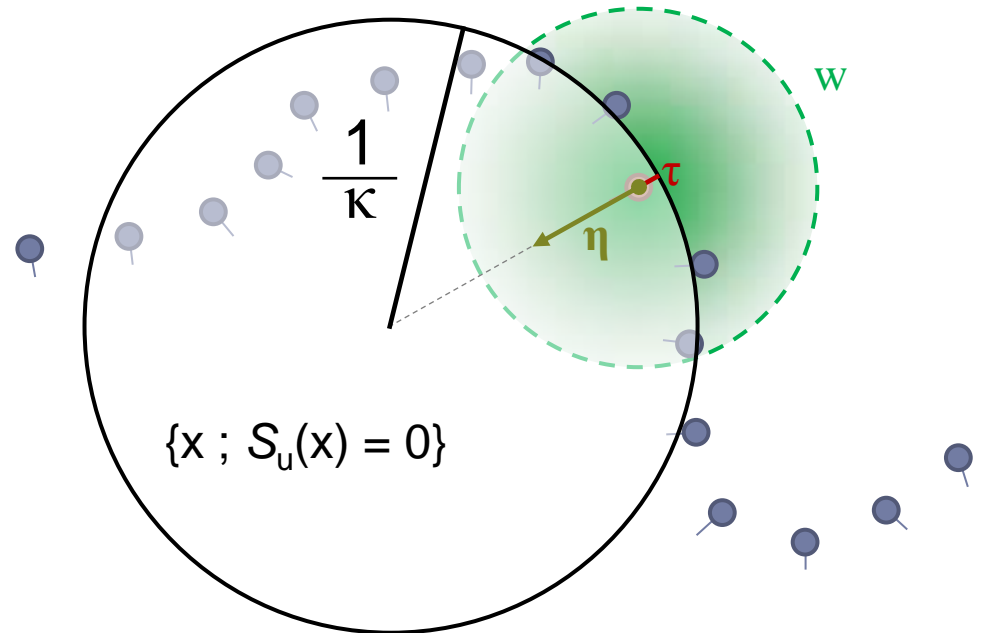
$$\mathbf{u} = [u_c \ \mathbf{u}_\ell \ u_q]^T$$

Geometric Descriptor – 1/3



- ▶ Algebraic hyper-sphere fitting procedure [GG2007]

- ▶ Normalization



$$\mathbf{u} = [u_c \ \mathbf{u}_\ell \ u_q]^T / \sqrt{\|\mathbf{u}_\ell\|^2 - 4u_c u_q}$$

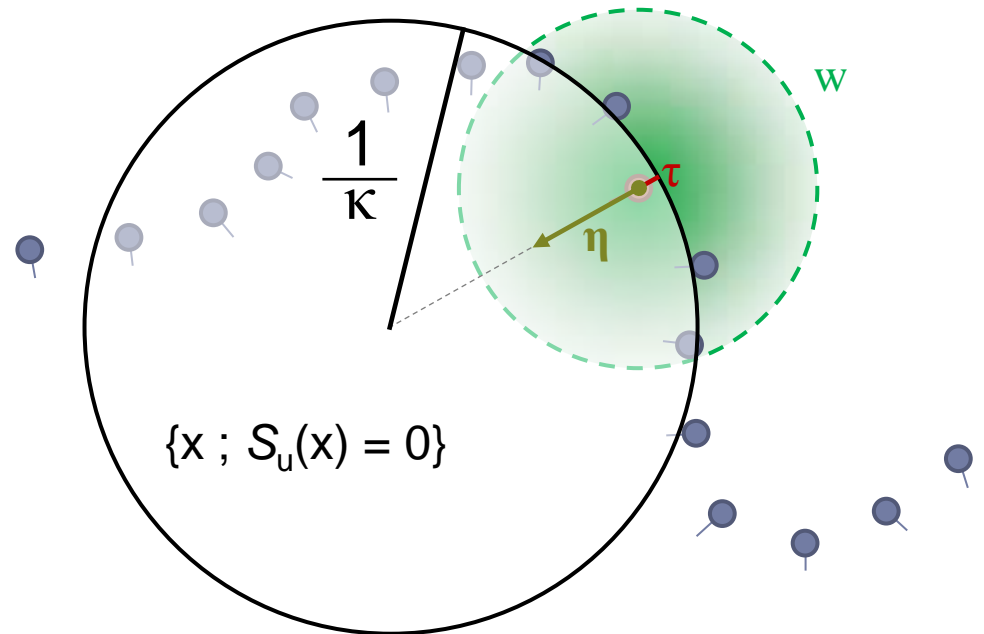
Geometric Descriptor – 1 / 3



- ▶ Algebraic hyper-sphere fitting procedure [GG2007]

- ▶ Normalization

- ▶ Re-parametrization



$$\mathbf{u} = [u_c \ \mathbf{u}_\ell \ u_q]^T / \sqrt{\|\mathbf{u}_\ell\|^2 - 4u_c u_q}$$

$$\hat{\mathbf{u}} = [\tau \ \eta \ \kappa]^T$$

Geometric Descriptor – 1/3

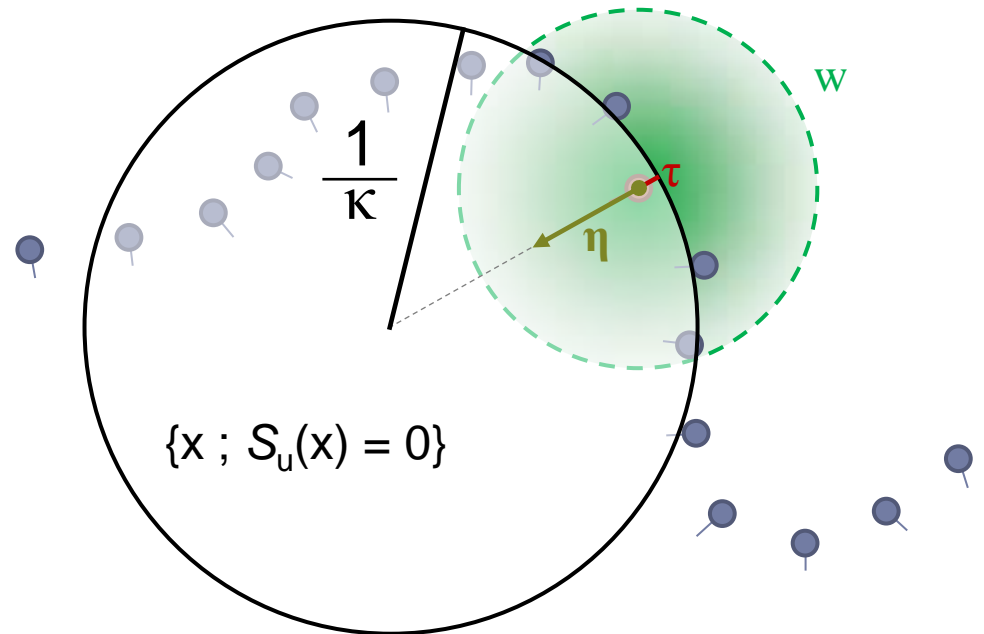


- ▶ Algebraic hyper-sphere fitting procedure [GG2007]

- ▶ Normalization

- ▶ Re-parametrization

- ▶ Scale Invariance



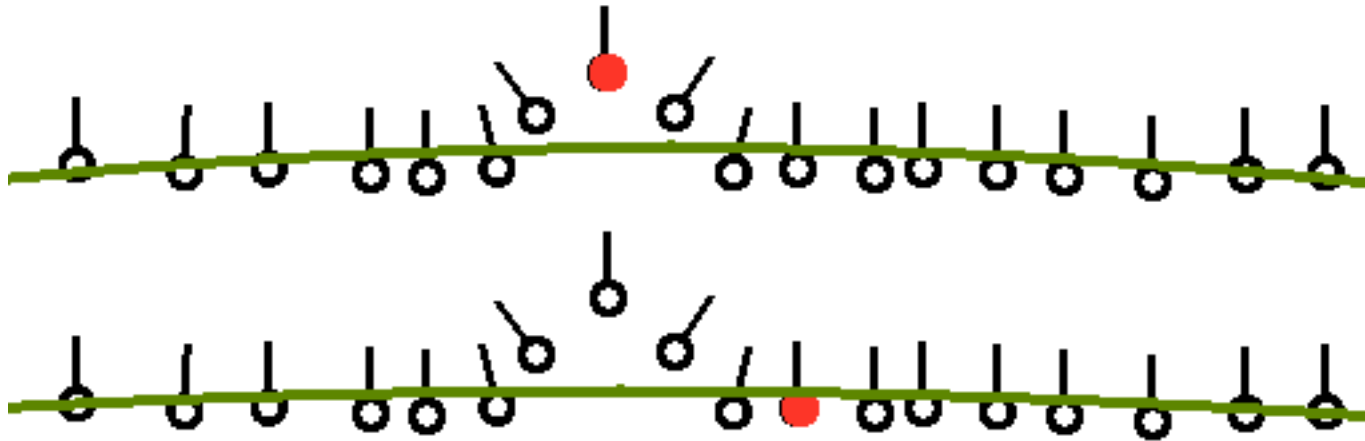
$$\mathbf{u} = [u_c \ \mathbf{u}_\ell \ u_q]^T / \sqrt{\|\mathbf{u}_\ell\|^2 - 4u_c u_q}$$

$$\hat{\mathbf{u}} = [\tau/t \ \eta \ t\kappa]^T$$

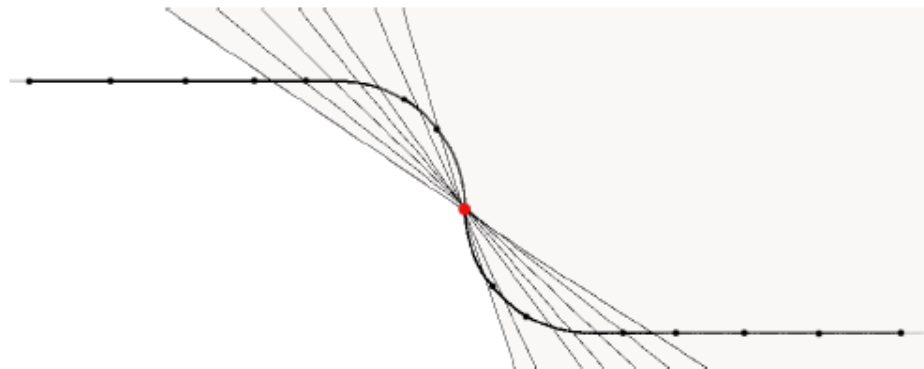
Geometric Descriptor – 2/3



► Impact of τ



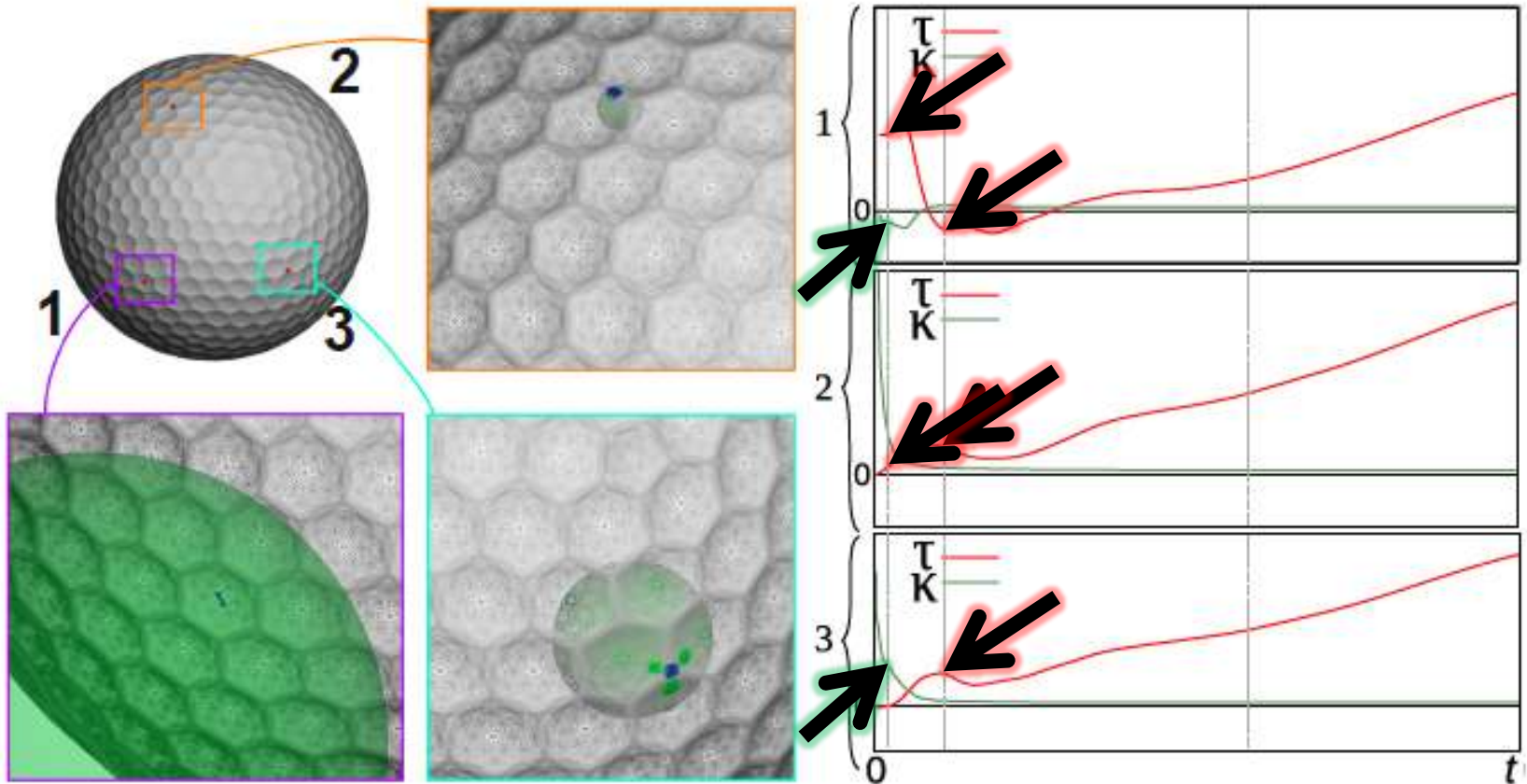
► Impact of η



Geometric Descriptor – 3/3



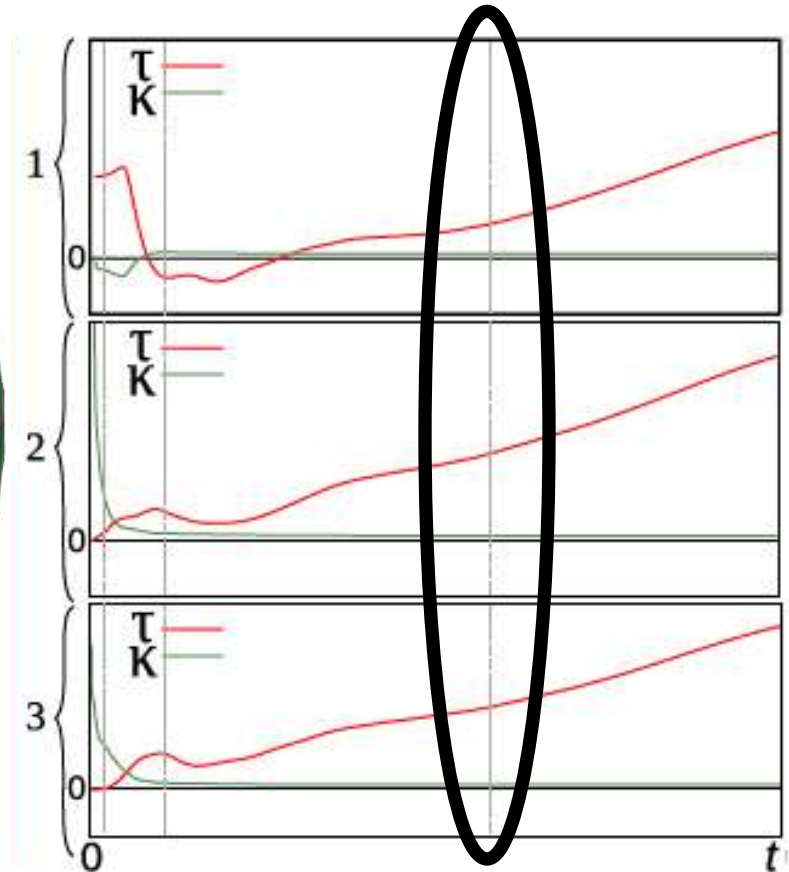
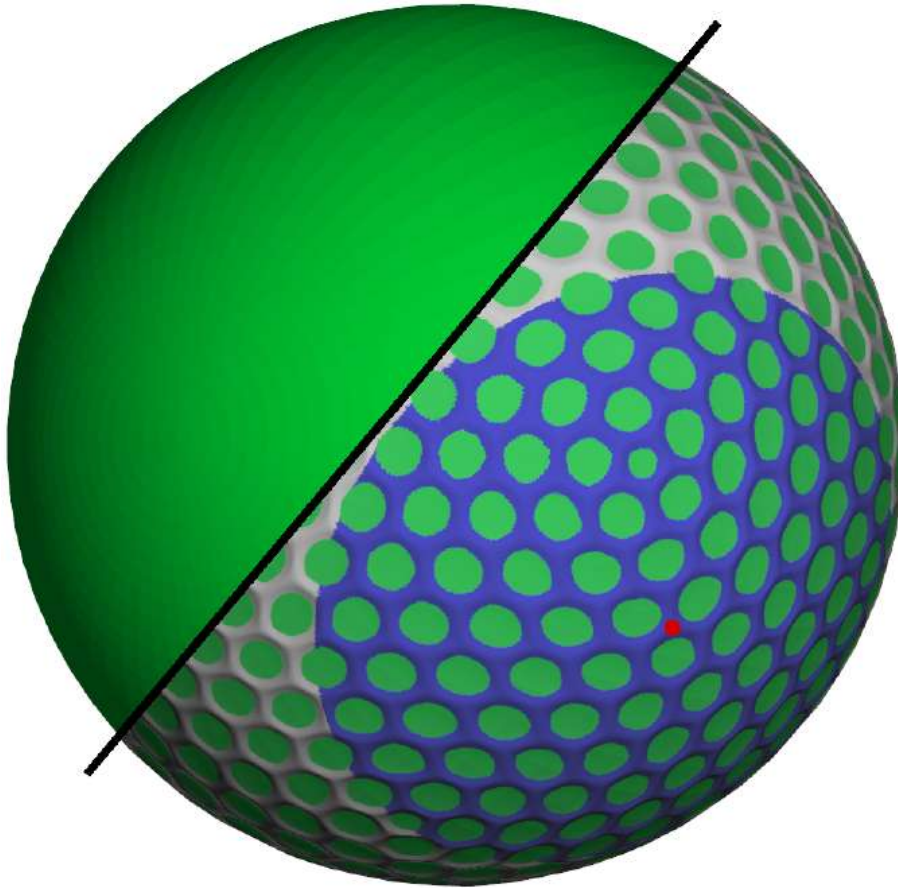
- ▶ An example in 3D



Geometric Descriptor – 3/3



- ▶ An example in 3D



Geometric Variation



- ▶ Given our descriptor $[\tau \ \eta \ \kappa]^T$

- ▶ Variation is related to $\left[\frac{d\tau}{dt} \ \frac{d\eta}{dt} \ \frac{d\kappa}{dt} \right]^T$

- ▶ Geometric Variation

$$v(\mathbf{p}, t) = \left(\frac{d\tau}{dt} \right)^2 + \left(t \frac{d\eta}{dt} \right)^2 + \left(t^2 \frac{d\kappa}{dt} \right)^2$$

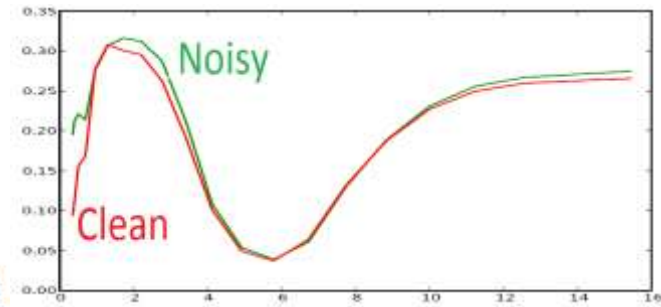
Summary

- ▶ Geometric Descriptor
- ▶ Geometric Variation



Results/Applications – 1/3

- ▶ Descriptor is robust to noise

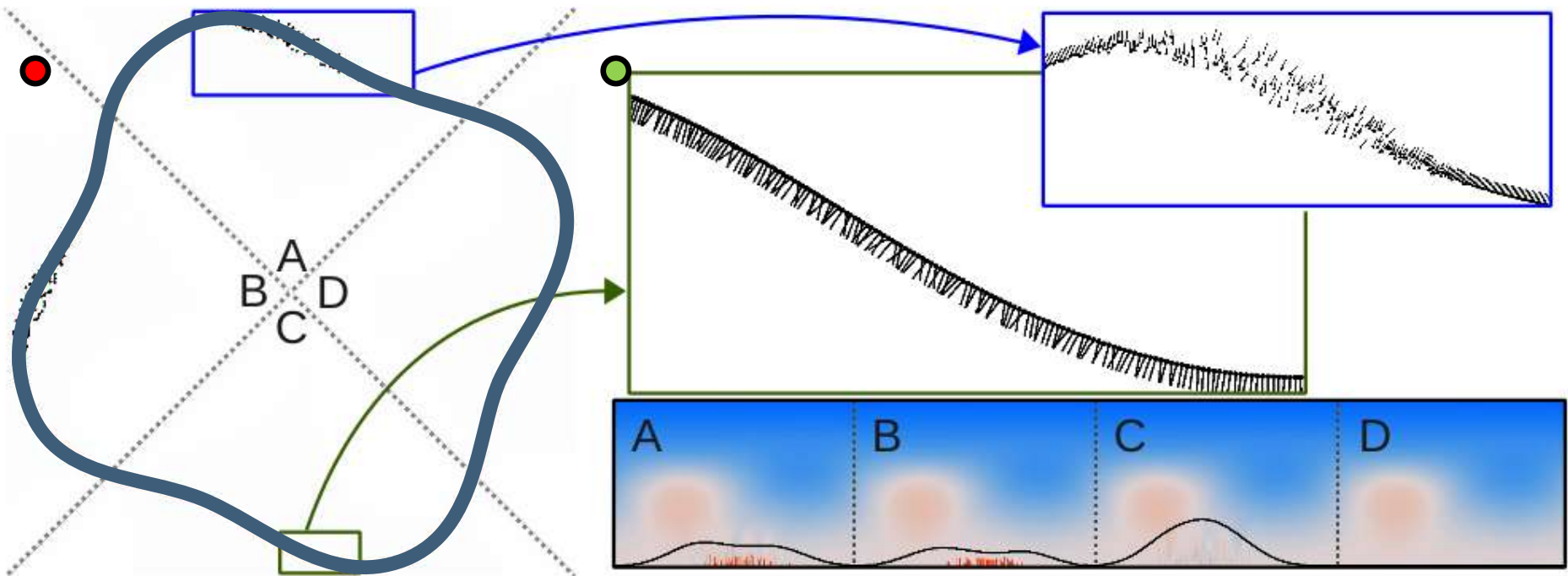


Gargoyle - 250k points
20 scales, 6.6s.

5% random noise

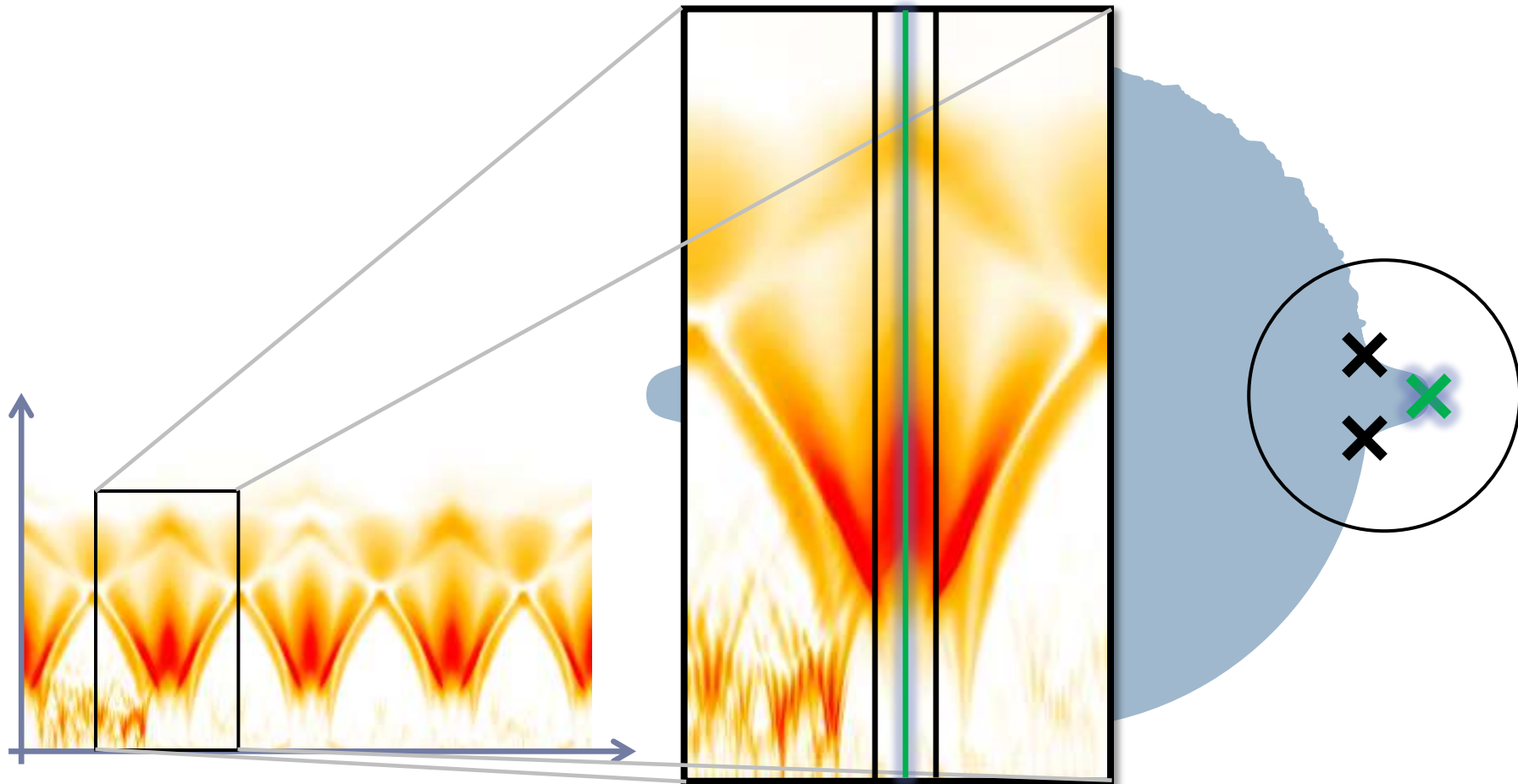
Results/Applications – 1/3

- ▶ Descriptor is robust to noise
- ▶ Can guide surface reconstruction



Results/Applications – 2/3

- ▶ Continuous detection of the feature area



Results/Applications – 2/3

- ▶ Continuous detection of the feature area

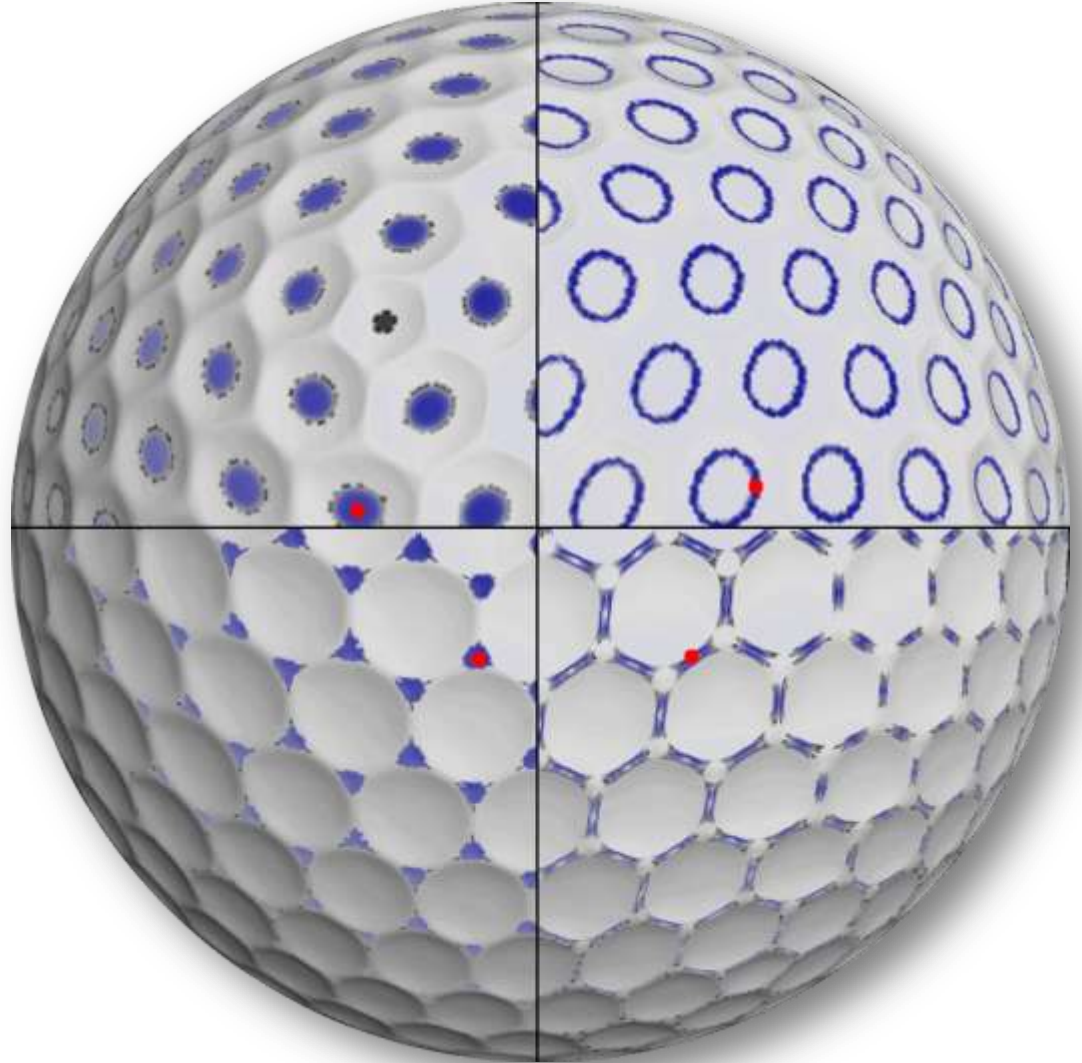


2d curve- 5k points
1000 scales, 3s.

Armadillo - 173k points
20 scales, 6.3s.

Results/Applications – 3/3

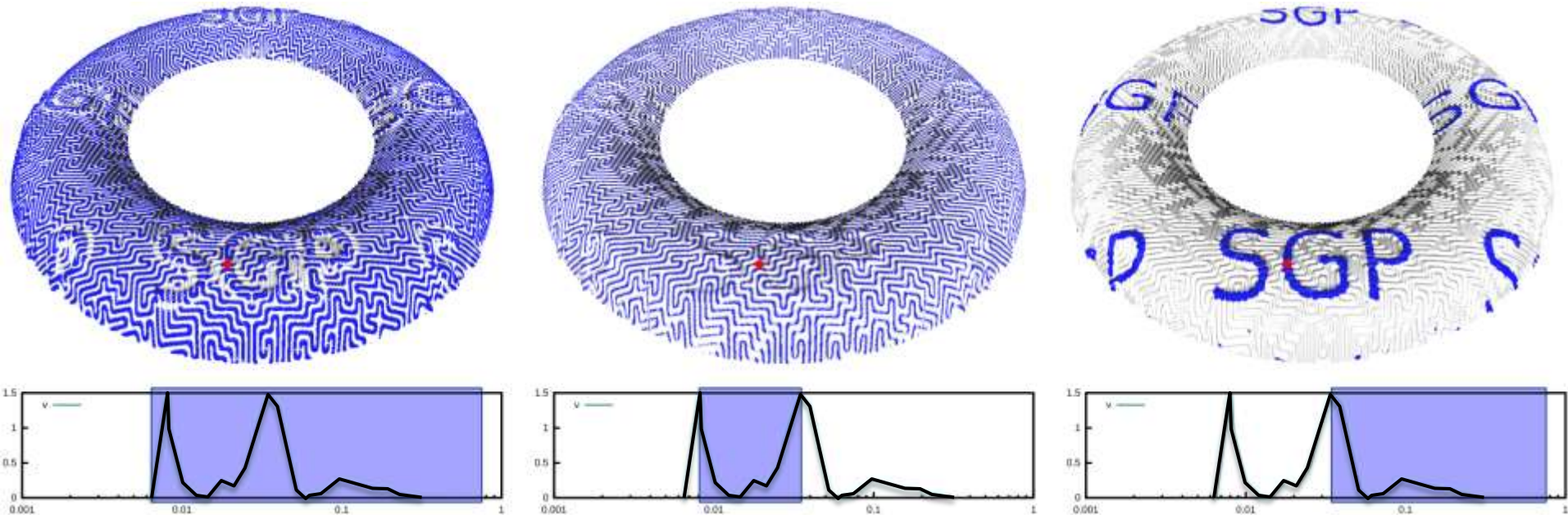
- ▶ Similarity detection,
 - ▶ Multi-scale profile



Results/Applications – 3/3

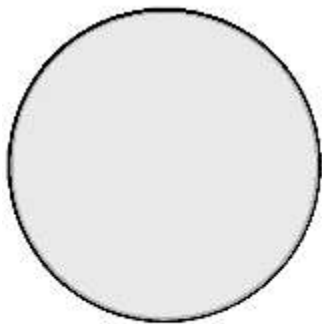
- ▶ Similarity detection,
 - ▶ Multi-scale profile
 - ▶ Specifics scale range

Torus- 500k points
20 scales, 42s.

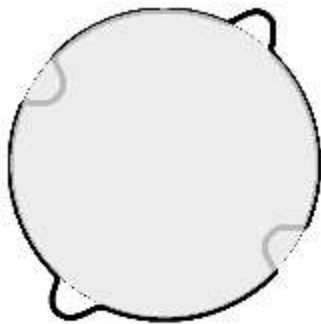


Discussion – 1/2

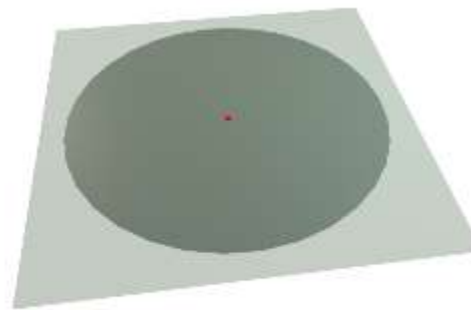
- ▶ Main limitation
 - ▶ isotropic,
- ▶ Analytic fitness measure
 - ▶ Squared Pratt Norm [GG2007]
 - ▶ $\varphi = \|\mathbf{u}_l\|^2 - 4u_c u_q$



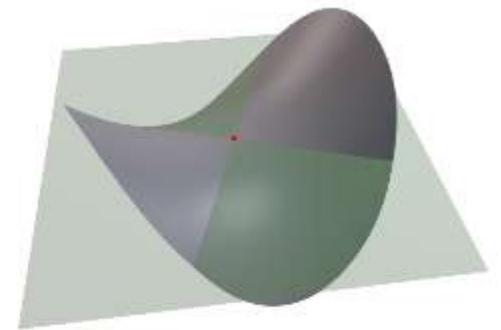
$$\varphi = 1$$



$$\varphi = 0.91$$



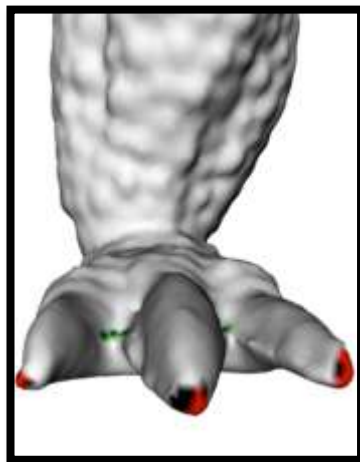
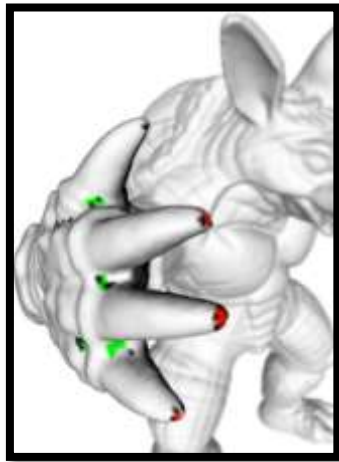
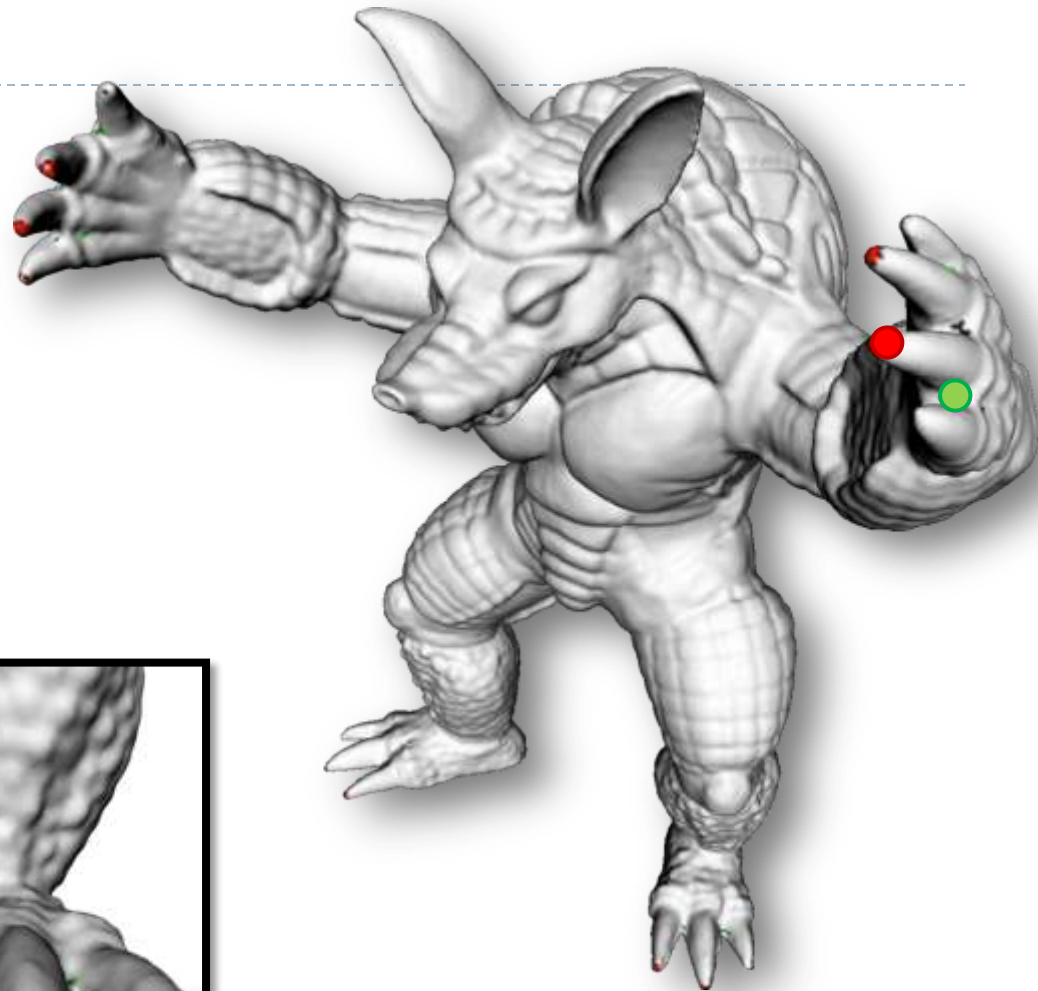
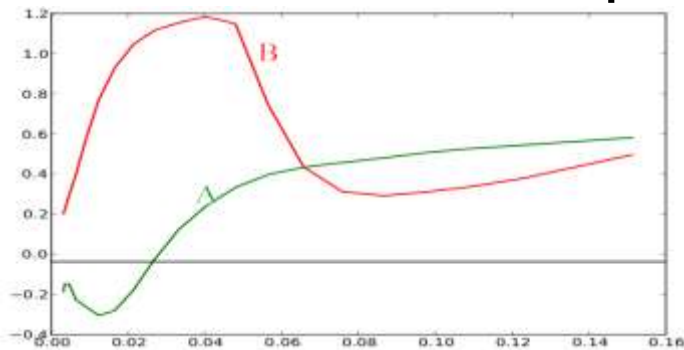
$$\varphi = 1$$



$$\varphi = 0.86$$

Discussion – 2/2

- ▶ Use multi-scale profile to characterize shape



Conclusions

▶ Contributions

- ▶ Stable second order descriptor for 2D/3D manifolds
- ▶ Continuous in scale and space from point-set input
- ▶ Continuous analysis to detect pertinent scales

▶ Future work:

- ▶ Use non-linear kernel to deal w/ additional attributes
- ▶ Analyze spatial variations to characterize anisotropy

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Thank you

Nicolas Mellado
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Pascal Barla
Patrick Reuter
Christophe Schlick

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<http://manao.inria.fr>
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