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Quality of Real-Time Streaming in Wireless Cellular Networks

Stochastic Modeling and Analysis

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Abstract

We present a new stochastic service model with capacity sharing and interruptions, appropriate for the evaluation of the quality of real-time streaming (RTS), like e.g. mobile TV, in wireless cellular networks. The general model takes into account multi-class Markovian process of call arrivals, (to capture different radio channel conditions, requested streaming bit-rates and durations) and allows for a general resource allocation policy saying which users are temporarily denied the requested fixed streaming bit-rates (put in outage) due to resource constraints. We give expressions for several important performance characteristics of the model, including mean time spent in outage and mean number of outage incidents for a typical user of a given class. These expressions involve only stationary probabilities of the (free) traffic demand process, which is a vector of independent Poisson random variables describing the number of users of different classes.

In order to analyze RTS in 3GPP Long Term Evolution (LTE) cellular networks, we specify our general model assuming orthogonal user channels with the peak bit-rates close to the theoretical Shannon's bound in the additive white Gaussian noise (AWGN) channel, which leads to the resource constraints in a multi-rate linear form. In this setting we consider a natural class of least-effort-served-first resource allocation policies, for which the characteristics of the model can be further evaluated using Fourier analysis of Poisson variables. Within this class we identify and evaluate an optimal and a fair policy, the latter being suggested by LTE implementations. We also propose some intermediate policies, which allow to solve the optimality/fairness tradeoff caused by unequal user radio-channel conditions. Our results can be used for the evaluation of the quality of RTS in LTE networks and dimensioning of these networks.

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Index Terms

Real-time streaming, stochastic model, mobile TV, LTE, quality of service, interruptions, outage, deep outage, capacity-sharing, Poisson process

I. Introduction

Wireless cellular networks offer nowadays possibility to watch TV on mobile devices, which is an example of a real-time content streaming. This type of traffic demand is expected to increase significantly in the future. In order to cope with this process, network operators need to implement in their dimensioning tools efficient methods allowing to predict the quality of this type of service. The quality of real-time streaming (RTS) is principally related to the number and duration of *outage incidents* — (hopefully short) periods when the network cannot deliver to a given user in real-time the requested content of the required quality. In this paper we propose a stochastic model allowing for an analytic evaluation of such metrics. It assumes a traffic demand with different radio conditions of calls, corresponding to the spatial distribution of the signal-to-interference-and-noise ratio (SINR) in a given network. It can also be specified to take into account all main characteristics of a given wireless cellular technology. We develop expressions for several important performance characteristics of this model, including the mean time spent in outage and the mean number of outage incidents for a typical streaming call *in function of its experienced SINR level*. These expressions involve only stationary probabilities of the (free) traffic demand process, which is a vector of independent Poisson random variables describing the number of users of different radio conditions.

We use this model to analyze RTS in 3GPP Long Term Evolution (LTE) cellular networks, assuming orthogonal user channels with the peak bit-rates (achievable when there are no other users) close to the theoretical Shannon's bound in the additive white Gaussian noise (AWGN) channel. These assumptions lead to a radio resource constraint in a multi-rate linear form. Namely, each user experiencing a given SINR value, requires a fixed fraction of the normalized radio capacity, related to the ratio between its requested and peak bit-rates. All users of a given configuration (with different SINR values) can be entirely satisfied if and

only if the total required capacity is not larger than one. [1]

In the above context of a multi-rate linear radio resource constraint, we analyse some natural parametric class of *least-effort-served-first* (LESF) service policies, which assign service to users in order of their increasing radio capacity demand, until the full capacity (possibly with some margin) is reached. The capacity margin may be used to offer some "lower quality" service to users temporarily in outage thus realizing some type of fairness with respect to unequal user radio-channel conditions. This class contains an optimal and a fair policy, the latter being suggested by LTE implementations.

In order to evaluate the quality of service metrics explicitly, we relate the characteristics of outage incidents induced by the LESF policies to the distribution functions of some linear functionals of the Poisson vector describing the state of the system. We calculate the Fourier transforms of these functions and use a well-known Fourier transform inversion method to obtain numerical values of the quantities of interest. Using this approach, we present a thorough study of the quality of RTS with LESF policies.

Let us now recollect a few *related works* on the performance evaluation of cellular networks. In early 80's, wireless cellular networks were carrying essentially voice calls, which require constant bit-rates (CBR) and are subject to admission control policies with blocking (at the arrival epoch) to guarantee these rates for calls already in service. An important amount of work has been done to propose efficient call admission conditions [3–5]. Policies with admission conditions in the multi-rate linear form have been considered e.g. in [6–8].

Progressively, cellular networks started carrying also calls with variable bit-rates (VBR), used to transmit data files. The available resources are (fairly) shared between such calls and when the traffic demand increases, the file transfer delays increase as well, but (in principle) no call is ever blocked. These delays may be evaluated analytically using multi-rate linear resource constraint in conjunction with multi-class processor sharing models; cf e.g. [8, 9].

Recently, users may access multimedia streaming services through their mobile devices [10]. They are provided via CBR connections, essentially without admission control, but they tolerate temporary interruptions, when network congestions occur. One may distinguish two

[1] Recall that in the case of voice calls and, more generally, constant bit-rate (CBR) calls the multi-rate linear form of the resource constraints has already proved to lead to efficient model evaluation methods, via e.g. Kaufman-Roberts algorithm [1, 2]. Despite some fundamental similarities to CBR service, the RTS gives rise to a new model, due to the fact that the service denials are not definitive for a given call, but have a form of temporal interruptions (outage) periods.

types of streaming traffic. In *real-time streaming (RTS)* (as e.g. in mobile TV), considered in this paper, the portions of the streaming content emitted during the time when the transmission to a given user is interrupted (is in outage) are definitely lost for him (unless a "secondary", lower-rate streaming is provided during these periods). In *non-real-time streaming (NRTS)* (like e.g., video-on-demand, YouTube, Dailymotion, etc), a user starts playing back the requested multimedia content after some initial delay, required to deliver and buffer on the user device some initial portion of it. If further transmission is interrupted for some time making the user buffer content drop to zero (buffer starvation) then the play-back is stopped until some new required portion of the content is delivered. Several papers study the effect of the variability of the wireless channel on the performance of a single streaming call; see for e.g. [11], [12]. In [13] VBR transmissions and RTS are considered jointly in some analytical model, however the number and duration of outage periods are not evaluated. In [14] the tradeoff between the start-up delay and the probability of buffer starvation is analyzed in a Markovian queuing framework for NRTS streaming.

The remaining part of this paper is organized as follows. In Section II we will present our model for the evaluation of the quality of RTS in wireless cellular networks. Technical proofs of the results presented in this section are postponed to the Appendix, where they are given in a more general context. Section III specifies our model to be compliant with the LTE cellular networks specification and presents numerical results regarding the quality of RTS in these networks.

II. Streaming in wireless cellular networks

In this section we present a new stochastic model of RTS in the context of wireless cellular networks.

A. System assumptions

We consider the following scenario of multi-user streaming in a cellular network.

a) Network layer: Geographically distributed users wish to obtain down-link wireless streaming of some (typically video) content, contacting base stations of a network at random times, for random durations, requesting some fixed streaming bit-rates. We consider a uni-cast traffic (as opposed to the broadcast or multi-cast case), i.e.; the content is delivered to all users via private connections. Different classes of users (calls) need to be distinguished, regarding their radio channel conditions, requested streaming bit-rates and mean streaming

times. Each user chooses one base station, the one with the smallest path-loss, independently of the configuration of users served by this station.

b) Data layer — streaming policies: If a given base station cannot cope with request of all its users at a given time, it temporarily stops streaming the requested content at the requested rate to users of some classes, according to some given policy (to be described), which is supposed to preserve a maximal subset of served users. We call these (classes of) users with the requested bit-rate temporarily denied *in outage*. The users in outage will not receive the part of the content which is emitted during their outage times (this is the principle of the RTS). We will also consider policies, which offer some "best-effort" streaming bit-rates for some classes of users in outage, thus allowing for example to keep receiving the requested content but of a lower quality. Users, which are (temporarily) denied even this lower quality of service are called *in deep outage*.

c) Medium access : In this paper we assume that users are connected to the serving antennas via orthogonal single-input-single-output (SISO) channels allowing for the peak-rate close to the theoretical Shannon's bound in the additive white Gaussian noise (AWGN) model.[2] We will also comment on how to model multiple-input-multiple-output (MIMO) and broadcast channels.

d) Physical layer: The quality of channel of a given user depends on the path-loss of the signal with respect to its serving base station, a constant noise, and the interference form other (non-serving) base stations. These three components determine its signal-to-interference-and-noise ratio (SINR). Our main motivation for considering a multi-class model is to distinguish users with different SINR values. In other words, even if we assume that all users require the same streaming times and rates, we still need a multi-class model due to (typically) different SINR's values of users in wireless cellular networks.

e) Performance characteristics: We will present and analytically evaluate performance of some (realistic) streaming policies in the context described above. We will be particularly interested in the following characteristics:

- fraction of time spent in outage and in deep outage during the typical call of a given class,
- number of outage incidents occurring during this call,

[2]Orthogonality of channels is an appropriate assumption for current LTE (Long Term Evolution) norm for cellular networks based on OFDMA, as well as for other multiple access techniques as FDA, TDMA, CDMA assuming perfect in-cell orthogonality, and even HDR neglecting the scheduler gain.

- mean throughput (average bit-rate) during such call, accounting for the requested bit-rates and for the "best-effort" bit-rate obtained during the outage periods.

B. Model description

1) Traffic demand: Consider $J \geq 1$ *classes of calls* (or, equivalently, users) characterized by different *requested streaming bit-rates* r_k, *wireless channel conditions* described by the signal-to-interference-and-noise ratio SINR_k with respect to the serving base-station, and *mean requested streaming times* $1/\mu_k$, $k = 1, \ldots, J$.

We assume that calls of class $k \in \{1, \ldots, J\}$ arrive in time according to a Poisson process with intensity $\lambda_k > 0$ (number of call arrivals per unit of time, per base station) and stay in the system (keep requesting streaming) for independent times, having some *general distribution* with mean $1/\mu_k < \infty$. [3] Different classes of calls are independent from each other. We denote by $X_k(t)$ the number of calls of a given class requesting streaming from a given BS at time t. Let $\boldsymbol{X}(t) = (X_1(t), \ldots, X_J(t))$; we call it the (vector of) user configuration at time t. The stationary distribution π of $\boldsymbol{X}(t)$ coincides with the distribution of the vector $(X_1, \ldots, X_J)$ of independent Poisson random variables with means $\mathbf{E}[X_k] := \rho_k = \lambda_k/\mu_k$, $k = 1, 2, \ldots, J$. We call ρ_k the *traffic demand* (per base station) of class k.

2) Wireless resource constraints: Users are supposed to be offered the requested streaming rates for the whole requested streaming times. However, due to limited wireless resources, for some configuration of users $\boldsymbol{X}(t)$, the requested streaming rates $\boldsymbol{r} = (r_1, \ldots, r_J)$ may be not achievable. Following the assumption of orthogonal AWGN SISO-like wireless channels available for users of a given station, we assume that the requested rates are achievable for all calls present at time t if

$$X_k(t) r_k = \lambda_k r_k^{\max}, \quad k = 1, \ldots, J, \tag{1}$$

for some non-negative vector $(\lambda_1, \ldots, \lambda_J)$, such that $\sum_{k=1}^{J} \lambda_k \leq 1$, where

$$r_k^{\max} = \gamma W \log(1 + \text{SINR}_k) \tag{2}$$

is the maximal (peak) bit-rate of a user of class k, whose channel conditions are characterized by SINR_k. (The rate $r_k^{\max}$ is available to a user of class k if it is the only user served by the base station.) Here W is the frequency bandwidth and γ (with $0 < \gamma \leq 1$) is

[3] All the results presented in this paper do not depend on the particular choice of the streaming time distributions. This property is often referred to in the queuing context as the insensitivity property.

a coefficient telling how close a given coding scheme approaches the theoretical Shannon's bound (corresponding to $\gamma = 1$); cf [15, Th .9.1.1]. [4] Note that the assumption (1) corresponds to the situation, when users neither hamper nor assist each other's transmission. They use channels which are perfectly separated in time, frequency or by orthogonal codes, nevertheless sharing these resources. [5]

We can interpret the ratio between the requested and maximal bit-rates $\varphi_k = r_k/r_k^{\max}$ as the *resource demand* of a user of class k. Note that the configuration of users $\boldsymbol{X}(t)$ can be entirely served if and only if the total resource demand satisfies the constraint

$$\sum_{k=1}^{J} \varphi_k X_k(t) \leq 1\,. \tag{3}$$

This is a *multi-rate linear resource constraint*.

3) Service policy: If the requested streaming rates are not achievable for a given configuration of users $\boldsymbol{X}(t)$ present at time t, then some classes of users will be temporarily put in outage at time t, meaning that they will receive some smaller bit-rates (whose values are not guaranteed and may depend on the configuration $\boldsymbol{X}(t)$). These smaller, "best-effort" bit-rates may drop to 0, in which case we say that users are in deep-outage. Let us recall that the times at which users are in outage and deep outage do *not* alter the original streaming times; i.e. the streaming content is not buffered, nor delayed during the outage periods.

We will define now a parametric family of service polices for which *classes with smaller resource demands have higher service priority*. In this regard, in the remaining part of the paper we assume (without loss of generality) that the resource demands of users from different classes are ordered $\varphi_1 < \varphi_2 < \ldots < \varphi_J$.

a) Least-effort-served-first policy: For a given configuration of users $\boldsymbol{X} = \boldsymbol{X}(t)$ requesting streaming at time t, *least-effort-served-first policy with δ-margin* (*LESF(δ)* for short)

[4]It was also shown in [16] how that the performance of AWGN *multiple input multiple output* (MIMO) channel can be approximated by taking values of $\gamma \geq 1$. Another possibility to consider MIMO channel is to use the exact capacity formula given in [17].

[5]From information theory point of view, the orthogonality assumption is not optimal. In fact, the theoretically optimal performance is offered by the *broadcast channel* model. It is known that in the case of AWGN broadcast channel the rates $\boldsymbol{r}$ are (theoretically) achievable for the configuration $\boldsymbol{X}$ if (and only if) there exists a vector $(\lambda_1, \ldots, \lambda_J)$, such that $\sum_{k=1}^{J} \lambda_k \leq 1$ and

$$X_k r_k = W \log\left(1 + \frac{\lambda_k}{1/\mathrm{SNR}_k + \sum_{i=1}^{k-1} \lambda_i}\right) \quad k = 1, \ldots, J,$$

where the classes of users are numbered such that $\mathrm{SINR}_1 \geq \mathrm{SINR}_2 \geq \ldots \geq \mathrm{SINR}_J$; cf [18, Eq. 6.29].

attributes the requested bit-rates to all users in classes $k = 1, \ldots, K$, where

$$K = K^{\delta}(\boldsymbol{X}) = \max\left\{k \in \{1, \ldots, J\} : \sum_{j=1}^{k-1} \varphi_j X_j + \varphi_k \sum_{j=k}^{J} X_j \mathbb{1}(\varphi_j \leq \varphi_k(1+\delta)) \leq 1\right\}, \tag{4}$$

where δ is a constant satisfying $0 \leq \delta \leq \infty$.

Remark 2.1: The LESF(0) policy is *optimal* in the following sense: given constraint (3) and the assumption that the classes with smaller resource demands have higher priority, this policy allows to serve the maximal subset of users present in the system. For the same reason any LESF(δ) policy with $\delta > 0$ is clearly sub-optimal. In order to explain the motivation for considering such policies, one needs to extend the model and explain what actually happens with classes of users which experience outage. In this regard, note that $C = \sum_{j=1}^{K} \varphi_j X_j \leq 1$ is the actual fraction of the server capacity consumed by the users which are not in outage. The remaining server capacity $1-C$ (which is not needed to serve users in classes $1, \ldots, K$) can be used to offer some "lower quality" service (e.g. streaming with lower video resolution, etc) to the users in classes $K+1, \ldots, J$ which are in outage. Note by (4) that the remaining server capacity under the policy LESF(δ) is at least

$$1 - C \geq \varphi_K \sum_{j=K+1}^{J} X_j \mathbb{1}(\varphi_j \leq \varphi_K(1+\delta)).$$

Hence, the server accepting the class K as the least-priority class being "fully" served, leaves enough remaining capacity to be able to make the same effort (allocate service capacity φ_K) for all users in outage in classes whose service demand exceeds φ_K by no more than $\delta \times 100\%$. These latter users will not have "full" required service (since this requires more resources, $\varphi_j > \varphi_K$, for the full service) but only some "lower quality" service (to be specified in what follows). Consequently, one can conclude that policies LESF(δ) with $\delta > 0$, being sub-optimal, ensure some *fairness*, in the sense explained above. Clearly the policy LESF(∞) (i.e., with $\delta = \infty$) is the most fair, in the sense that it reserves enough remaining capacity to offers the "lower quality" service for *all* users in outage (no deep outage). Thus, we will call LESF(∞) the *LESF fair* policy.

b) Best-effort service for users in outage: We will specify now a natural model for the "best-effort" streaming bit-rates that can be offered for users in outage in association with a given LESF(δ) policy. For $k > K = K^{\delta}(\boldsymbol{X})$ denote

$$r'_k = r'^{\delta}_k(\boldsymbol{X}) = r_k^{\max} \frac{1 - \sum_{j=1}^{K} x_j \varphi_j}{\sum_{j=K+1}^{J} x_j \mathbb{1}(\varphi_j \leq (1+\delta)\varphi_K)} \quad \text{if } \varphi_k \leq (1+\delta)\varphi_K \text{ and } 0 \text{ otherwise.} \tag{5}$$

Note that the rates $(r_1, \ldots, r_K, r'_{K+1}, \ldots, r'_J)$ are achievable for the configuration $\boldsymbol{X}$ under resource constraint (3). Note also that users in classes j such that $\varphi_j > (1+\delta)\varphi_K$ do not receive any positive bit-rate. We say, they are in *deep outage*.

4) Performance metrics: Configuration of users $X(t)$ evolves in time, it changes at arrival and departure times of users. At each arrival or departure epoch the base station applies the outage policy to the new configuration of users to decide which classes of users receive requested streaming rates and which are in outage (or deep outage).

Let us introduce the following characteristics of the *typical call (user)* of class $k = 1, \ldots, J$.

- P_k denotes the *probability of outage at the arrival epoch for class* k. This is the probability that the typical call of this class is put in outage immediately at its arrival epoch.
- D_k denotes the *mean total time spent in outage during the typical call of class* k.
- M_k denotes the *mean number of outage incidents experienced during the typical call of class* k.

More formal definitions of these characteristics, as well as other *system* characteristics (as e.g. the intensity of outage incidents) is given in the Appendix. We also introduce two further characteristics related to the mean *throughput* obtained during the typical call of class $k = 1, \ldots, J$.

- Denote by T_k the *mean throughput during the typical call of class* k. This is the mean bit-rate obtained during such a call, taking into account the bit-rate r_k when the call is not in outage and the best-effort bit rate r'_k obtained during the outage periods, averaged over call duration.
- Let T'_k be the *part of the throughput obtained during the outage periods of the typical call of class* k. This is the mean best-effort bit-rate of such call averaged over outage periods.

C. Model evaluation

1) Results: We will show how the performance metrics regarding outage incidents and duration, introduced in Section II-B4, can be expressed using probability distribution functions of some *linear* functionals of the random vector $X_1, \ldots, X_J$ of independent Poisson random variables with parameters ρ_j, respectively. Recall that these random variables correspond to the number of calls of different classes present in the stationary regime of our streaming model.

Specifically, for given $\delta > 0$, $k = 1, \ldots, J$ and $t \geq 0$ denote

$$F_k^\delta(t) := \mathbf{P}\left\{\sum_{j=1}^{k} X_j^{\delta,k}\varphi_j \leq t\right\},$$

where $X_j^{\delta,k} = X_j$ for $j = 1, \ldots, k-1$ and $X_k^{\delta,k} = \sum_{j=k}^{J} X_j \mathbb{1}(\varphi_j \leq \varphi_k(1+\delta))$.

The following results follow from the analysis of a more general model presented in the Appendix.

Proposition 2.2: *The probability of outage at the arrival epoch for user of class k is equal to*

$$P_k = 1 - F_k^\delta(1-\varphi_k) \qquad k = 1, \ldots, J\,. \tag{6}$$

The mean total time spent in outage during the typical call of class k is equal to

$$D_k = \frac{P_k}{\mu_k} = \frac{1 - F_k^\delta(1-\varphi_k)}{\mu_k} \qquad k = 1, \ldots, J\,. \tag{7}$$

The mean number of outage incidents experienced during the typical call of class k (after its arrival) is equal to

$$M_k = \frac{1}{\mu_k}\sum_{j=1}^{J}\lambda_j\Big(F_k^\delta(1-\varphi_k) - F_k^\delta(1-\varphi_k-\varphi_j)\Big) \qquad k = 1, \ldots, J\,. \tag{8}$$

Proof: The expression (6) follows from Proposition A.3 and the definition of LESF(δ) policy. Indeed, note that

$$1 - F_k^\delta(1-\varphi_k) = \mathbf{P}\left\{\sum_{j=1}^{k} X_j^{\delta,k}\varphi_j > 1-\varphi_k\right\}$$

is probability that for the steady state of the system appended with one user of class k, $\boldsymbol{X}' = (X_1, \ldots, X_k + 1, \ldots, X_J)$, we have the class k is in outage, i.e.; $k > K^\delta(\boldsymbol{X}')$. Similarly (7) follows from Proposition A.4 and (8) follows from Proposition A.5. ■

Regarding the throughput characteristics, we have the following result.

Proposition 2.3: *The mean throughput during the typical call of class k is equal to*

$$T_k = r_k(1-P_k) + T_k' = r_k F_k^\delta(1-\varphi_k) + T_k'\,,$$

where

$$T_k' = \mathbf{E}\left[r_k'^{\delta}(X_1, \ldots, X_k+1, \ldots, X_J)\mathbb{1}\Big(K^\delta(X_1, \ldots, X_k+1, \ldots, X_J) < k\Big)\right], \tag{9}$$

with the best-effort rate $r_k'(\cdot)$ given by (5) and the least-priority class $K^\delta(\cdot)$ begin served by the $LESF(\delta)$ policy given by (4), is the part of the throughput obtained during the outage periods.

Proof of this proposition is given in the Appendix.

Remark 2.4: Recall from (5) that the variable rates r'_k are obtained by the user of class k when he is in outage, i.e., $k > K$. They are non-null, $r'_k > 0$, only if $\varphi_k \leq (1+\delta)\varphi_K$. In the case of equal requested rates r_k, the intersection of the two conditions $0 < r'_k$ and $k > K$ is equivalent to

$$(1 + \mathrm{SINR}_K)^{1/(1+\delta)} - 1 \leq \mathrm{SINR}_k \leq \mathrm{SINR}_K \,. \tag{10}$$

2) Remarks on numerical evaluation: In order to be able to use the expressions given in (2.2) we need to evaluate the distribution functions $F_k^\delta(t)$. In what follows we show how this can be done using Laplace transforms. Regarding the throughput in outage T'_k, expressed in (9) as the expectation of a *non-linear* functional of the vector $(X_1, \ldots, X_J)$, we will use Monte Carlo simulations to obtain numerical values for this expectation.

Denote by $\mathcal{L}_k^\delta(\theta) := \int_0^\infty e^{-\theta s} F_k^\delta(s) \mathrm{d}s$ the Laplace transform of the function $F_k^\delta(t)$.

Fact 2.5: We have

$$\mathcal{L}_k^\delta(\theta) = \frac{1}{\theta} \exp\left[\sum_{j=1}^{k} \rho_j^{\delta,k} \left(e^{-\theta\varphi_j} - 1\right)\right] ,$$

where $\rho_j^{\delta,k} = \rho_j$ for $j = 1, \ldots, k-1$ and $\rho_k^{\delta,k} = \sum_{j=k}^{J} \rho_j \mathbb{1}(\varphi_j \leq \varphi_k(1+\delta))$.

Proof: Note that for given $\delta > 0$, $k = 1, \ldots, J$ the random variables $X_1^{\delta,k}, \ldots, X_k^{\delta,k}$ are independent, of Poisson distribution, with parameters $\rho_1^{\delta,k}, \ldots, \rho_k^{\delta,k}$, respectively. The result follows from [19, Proposition 1.2.2] and a general relation $\int_0^\infty e^{-\theta s} F(s)\, \mathrm{d}s = \frac{1}{\theta} \int_0^\infty e^{-\theta s} F(\mathrm{d}s)$. ■

The probabilities $F_k^\delta(\cdot)$ may be retrieved from $\mathcal{L}_k^\delta(\cdot)$ using standard techniques. For example [20, with the algorithm implemented by Hollenbeck [21] in Matlab]. In what follows we present a more explicit result based on the Bromwich contour inversion integral. In this regard, denote $\overline{\mathcal{L}}_k^\delta(\theta) = 1/\theta - \mathcal{L}_k^\delta(\theta)$ (which is the Laplace transform of complementary distribution function $1 - F_k^\delta(t)$). Also, denote by $\mathcal{R}(z)$ the real part of the complex number z.

Fact 2.6: We have

$$F_k^\delta(t) = 1 - \frac{2e^{at}}{\pi} \int_0^\infty \mathcal{R}\left(\overline{\mathcal{L}}_k^\delta(a+iu)\right) \cos ut \, \mathrm{d}u \,, \tag{11}$$

where $a > 0$ is an arbitrary constant.

Proof: See [22]. ■

Remark 2.7: As shown in [22], the integral in (11) can be numerically evaluated using the trapezoidal rule, with the parameter a allowing to control the approximation error. Specifically,

for $n = 0, 1, \ldots$ define

$$h_n(t) = h_n(t; a, k, \delta) := \frac{(-1)^n e^{a/2}}{t} \mathcal{R}\left(\overline{\mathcal{L}}_k^\delta\left(\frac{a + 2n\pi i}{2t}\right)\right),$$

$S_n(t) := \frac{h_0(t)}{2} + \sum_{i=1}^n h_i(t)$, and $S(t) = \lim_{n\to\infty} S_n(t)$. Then $\left|F_k^\delta(t) - (1 - S(t))\right| \leq e^{-a}$. Finally, the (alternating) infinite series $S(t)$ can be efficiently approximated using for example the Euler summation rule

$$S(t) \approx \sum_{i=0}^{M} \binom{M}{i} 2^{-M} S_{N+i}(t)$$

with a typical choice $N = 15$, $M = 11$.

III. Quality of real-time streaming in LTE

In this section we will use the model developed in Section II to evaluate the quality of RTS in LTE networks.

A. LTE model and traffic specification

1) SINR distribution: Recall that the main motivation for considering a multi-class model was the necessity to distinguish users with different radio conditions, related to different values of the SINR they have with respect to the serving base stations. In order to choose representative values of SINR in a given network and to know what fraction of users experience a given value, we need to know the *(spatial) distribution of the SINR* (with respect to the serving base station) experienced in this network (possibly biased by the spatial repartition of arrivals of streaming calls). This distribution can be obtained from real-network measurements, simulations or analytic evaluation of an appropriate spatial, stochastic model.[6] In this paper we will use the distribution of SINR obtained from the simulation compliant with the 3GPP recommendation in the so-called calibration case (to be explained in what follows). At present, assume simply, that we are given a cumulative distribution function (CDF) of the SINR expressed in dB, $F(x) := \mathbf{P}\{10\log_{10}(\text{SINR}) \leq x\}$, obtained from either of these methods. In other words, $F(x)$ represents the fraction of mobile users in the given network which experience the SINR (expressed in dB) not larger than x.

[6]For this latter possibility, we refer the reader to a recent paper on Poisson modeling of real cellular networks subject to shadowing [23], as well as to [24], completed in [25], where the distribution of the the SINR in Poisson networks is evaluated explicitly.

Consider a discrete probability mass function

$$p_k := F\Big(\frac{x_{k+1}+x_k}{2}\Big) - F\Big(\frac{x_k+x_{k-1}}{2}\Big) \qquad k = 1, 2, \ldots, J\,, \tag{12}$$

with $x_0 = -\infty$, $x_{J+1} = \infty$. We define the class $k = 1, \ldots, J$ of users as all users having the SINR expressed in dB in the interval $\big((x_k + x_{k-1})/2, (x_{k+1} + x_k)/2\big]$, and approximate their SINR by the common value $\text{SINR}_k = 10^{x_k/10}$. Clearly p_k is the fraction of mobile users in the given network which experience the SINR close to SINR_k. Hence, in the case of a homogeneous streaming traffic (the same requested streaming rates and mean streaming times, which will be our default assumption in the numerical examples) we can assume the intensity of arrivals λ_k of users of class k to be equal to $\lambda_k = p_k\lambda$ where $\lambda = \sum_{i=k}^{J} \lambda_k$ is the total arrival intensity (per unit of time per serving base station) to be specified together with the CDF F of the SINR.

a) CDF of the SINR for 3GPP recommendation: We obtain the CDF F of SINR from the simulation compliant with the 3GPP recommendation in the so-called calibration case, (compare to [26, Figure A.2.2-1(right)]). More precisely, we consider the geometric pattern of BS placed on the 6×6 hexagonal lattice. In the middle of each hexagon there are three symmetrically oriented BS antennas, which gives a total of 108 BS antennas. The distance between the centers of two neighboring hexagons is $0.5\,$km. Each BS antenna is characterised by the following horizontal pattern $A(\phi) = -\min(12(\phi/\theta)^2, A_m)$, where ϕ is the angle in degrees, with $\theta = 70°$, $A_m = 20dB$, and uses transmission power $P = 60$dBm (including omnidirectional gain of 14dBi). The distance-loss model (corresponding to the frequency carrier 2GHz) is $L(r) = 128.1 + 37.6\log_{10}(r)$[dB] where r is the distance in km. A supplementary penetration loss of 20dB is added. The shadowing is modeled as a centered log-normal random variable of standard deviation 8dB. The noise power equals -95dBm. In order to obtain the empirical CDF of the SINR we generate 3600 random user locations uniformly in the network (100 user locations per hexagon on average). Each user is connected to the antenna with the strongest received signal (smallest propagation-loss including distance, shadowing and antenna pattern) and the SINR is calculated. The obtained empirical CDF F of the SINR is shown on Figure 1.

2) Link characteristics: 3GPP shows in [27, §A.2] that there is a 25% gap between the practical coding schemes and the Shannon's limit for the AWGN channel. Moreover, some of the transmitted bits are used for signaling, which induces a supplementary capacity loss of about 30% (see [28, §6.8]). This made us assume $\gamma = 0.5(\approx 0.75(1-0.3))$ in (2). The

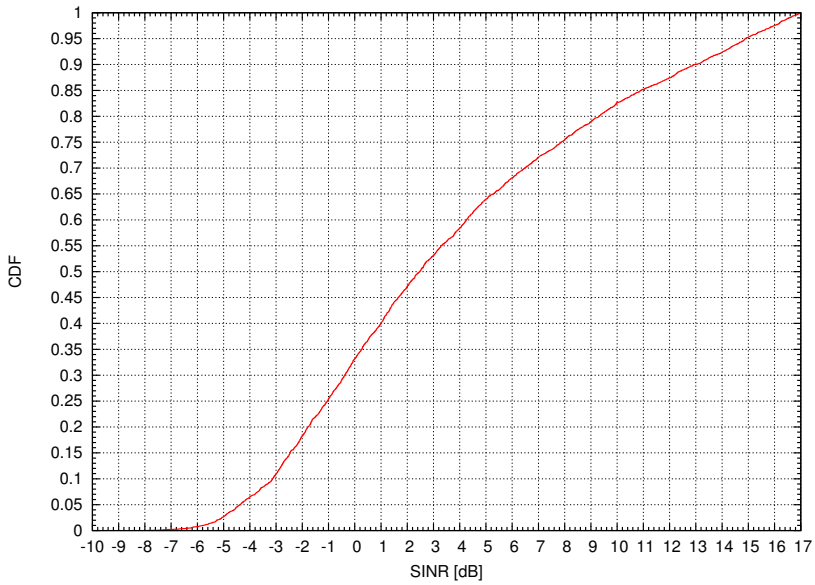

Fig. 1. Cumulative distribution function of the SINR obtained according to 3GPP specification; see Section III-A1a.

system bandwidth is $W = 10$MHz.

3) Streaming traffic: We assume that all calls require the same streaming rate $r_k = 256$ kbit/s and have the same streaming call time distribution. We split them into $J = 100$ user classes characterized by values of the SINR falling into different intervals regularly approximating the SINR domain from $x_1 = -10$dB to $x_J = 17$dB as explained in Section III-A1. In our performance evaluation we will consider two values of the spatially uniform traffic demand: 900 and 600 Erlang/km^2. (All results presented in what follows do not depend on the mean streaming time but only on the traffic demand). Consequently, kth class traffic demand per unit of surface is equal to, respectively, $p_k \times 900$ and $p_k \times 600\,\text{Erlang/km}^2$, where p_k are given by (12). Multiplying by the surface served by one base station equal to $\sqrt{3} \cdot (0.5\,\text{km})^2/6 \approx 0.0722\,\text{km}^2$ we obtain the traffic demand per cell, per class, equal to $\rho_k = p_k \times 900 \times 0.0722 \approx p_k \times 64.9$ Erlang and $\rho_k = p_k \times 600 \times 0.0722 \approx p_k \times 43.3$ Erlang, respectively, for the two studied scenario.

B. Performance evaluation

Assuming the LTE and traffic model described above, we consider now streaming policies LESF(δ) defined in Section II-B3. Recall that in doing so, we assume that users are served by the antenna offering the smallest path-loss, and dispose orthogonal down-link channels,

with the maximal rates $r_k^{\max}$ depending on the value of the SINR (interference comes from non-serving BS) characterizing class k. Roughly speaking, LESF(δ) policy assigns the total requested streaming rate $r_k = 256$kbit/s for the maximal possible subset of classes in the order of decreasing SINR, leaving some capacity margin to offer some "best-effort" streaming rates for (some) users remaining in outage. These streaming rates r'_k given by (5) depend on the current configuration of users and are non-zero for users with SINR within the interval $(1 + \text{SINR}_K)^{1/(1+\delta)} - 1 \leq \text{SINR} \leq \text{SINR}_K$, where SINR_K is the minimal value of SINR for which users are assigned the total requested streaming rate; cf Remark 2.4. In particular, LESF(0), called the *optimal* policy, leaves no capacity margin for users in outage, while LESF(∞), called the *fair* one, offers a "best-effort" streaming rate for all users in outage at the price of assigning the full requested rate 256kbit/s to a smaller number of classes (higher value of SINR_K) [7]. In what follows, we use our results of Section II-C to evaluate performance of these streaming policies in the LTE network model.

1) Outage time: Figure 2 shows the mean time of the streaming call spent in outage normalized by call duration, $\mu_k D_k$, evaluated using (7), in function of the SINR value characterizing class k, for the traffic 900 Erlang/km^2 and different policies LESF(δ). Figure 3 shows the analogous results assuming traffic load of 600 Erlang/km^2. The main observations are as follows:

- All LESF policies exhibit a cut-off behaviour: the fraction of time in outage drops rapidly from 100% to 0% when SINR transgresses some critical values. This cut-off is more strict for the optimal policy.
- For the traffic of 900 Erlang/km^2, users with SINR$\geq$ 3dB are practically never in outage, when the optimal policy is used. The same holds true for users with SINR$\geq$ 13dB, when the fair policy is used.
- When the traffic drops to 600 Erlang/km^2, these critical values of SINR decrease by 2dB and 5dB, respectively, for the optimal and the fair policy. Note that the fair policy is more sensitive to higher traffic load.

2) Number of outage incidents: Figure 4 shows the mean number of outage incidents per streaming call, M_k evaluated using (8), in function of the SINR value characterizing class k, for the traffic 900 Erlang/km^2 and different policies LESF(δ). (Recall that we assume the same streaming time distribution for all users, and hence $\lambda_j/\mu_k = \rho_j$ making the expression

[7]The LESF fair policy seems to be adopted in some implementations of the LTE.

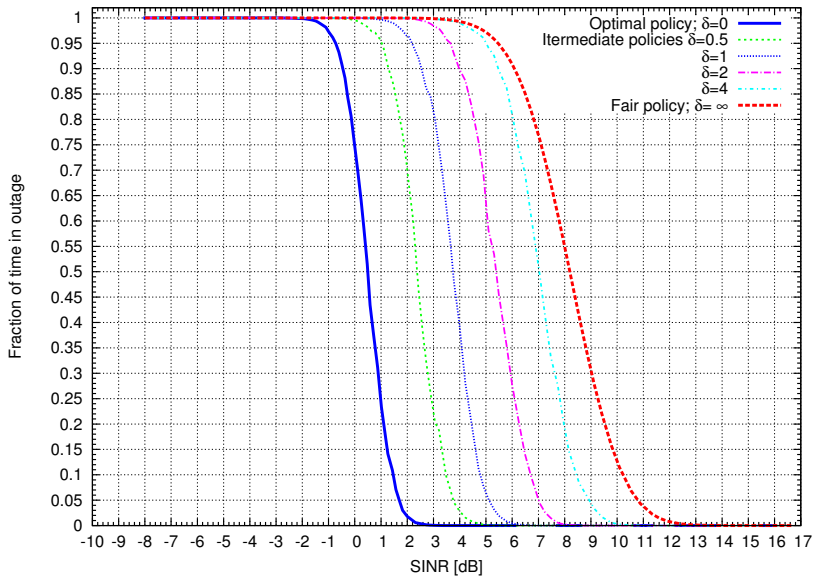

Fig. 2. Mean fraction of the requested streaming time in outage, in function of the user SINR for different policies LESF(δ); traffic 900 Erlang/km^2.

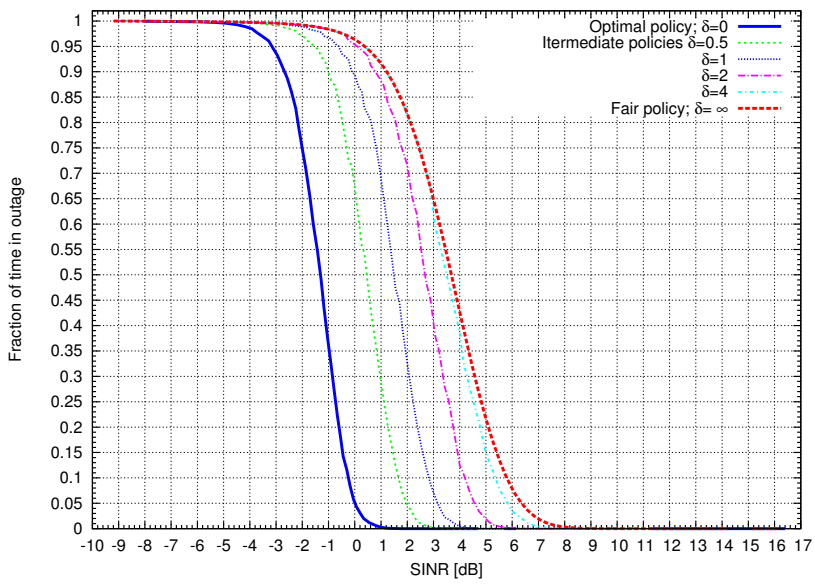

Fig. 3. Fraction of time in outage as on Figure 2 for traffic 600 Erlang/km^2.

in (8) depend only on the vector of traffic demand per class.) Figure 5 shows the analogous results assuming traffic demand of 600 Erlang/km^2. The main observations are as follows:

- For all policies, the number of outage incidents (during the service) is non-zero only for users with the SINR close to the critical values revealed by the analysis of the outage times. Users with SINR below these values are constantly in outage while users with SINR above them never in outage.
- More fair policies generate slightly more outage incidents. The worst values are 2 to

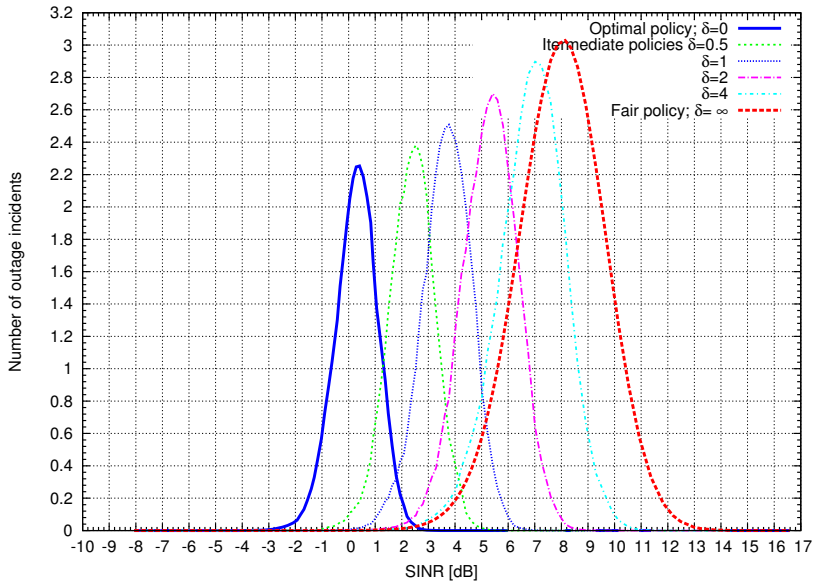

Fig. 4. Number of outage incidents during the requested streaming time, in function of the user SINR for different policies LESF(δ); traffic 900 Erlang/km^2.

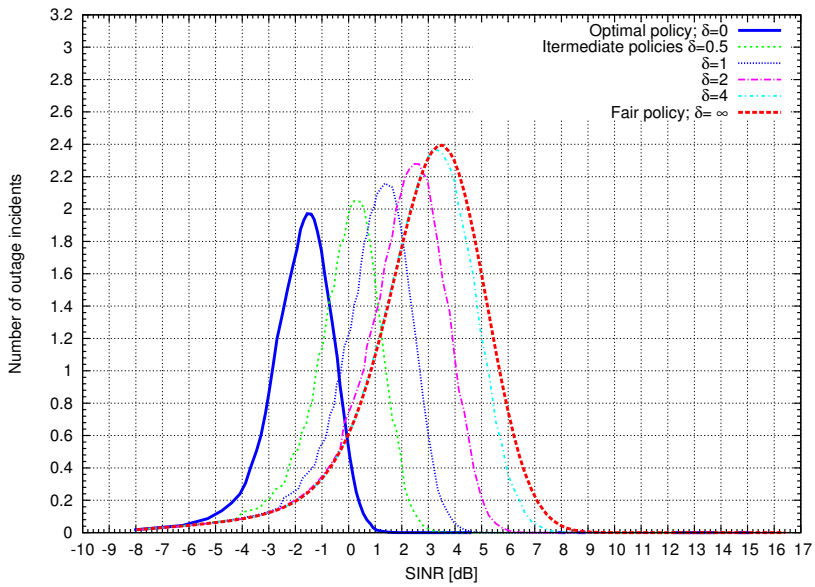

Fig. 5. Number of outage incidents as on Figure 4 for traffic 600 Erlang/km^2.

2.2 interruptions per call for the optimal policy, depending on the traffic value, and 2.4 to 3 interruptions per call for the fair policy.

Studying outage times and outage incidents we do not see apparent reasons for considering fair policies. This motivates our study of the best-effort service in outage.

3) The role of the "best effort" service: Figure 6 shows the fraction of time spent in deep outage in function of the SINR, assuming traffic 900 Erlang/km^2. These values should be compared to the fraction of time spend in outage (for convenience copied on Figure 6 from

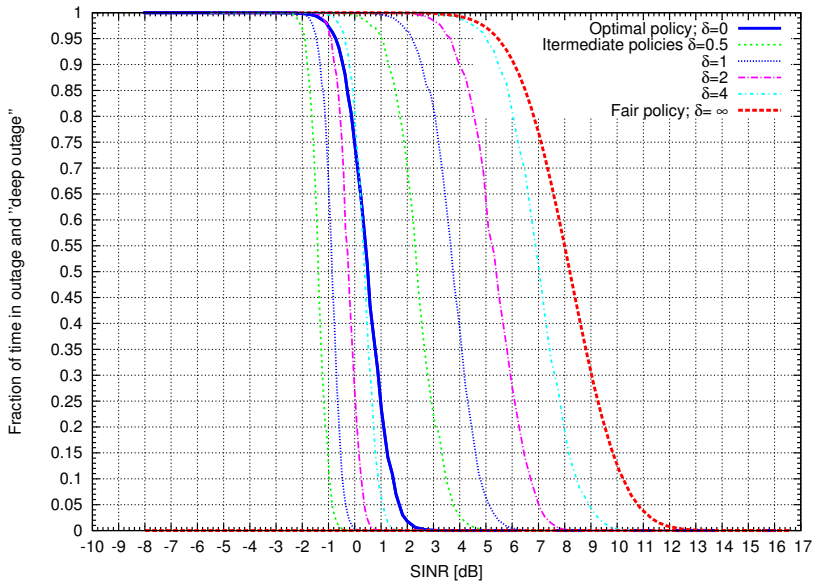

Fig. 6. Deep outage versus outage time. For any policy LESF(δ), with $0 < \delta < \infty$, the left curve of a given style represents the fraction of time spent in deep outage. The right curve of a given style recalls the fraction of time spent in outage (already plotted on Figure 2). The optimal policy ($\delta = 0$) does not offer any "best effort" service. The fair policy ($\delta = \infty$) offers this service for all users in outage.

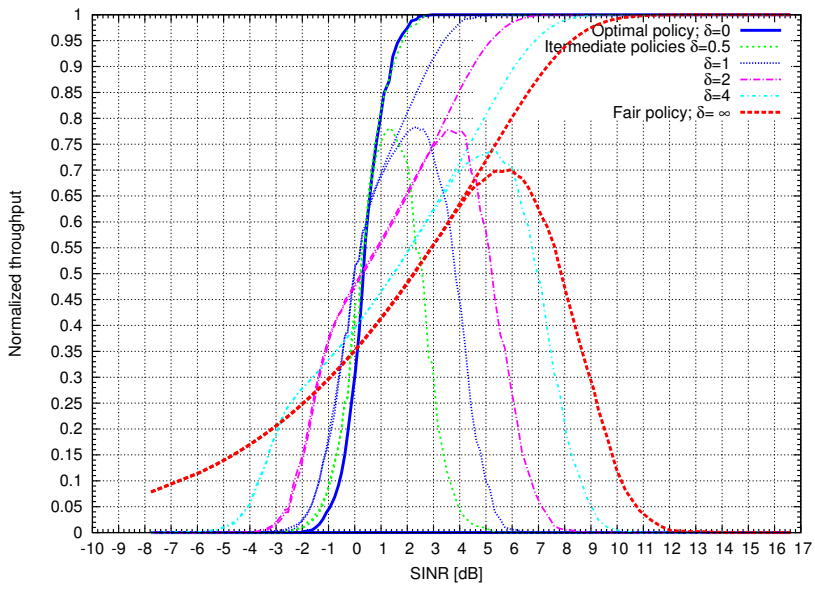

Fig. 7. Mean total throughput normalized to its maximal value 256kbit/s obtained during the service time (upper curves) and its fraction obtained when user are in outage (lower curves) for different policies LESF(δ) traffic 900 Erlang/km^2.

Figure 2). Recall, users in outage do not receive the full requested streaming rate (assumed 256kbit/s in our example), however they do receive some non-null "best effort" rates given by (5), unless they are in deep outage — have SINR to small; cf Remark 2.4. Considering users in outage but not in deep outage as "partially satisfied", increasing fairness margin δ allows to (at least) partially satisfy users with decreasing SINR values. Obviously the level of

the "partial satisfaction" depends on the throughput obtained in outage periods, which is our quantity of interest on Figure 7. It shows also two curves for all policies LESF(δ) assuming traffic 900 Erlang/km^2. The upper ones represent the mean total throughput realized during the service, normalized to its maximal value; i.e., $T_k/(256\text{kbit/s})$, in function of the SINR value characterizing class k. The fractions of this throughput realized during outage periods, $T'_k/(256\text{kbit/s})$, are represented by the lower curves.

Figures 7 and 6 teach us that the role of the LESF(δ) policies with $\delta > 0$ may be two-fold.

- LESF(δ) policies with small values of δ, e.g. $\delta = 0.5$, *improve "temporal homogeneity" of service with respect to the optimal policy, for users having SINR near the critical value.* For example, a user having SINR equal to 1dB is served by the optimal policy during 80% of time with the full requested streaming rate (cf. Figure 6). However, for the remaining 20% of time it does not receive any service (deep outage, rate 0bits/s). The policy LESF(0.5) offers to such a user 80% of the requested streaming rate during the whole streaming time (cf. Figure 7), with no deep outage periods (cf. Figure 6). The price for this is that a slightly higher SINR is required to receive the full requested streaming rate (at least 5dB, instead of 3dB for the optimal policy).
- The fair policy LESF(∞) *improves the spatial homogeneity of service*. It leaves no user in deep outage, however a much larger SINR$=$ 13dB is required for not to be in outage (cf. Figure 6). Moreover, the throughput of all users in outage but not in deep outage is substantially reduced e.g. from 80% to 40% for SINR$=$ 1dB, with respect to some intermediate LESF(δ) policies (with $0 < \delta < \infty$). These intermediate policies can offer an interesting compromise between the optimality and fairness.

IV. Conclusions

In this paper, a real-time streaming (RTS) traffic, as e.g. mobile TV, is analyzed in the context of wireless cellular networks. An adequate stochastic model is proposed to evaluate user performance metrics, such as frequency and number of interruptions during RTS calls in function of user radio conditions. Despite some fundamental similarities to the classical Erlang loss model, a new model was required for this type of service, where the service denials are not definitive for a given call, but only temporal – having the form of, hopefully short, interruptions (outage) periods. Our model allows to take into account realistic implementations of the RTS service, e.g. in the LTE networks. In this latter context, several

numerical demonstrations are given, presenting the quality of service metrics in function of user radio conditions.

APPENDIX
A GENERAL REAL-TIME STREAMING (RTS) MODEL

In this section we will present a general stochastic model for real-time streaming. An instantiation of this model was used in the main body of the paper to evaluate the real-time streaming in wireless cellular networks. This model comprises Markovian, multi-class process of call arrivals and their independent, arbitrarily distributed streaming times. These calls are served by a server whose service capacity is limited. Depending on numbers of calls of different classes present in the system, the server may not be able to serve some classes of users. If such a congestion occurs, these classes are temporarily denied the service, until the next call arrival or departure, when the situation is reevaluated. These service denial periods, called outage periods, do not alter the call sojourn times in the system. Our model allows for a very general service (outage) policy saying which classes of users are temporarily denied the service due to insufficient service capacity. We will evaluate key characteristics of this model using the formalism of point processes and their Palm theory, often used in the modern approach to stochastic networking [29]. Specifically, we are interested in the intensity of outage incidents, the mean inter-outage times and the outage durations of a given class, seen from the server perspective, as well as the probability of outage at the arrival epoch, mean total time in outage and mean number of outage incidents experienced by a typical user of a given class. The expressions developed for these characteristics involve only stationary probabilities of the (free) traffic demand process, which in our case is a vector of independent Poisson random variables. Recall that such a representation is possible e.g. for the well known Erlang-B formula, giving the blocking probability in the classical (possibly multi-class) Erlang's loss model. Indeed, our model can be seen as an extension of the classical loss model, where the losses (i.e., service denials) are not definitive for a given call, but only temporal — having the form of outage periods.

A. *Traffic demand*

Consider $J \geq 1$ classes of users identified with calls. We assume that users of class $k \in \{1, \ldots, J\}$ arrive in time according to a Poisson process $N_k = \{T_n^k : n\}$ with intensity $\lambda_k > 0$ and stay in the system for independent requested streaming times W_n^k having some

general distribution with mean $1/\mu_k < \infty$. All the results presented in what follows do not depend on the particular choice of the streaming time distributions — the property called in the queueing-theoretic context *insensitivity property*. Denote by $\tilde{N}_k = \{(T_n^k, W_n^k) : n\}$ the process of arrival epochs and streaming times (call durations) of users of class k. We assume that $\tilde{N}_k$ are independent across $k = 1, \ldots, J$. Denote by $X_k(t) = \sum_n \mathbb{1}_{[T_n^k, T_n^k + W_n^k)}(t)$, with $\mathbb{1}_A(x) = 1$ being the indicator function of set A, the number of users of class k present in the system at time t and let $\boldsymbol{X}(t) = (X_1(t), \ldots, X_J(t))$; we call it the (vector of) user configuration at time t. The stationary distribution π of $\boldsymbol{X}(t)$ coincides with the distribution of a vector of independent Poisson random variables $(X_1, \ldots, X_J)$ with means $\mathbf{E}[X_k] := \rho_k = \lambda_k/\mu_k$, $k = 1, 2, \ldots, J$. We call ρ_k the *traffic demand* of class k.

We adopt the usual convention for the numbering of the arrival epochs $T_0^k \leq 0 < T_1^k$. The same convention is used with respect to all point processes denoting some time epochs.

B. Resource constraints and outage policy

For class $k = 1, \ldots, J$, let a subset of user configurations $\mathcal{F}_k \subset \bar{\mathbb{N}}^J$ be given, where $\bar{\mathbb{N}} = \{0, 1, \ldots\}$, such that all X_k users of class k present in the configuration $\boldsymbol{X} = (X_1 \ldots, X_k, \ldots, X_J)$ are served if and only if $\boldsymbol{X} \in \mathcal{F}_k$ and no user of class k is served (we say it is in *outage*) if $\boldsymbol{X} \notin \mathcal{F}_k$. We call $\mathcal{F}_k$ the *kth class (service) feasibility set*. Denote by $\pi_k = \pi(\mathcal{F}_k)$ the probability that the stationary configuration of users is in kth class feasibility set.

We assume that, upon each arrival or departure of a user, the system updates its decision and, for any class k, it assigns the service to all users of class k if the updated configuration of users is in $\mathcal{F}_k$. All users of any class j for which the updated configuration is in $\mathcal{F}'_k = \bar{\mathbb{N}}^J \setminus \mathcal{F}_k$ will be placed in outage (at least) until the next user arrival or departure.

In what follows we will assume that no user departure can cause outage of any class of users i.e., switch a given configuration from $\mathcal{F}_k$ to $\mathcal{F}'_k$. (However a user departure may make some class j switch from $\mathcal{F}'_j$ to $\mathcal{F}_j$.)

Denote by $\tilde{X}_k(t) := X_k(t)\mathbb{1}_{\mathcal{F}_k}(\boldsymbol{X}(t))$ the number of users of class k *not in outage* at time t. Denote by $\tilde{\boldsymbol{X}}(t) = (\tilde{X}_i(t), \ldots, \tilde{X}_J(t))$ the configuration of users *not* in outage at time t.

C. Performance metrics

In what follows we will be interested in the following characteristics of the model.

1) Virtual system metrics: During its time evolution, the user configuration $\boldsymbol{X}(t)$ alternates visits in the feasibility set $\mathcal{F}_k$ and its complement $\mathcal{F}'_k$, for each class $k = 1, \ldots, J$. We are interested in the expected visit durations in theses sets as well as the intensities (frequencies) of the alternations. More formally, for each given $k = 1, \ldots, J$, we define the point process $B_k := \{\tau_n^k : n\}$ of exit epochs of $\boldsymbol{X}(t)$ from $\mathcal{F}_k$; i.e., all epochs t such that $\Big(\boldsymbol{X}(t-), \boldsymbol{X}(t)\Big) \in \mathcal{F}_k \times \mathcal{F}'_k$ (with the convention $\tau_0^k \leq 0 < \tau_1^k$). These are epochs when all users of class k present in the system (if any) have their service interrupted.

Denote by $\sigma_n'^k := \sup\{t - \tau_n^k : \boldsymbol{X}(s) \in \mathcal{F}'_k \ \forall s \in [\tau_n^k, t)\}$ the duration of the n th visit of the process $\boldsymbol{X}(t)$ in $\mathcal{F}'_k$ and by $\sigma_n^k := \tau_{n+1}^k - \tau_n^k - \sigma_n'^k$ the duration of the n th visit of the process $\boldsymbol{X}(t)$ in $\mathcal{F}_k$. We define for each class $k = 1, \ldots, J$:

- *The intensity of outage incidents of class k*, i.e., the mean number of outage incidents of this class per unit of time

$$\Lambda_k := \lim_{T\to\infty} \frac{1}{T} \sum_n \mathbb{1}_{[0,T)}(\tau_n^k)\,.$$

  Obviously Λ_k is also the intensity of entrance to the k th class feasibility set $\mathcal{F}_k$.

- *The mean service time between two outage incidents of class k*

$$\bar{\sigma}_k := \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \sigma_n^k\,.$$

- *The mean outage duration of class k*

$$\bar{\sigma}'_k := \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \sigma_n'^k\,.$$

Note that the above metrics characterize a "virtual" quality of the service, since some visits in $\mathcal{F}_k$ and $\mathcal{F}'_k$ may occur when there is no k th class user in the system (in the latter case the outage of this class is not experienced by any user).

2) User metrics: We adopt now a user point of view on the system. We define for each class $k = 1, \ldots, J$:

- *The probability of outage at the arrival epoch for user of class k*

$$P_k = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}_{\mathcal{F}'_k}(\boldsymbol{X}(T_n^k))\,.$$

- *The mean total time in outage of user of class k*

$$D_k = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \int_{[T_n^k, T_n^k + W_n^k)} \mathbb{1}_{\mathcal{F}'_k}(\boldsymbol{X}(t))\,\mathrm{d}t\,.$$

- *The mean number of outage incidents experienced by user of class k after its arrival*

$$M_k = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \sum_{m} \mathbb{1}_{(T_n^k, T_n^k + W_n^k)}(\tau_m^k) \,.$$

  Note that eventual outage experienced at the arrival of a given user is not counted in M_k. The mean total number of outage incidents (including possibly at the arrival epoch) is hence $P_k + M_k$.

For a given class $k = 1, \ldots, J$, denote by $\varepsilon_k = (0, \ldots, 1, \ldots, 0) \in \bar{\mathbb{N}}^J$ the unit vector having its kth component equal to 1. Hence $\boldsymbol{x} + \varepsilon_k$ represents adding one user of class k to the configuration of users $\boldsymbol{x} \in \bar{\mathbb{N}}^J$. Denote by $\mathbf{P}$ the probability under which $\{\boldsymbol{X}(t) : t\}$ is stationary and by $\mathbf{E}$ the corresponding expectation. Recall that $\pi\{\boldsymbol{x} \in \cdot\} = \mathbf{P}\{\boldsymbol{X}(t) \in \cdot\}$ is the distribution of the stationary configuration of users $\boldsymbol{X}(t)$ (it corresponds to independent Poisson variables of mean ρ_k).

D. General results

We present first results regarding the virtual system metrics. These results will be next used to evaluate the user metrics.

Lemma A.1: *The intensity of outage incidents of class k is $\mathbf{P}$-almost surely equal to*

$$\Lambda_k = \sum_{j=1}^{J} \lambda_j \pi \left\{ \boldsymbol{x} \in \mathcal{F}_k, \boldsymbol{x} + \varepsilon_j \in \mathcal{F}'_k \right\} \qquad k = 1, \ldots, J.$$

Proof: Let $N = \sum_{j=1}^{J} N_j$ be the point process counting the arrival times of users of all classes. By independence, N is the Poisson point process of intensity $\lambda = \sum_{j=1}^{J} \lambda_j$. Then, by the ergodicity of the process $\{\boldsymbol{X}(t) : t\}$ and the fact that the exists from $\mathcal{F}_k$ can take place only at some user arrival epoch we have by the Campbell formula [cf. e.g. 29, Equation (1.2.19)[8]],

$$\Lambda_k = \mathbf{E}\left[\int_{[0,1)} \mathbb{1}_{\mathcal{F}_k \times \mathcal{F}'_k} \left(\boldsymbol{X}\left(t-\right), \boldsymbol{X}\left(t\right) \right) N\left(\mathrm{d}t\right) \right] = \lambda \mathbf{P}_N^0 \left\{ \boldsymbol{X}(0-) \in \mathcal{F}_k, \boldsymbol{X}(0) \in \mathcal{F}'_k \right\} ,$$

where $\mathbf{P}_N^0$ designates the Palm probability associated to N (which is, roughly speaking, the conditional probability given an arrival at time 0). By PASTA (Poisson Arrivals See Time Averages) property [cf. 29, Equation (3.3.4)] the configuration of users $\boldsymbol{X}(0-)$ under $\mathbf{P}_N^0$ has distribution π. Moreover, $\boldsymbol{X}(0) = \boldsymbol{X}(0-) + \varepsilon_\xi$ where $\xi \in \{1, \ldots, J\}$ is under $\mathbf{P}_N^0$ independent of $X(0-)$ and takes value j with probability λ_j/λ. This completes the proof. ■

[8] with $Z_n := (\boldsymbol{X}(T_n-), \boldsymbol{X}(T_n))$ and $f(t,z) = \mathbb{1}_{[0,1)}(t)\mathbb{1}_{\mathcal{F}_k \times \mathcal{F}'_k}(z)$

Lemma A.2: *The mean service time between two outage incidents and the mean outage duration of class k are $\mathbf{P}$-almost surely equal to, respectively,*

$$\bar{\sigma}_k = \frac{\pi\left(\mathcal{F}_k\right)}{\Lambda_k}, \quad \bar{\sigma}'_k := \frac{\pi(\mathcal{F}'_k)}{\Lambda_k} \qquad k = 1, \ldots, J,$$

where Λ_k is given in Lemma A.1.

Proof: First we prove the expression for $\bar{\sigma}_k$. By ergodicity $\bar{\sigma}_k = \mathbf{E}^0_{B_k}\left[\sigma^k_0\right]$ $\mathbf{P}$-almost surely, where $\mathbf{E}^0_{B_k}$ designates the expectation with respect to the Palm probability associated to B_k, and $\mathbf{E}^0_{B_k}\left[\tau^k_0\right] = 1/\Lambda_k$; [see e.g. 29, Equation (1.6.8) and Equation (1.2.27)]. Applying the mean value formula [see 29, Equation (1.3.2)[9]] we get $\pi(\mathcal{F}_k) = \Lambda_k \mathbf{E}^0_{B_k}\left[\sigma^k_0\right]$, which completes the proof of the expression for $\bar{\sigma}_k$. For the other expression, note by the definition of the sequence σ^k_n, σ'^k_n and τ^k_n that $\mathbf{P}$-almost surely,

$$\bar{\sigma}'_k = \mathbf{E}^0_{B_k}\left[\sigma'^k_0\right] = \mathbf{E}^0_{B_k}\left[\tau^k_1 - \sigma^k_0\right] = \frac{1}{\Lambda_k} - \frac{\pi(\mathcal{F}_k)}{\Lambda_k} = \frac{\pi(\mathcal{F}'_k)}{\Lambda_k},$$

which completes the proof. ■

Proposition A.3: *The probability of outage at the arrival epoch for user of class k is equal to*

$$P_k = \pi\left\{\boldsymbol{x} + \varepsilon_k \in \mathcal{F}'_k\right\} \qquad k = 1, \ldots, J \tag{13}$$

$\mathbf{P}$-almost surely.

Proof: By ergodicity we have $P_k = \mathbf{P}^0_{N_k}\left\{\boldsymbol{X}(0) \in \mathcal{F}'_k\right\}$, where $\mathbf{P}^0_{N_k}$ designates the Palm probability associated to N_k (arrival process of the users of class k). By PASTA property the configuration of users $\boldsymbol{X}(0-)$, just before arrival of the user of class k at time 0, has distribution π. Once the user enters the system, the users configuration becomes $\boldsymbol{X}(0-)+\varepsilon_k$, whence the result. ■

Proposition A.4: *The mean total time in outage of user of class k is $\mathbf{P}$-almost surely equal to*

$$D_k = \frac{1}{\mu_k}\pi\left\{\boldsymbol{x} + \varepsilon_k \in \mathcal{F}'_k\right\} \qquad k = 1, \ldots, J\,.$$

Proof: Again using the ergodicity of $\{\boldsymbol{X}(t)\}$ we can write

$$D_k = \mathbf{E}^0_{N_k}\left[\int_{[0,W^k_0)} \mathbb{1}_{\mathcal{F}'_k}(\boldsymbol{X}(t))\,\mathrm{d}t\right].$$

Denote by $\boldsymbol{Y}(t) := \boldsymbol{X}(t) - \varepsilon_k \mathbb{1}_{[T^k_0, T^k_0+W^k_0)}(t)$ the process of configurations of users other than the user number 0 of class k (which arrives at time 0 under $\mathbf{E}^0_{N_k}$). By Slivnyak theorem [see

[9] with $Z_k(t) = \mathbb{1}_{\mathcal{F}_k}(\boldsymbol{X}(t))$

e.g. 19, Theorem 1.13] the distribution of the process $\{\boldsymbol{Y}(t):t\}$ under $\mathbf{E}^0_{N_k}$ is the same as this of $\{\boldsymbol{X}(t):t\}$ under $\mathbf{P}$. Using the fact that W_0^k and $\boldsymbol{Y}(t)$ are independent under $\mathbf{P}^0_{N_k}$ with $\mathbf{E}^0_{N_k}[W_0^k]=1/\mu_k$ we obtain

$$D_k=\int_0^\infty \mathbf{E}^0_{N_k}\left[\mathbb{1}_{[0,W_0^k)}(t)\mathbb{1}_{\mathcal{F}'_k}(\boldsymbol{Y}(t)+\varepsilon_k)\right]\,\mathrm{d}t=\frac{1}{\mu_k}\,\pi\left\{\boldsymbol{x}+\varepsilon_k\in\mathcal{F}'_k)\right]\,,$$

which completes the proof. ■

Proposition A.5: *The mean number of outage incidents experienced by user of class k after its arrival is $\mathbf{P}$-almost surely equal to*

$$M_k=\frac{1}{\mu_k}\sum_{j=1}^{J}\lambda_j\pi\left\{\boldsymbol{x}+\varepsilon_k\in\mathcal{F}_k,\boldsymbol{x}+\varepsilon_k+\varepsilon_j\in\mathcal{F}'_k\right\},\qquad k=1,\dots,J\,. \tag{14}$$

Proof: Again using the ergodicity of $\{X(t)\}$ we know that, $\mathbf{P}$-almost surely,

$$M_k=\mathbf{E}^0_{N_k}\left[\int_{(0,W_0^k)}B_k(\mathrm{d}t)\right]\,.$$

Using the same arguments as these used in the proof of Proposition A.4 we obtain

$$M_k=\mathbf{E}^0_{N_k}\left[B_k^*(0,W_0^k)\right]=\frac{\Lambda_k^*}{\mu_k}$$

where $B_k^*=:\{\tau_n^{*k}:n\}$ is the point process of exit epochs of $\boldsymbol{X}(t)$ from $\mathcal{F}_k^*=\{\boldsymbol{x}:\boldsymbol{x}+\varepsilon_k\in\mathcal{F}_k$ and Λ_k^* its intensity. Using Lemma A.1 with $\mathcal{F}_k$ replaced by $\mathcal{F}_k^*$ concludes the proof. ■

We will now prove the result regarding the throughput of the typical call of class k.

Proof of Proposition 2.3: We have

$$T_k=T_k^\delta=\mu_k\mathbf{E}^0_{N_k}\left[\int_{[0,W_0^k)}r_k\mathbb{1}(\boldsymbol{X}(t)\in\mathcal{F}_k^\delta)+r_k'^{\delta}(\boldsymbol{X}(t))\mathbb{1}(\boldsymbol{X}(t)\notin\mathcal{F}_k^\delta)\,\mathrm{d}t\right]\,.$$

It is easy to see, as in the proof of Proposition A.4, that $T_k=r_k\pi\left\{\boldsymbol{x}+\varepsilon_k\in\mathcal{F}_k^\delta\right\}+T'_k$, where

$$T'_k=\mathbf{E}\left[r_k'^{\delta}(\boldsymbol{X}(t)+\varepsilon_k)\mathbb{1}((\boldsymbol{X}(t)+\varepsilon_k)\notin\mathcal{F}_k^\delta)\right] \tag{15}$$

is the part of the throughput obtained by user of class k during its outage time. ■

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