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► **To cite this version:**

Olivier Bouré, Nazim Fatès, Vincent Chevrier. First steps on asynchronous lattice-gas models with an application to a swarming rule. [Research Report] 2012, pp.10. hal-00687987v1

HAL Id: hal-00687987

<https://inria.hal.science/hal-00687987v1>

Submitted on 16 Apr 2012 (v1), last revised 14 Jun 2012 (v2)

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First steps in asynchronous lattice-gas models: application to a swarming rule

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April 16, 2012

Abstract

Taking over studies on asynchronous cellular automata, the perfect synchrony of lattice-gas models is now questioned, in which additional constraints impose to redefine asynchronism according to the model requirements, such as conservation of the number of particles. Several definitions of an asynchronism LGCA adapted to these hypotheses are proposed, and applied to a model of swarming, in order to explore the robustness of the original behaviour under those novel dynamics.

keywords: Asynchronous cellular automata, Lattice-Gas cellular automata, Robustness, Swarming behaviour

1 Introduction

In the field of discrete dynamical systems, cellular automata (CA) are widely used as a tool for the simulation of natural phenomena and as a model of parallel computing. Their discrete and spatially-extended structure makes them a computationally simple model yet capable of displaying a wide range of complex behaviours. While initial studies only considered a simultaneous updating of all components, recent years have seen an increasing interest in asynchronous CA models, where the perfect synchrony hypothesis is relaxed. This change in hypotheses widened the study of CA to novel properties. We are here particularly interested in *robustness*, which characterises the tolerance of the behaviour to external disturbances and structural changes. In the case of asynchronism, robustness boils down to exploring how much of the CA behaviour is related to the synchronous scheme, and how much comes intrinsically from the individual rule [10]. Thus, we observed in previous works a variety of behaviours that arise from different types of updating scheme, including phase transitions [4, 8].

In particular, studies on asynchronous CA have revealed the occasional appearance of synchrony-dependent behaviours, which we call *synchronous singularities*, that is, a behaviour which stability depends directly on the synchronic-

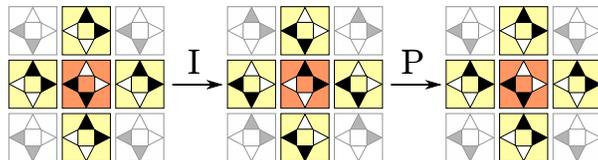


Figure 1: Synchronous singularity in a lattice-gas cellular automaton. Black represents one particle in a channel, and white an empty channel.

ity of the model. Examples of such phenomena include the periodic patterns of the *Game of Life* [2, 7], several rules of Elementary Cellular Automata [8], or even the spatially-extended version of Prisoner’s Dilemma [9], which behaviour is qualitatively altered when noise is introduced in the dynamics. Considering those examples, one may wonder whether these phenomena can be observed only in discrete dynamical systems where the individual rule is deterministic or whether they are more general.

To tackle this question, we focus on a peculiar metastable pattern, uncovered in previous work [3]: in this CA-like model, particles are traveling through the cell grid, according to a synchronous updating scheme which consists of the successive application of two rules, the interaction I – which reorganises the particles within a cell to maximise their alignment with the particles of the neighbours – and the propagation P – which displaces particles towards the corresponding neighbours. Paradoxically in this pattern, particles are all opposing the directions of their direct neighbours, in spite of the alignment-favouring local rule (see Fig. 1).

A step-by-step look at the evolution of this pattern may let us think that this behaviour requires a deterministic updating rule. However, what is outstanding is that this counter-intuitive pattern has been observed to self-organise (a) with a stochastic local rule, (b) from a random initial configuration. Thus the question: if such a pattern can form outside of a determinism model, how is it related to the perfect synchrony of the system? And how can we question this property in lattice-gas models in order to probe the robustness to asynchronism?

In Sec. 2, we confront the notion of asynchronism to the constraints and hypotheses of lattice-gas cellular automata and propose several definitions of an asynchronous updating scheme. In Sec. 3, we precise further the individual rules considered for our model and compare the observed patterns in the synchronous and asynchronous versions. Finally in Sec. 4, we shortly discuss on the role of asynchronism in discrete dynamical systems, based on our definitions and experimental results.

2 Defining asynchronous lattice-gas cellular automata

There exist a wide range of interpretations and definitions given to asynchronism, and even more diverse resulting behaviours (see for example [1, 4] and ref. therein). The problem we are tackling here is that the system we want to “make asynchronous” is a lattice-gas cellular automaton which, in spite of its compatibility with the classical CA definition, includes additional hypotheses that may not be conserved when changing the updating scheme. Let us now examine a definition of this type of model.

2.1 Definition of lattice-gas cellular automata

The lattice-gas cellular automaton (LGCA) we are interested in is a particular CA, where:

- the *cellular space* $\mathcal{L} = \{\mathbb{Z}/L \cdot \mathbb{Z}\}^2$ is a 2-dimensional square lattice of length L with periodic boundary conditions.
- the *neighbourhood* $(n_i)_{i \in \llbracket 1, 4 \rrbracket} = \{(1, 0); (0, 1); (-1, 0); (0, -1)\}$ associates to a cell $c \in \mathcal{L}$ the set of its 4 nearest neighbours.

In a LGCA, each cell is connected to its neighbours via *channels* through which particles can travel. Cell configurations \vec{x}_c are therefore represented as a vector of the numbers of particles contained by each of its channels $x_i(c)$. The dynamics of LGCA is determined by the successive application of two global transitions:

1. The *interaction step* reorganises particles within cells according to a transition function f_I , which takes the state of a cell and its neighbourhood:

$$\forall c \in \mathcal{L}, \vec{x}_c^I = f_I(\vec{x}_c, \vec{x}_{c+n_1}, \dots, \vec{x}_{c+n_4}). \quad (1)$$

2. The *propagation step* displaces particles from the channels of a cell to the corresponding neighbours:

$$\forall c \in \mathcal{L}, \forall i \in \llbracket 1, 4 \rrbracket, x_i^P(c) = x_i^I(c - n_i). \quad (2)$$

In addition, we consider two fundamental hard constraints which we want to keep valid under an asynchronous updating scheme:

particle exclusion : channels may contain at most one particle, and are therefore considered as either empty (state 0) or full (state 1).

particle conservation : particles must always be conserved when they interact and propagate.

2.2 Which asynchronism?

For this work, we will consider α -*synchronism* [8, 4] (also referred to as *Random Independent Scheme* by Cornforth *et al.* [5]) which follows the general definition: at any time step, each component updates according to the regular transition function with probability α , or remains unchanged using the identity function with probability $1 - \alpha$. This updating scheme provides us with a useful control parameter, the *synchrony rate* α , allowing for a continuous control of the updating scheme from evolution in perfect synchrony ($\alpha = 1$) to quasi-sequential updates ($\alpha \rightarrow 0$).

The global transition function of LGCA from a time t to $t+1$ is constituted of two steps, interaction and propagation, applied successively to all cells. When we want to apply asynchronism, we need to properly define the meanings of “updating” and “remaining unchanged” in terms of computation. Does asynchronism apply to the sole interaction? The propagation? Shall we consider these transitions as correlated, independent?

To address this issue, we propose as a starting point three possible implementations of asynchronism:

- (1) **Asynchronous interaction** : the interaction transition function is applied with a probability α_I , called the *interaction rate*.
- (2) **Asynchronous propagation** : the interaction is always applied, but the propagation is applied with a probability α_P , called the *propagation rate*.
- (3) **Correlated interaction and propagation.** : for each cell, either interaction and propagation are both applied with a probability α , otherwise none of them is applied.

However, if the asynchronous interaction (1) can be implemented without problem as the interaction rule itself is particle-conserving, desynchronising the propagation (2, 3) is not a straightforward operation and requires further reflexion.

2.2.1 Conserving particles during the propagation.

Let us consider the case where asynchronous propagation is simply determined as the application of the transition rule with probability α_P , and identity function with probability $1 - \alpha_P$. In CA models, there is no actual means for a given cell to know whether its neighbours are selected for update or not. This is problematic as we need this type of information to decide whether to propagate the state of a channel or not (see Fig. 2-b and -c).

There exist several model adaptations that can solve this issue: for instance, by adding specific states which keep cells synchronized [11] or by using additional steps to perform validity checks (*e.g.* transactional CA [12]). Similarly, our proposition here consists in modifying the point of view of the asynchronous propagation function by considering no longer the cells but the channels as the base components of the system. We intend to first select randomly the cells which are to be updated, and secondly to determine from non-selected cells

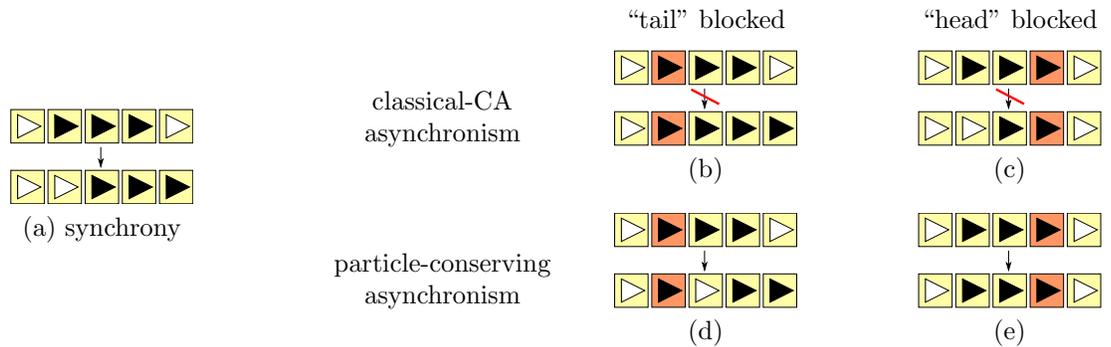


Figure 2: Representation of the propagation step along one channel direction in a sample of five cells, for different updating schemes and situations of particles. Darker cells are not selected for update. (a) is the synchronous classical case, (b,c) the asynchronous case without the particle-conserving changes and (d,e) our proposition for an asynchronous propagation.

which channels are *free*, and thus updating, and which are *blocked* and remaining unchanged. We will now specify formal definitions of asynchronous interaction and propagation, which may be later correlated using selection functions.

2.3 Definition of an asynchronous LGCA

2.3.1 Interaction step.

We introduce a *selection function* $\Delta_I : \mathbb{N} \rightarrow \mathcal{P}(\mathcal{L})$ which associates to a time t the subset of cells to be updated during the interaction step, where each cell has a probability α_I to be selected. The interaction transition function becomes $\forall t \in \mathbb{N}, \forall c \in \mathcal{L}, \mathcal{N} = \{n_1, \dots, n_4\}$:

$$\vec{x}_c^I = f_{\Delta_I}(\vec{x}_c, \vec{x}_{c+n_1}, \dots, \vec{x}_{c+n_4}) = \begin{cases} f_I(\dots) & \text{if } c \in \Delta_I(t) \\ \vec{x}_c & \text{otherwise.} \end{cases} \quad (3)$$

When $\alpha_I = 0$, the system is ballistic, *i.e.* particles always go straight.

2.3.2 Propagation step.

First, we need to determine which cells update, and which remain unchanged. Similarly to asynchronous interaction, we introduce a *selection function* $\Delta_P : \mathbb{N} \rightarrow \mathcal{P}(\mathcal{L})$ which returns for time t the subset of cells to be updated during the propagation step – where each cell has a probability α_P to be selected. In addition, we define $\mathcal{B}^t \subset \llbracket 1, 4 \rrbracket \times \mathcal{L}$ as the set of *blocked channels*, which will remain unchanged times t and $t + 1$.

To build this set, we state that a channel $(i, c)^t$ (channel i , cell c , time t) is blocked if it contains a particle ($x_i^t(c) > 0$), and if one of the following criteria is

true: (C1) its containing cell c is not selected for update, or (C2) its destination channel is blocked. We thus have:

$$(i, c)^t \in \mathcal{B}^t \text{ if } (x_i^t(c) > 0) \text{ and } \begin{cases} c \notin \Delta_P(t) & (C1 : \text{cell is not updated}) \\ \text{or} \\ (i, c + n_i)^t \in \mathcal{B}^t & (C2 : \text{destination is blocked}) \end{cases}$$

The construction of \mathcal{B}^t describes a general relation between particles. It can be implemented with a recursive algorithm: once a channel is blocked because of criterion (C1), the chain of channels that “point” to this one must be also blocked if they contain a particle because of criterion (C2) (see Fig. 2-e).

The propagation transition function therefore becomes:

$$x_i^P(c) = \begin{cases} x_i^I(c) & \text{if } (i, c) \in \mathcal{B}^t \\ 0 & \text{if } (i, c) \notin \mathcal{B}^t \text{ and } (i, c - n_i) \in \mathcal{B}^t \\ x_i^I(c - n_i) & \text{otherwise.} \end{cases} \quad (4)$$

When $\alpha_P = 0$, the particles never propagate and thus never leave their cell.

3 Observation of the asynchronous behaviour

The model of swarming we present is extracted from a compilation of bio-inspired models by Deutsch *et al.* [6]. It describes a probabilistic swarming interaction rule in which the particles of a cell tend to align themselves with the neighbourhood predominant directions. The probability for a given cell configuration \vec{x}_c to reorganise into \vec{x}_c^I is given by:

$$P(\vec{x}_c \rightarrow \vec{x}_c^I) = \frac{1}{Z} \exp \left[\sigma \cdot \vec{J}_c(\vec{x}^I) \cdot \vec{D}_c(\vec{x}) \right] \cdot \delta(\vec{x}_c, \vec{x}_c^I) \quad (5)$$

where:

- Z is the normalisation factor, so that $\sum_{\vec{x}_c^I} P(\vec{x}_c \rightarrow \vec{x}_c^I) = 1$
- $\sigma \in \mathbb{Z}$ is the alignment sensitivity.
- $\vec{J}_c(\vec{x}^I)$ is the *flux* in cell c , the resulting vector of particle directions.
- $\vec{D}_c(\vec{x})$ is the *director field*, the sum of cell \vec{x}_c neighbours’ flux.
- $\delta(\vec{x}_c, \vec{x}_c^I) = 1$ if cell configurations \vec{x}_c and \vec{x}_c^I have same number of particles, 0 otherwise. This ensures conservation of particles.

Starting from a random initial configuration of density ρ , where ρ is the probability for each channel to contain a particle, we need only to set the parameter σ to determine the local interaction rule. This limits the parametrical space to the σ/ρ plane, allowing for a complete exploration of the different qualitative behaviours that the model may display.

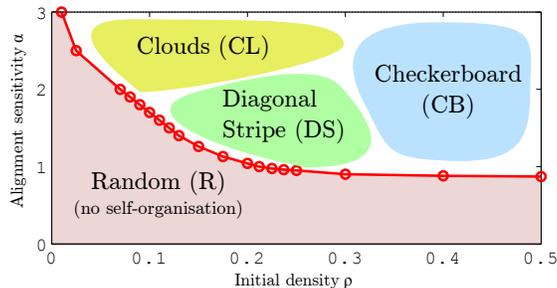


Figure 3: Spatial distribution of the different patterns in the sensitivity-density parametrical plane.

3.1 Observation of the synchronous behaviour

In a previous work, we reported that this model displays a variety of qualitatively different behaviours, in spite of a simple and stochastic interaction rule. In fact using the two control parameters σ and ρ , two phases may emerge, chaotic and organised [6]. In a recent work, we showed that the organised phase can display several stable patterns of global particle organisation [3] (see Fig. 3):

Diagonal stripe pattern (DS) : a traveling compact diagonal shape of particles in two orthogonal directions, that loops over the boundaries of the lattice.

Checkerboard pattern (CB) : local groups of anti-aligned particles. Typically occurs for high densities (typically $\rho \approx 0.5$).

Clouds pattern (CL) : traveling competitive groups of aligned particles. Typically occurs for a high sensitivity ($\sigma \approx 3 - 4$).

It is interesting to note that these behaviours have been observed for systems of limited size and time, and can therefore be considered as metastable attraction basins. This means that once a behaviour is reached, the system will remain in such a configuration, until random fluctuations make it “escape” this behaviour and reach another.

3.2 Exploration of the asynchronous behaviour

We are interested whether these patterns are robust under an asynchronous updating scheme. As a preliminary study, we now propose to consider each of the “main” patterns independently by setting the system in adequate initial condition, iterate the system for a large number of step ($t = 2000$) and compare observations for different values of the probabilities α_I and α_P of the two transition steps.

3.2.1 Asynchronous interaction.

Two types of observations can be made (see Fig. 4-a):

1. For the diagonal stripe pattern as well as the clouds pattern, the resulting behaviour is less stable but qualitatively still valid, until a certain value of the synchrony rate α_I is reached. The behaviour then switches to lower-sensitivity patterns (diagonal stripe pattern and then randomness).
2. However, a small but not infinitesimal amount of asynchronism ($\alpha_I \approx 0.95$) is sufficient to switch the checkerboard behaviour to a diagonal stripe pattern which covers the lattice almost entirely.

As a result, the asynchronous interaction seems to act like an additional noise to the interaction transition function. Although low-density patterns (DS and CL) seem robust to this perturbation, the checkerboard pattern turns almost immediately into a diagonal stripe already for a high synchrony rate.

3.2.2 Asynchronous propagation.

All tested patterns displayed a sudden change of behaviour for a small amount of asynchronism (see Fig. 4-b).

1. The diagonal stripe occasionally appears for high α_P but is soon replaced by semi-random configurations, where a relative consensus regularly appears and disappears, but never stabilises.
2. The checkerboard pattern might be relatively stable for very high values of α_P but the behaviour usually shifts to a global consensus after some time.
3. Finally, the clouds pattern forms one or several compact stripes of aligned particles, seemingly for any value of the probability $\alpha_P < 1$.

These observations suggest a destructive effect of asynchronous propagation on all synchronous behaviours, even for a high propagation rate $\alpha_P \approx 0.95$. However, this observation is not valid for infinitesimal values $\alpha_P = 1^-$, which let us think that our patterns have a minimum robustness to asynchronous propagation.

3.2.3 Correlated interaction and propagation.

Similar experiments have been applied to the correlated updating scheme. The resulting behaviour is a mix of asynchronous interaction and asynchronous propagation:

1. For higher values of the synchronism rate ($\alpha \approx 0.95$), the behaviour follows the sudden changes observed in the asynchronous propagation.

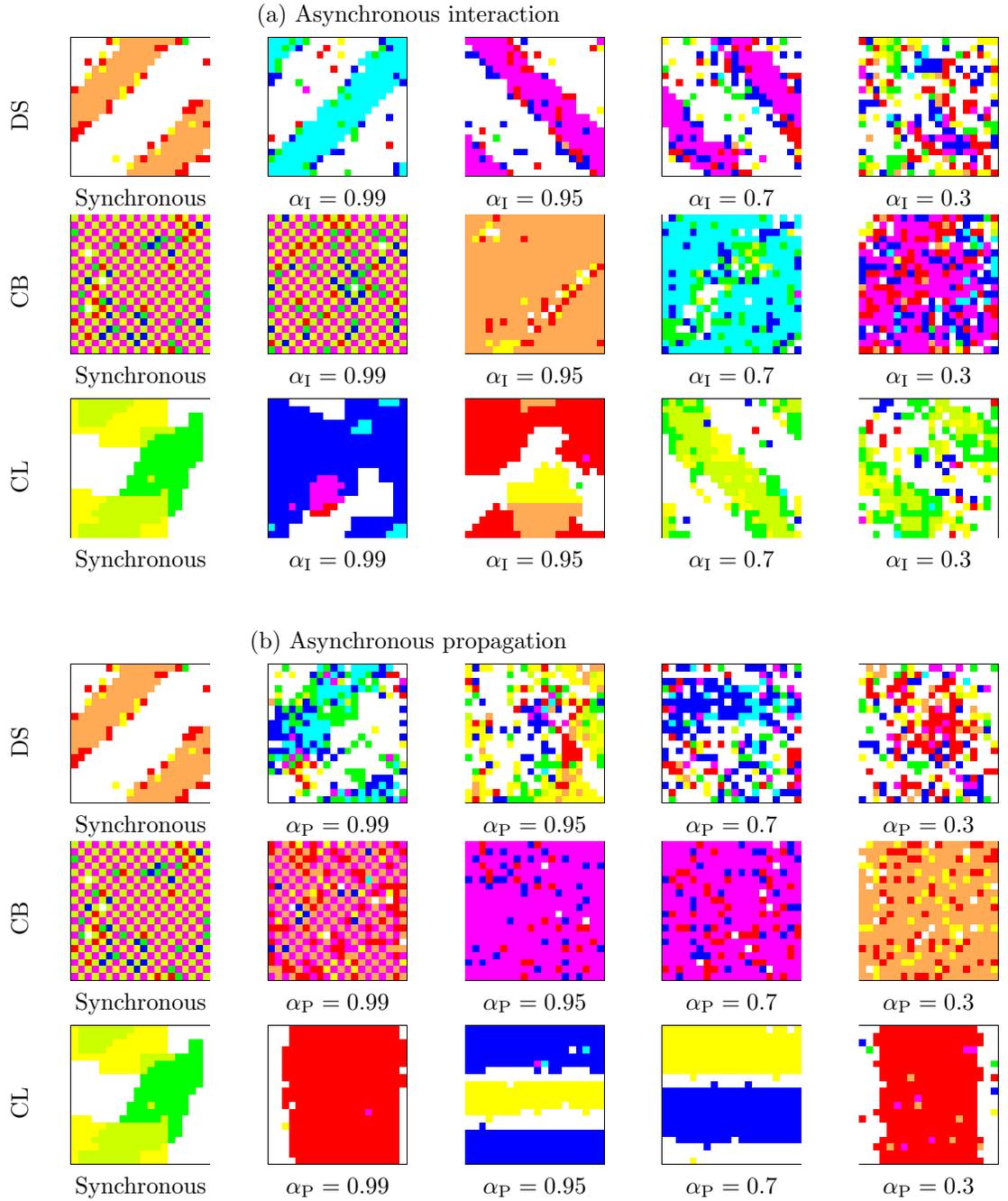


Figure 4: Evolution of the main patterns for different interaction (propagation) rate values α_I (α_P , respectively). Configurations are obtained from random initial configurations run for 5000 steps for typical values of σ and ρ (DS: $\{\sigma = 1.5; \rho = 0.2\}$, CB: $\{\sigma = 1.5; \rho = 0.45\}$, CL: $\{\sigma = 4; \rho = 0.2\}$). The simulations and visualisations are realised with *FiatLux*, a CA simulator in Java (<http://fiatlux.loria.fr>).

2. From lower values ($\alpha \approx 0.5$), the swarming and identity interaction rules become equiprobable and, as particles do not gather in clusters any more, the system enters a random phase.

Considering how differently asynchronous interaction and asynchronous propagation affect the system dynamics, and the synchrony rates for those changes were observed, this result is somehow “expected”: the behaviour changes correspond more or less to the changes observed for each type of asynchronism.

3.2.4 Synthesis.

This model, which seemed rather robust in the first place due to the stochastic rule and self-organisation phenomena, ends up being sensitive to modifications of the updating scheme, depending on which pattern and asynchronism are considered. For instance, we observed that there exist (1) differences between patterns in their robustness to asynchronism (*e.g.* diagonal stripe versus checkerboard under asynchronous interaction) and (2) differences between types of asynchronism in their effects on the system’s behaviour (*e.g.* diagonal stripe under asynchronous interaction and propagation). These observations are interesting in the sense that they show the dependence between the type of asynchronism applied to the system and the stability of each pattern. However, the quantitative aspect of those changes remains to be investigated in deeper details, as our simulations were limited in space and time and therefore may not be representative of the behaviour in infinite space.

Let us return to the question of the dependence on perfect synchrony of the checkerboard pattern. Intuitively, an infinitesimal desynchronisation of the update should not fundamentally change the mechanisms intrinsic to the patterns, as they undergo constant noise from the local update rule. However, our experiments failed to confirm this hypothesis and we are unable at this stage to state whether the checkerboard pattern is indeed a “synchronous singularity”, that is, whether there exists a qualitative change of behaviour between perfect synchrony ($\alpha = 1$) and quasi-synchrony ($\alpha = 1^-$).

For now, additional experiments must be conducted in order to quantify those changes for different parameter intervals, size and time scales, etc. and to give a proper estimation of the robustness of this model.

4 Discussion

This paper presented a first definition of an asynchronous LGCA, which means to extract from classical CA the idea of questioning the perfect synchrony, and adapt it to the definition and hypotheses of a lattice-gas model of swarming.

From the modelisation point of view, we learned from this specific example that a naive expression of asynchronism – for instance reproducing the definition for classical CA – could compromise some constraints of the model and therefore requires us to adapt the modeling point of view. In our case, the model uses a CA to represent traveling particles on discrete space, which imposes us

to change the paradigm and consider *channels* as the base components of the updating scheme. More generally, this means that for models which have specific associated hypotheses, such as conservation of particles, applying the same approach may require to adapt the description of the model to these constraints so they are not conflicted by the perturbation.

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