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# $H_\infty$ Control Design for Novel Teleoperation System Scheme: A Discrete Approach

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**Abstract:** This paper addresses the problem that, the discretization of stabilizing the continuous bilateral teleoperation controllers for digital implementation may lead to instable teleoperation or poor performance. With this problem, a discrete approach for the novel proxy teleoperation control scheme under time-varying delays is considered in the paper. The principle results involve sufficient conditions in terms of discrete Lyapunov-Krasovskii functionals (LKF) and  $H_\infty$  control theory, which are resolved by Linear Matrix Inequality (LMI). The simulations of different working conditions are performed to verify the effectiveness of discretization for the continuous bilateral teleoperation system.

*Keywords:* Teleoperation; Time-varying delay;  $H_\infty$  control; Discrete Lyapunov-Krasovskii functionals; Linear Matrix Inequality

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## 1. INTRODUCTION

In bilateral master/slave teleoperation system, the motion or force information of the master robot handled by the human operator are transmitted from the master to the slave, and the corresponding information of the slave robot or environment are also transmitted back to the master. Besides, the master and slave are coupled via the communication medium, which incurs the additional dynamics represented by time-varying delays [Richard (2003)], they deteriorate the system performance, even make the system instable. So, these delays should be considered in the design of teleoperation [Anderson and Spong (1989)].

Recently, in continuous-time domain, many methods are proposed to address the stability and performance problem under time delays, *e.g.* nonlinear methods (passivity-based control, input to state stability, sliding mode control), robust control ( $H_\infty$  design,  $\mu$ -synthesis), adaptive control, predictive control. For more details of control strategies, the readers could refer to [Arcara and Melchiorri (2002), Nuño et al. (2011), Chiasson and Loiseau (2007)] and references therein. Accordingly, some approaches for the controller design in discrete-time domain, not as many as in continuous-time domain, are also presented, [Kapila and Haddad (1998)] introduced a memoryless  $H_\infty$  controller design method based on a modified Riccati equation in order to deal with constant delays. Similarly, for the constant delays, the discretization approach of stabilizing continuous bilateral teleoperation controller was designed in [Tavakoli et al. (2008), Tavakoli et al. (2007)]. Recently, [Moezzi et al. (2008)] presents an adaptive switching supervisory control scheme for highly uncertain discrete systems with time-varying state delays and time-varying parameters, but with an assumption that the system is subject to the external bounded disturbances. A discrete passivity-based control method is utilized to handle

time-varying delays, but without the performance guarantee [Beresteky et al. (2004)].

Our latest research [Zhang et al. (2011b)] presented a novel continuous teleoperation control scheme, force-reflecting proxy control scheme, designed by Lyapunov-Krasovskii functionals (LKF) and  $H_\infty$  control theory. Firstly, our control architecture can guarantee the stability of whole system, and further, realize the position tracking and force tracking between the master and the slave under time-varying delays. Because of the wealth of our continuous control scheme, in this paper, we will use a discrete approach to analyze the control scheme and obtain better system performance than the approach in continuous-time domain. Specifically, we present a rigorous development of the controllers in the form of Linear Matrix Inequality (LMI) for discrete teleoperation by using discrete LKF and  $H_\infty$  control (the readers could refer to [Meng et al. (2010), Hetel et al. (2008), Fridman and Shaked (2005), Rehm and Allgower (2002)] for more details about the stability analysis of discrete delay systems by Lyapunov methods). Our work in this paper is valuable to the digital implementation on the experimental test-bench and switch controller design [Kruszewski et al. (2011)].

Based on the discrete theorems, various simulations are made and the results are compared to the other approaches in order to verify that, our discrete approach can ensure the system stability and optimizes the system performance.

This paper is organized as follows: Section 2 introduces the modeling and stability analysis of discrete delay system. The main results are given in Section 3. In Section 4, the simulations are presented. Finally we conclude in Section 5.

## 2. STABILITY ANALYSIS OF DISCRETE DELAY SYSTEM

This section is devoted to introduce the discrete stability theorem with  $H_\infty$  control performance index, the system can be modeled as follow,

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$$\begin{cases} x(k+1) = \sum_{i=0}^n A_i x(k - \tau_i(k)) + Bw(k), \\ z(k) = Cx(k), \end{cases} \quad (1)$$

where,  $x(k) \in \mathbf{R}^n$ ,  $w(k) \in \mathbf{R}^l$  is defined as the exogenous disturbance signal, and  $z(k) \in \mathbf{R}^q$  is seen as the objective control output,  $A_i$ ,  $i = 1, 2, \dots, n$ ,  $B$  and  $C$  are constant matrices.  $\tau_0(k) \equiv 0$ , the time-varying delays  $\tau_i(k)$  are the positive integers, which can be modeled as  $\tau_i(k) \in [h_1, h_2]$ ,  $i = 1, 2, \dots, n$ ,  $h_1, h_2$  are the positive integers,  $h_1 \geq 0$ ,  $h_2 \geq h_1$ . Considering the following discrete Lyapunov-Krasovskii functional with the notation of  $y(k) = x(k+1) - x(k)$ ,

$$\begin{aligned} V(k, x(k)) &= x(k)^T P x(k) \\ &+ \sum_{i=k-h_2}^{k-1} x(i)^T S_a x(i) + \sum_{i=k-h_1}^{k-1} x(i)^T S x(i) \\ &+ h_1 \sum_{i=-h_1}^{-1} \sum_{j=k+i}^{k-1} y(j)^T R y(j) \\ &+ \sum_{i=1}^n (h_2 - h_1) \sum_{j=-h_2}^{-h_1-1} \sum_{l=k+j}^{k-1} y(l)^T R_{ai} y(l). \end{aligned} \quad (2)$$

In order to guarantee the improvement of the overall performance, we define  $H_\infty$  control performance index with a positive scalar  $\gamma$ ,

$$J(w) = \sum_{i=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k)] < 0. \quad (3)$$

Then, we obtain the following theorem.

**Theorem 1.** Suppose there exist  $n \times n$  matrices  $P > 0$ ,  $R > 0$ ,  $S > 0$ ,  $S_a > 0$ ,  $R_{ai} > 0$ ,  $P_2, P_3, Y_1, Y_2$ ,  $i = 1, 2, \dots, n$ , and a positive scalar  $\gamma$ , such that the condition (4) with notations (5) is feasible, then the system (1) is asymptotically stable and  $J(w) < 0$  for time-varying delays  $\tau_i(k) \in [h_1, h_2]$ ,  $h_1 \geq 0$ ,  $h_2 \geq h_1$ ,  $i = 1, 2, \dots, n$ .

**Proof.** According to  $H_\infty$  control theory, we consider the condition,

$$\Delta V(k, x(k)) + z(k)^T z(k) - \gamma^2 w(k)^T w(k) < 0, \quad (6)$$

where,  $\Delta V(k, x(k)) = V(k+1, x(k+1)) - V(k, x(k))$ . From [Zhang et al. (2011b)], we can see that,  $J(w) < 0$  can be assured if Eq. 6 is negative. One obtains,

$$\begin{aligned} &\Delta V(k, x(k)) + z(k)^T z(k) - \gamma^2 w(k)^T w(k) \\ &= x(k)^T (S + S_a) x(k) + x(k+1)^T P x(k+1) - x(k)^T P x(k) \\ &- x(k-h_1)^T S x(k-h_1) - x(k-h_2)^T S_a x(k-h_2) \\ &+ y(k)^T [h_1^2 R + (h_2 - h_1)^2 \sum_{i=1}^n R_{ai}] y(k) \\ &- h_1 \sum_{i=k-h_1}^{k-1} y(i)^T R y(i) - (h_2 - h_1) \sum_{j=k-h_2}^{k-h_1-1} y(j)^T \sum_{i=1}^n R_{ai} y(j) \\ &+ z(k)^T z(k) - \gamma^2 w(k)^T w(k). \end{aligned} \quad (7)$$

Substituting for  $z(k)$  and applying the Jensen's inequality [Gu et al. (2003)] yield,

$$\begin{aligned} &\Delta V(k, x(k)) + z(k)^T z(k) - \gamma^2 w(k)^T w(k) \\ &\leq x(k)^T (S + S_a) x(k) + x(k+1)^T P x(k+1) - x(k)^T P x(k) \\ &- x(k-h_1)^T S x(k-h_1) - x(k-h_2)^T S_a x(k-h_2) \\ &+ y(k)^T [h_1^2 R + (h_2 - h_1)^2 \sum_{i=1}^n R_{ai}] y(k) \\ &- [x(k)^T - x(k-h_1)^T] R [x(k) - x(k-h_1)] \\ &- \sum_{i=1}^n v_{1i}^T R_{ai} v_{1i} - \sum_{i=1}^n v_{2i}^T R_{ai} v_{2i} \\ &+ z(k)^T z(k) - \gamma^2 w(k)^T w(k), \end{aligned} \quad (8)$$

where,

$$v_{1i} = \sum_{i=k-\tau_i(k)}^{k-h_1-1} y(i), \quad v_{2i} = \sum_{i=k-h_2}^{k-\tau_i(k)-1} y(i), \quad i = 1, 2, \dots, n. \quad (9)$$

In  $\Delta V(k, x(k))$ ,  $x(k+1)^T P x(k+1) - x(k)^T P x(k)$  is replaced by  $y(k)^T P y(k) + x(k)^T P y(k) + y(k)^T P x(k)$ . Introducing free weighting matrices  $P_2, P_3, Y_1, Y_2$  as in [Zhang et al. (2011b)], we get,

$$\begin{aligned} 0 &= 2[x(k)^T P_2^T + y(k)^T P_3^T] \\ &[A_0 x(k) + Bw(k) + \sum_{i=1}^n A_i x(k-h_1) - \sum_{i=1}^n A_i v_{1i} - y(k) - x(k)], \\ 0 &= 2[x(k)^T Y_1^T + \dot{x}(k)^T Y_2^T] \\ &[nx(k-h_2) + \sum_{i=1}^n v_{1i} + \sum_{i=1}^n v_{2i} - nx(k-h_1)]. \end{aligned} \quad (10)$$

Using notation,

$$\eta(k) = \text{col}\{x(k), y(k), x(k-h_1), x(k-h_2), v_{11}, v_{12}, \dots, v_{1n}, v_{21}, v_{22}, \dots, v_{2n}, w(k)\}. \quad (11)$$

Finally, if the LMI in Eq. 4 is feasible, we obtain,

$$\Delta V(k, x(k)) + z(k)^T z(k) - \gamma^2 w(k)^T w(k) \leq \eta(k)^T \Gamma^1 \eta(k) < 0. \quad (12)$$

Specially, in Eq. 1, when the system is delay free ( $n = 0$ ) as follow,

$$\begin{cases} x(k+1) = A_0 x(k) + Bw(k), \\ z(k) = Cx(k). \end{cases} \quad (13)$$

From *Theorem 1*, we can get the following theorem.

**Theorem 2.** Suppose there exist  $n \times n$  matrices  $P > 0$ ,  $P_2, P_3$  and a positive scalar  $\gamma$ , such that the condition (14) with the notation (15) is feasible, then the delay-free system (13) is asymptotically stable and  $J(w) < 0$ .

$$\Gamma^2 = \begin{pmatrix} \Gamma_{11}^2 & P - P_2^T - P_3^T + A_0^T P_3 & P_2^T B \\ * & -P_3 - P_3^T & P_3^T B \\ * & * & -\gamma^2 I \end{pmatrix} < 0, \quad (14)$$

$$\Gamma_{11}^2 = A_0^T P_2 + P_2^T A_0 - P_2 - P_2^T + C^T C. \quad (15)$$

$$\Gamma^1 = \begin{pmatrix} \Gamma_{11}^1 & \Gamma_{12}^1 & R + \sum_{i=1}^n P_2^T A_i - n Y_1^T & n Y_1^T & -P_2^T A_1 + Y_1^T & \dots & -P_2^T A_n + Y_1^T & Y_1^T & \dots & Y_1^T & P_2^T B \\ * & \Gamma_{22}^1 & \sum_{i=1}^n P_3^T A_i - n Y_2^T & n Y_2^T & -P_3^T A_1 + Y_2^T & \dots & -P_3^T A_n + Y_2^T & Y_2^T & \dots & Y_2^T & P_3^T B \\ * & * & -S - R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_{a1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \dots & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R_{an} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -R_{a1} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \dots & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -R_{an} & 0 \\ * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{pmatrix} < 0, \quad (4)$$

$$\Gamma_{11}^1 = S + S_a - R + A_0^T P_2 + P_2^T A_0 - P_2 - P_2^T + C^T C, \quad \Gamma_{12}^1 = P - P_2^T - P_3^T + A_0^T P_3, \quad (5)$$

$$\Gamma_{22}^1 = P - P_3 - P_3^T + h_1^2 R + (h_2 - h_1)^2 \sum_{i=1}^n R_{ai}.$$

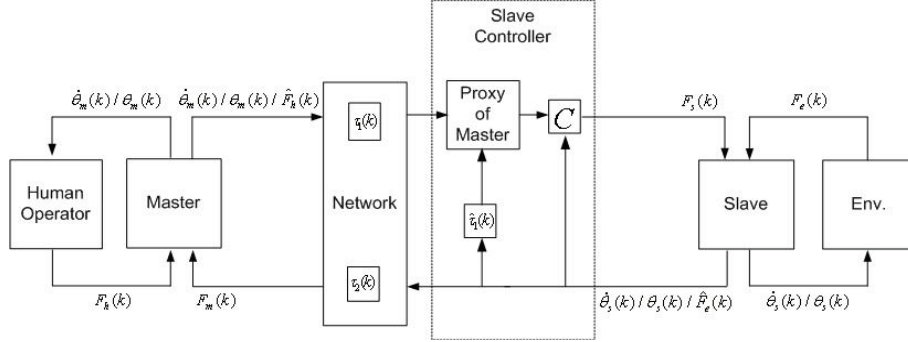


Fig. 1. Discrete force-reflecting proxy control scheme.

### 3. MAIN RESULTS

#### 3.1 System Description and Problem Formulation

The discrete force-reflecting proxy control scheme is presented in Figure 1, in which:  $F_m(k)$  and  $F_s(k)$  are the actuated inputs of the master and the slave;  $F_h(k)$  and  $F_e(k)$  are the forces of the human operator and environment on the system;  $\hat{F}_h(k)$  and  $\hat{F}_e(k)$  are the estimations of these two forces, which can be obtained by adding the perturbation observers in reality.

$\tau_1(k)$  (from the master to the slave) and  $\tau_2(k)$  (from the slave to the master) are the time-varying delays, which are modeled as the previous section,  $\tau_1(k), \tau_2(k) \in [h_1, h_2]$ . Master and slave clocks are synchronized thanks to time-stamped data packet exchanges between them, using a network time protocol as in [Kruszewski et al. (2011)]. Therefore, the estimated network delay between the master and slave,  $\hat{\tau}_1(k)$ , is available at slave's side:  $\hat{\tau}_1(k) = \tau_1(k)$ .

From the master to slave, the information transferred are the velocity/position of the master and the estimated force  $\hat{F}_h(k)$ . However, from the slave to the master, only the estimated force  $\hat{F}_e(k)$  is transferred, so the force tracking,  $F_m(k) = \hat{F}_e(k - \tau_2(k))$ , is realized, if the stability of the whole system is verified.

$\dot{\theta}_m(k)/\theta_m(k)$  and  $\dot{\theta}_s(k)/\theta_s(k)$  are the velocities/positions of the master and slave, and their models are described as follows,

$$(\Sigma_m^d) \quad x_m(k+1) = (A_{md} - B_{md}K_{md}^0)x_m(k) + B_{md}(F_m(k) + F_h(k)), \quad (16)$$

$$(\Sigma_s^d) \quad x_s(k+1) = (A_{sd} - B_{sd}K_{sd}^0)x_s(k) + B_{sd}(F_s(k) + F_e(k)), \quad (17)$$

where,  $x_m(k) = \dot{\theta}_m(k)$ ,  $x_s(k) = \dot{\theta}_s(k)$ .  $K_{md}^0$ ,  $K_{sd}^0$  are the local controllers ensuring the speed stability.

The proxy of master and the controller  $C$  (together defined as the slave controller) will be designed in sequential steps in the following. Note that, the proxy of master is like a remote observer of the master, which is used at the slave side to reduce the impact of the time-varying delays. We consider the discrete model of proxy ( $\dot{\theta}_p(k)$ ,  $\theta_p(k)$  is the velocity/position of the proxy of master),

$$(\Sigma_p^d) \quad x_p(k+1) = (A_{md} - B_{md}K_{md}^0)x_p(k) - B_{md}F_p(k) + B_{md}(\hat{F}_e(k - \hat{\tau}_1(k)) + \hat{F}_h(k - \tau_1(k))). \quad (18)$$

Because the proxy acts as a remote observer of the master, the proxy model is same with the master,  $x_p(k) = \dot{\theta}_p(k)$ .  $L_d = (L_{d1} \ L_{d2} \ L_{d3})$  is the gain of proxy that will be designed to synchronize the position between the master and the proxy of master,

$$F_p(k) = L_d \begin{pmatrix} \dot{\theta}_p(k - \hat{\tau}_1(k)) \\ \dot{\theta}_m(k - \tau_1(k)) \\ \theta_p(k - \hat{\tau}_1(k)) - \theta_m(k - \tau_1(k)) \end{pmatrix}, \quad (19)$$

next,  $K_d = (K_{d1} \ K_{d2} \ K_{d3})$  is the gain of the controller  $C$ ,

$$F_s(k) = -K_d \begin{pmatrix} \dot{\theta}_s(k) \\ \dot{\theta}_p(k) \\ \theta_s(k) - \theta_p(k) \end{pmatrix}. \quad (20)$$

#### 3.2 Design of Proxy of Master

Firstly, we design the proxy of master and consider the whole system of master and proxy as follow,

$$(\Sigma_{mp}^d) \begin{cases} x_{mp}(k+1) = A_{mpd}^0 x_{mp}(k) + A_{mpd}^1 x_{mp}(k - \tau_1(k)) \\ \quad + B_{mpd} w_{mp}(k), \\ z_{mp}(k) = C_{mpd} x_{mp}(k), \end{cases} \quad (21)$$

where,

$$x_{mp}(k) = \begin{pmatrix} \hat{\theta}_p(k) \\ \hat{\theta}_m(k) \\ \theta_p(k) - \theta_m(k) \end{pmatrix}, \quad z_{mp}(k) = (\theta_p(k) - \theta_m(k)), \quad (22)$$

$$w_{mp}(k) = \begin{pmatrix} \hat{F}_e(k - \hat{\tau}_1(k)) + \hat{F}_h(k - \tau_1(k)) \\ F_m(k) + F_h(k) \end{pmatrix}.$$

$A_{mpd}^0$ ,  $B_{mpd}$  is discretized by  $A_{mp}^0$ ,  $B_{mp}$  in [Zhang et al. (2011b)],  $C_{mpd} = (0 \ 0 \ 1)$  and,

$$A_{mpd}^1 = \begin{pmatrix} -B_{md}L_{d1} & -B_{md}L_{d2} & -B_{md}L_{d3} \\ 0 & 0 & 0 \end{pmatrix} = -B_L L_d, \quad (23)$$

where,  $B_L = (B_{md}^T \ 0 \ 0)^T$ . Then we design  $L_d$  by the following stability theorem.

**Theorem 3.** Suppose there exist matrices  $P > 0$ ,  $R > 0$ ,  $S > 0$ ,  $S_a > 0$ ,  $R_{a1} > 0$ ,  $P_2$ ,  $Y_1$ ,  $Y_2$ ,  $M$ , and positive scalars  $\gamma$  and  $\xi$ , such that the condition (25) with notations (26) is feasible, then the system (21) is asymptotically stable and  $J(w) < 0$  for time-varying delays  $\tau_1(k) \in [h_1, h_2]$ . The control gain of the proxy is given by,

$$L_d = M P_2^{-1}. \quad (24)$$

**Proof.** We use *Theorem 1* on system Eq. 21. Inspired from [Fridman and Shaked (2001)], a series of steps is made to deal with nonlinear matrix terms,  $P_2^T B_L L_d$ ,  $P_3^T B_L L_d$ : Multiplying  $\Gamma^1$  by  $\text{diag}\{P_2^{-T}, \dots, P_2^{-T}, I\}$  at the left side,  $\text{diag}\{P_2^{-1}, \dots, P_2^{-1}, I\}$  at the right side; choosing  $P_3 = \xi P_2$ ; defining  $M = L_d P_2$ ; applying the Schur formula, and then, the result follows.

### 3.3 Design of Controller C

The position tracking between the master and the proxy of master has been achieved. And then, the position tracking between the proxy of master and slave is achieved by the controller  $C$ . The discrete model of the proxy, the controller  $C$  and the slave is described as follows,

$$(\Sigma_{ps}^d) \begin{cases} x_{ps}(k+1) = A_{psd} x_{ps}(k) + B_K F_s(k) + B_{psd} w_{ps}(k), \\ z_{ps}(k) = C_{psd} x_{ps}(k), \end{cases} \quad (27)$$

where,  $A_{psd}$ ,  $B_{psd}$  is transformed from  $A_{ps}^0$ ,  $B_{ps}$  in [Zhang et al. (2011b)], and  $B_K = (B_{sd}^T \ 0 \ 0)^T$ ,  $C_{psd} = (0 \ 0 \ 1)$ , and,

$$x_{ps}(k) = \begin{pmatrix} \hat{\theta}_s(k) \\ \hat{\theta}_p(k) \\ \theta_s(k) - \theta_p(k) \end{pmatrix}, \quad z_{ps}(k) = (\theta_s(k) - \theta_p(k)), \quad (28)$$

$$w_{ps}(k) = \begin{pmatrix} F_e(k) \\ \hat{F}_e(k - \hat{\tau}_1(k)) + \hat{F}_h(k - \tau_1(k)) - F_p(k) \end{pmatrix}.$$

Thus, the system transformation is made to apply LMI,

$$(\bar{\Sigma}_{ps}^d) \begin{cases} x_{ps}(k+1) = (A_{psd} - B_K K_d) x_{ps}(k) + B_{psd} w_{ps}(k), \\ z_{ps}(k) = C_{psd} x_{ps}(k). \end{cases} \quad (29)$$

Then, we obtain the following theorem.

**Theorem 4.** Suppose there exist matrices  $P > 0$ ,  $P_2$ ,  $W$ , and positive scalars  $\gamma$  and  $\xi$ , such that the condition (31) with notations (32) is feasible, then the system (27) is asymptotically stable and  $J(w) < 0$ . The control gain of the controller  $C$  is given by,

$$K_d = W P_2^{-1}. \quad (30)$$

$$\Gamma^4 = \begin{pmatrix} \Gamma_{11}^4 & \Gamma_{12}^4 & B_{psd} & P_2^T C_{psd}^T \\ * & \Gamma_{22}^4 & \xi B_{psd} & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{pmatrix} < 0, \quad (31)$$

$$\Gamma_{11}^4 = P_2^T A_{psd}^T + A_{psd}^T P_2 - B_K W - W^T B_K^T - P_2 - P_2^T, \quad (32)$$

$$\Gamma_{12}^4 = P - P_2 - \xi P_2 + \xi P_2 A_{psd}^T - \xi W^T B_K^T,$$

$$\Gamma_{22}^4 = P - \xi P_2 - \xi P_2^T.$$

**Proof.** We apply system Eq. 29 in *Theorem 2*, and  $K_d$  is obtained by the similar process of *Theorem 3*.

### 3.4 Global System Stability

The global system stability should be verified based on the position tracking between the master and proxy, the proxy and slave in discrete-time domain. The discrete whole system is described,

$$(\Sigma_{mps}^d) \begin{cases} x(k+1) = A_{mps} x(k) + B_{mps}^K F_s(k) \\ \quad - B_{mps}^L F_p(k - \tau_1(k)) + B_{mps} w(k), \\ z(k) = C_{mps} x(k). \end{cases} \quad (33)$$

Here,  $x(k)$ ,  $w(k)$ ,  $z(k)$  and  $A_{mps}$ ,  $B_{mps}$  are the discretization of continuous system  $(x(t), w(t), z(t), A_0, B)$  in [Zhang et al. (2011b)]. We redefine,

$$F_s(k) = -\bar{K}_d x(k) = - \begin{pmatrix} K_{d1} & K_{d2} & 0 & K_{d3} & 0 \end{pmatrix} x(k),$$

$$F_p(k) = \bar{L}_d x(k - \tau_1(k)) = \begin{pmatrix} 0 & L_{d1} & L_{d2} & 0 & L_{d3} \end{pmatrix} x(k - \tau_1(k)), \quad (34)$$

$$B_{mps}^K = \begin{pmatrix} B_{sd}^d \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad B_{mps}^L = \begin{pmatrix} 0 \\ B_{md}^d \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Thus, we suppose  $A_0^d = A_{mps} - B_{mps}^K \bar{K}_d$ ,  $A_1^d = -B_{mps}^L \bar{L}_d$ , the whole system in Eq. 33 is transformed as follow,

$$(\bar{\Sigma}_{mps}^d) \begin{cases} x(k+1) = A_0^d x(k) + A_1^d x(k - \tau_1(k)) + B_{mps} w(k), \\ z(k) = C_{mps} x(k). \end{cases} \quad (35)$$

By *Theorem 1*, the global system stability is verified, and the force tracking,  $F_m(k) = \hat{F}_e(k - \tau_2(k))$ , can be achieved on the basis of the control scheme.

## 4. ILLUSTRATIVE EXAMPLE

In this section, the simulations are performed in different working conditions so to evaluate the performance of the proposed approach. The master, the proxy of master and the slave models can be described as integrators,  $1/s$ ,  $1/s$  and  $2/s$ . Besides, the poles of the master, the proxy and the slave are given as  $[-100.0]$  in continuous-time domain. By the discrete approach, the discrete system parameters can be obtained.

$$\Gamma^3 = \begin{pmatrix} \Gamma_{11}^3 & \Gamma_{12}^3 & R - B_L M - Y_1^T & Y_1^T & Y_1^T + B_L M & Y_1^T & B_{mpd} & P_2^T C_{mpd}^T \\ * & \Gamma_{22}^3 & -\xi B_L M - Y_2^T & Y_2^T & Y_2^T + \xi B_L M & Y_2^T & \xi B_{mpd} & 0 \\ * & * & -S - R & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S_a & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_{a1} & * & 0 & 0 \\ * & * & * & * & * & * & -R_{a1} & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \\ * & * & * & * & * & * & * & -I \end{pmatrix} < 0, \quad (25)$$

$$\begin{aligned} \Gamma_{11}^3 &= S + S_a - R + P_2^T A_{mpd}^0{}^T + A_{mpd}^0 P_2 - P_2 - P_2^T, & \Gamma_{12}^3 &= P - P_2 - \xi P_2 + \xi P_2^T A_{mpd}^0{}^T, \\ \Gamma_{22}^3 &= P - \xi P_2 - \xi P_2^T + h_1^2 R + (h_2 - h_1)^2 R_{a1}. \end{aligned} \quad (26)$$

The constant sampling time is  $T = 0.001s$ , and  $h_1 = 1$ ,  $h_2 = 100$  (in continuous-time domain,  $h_2 = 0.1s$ ).  $L_d$ ,  $K_d$  with  $\gamma_{min}^{L_d}$ ,  $\gamma_{min}^{K_d}$  and the global system stability with  $\gamma_{min}^g$  are obtained as follows,

$$\begin{aligned} L_d &= \begin{pmatrix} 4.6815 & -5.1390 & 540.7828 \end{pmatrix}, & \gamma_{min}^{L_d} &= 0.0051, \\ K_d &= \begin{pmatrix} 273 & -127 & 10961 \end{pmatrix}, & \gamma_{min}^{K_d} &= 2.9568 \times 10^{-4}, \\ \gamma_{min}^g &= 0.0327. \end{aligned} \quad (36)$$

Note that, the  $\xi$  in the theorems mentioned above is an important variable when verifying the LMI conditions. A query loop algorithm is considered to search for the best value of  $\xi$ .

#### 4.1 Tracking in Abrupt Changing Motion

In this working condition, the human operator ( $F_h(k)$ ) is modeled as the pulse generator. Besides, for comparison reasons, we have applied the proposed discrete approach in this paper to our another control scheme from [Zhang et al. (2011a)] (bilateral state feedback control scheme) and obtained the results in discrete-time domain. Figure 2 shows the position tracking between the master and slave respectively for discrete bilateral state feedback control scheme [Zhang et al. (2011a)], continuous force-reflecting proxy control scheme [Zhang et al. (2011b)] and our result in this paper.

The middle and lower parts of Figure 2 have amplified the mutation point of the position, we can see that, our discrete approach ensures the system stability and makes better position tracking, which can be illustrative by  $\gamma_{min}$ . Besides, in discrete-time domain, the performance of the force-reflecting proxy control scheme used in this paper is better than the bilateral state feedback control scheme.

#### 4.2 Tracking in Wall Contact Motion

Similarly, the position tracking of wall contact motion in discrete-time domain is presented in Figure 3, here the slave is driven to the hard wall with a stiffness of  $K_e = 30kN/m$  located at the position  $x = 1.0m$ . Based on the characteristics of force-reflecting proxy control scheme, the force tracking,  $F_m(k) = \hat{F}_e(k - \tau_2(k))$ , can be seen in Figure 4. Because of the sudden changes of the force of environment, there exist some estimation errors, which are difficult to avoid.

### 5. CONCLUSIONS

In this paper, we present a discrete approach to address the stability and performance problem under time-varying delays, this is valuable to the digital implementations and switch controller design, which are being studied in our experimental testbench. Based on LMI, LKF and  $H_\infty$  control, several discrete LMI-based theorems have been presented.

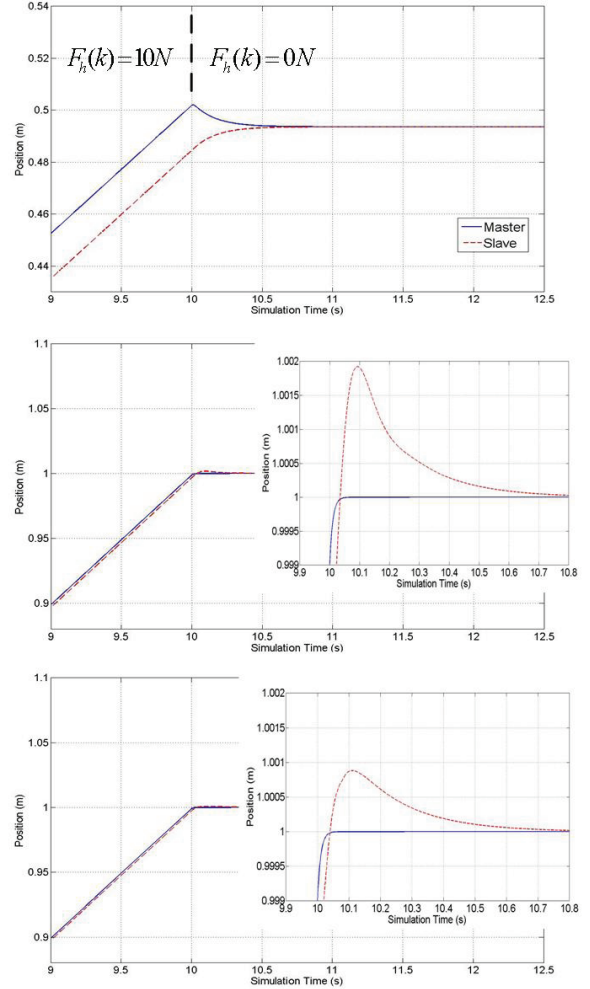


Fig. 2. Position response in abrupt changing motion (upper: discrete bilateral state feedback control scheme [Zhang et al. (2011a)]; middle: continuous force-reflecting proxy control scheme [Zhang et al. (2011b)]; lower: our result).

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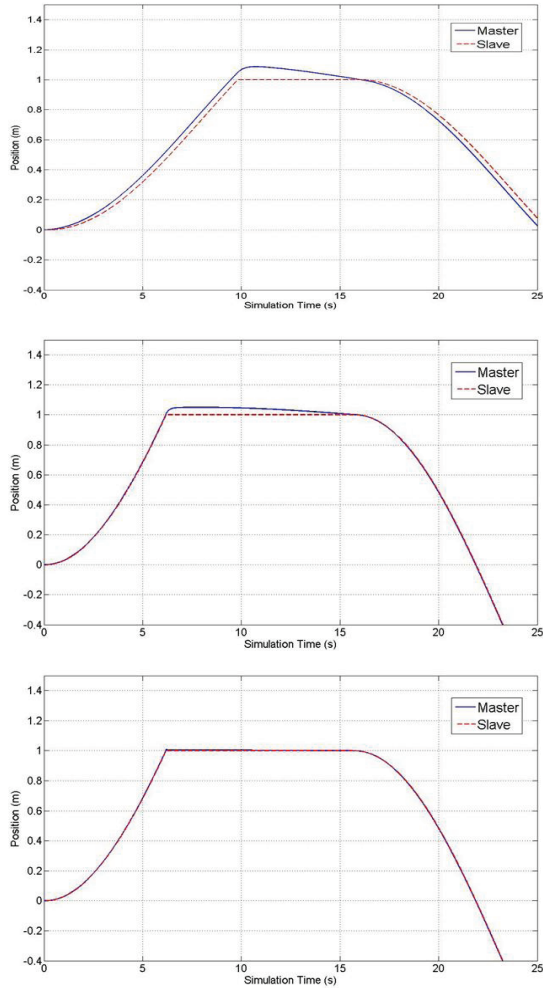


Fig. 3. Position response in wall contact motion (upper: discrete bilateral state feedback control scheme [Zhang et al. (2011a)]; middle: continuous force-reflecting proxy control scheme [Zhang et al. (2011b)]; lower: our result).

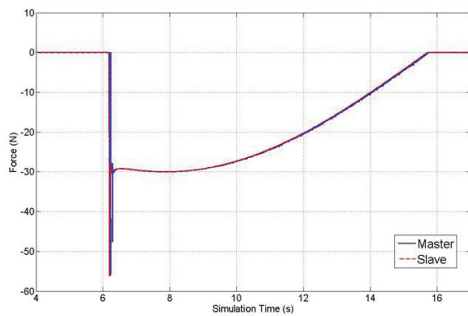


Fig. 4. Force response in wall contact motion ( $F_m(k)$ ;  $\hat{F}_e(k)$ ).

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