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Eitan Altman. Game theoretic approaches for studying competition over popularity and over advertisement space in social networks. [Research Report] 2012. hal-00683781

HAL Id: hal-00683781
https://inria.hal.science/hal-00683781

Submitted on 29 Mar 2012

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Game theoretic approaches for studying competition over popularity and over advertisement space in social networks

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March 29, 2012

Abstract

Various tools are available for increasing the speed of content dissemination such as embeddinigs in some popular web pages, sharing in some other social networks, and advertisement. In particular, when individuals pass through a conotent provider to distribute contents, they can benefit from tools such as recommendation systems. The content provider can give a preferencial treatment to individuals who pay for advertisement. In this paper we study competition between several contents, each characterized by some given potential popularity. We study competition through advertisements that are placed at the beginning of the dissemination of contents. We answer the question of when is it worthwhile to invest in adveretisement as a function of the potential popularity of a content as well as its competing contents. The competition between similar contents (e.g. news channels) over a finite set of potential destinations. We then consider a second model in which there is also competition on advertisement space. We compute the equilibrium strategy and identify its structre and properties for each one of the situations.

1 Introduction

We consider in this paper competition between individuals (that we call "seeds") who create contents and wish to disseminate their content using the some social network. We assume that it is either the content that the seeds sell to interested peers or that their content provides information to some other product that peers are intnerested in. We assume that a seed can increasae its popularity using various costly actions. We call these dissemination acceleration actions. In particular, one can propose to pay the network provider in norder to receive a preferential treatment to one's content and have its rate of dissemination increased. This paper focuses on the study of acceleration acations at the beginning of the dissemination process.

We study two types of actions.

- The first are user independent actions. These are actions that several users can take, independently of each other. For example, the owner of a hotel can put an advertisement in cites specializing in proposing tourist packages.

- The second type consists of competition over an exclusive acceleration resource that is only available to one content source. We assume that only one user among those that propose a similar content can receive the preferential treatment.

As an example of actions of the first kind, observe Figure 3 that concerns a video clip of Obama over Youtube. Below the screen we can see the popularity curve (in terms of number of downloads) of the content as a function of time. We can also see some initial actions that increased the popularity of the clip. For example, action D consisted of embedding the video in the WEB site of the White-House. This action brought 31,008 viewers to this video as is seen in the table below the curve.

When some video makes it to the first position in the recommendation list related to a given set of tags, then it gets a higher visibility than the others that appear in that list, and therefore the speed of propagation is expected to increase. The first position in the list is available for seeds that pay for appearing there.

As an example of the second type of actions, observe Fig 2 that shows the computer screen that I had when watching a video clip on music by Piazzola using Youtube. One can observe three types of advertisements. There is an advertisement for EFS at the bottom of the large dark rectangle which is the screen that shows the video. If one wishes to watch the video then the dark rectangle will occupy the whole computer screen and then this advertisement will be the only one you would see. There is a second advertisement at the top right part of the screen - for courses in Piano Jazz. The first two advertisements are not special for content. We are interested in a third type of advertisement: To the right we see the first five video clips in a recommendation list provided by Youtube. The first in the list has a tag "Ad". It is a video clip that received a priority in the recommendation list. Only one clip is reserved in the recommendation graph for advertisement purposes.

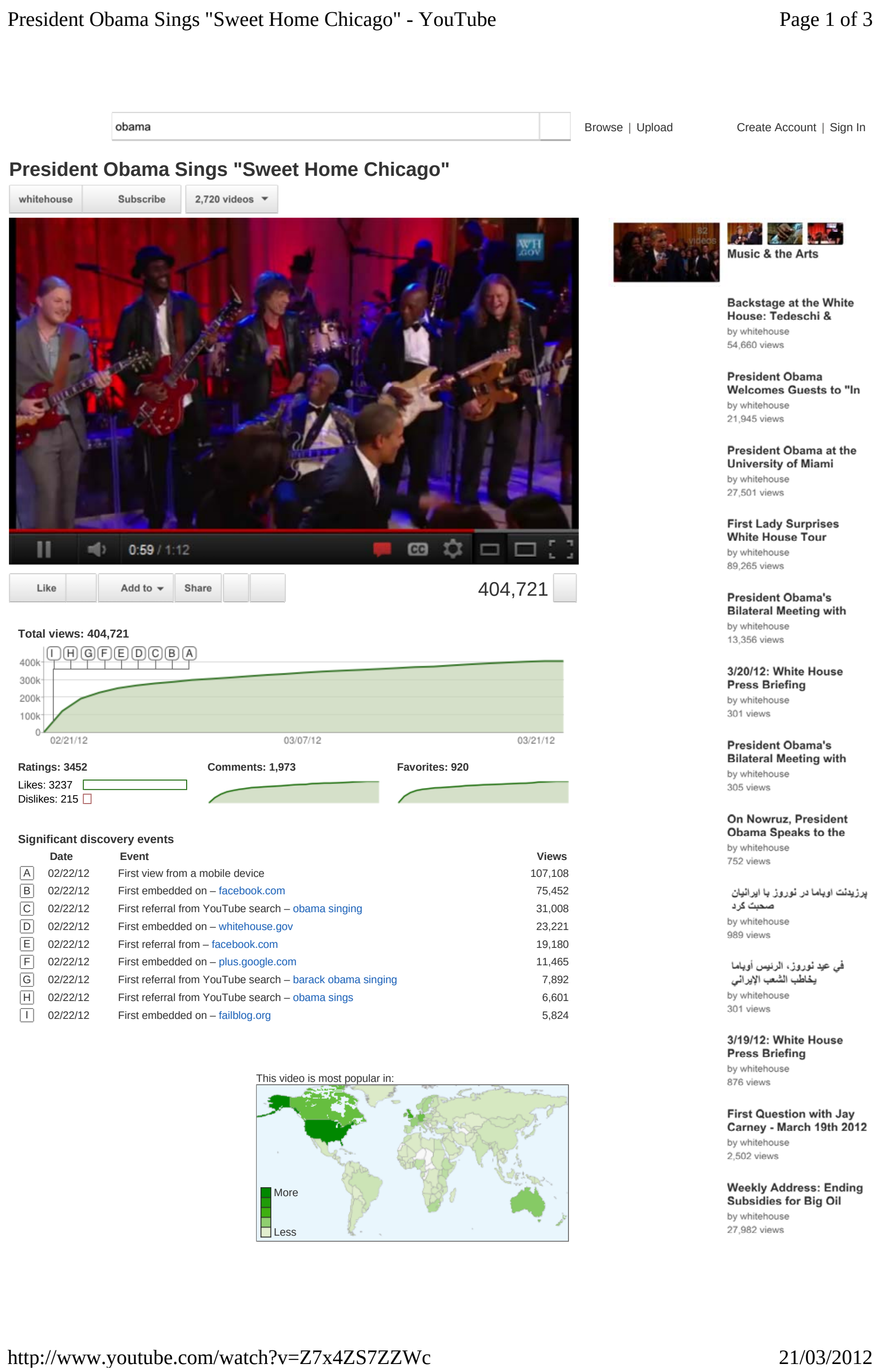

Figure 1: Actions for increasing the popularity of content: embedding and sharing

Figure 2: Publicity in Youtube

In addition to a competition on the preferencial service we consider also a competition over a common consumer population target. More precisely, we assume that there is a finite population that are potentially interested by content proposed by each one of several competing seeds. Any individual within this set will purchasse the content from the first seed that it becomes aware of. Thus paying for speeding up the rate of dissemination would then allow a seed not only to reach its target population faster but also decrease the number of peers that the competitors would reach. We formulate this decision problem as a game with finite state and action spaces. The solution of the problem allows us to provide guidelines for individual's advertisement strategies.

Related work: Epidemic propagation of information has been extensively studied in the context of marketing, and various related games have been formulated and solved. See [10] for a survey as well as [7, 8].

In contrast with previous work, our models focus on decision making by individuals who create contents and who compete over consumers, and their interaction with advertisement opportunities proposed either by the owner of the social network or by other coupled social networks. In a previous related work [3], we studied a stochastic game for dynamic advertisement strategies over a social network. There, new advertising opportunities were assumed to be available at each time epoch. Various information structures were considered. The state dependent equilibrium was computed and showen to be of a threshold type under some assumptions on the cost. More precisely, it was shown that advertisement effort is larger at equilibrium for seeds of more popular contents. Moreover, it is increasing in the amount of destinations that have not yet received any content. In contrast, the present work considers two types of advertizing that occur when initiating the spreading

We restrict our attention in this paper to strategic interaction between similar types of content. Some initial work on the question of competition between different types of content can be found in [2]. Other aspects of competition between both service providers as well as content providers can be found in [1, 4, 6, 9, 5].

2 Model dissemination in a competing environment

of the content.

We begin this paper by introducing in Section 2 a dynamic model that describes the impact of competition over the the population target on the dissemination of the competing contents. We then study in Sections 3 and 4 the equilibria obtained in advertising games when using the two type of actions, respectively (the user independent case and the exclusive case).

Assume that there is a set $\mathcal{N}$ of $N+1$ competing contents among the subscribers of a social network (the content may correspond to, say, some editting softwares that are sold over the network). N is assumed to be a random varable with Poisson distribution with parameter θ.

Let M the number of peers in the network that are interested in the content originating in $\mathcal{N}$.

We assume that opportunities for accessing a content n arrive at destination m according to a Poisson process with parameter λ_n starting at time $t = 0$. Hence if at time $t = 0$ destination m were interested in content n, it would have to wait some time which is exponentially distributed with parameter some parameter λ_i.

Let $X_i(t)$ be the number of destinations that have obtained by time t content from seed i. Let $x_i(t) := E[X_i(t)]$ and $x(t) := \sum_{i=1}^{N} x_i(t)$. Then

$$\dot{x}_i(t) = \lambda_i(M - x(t)) \tag{1}$$

Taking the summation over i in (1), we get,

$$\dot{x}(t) = \dot{x}(t) = \lambda(M - x(t)) \tag{2}$$

where $\lambda = \sum_{i=1}^{N} \lambda_i$.

The solution of (2) is

$$x(t) = M - (M - x(0))\,(1 - \exp(-\lambda t))$$

Thus $x_i(t)$ is the solution of

$$\dot{x}_i = \frac{\lambda_i}{\lambda} y(t) \tag{3}$$

The solution of (3) is

$$x_i(t) = x_i(0) + (M - x_i(0))\frac{\lambda_i}{\lambda}\,(1 - \exp(-\lambda t))$$

Assuming $x_i(0) = 0$ for all i, we get finally

$$x_i(t) = M\frac{\lambda_i}{\lambda}(1 - \exp(-\lambda t)) \tag{4}$$

Althogh the above model is quite simplistic, the form of the last equation is quite similar to the form of the curves describing the evolution of popularity in Youtube videos. As an example, observe the curve

$$x_i(t) = 420000(1 - \exp(-0.3t)$$

depicted in figure 3. It is obtained from the last equation by identifying $\lambda = 0.3$ and $M\frac{\lambda_i}{\lambda} = 420000$. With these estimations we see that the download curve is very similar to the one obtained using the youtube statistics in Figure 3.

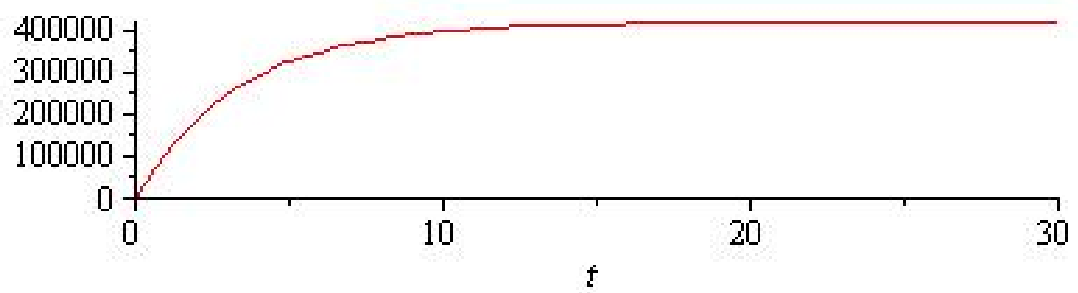

Figure 3: A mathematical model for the evolution of the popularity of Obama

In the next Sections we shall build on the results of this Section. We shall especially make use of the fact that as $t \to \infty$, $x_i(t)$ converges (monotonically) to $M\frac{\lambda_i}{\lambda}$.

3 The advertisement game of user independent actions

Next, we assume that λ_j are decision variables. Thus any player j can choose λ_j satisfying $\lambda_i \geq \phi_i$. ϕ_i corresponds to deciding not to accelerate. λ_j, $j = 1, ..., N$ is an equilibrium if for each j, λ_j maximizes $W_j(x)$ over $x_j \geq \phi_j$ where

$$W_j(\mathbf{x}) := \frac{x_j}{\sum_k x_k} - \gamma(x_j - \phi_j)$$

$\gamma > 0$ is the acceleration cost. Note that this utility function is concave in x_j.

We write the Lagrangian for player j as

$$L_j(\mathbf{x}) = W_j(\mathbf{x}) + \beta_j(x_j - \phi_j)$$

where the Lagrange multipliers β_i are non-positive.

The best response: we solve

$$0 = \frac{\partial L_j}{\partial x_j} = \frac{\sum_{n \neq j} x_n}{\left(\sum_{n=1}^N x_n\right)^2} - \gamma - \beta_j$$

which gives the quadratic equation in x_j:

$$x_j^2 + 2\left(\sum_{n \neq j} x_n\right) x_j + \left(\sum_{n \neq j} x_n\right)^2 - \frac{\sum_{n \neq j} x_n}{\gamma + \beta_j} = 0$$

The non-negative solution is given by

$$x_j = -\sum_{n \neq j} x_n + \sqrt{(\beta_j + \gamma)^{-1}\left(\sum_{n \neq j} x_n\right)} \tag{5}$$

Denote $\psi = \sum_{i=1}^N x_i$. Then (5) implies that for all j,

$$\psi^2(\beta_j + \gamma) = \sum_{n \neq j} x_n = \psi - x_j \tag{6}$$

Taking the sum over all j we get we get the quadratic equation in ψ:

$$\psi^2 \sum_j (\beta_j + \gamma) = (N-1)\psi$$

so finally

$$\psi = \frac{N-1}{\sum_j(\gamma + \beta_j)} \tag{7}$$

3.1 The symmetric case

In the symmetric case, when $\phi = \phi_j$ are the same for all j, we have by (7)

$$x_j = \max\left(\phi, \frac{N-1}{N^2\gamma}\right)$$

This equals ϕ as long as

$$\gamma \geq \frac{N-1}{N^2\phi} =: \gamma_0$$

Assuming that the gain of the network is given by the advertisement cost, we have for any $\gamma \geq \gamma_0$ zero gain. For any $\gamma < \gamma_0$, on the other hand, the gain of the network is

$$N\gamma(x_j - \phi) = N(1 - \gamma\phi) - 1.$$

This can be made arbitrarily close to N by choosing γ sufficiently small.

The globally (social) optimum is obtained for $x_j = \phi$.

The price of anarchy for a given γ_0 is given by 1 for $\gamma \geq \gamma_0$, and by $N(1 - \gamma\phi)$ otherwize. Thusu

$$PoA = \max\left(1, N(1 - \gamma\phi)\right)$$

which can be made arbitrarily close to N by choosing γ small enough.

3.2 The general case

Substituting (7) in (6) we obtain

$$x_j = \psi(1 - \psi(\gamma + \beta_j)). \tag{8}$$

Due to the complementarity conditions on β_j, whenever $x_j > \phi_j$ then $\beta_i = 0$.

This, together with (8) imply that for all j for which $x_j > \phi_j$, we have

$$x_j = \psi(1 - \psi\gamma) \tag{9}$$

and the rest do not accelerate, i.e. $x_j = \phi_j$.

Hence there is some α such that $x_i = \phi_i$ for all i such that $\phi_i > \alpha$ and for all other i's, x_i are equal and given by (9). For all these, the equilibrium value does not depend on ϕ_i. Thus the strcture of the equilibrium policy is either not to accelerate, or to accelerate till some target is reached. This target is the same for all those who accelerate and does not depend directly on the propagation rates ϕ_i before accelerating. (The values ϕ_i only determine whether there will be an acceleration of the ith content).

For all j for which $x_j = \phi_j$ we have by (8)

$$\gamma + \beta_j = \frac{1}{\psi}\left(1 - \frac{\phi_j}{\psi}\right)$$

In particular, assume that

$$\frac{N-1}{N^2\gamma} > \max_{j=1,\ldots,N} \phi_j. \tag{10}$$

Then

$$x_i = \frac{N-1}{N^2\gamma} \tag{11}$$

satisfies $x_i > \phi_i$ implying $\beta_i = 0$ for all i. We conclude that x_i given in (11) is the equilibrium if (10) holds.

Next assume it does not hold. Reorder the players such that ϕ_i is decreasing in i. Then there is a group $\mathbf{M}$ of, say m players, who do not accelerate and for whome $x_i = \phi_i$.

Assume that m is the largest integer such that $x_m = \phi_m$. Then from (9),

$$\psi = (N-m)\psi(1-\psi\gamma) + Z[m]$$

where $Z[m] = \sum_{i=1}^{m} \phi_i$. Thus

$$(N-m)\gamma\psi^2 + \psi(1-N+m) - Z[m] = 0$$

so that

$$\psi[m] = \frac{1}{2(N-m)\gamma}\left((N-m-1) + \sqrt{(N-m-1)^2 + 4(N-m)\gamma Z[m]}\right)$$

4 The case of a single advertisement opportunity

We assume that the contents of the N seeds appear in some ordered recommendation graph. Thanks to its position in the recommendation list, the first in the list, say the content of seed i, is assumed to be more visible than the others. We assume that this translates to a larger value of λ_i More precisely, we consider the situation where λ_j are all equal to some constant η, except for λ_i which is taken to be a times that value.

We assume that each seed can make a bid in order to be in the top of the list. There is a cost c for bidding, and in addition, the one that wins has to pay some fixed cost d. If no one makes any bid then all seeds are equally likely to find their content in the head of the line, and no seed has to pay anything. If k seeds bid for being the first in the line then that place will be attricuted to one of them with equal probabililties.

The expected utility of player j is given by

$$U_j = E[X_j] - \gamma E[C_j]$$

where

$$X_j = \lim_{t\to\infty} X_j(T)$$

is the number of destinations that will receive have the content of seed j, and C_j is the cost for bidding.

We have

$$R(f,n) = \frac{aM}{n-1+a}.$$

This is the dissemination utility for the seed that is the first in the recommendation list given that there are $n-1$ other seeds. The dissemination utility for every other seed is given by

$$R(o,n) = \frac{1}{n-1+a}M.$$

Theorem 1. *(i) $p = 0$ is an equilibrium if*

$$\frac{M}{n} \geq \frac{aM}{n-1+a} - d - c.$$

The value at equilibrium for each player is then M/n.
(ii) $p = 1$ is an equilibrium if

$$\frac{M}{n} - d - \frac{c}{n} \geq \frac{M}{n-1+a}.$$

The value at equilibrium for each player is then $M/n - d - \frac{c}{n}$.
(iii) Otherwize there is a mixed equilibrium p such that at equilibrium, player is indifferent between bidding or not. p is thus given by the solution of $U_i(B,p) = J_i(A,p)$ where

$$J_i(A,p) = (1-p)^{n-1}\frac{M}{n} + \left(1-(1-p)^{n-1}\right)\frac{M}{n-1+a}$$

and

$$U_i(B,p) = \left[\frac{M}{n-1+a} + c\right]\left(1 + \frac{(a-1)(n-1)}{np}(1-(1-p)^n)\right) + d.$$

Proof. We have:

$$U_i(B,0) = \frac{Ma}{n-1+a} - d - c, \qquad U_i(A,0) = \frac{M}{n}.$$

This implies (i).

For $p = 1$, we have

$$U_i(B,1) = \frac{M}{n} - d - \frac{c}{n} \qquad U_i(A,1) = \frac{M}{n-1+a}.$$

This implies (ii).

Assume that each seed bids with probability p except for player i. Let ν be the number of those that bid not including player i.

If player i abstains (does not bid), then

$$J_i(A,p) = ME\left[1\{\nu=0\}\frac{M}{n} + 1\{\nu>0\}\frac{M}{n-1+a}\right]$$
$$= (1-p)^{n-1}\frac{M}{n} + \left(1-(1-p)^{n-1}\right)\frac{M}{n-1+a}.$$

If it bids and there are ν others that bid as well, then

$$J_i(B,p) = E\left[\frac{M}{n-1+a}\frac{\nu+a}{\nu+1}\right]$$
$$= E\left[\frac{M}{n-1+a}\left(1+\frac{a-1}{\nu+1}\right)\right]$$

ν has a Binomial distribution with parameter $(n-1,p)$. Thus,

$$E[\frac{1}{\nu+1}] = \sum_{i=0}^{n-1} P(\nu=i)\frac{1}{i+1}] = \sum_{i=0}^{n-1}\frac{(n-1)!}{i!(n-1-i)!}\frac{1}{i+1}p^i(1-p)^{n-1-i}$$
$$= \frac{1}{np}\sum_{i=0}^{n-1}\frac{n!}{(i+1)!(n-(i+1))!}p^{i+1}(1-p)^{n-(i+1)} = \frac{1}{np}(1-(1-p)^n)$$

Taking into account the bidding costs we have

$$U_i(B,p) = J_i(B,p) - d - c = E\left[\frac{\nu-1+a}{\nu}\right]$$
$$= \left[\frac{M}{n-1+a} + c\right]\left(1 + \frac{a-1}{np}(1-(1-p)^n)\right) + d$$

(iii) now follows since, if for some $p \in [0,1]$, $E[U_i(A,p)] = E[U_i(B,p)]$ then p is an equilibrium.
■

5 Concluding remarks

We have presented in this paper two types of competitive interactions between content producers (seeds) who use a social network to disseminate their content. In both there is a competition over a limited common set of potential destinations. The decisions available to the seeds are related to costly advertisement and their decisions may depend on the level of popularity of their contents. In the first scenario we considered situations where each of several seeds can increase its dissemination rate by some actions such as advertisement, sharing and embedding. In the second we considered situations in which the competition is also on the limitted advertisement opportunities. We characterized in both cases the equilibrium advertisement policies and identified their structure. We now extend these results to some other information structures.

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