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# The Effect of the *Back* Button in a Random Walk: Application for PageRank

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## ABSTRACT

Theoretical analysis of the Web graph is often used to improve the efficiency of search engines. The PageRank algorithm, proposed by [5], is used by the Google search engine [4] to improve the results of the queries.

The purpose of this article is to describe an enhanced version of the algorithm using a realistic model for the *back* button. We introduce a limited history stack model (you cannot click more than  $m$  times in a row), and show that when  $m = 1$ , the computation of this *Back* PageRank can be as fast as that of a standard PageRank.

## Categories and Subject Descriptors

F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems—*Computations on matrices*

## General Terms

Algorithms, Measurement

## Keywords

Web analysis, PageRank, Random walk, flow, back button

## 1. INTRODUCTION

Since the introduction of the *PageRank* algorithm in 1998, numerous enhancements were made in both implementation and theoretical efficiency. Using the stochastic aspect of the PageRank algorithm, the concept of *backoff process* was introduced by *Fagin et al.* [3] as an idealized model of browsing the web using both hyperlinks and the *back* button. This model allows the history stack to grow unboundedly. We introduce a bounded history stack, and show that in the special case of a one page history, there is an explicit and fast algorithm for computing the PageRank.

## 2. NOTATIONS

Let  $G = (V, E)$  be a web graph, that is a set  $V$  of web pages linked to each other by a set  $E$  of edges.

If  $G$  is aperiodic and strongly connected, it is well known [6] that the iterative process

$$\forall v \in V, n \in \mathbb{N}, P_{n+1}(v) = \sum_{w \rightarrow v} \frac{P_n(w)}{d(w)}, \quad (1)$$

where  $d(v)$  is the out-degree of  $v \in V$ , converges towards an unique probability  $P$  for any given probability  $P_0$ .

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However the web graph is far from being strongly connected [2]. One solution is to introduce a dumping factor  $d$ . The principle of the dumping factor is to “dump” the iterative process:

$$\forall v \in V, n \in \mathbb{N}, P_{n+1}(v) = d \sum_{w \rightarrow v} \frac{P_n(w)}{d(w)} + (1-d)S(v), \quad (2)$$

where  $S$  is a given probability on  $V^1$ .

A dumping factor is equivalent to working on a weighted strongly connected graph. If  $G$  is leafless, the limit  $P$  of (2) exists. Otherwise, normalization is needed.

## 3. BACK BUTTON MODEL

We suggest to refine the PageRank model by inserting the possibility to *return*. We choose a bounded history stack, so the PageRank algorithm is equivalent to a Markov chain with finite memory  $m$ . Potentially, this leads to consider all the possible paths in  $G$  of length  $m$ . For  $m = 1$ , this corresponds to the set  $E$  of the hyperlinks. We introduce two intuitive models for  $m = 1$ , one of them collapsing the working space from  $E$  to  $V$ . To begin with and for simplicity, we examine our *Back* button process without dumping.

### 3.1 Reversible *back*

In this model, we suppose that the web user can click at each state either on the links or on the *Back* button (the *Back* button is then considered as an outgoing link like the others). The probability of using the *Back* button is the same as that of using a given link. Using *Back* button brings the user back to the previous state<sup>2</sup>.

Let  $P_n^{rb}(w, v)$  be the probability of being in  $v$  in the instant  $n$  coming from  $w$  in the instant  $n - 1$ .  $P_n^{rb}(w, v)$  is defined if  $(w, v) \in E$  or  $(v, w) \in E$ . We can express the probability  $P_n(v)$  of being in  $v$  at the instant  $n$  as follows:

$$P_n(v) = \sum_{w \leftrightarrow v} P_n^{rb}(w, v) \quad (3)$$

Note that because of the *Back* process, we work on the non-directed graph induced by  $G$ .

Working on the same principle, we deduce an equation expressing  $P_n^{rb}(w, v)$ : if  $(w, v) \notin E$  (but  $(v, w)$  is), going from  $w$  to  $v$  implies using the *Back* button; then we were previously in  $w$  coming from  $v$ . Otherwise, either the *Back* button or the regular link can be used. Thus we have:

<sup>1</sup>Most of the time,  $S \equiv \frac{1}{|V|}$ , but some have suggested that it would be better to “personalize” it [1].

<sup>2</sup>Thus two consecutive uses of the *Back* button cancel each other.

$$P_{n+1}^{rb}(w, v) = \begin{cases} \frac{1}{d(w)+1}(P_n(w) + P_n^{rb}(v, w)) & \text{if } (w, v) \in E, \\ \frac{P_n^{rb}(v, w)}{d(w)+1} & \text{otherwise.} \end{cases} \quad (4)$$

Using (3) and (4) gives an iterative process for computing the new PageRank, but if  $G' = (V, E')$  is the non-oriented graph induced by  $G$ , we have to use  $|V| + |E'|$  variables instead of  $|V|$  for the standard PageRank.

### 3.2 Irreversible Back

We now consider that the *Back* button cannot be used twice consecutively. This model, which seems more complex, as however three important advantages. First, it significantly reduces the stored PageRank by “greenhouse effect” in the end-nodes. Second, it is more appropriate to the insertion of a dumping factor (see 3.3). Finally it is less heavy on resource.

For  $(w, v) \in E$ , let  $P_n^{ib}(w, v)$  be the probability to arrive at  $v$  using an hyperlink in  $w$ , and  $\bar{P}_n^{ib}(v)$  the probability to arrive at  $v$  using the *Back* button.  $\bar{P}_{n+1}^{ib}$  can be deduced from  $P_n^{ib}$ :

$$\bar{P}_{n+1}^{ib}(v) = \sum_{w \leftarrow v} \frac{P_n^{ib}(v, w)}{d(w)+1} \quad (5)$$

Then we can tell  $P_{n+1}^{ib}$  from  $P_n^{ib}$  and  $\bar{P}_n^{ib}$ :

$$P_{n+1}^{ib}(w, v) = \frac{1}{d(w)+1} \sum_{u \rightarrow w} P_n^{ib}(u, w) + \frac{\bar{P}_n^{ib}(w)}{d(w)} \quad (6)$$

We can note that  $P_{n+1}^{ib}(w, v)$  does not depend on the arrival node  $v$ . We can then use  $P_n^{ib}$  on  $V$  instead of  $E$ , specifying only the departure node.

Equations (5) and (6) can now be written:

$$\bar{P}_{n+1}^{ib}(v) = P_n^{ib}(v) \sum_{w \leftarrow v} \frac{1}{d(w)+1} \quad (7)$$

$$P_{n+1}^{ib}(v) = \frac{1}{d(v)+1} \sum_{w \rightarrow v} P_n^{ib}(w) + \frac{\bar{P}_n^{ib}(v)}{d(v)} \quad (8)$$

### 3.3 Back button and dumping

For a real graph, insertion of the *Back* button ensures there is virtually no leaf, but the process may still not be irreducible, so we need to introduce a dumping factor. We made the choice to deactivate the *back* button after a crossing<sup>3</sup>. We can then merge (2), (7) and (8) to obtain:

$$\bar{P}_{n+1}^{ib}(v) = dP_n^{ib}(v) \left( \sum_{w \leftarrow v} \frac{1}{d(w)+1} \right) + (1-d)S(v) \quad (9)$$

$$P_{n+1}^{ib}(v) = d \left( \frac{1}{d(v)+1} \sum_{w \rightarrow v} P_n^{ib}(w) + \frac{\bar{P}_n^{ib}(v)}{d(v)} \right) \quad (10)$$

<sup>3</sup>Deactivating the *back* button after a crossing avoids to consider the  $V \times V$  crossing transitions in the *Back* process.

## 4. EFFECTIVE COMPUTATION

### 4.1 Convergence

The process we made is stochastic (there is no blind way), aperiodic and irreducible (because of the dumping factor). The Perron-Frobenius theorem applies and ensures that the iterative process converges towards an unique fixed point.

### 4.2 Optimization

Using (9) and (10), we get an iterative way of calculating  $P_n^{ib}$ , and  $P_{n+1}^{ib}(v)$  is equal to:

$$\sum_{w \rightarrow v} \frac{dP_n^{ib}(w)}{d(v)+1} + \sum_{w \leftarrow v} \frac{d^2 P_{n-1}^{ib}(v)}{d(v)(d(w)+1)} + \frac{d(1-d)S(v)}{d(v)} \quad (11)$$

Equation (11) is a two terms recurrence, but as we want to compute a fix point, the Gauss-Seidel method allows to use  $P_n^{ib}$  instead of  $P_{n-1}^{ib}$ ; indeed one can approximate  $P_{n+1}^{ib}(v)$  by:

$$\sum_{w \rightarrow v} \frac{dP_n^{ib}(w)}{d(v)+1} + \sum_{w \leftarrow v} \frac{d^2 P_n^{ib}(v)}{d(v)(d(w)+1)} + \frac{d(1-d)S(v)}{d(v)} \quad (12)$$

We remark that this iterative process has the same complexity that the standard PageRank computation.

Once  $P_n^{ib}$  has converged toward a vector  $P^{ib}$ , we obtain easily the asymptotic probability of presence  $P$  as follows:

$$P(v) = \sum_{w \rightarrow v} P^{ib}(w) + \bar{P}^{ib}(v) \quad (13)$$

## 5. CONCLUSION

We have proposed an alternative PageRank that can be obtained as easily that the standard PageRank and that should offer a better modelization of the web users. Computations made on a 8 millions pages graph showed that the top ranked pages differ from one model to another, yet both seemed interesting. We still have to merge this algorithm with a semantic pertinence-sort to be able to test this new model in the “real life”.

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