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Optimal power division amongst different user speed classes in Small Cell Networks

Veeraruna Kavitha and Veronique

Abstract

Small cell networks promise to boost the capacity and provide good QoS (Quality of Service) to cell edge users. However these pose serious challenges, especially in the case of high speed unidirectional (e.g., users traveling in high ways or in metros) users in the form of frequent handovers. The extent of the handover losses depend greatly upon the speed of the user and we showed in our previous paper that the cell size increases monotonically with speed. Thus in scenarios with diverse users (speeds spanning over large ranges), it would be inefficient to serve all of them using base stations spaced at the same distance and it is practically infeasible to design different cell sizes to different speeds. In this paper, we alternatively propose to allocate/divide the power to a user based on his speed. Higher power virtually increases the cell size. We provide an optimal power division, optimal for busy probability (optimizes the trade-off between better QoS and handover losses), for a given power budget and cell size. We further establish the need for higher power budgets for higher cell sizes, which we called as beta + power scaling, where beta is the pathloss factor. We finally obtain optimal cell size, after beta + scaling and optimal power division.

I. INTRODUCTION

Short radius cells appear as a promising solution to boost the capacity of mobile networks and to respond to the increasing demand for video oriented bandwidth demanding services (see [3], [4]). With smaller cell sizes, the cell edge users are also closer to the base station, good Quality of Service (QoS) can be achieved for all the users. One can serve the users at higher data rates (resulting in smaller service times for the same information exchange) while operating with reasonable transmission power. However, operating with short radius cells (Small Cells (SC) or Pico Cells (PC)) raises major issues in terms of mobility management. Indeed, as the cell size decreases, the handover rate critically increases especially for high or medium speed users which results in un-acceptable rate of call drops: the frequency of handovers increases and the risk for un-sufficient resources/unavailable servers gets higher. In these conditions, the calls drop before completion of the service with higher probabilities.

In [5], we studied the above mentioned trade-offs (associated with mobile users and handovers) and obtained a cell size that is optimal for appropriate metrics, like expected service time, drop or busy probability etc. These optimal cell sizes are derived as a function of various parameters like, user velocity profile, wireless medium characters (in terms of propagation co-efficient), the transmission power etc. One of the important conclusions of this study is that the optimal cell size for larger velocities is larger. This result could be understood because of the following: a) with increase in the user speed, the user crosses a cell faster thus resulting in a smaller amount of information communicated per cell; b) the amount of information required for handover remains the same; c) thus the percentage of the time for useful information transfer, decreases with increase in user speed unless the cell size is increased with the speed of the user; d) thus the optimal cell size increases with increase in user speed.

Based on the results in [5], it is suggested in [2] to also boost the power for high or medium speed users. In this paper, we work further on this idea. The results in [5] actually show that the cell size increases monotonically with increase in user speed. Thus one can achieve better performance if the cell sizes are varied based on the speed at which the user is moving. However, it would be a practically infeasible procedure to vary the physical cell size based on the user speed. We instead propose to vary the power based on the speed of the user. The higher power virtually increases the cell size. In fact in [5] (see the dependency of L^* on P in Theorem 3), it is shown that the optimal cell size decreases with increase in power. In this paper, we divide the available power among the various user speed regimes according to an appropriate optimization criterion, which considers the trade-off between better service rate and frequent handovers, when the same cell size is used for all the users. Further we obtain a common cell size which again optimizes the above trade-off, after optimal power division among user speed classes.

If users in speed regime V_i , which occur with probability p_i , are allocated power P_i then the average power spent in a cell will be equal to $\sum_i p_i P_i$. The power budget (per cell) specifies the maximum value that this average power can take. To maintain power budget per unit length constant (which maintains the total power budget constant) one has to scale the power budget linearly with cell size (in case of one dimensional cells). However this linear scaling will not be sufficient as will be seen shortly (see [1] also for more details). For small cell networks, the propagation losses are majorly due to distance based path losses. The distance based attenuation factor scales down by the path loss factor, beta. If the power budget is increased only linearly, the maximum possible rate (capacity) for a cell edge user decreases monotonically with increase in cell size. Thus with constant or linearly increasing (in cell size) power budget, the optimal cell size will always be zero, the trivial solution (see [1]). One needs to scale the power budget by an index that is strictly greater than the path loss factor, beta, to ensure that the rates

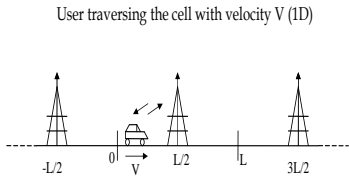


Fig. 1. User moving in a car while deriving service

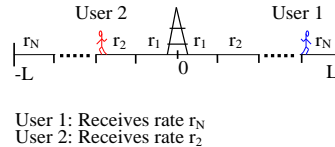


Fig. 2. A one dimensional cell, rate partitioning and user's movement

of the cell edge users either increase or remain the same with cell size, which further avoids the trivial solution. We call this as beta+ power scaling. In this paper, we explain and illustrate the need for beta+ scaling and obtain the optimal cell size when the power is boosted according to beta+ scaling.

The method and the tools used for the analysis in this paper could overlap largely with the ones used in our previous papers ([5] and [1]). But there are significant differences between the three papers. This paper and the results in [1] consider a radically different situation to that in [5], that of finite number of transmission rates. We also considered a total different category of users (randomly wandering users) in [1] while in this paper we propose a very different and new concept that of dividing the power based on the user speed regimes, in an optimal way.

II. SYSTEM MODEL

We consider users moving in a fixed direction. The users are moving in one direction (in a one dimensional line $[-D, D]$) and at high speeds, which vary negligibly during one call. One such example (see figure 1) is when a user driving in a car derives his service from portable base stations which are installed on street infrastructure (like lamp posts). The users can move in one of the two directions with equal probability, i.e., with half probability. We assume symmetry in both the directions and hence any performance (e.g., busy probability, drop probability etc.), conditioned on the direction of the user, will be equal for both the directions. Thus, unconditional performance would be the same as the performance given a direction, say left to right. *Without loss of generality we assume the users are moving from left to right.* The user moves with constant speed V , where V is uniformly distributed. We assume uniform arrivals. In case of small cell networks, the (adjacent) base stations are reasonably close and hence one has to design systems with good reuse factor to avail the advantages of small base station separations. In this work, we indeed assume this is the case, i.e., we assume no interference from the other cells.

Rate Regions: The cell is divided into $2N$ disjoint segments (based on the distance from Base station¹) such that the users in a segment are served with the same transmission rate. Let $\{\mathbb{A}_n\}_{n \in \mathbb{N}}$ represent these

¹In small cell networks (transmission at small distances), distance based propagation losses would be sufficient for deciding the theoretical rate limits as well as the practical transmission rates.

(see figure 2)

$$\mathbb{A}_n := \left[\frac{(n-1)L}{N}, \frac{nL}{N} \right] 1_{\{n>0\}} + \left[\frac{nL}{N}, \frac{(n+1)L}{N} \right] 1_{\{n<0\}} \text{ and } \mathbb{N} := \{-N, \dots, -1, 1, \dots, N\}. \quad (1)$$

User in region n receives service at rate $r_{|n|}$. Let $\mathbb{R} := \{r_1, \dots, r_N\}$ (which is arranged in decreasing order) represent the ensemble of all possible transmission rates. In Figure 2, both the segments $\mathbb{A}_n, \mathbb{A}_{-n}$ placed at the same distance on either side of BS are served with common rate $r_{|n|}$ and these common rates decrease as the distance from the BS increases. The rate at which the service is offered changes once the user switches the region.

Arrivals: There are two types of arrivals: 1) arrivals from external world (represented by subscript e and this is done only when there is ambiguity) modeled as Poisson arrivals with parameter λ ; 2) handover arrivals (always indicated using subscript h) modeled again as Poisson arrivals², but this stream is derived from a fraction of the stream 1 (stream of external arrivals) whose service is not completed at a cross over. The rate of arrivals into the cell of interest depends upon the cell dimension L and this is shown by either λ_L (for external arrivals) or $\lambda_{h;L}$ (for handover arrivals). For external arrivals, we assume³ $\lambda_L = \lambda L$ while $\lambda_{h;L}$ will be calculated in later sections. Every arrival, brings along with it Marks (Φ, S) , where $\Phi \in \mathbb{N}$ is the position of arrival with distribution $\Pi := \{\pi_n\}$ and S , the number of bytes to be transmitted, is exponentially distributed, i.e., $S \sim \mu \exp^{-\mu t} dt$ for some $\mu > 0$.

Resources: A cell can attend K parallel calls. The power per transmission, P , can depend upon L the cell dimension and this dependency will be discussed later.

An example of \mathbb{R} : One can choose the set of possible transmission rates, \mathbb{R} and N based on the practical channel coding schemes that would be used in the network design. The analysis presented can be utilized to study a system with any given \mathbb{R} and N . However, in this paper, we consider a specific example obtained using low SNR approximation of the following theoretical (capacity) rate⁴: $r(d) := P \left(1_{\{d \leq d_0\}} + r_0 |d|^{-\beta} 1_{\{d > d_0\}} \right)$ with $r_0 = d_0^\beta$. Here $r(d)$ is the rate at distance d , d_0 is a small lossless distance⁵ while β is the propagation co-efficient. We consider a specific system which supports

²This is a commonly made assumption, for example see [9], [10].

³If the arrivals in the entire line segment $[-D, D]$ occur at rate λ' those in segment $[-L, L]$ occur at a smaller rate $\lambda_L = \lambda' \text{Prob}(\text{arrival in } [-L, L])$. For the special case of uniform arrivals (i.e., arrivals landing uniformly in $[-D, D]$) $\lambda_L = \lambda L$.

⁴For unit noise variance, capacity equals $\log(1 + SNR)$, where signal to noise ratio $SNR = PA$, attenuation $A = 1_{\{d \leq d_0\}} + (d/d_0)^{-\beta} 1_{\{d > d_0\}}$. For low SNRs, $\log(1 + SNR) \approx SNR$ and hence capacity equals PA .

⁵Typically d_0 is very small and in this paper we consider optimizing over cell sizes $L > d_0 N$ so that $r(d) = r_0 P d^{-\beta}$ always.

transmission at the maximum possible rate for the entire region. For example in \mathbb{A}_n the farthest user will be at distance $|n|L/N$ and hence the rate that can be allocated, equals

$$r_n = r(|n|L/N) = r_0 P N^\beta L^{-\beta} |n|^{-\beta}. \quad (2)$$

Alternatively, if the system under consideration can design modulation and or channel coding schemes so as to achieve (almost) ν percent of the theoretical rates where $\nu < 1$ is a fixed coefficient, then again the above rate structure is applicable (after absorbing ν into r_0 of (2)).

Handovers: Whenever the user reaches the boundary $\{|x| = L\}$ the call is handed over to the neighboring cell. Every new connection requires s_h extra bytes to be exchanged to initiate it. The effect of these bytes (on the system performance) for a new call will be negligible (as it would be once), however one needs to consider their effect on handover calls. These bytes are usually very small in proportion to the actual bytes to be transmitted, i.e., $s_h \ll S$ with high probability. *We assume that these s_h bytes are exchanged with probability one, while the user is wandering in the last rate region (e.g., r_N) itself.*

Notations: We denote the transpose by t . Calligraphic letters represent matrices. Mathbb letters represent sets (e.g., \mathbb{N} - set of segment numbers \mathbb{R} - set of all possible transmission rates, \mathbb{A}_n - rate region n). The contents inside two flower brackets represent either a set or an ordered tuple (as according to convenience): for example $\{r_n\}$ represents the set \mathbb{R} while $\{\pi_n\}$ represents the ordered tuple Π . Lower case letters represent time index (k) or the segment index (n). Lower case bold letters represent the vectors.

Upper case letters either represent system parameters (e.g., D - dimension of Macrocell, L - dimension of small cell, P - Power per transmission, $\Pi = \{\pi_n\}_n = \{Prob(\text{Arrival in segment } n)\}_n$ - Vector of arrival probabilities etc.) or represent random variables (W - wandering time, S - number of bytes to be transferred, Φ - the segment in which the user is wandering etc.).

This scenario is exactly similar to the case in our previous work ([5]), but for one major difference. In [5], it is assumed that the rate of communication can be changed continually. This in some sense gives a "maximal" performance: if one can change rate of communication continually and that too, to the maximum possible one (i.e., capacity) then one obtains the best performance. But in reality this is not possible and we now consider similar situation but with maximum N different possible rates of communication as in the previous sections. This change makes an enormous changes in the results like one requires $\beta+$ power scaling where β represents the propagation co-efficient for an optimal cell size that is not zero (see also [1]).

III. HIGH SPEED USERS (CARS) MOVING IN ONE DIRECTION: ANALYSIS

A. Derivation of appropriate user speed dependent Performance measures

We consider Cell 0, without loss of generality and also the users moving from left to right. Let ψ_n represent the probability that a call that originated in rate segment n , \mathbb{A}_n , is completed before reaching the boundary L . The user would be serviced at rate $r_{|n|}$ (hence a total of $(L/N)(1/V)r_{|n|}$ bytes are transferred) till he crosses, \mathbb{A}_n then with rate $r_{|n+1|}$ and so on till he reaches the boundary point L . So, the number of bytes communicated during this transit would be $\sum_{m=n}^N r_m (L/N)(1/V)$ and hence the user will not complete his call (transfer of S bytes of information) before leaving the cell with probability:

$$\psi_n = \text{Pr} \left(S < \frac{L}{NV} \sum_{m \geq n}^N r_m \right) = 1 - E \left[e^{-\frac{\mu L \sum_{m=n}^N r_m}{NV}} \right] \text{ as } S \text{ is exponential.}$$

It is difficult to obtain the Laplace transform and hence ψ_n for velocities that are uniformly distributed. However the users are moving at high speeds (which are more or less constant), we are further dividing the users into different classes based on their speeds and so it is appropriate to assume that V is uniform between V_{max} and V_{min} , with V_{max} close to V_{min} and both away from 0. With these assumptions (for all realizations of V) one can approximate,

$$1 - \psi_n \approx 1 \quad \text{and} \quad \psi_n \approx \frac{\mu L \sum_{m=n}^N r_m}{N} E[1/V].$$

Let $P_{e,ho}$ ($P_{h,ho}$) represent the probability that a new (handed over) call is handed over (again) to the neighboring cell. Recall here that we are modeling the handovers also as Poisson arrivals. A new call can arrive in any \mathbb{A}_n with probability π_n while an handed over call always occurs at $-L$, i.e., in rate region \mathbb{A}_{-N} (note we consider only calls from left to right without loss of generality). Then by unconditioning (w.r.t. the event of arrival being in \mathbb{A}_n),

$$P_{e,ho} = 1 - \sum_{n=-N}^N \pi_n \psi_n = 1 - PL^{1-\beta} E[1/V] C_{e,ho} \quad \text{and} \quad P_{h,ho} = 1 - \psi_{-N} = 1 - PL^{1-\beta} E[1/V] C_{h,ho} \quad \text{with}$$

$$C_{e,ho} := \frac{\mu}{N} r_0 N^\beta \sum_{n=-N}^N \pi_n \sum_{m=n}^N |m|^{-\beta} \quad \text{and} \quad C_{h,ho} := \frac{\mu}{N} r_0 N^\beta \sum_{n=-N}^N |n|^{-\beta}. \quad (3)$$

Let b_n represent the average amount of time for which service is received in a cell, given that the call is originated in region n . Since these are high speed users and small cells, with high probability a call is not completed in one cell and hence one can approximate b_n with the time taken to reach the boundary:

$$b_n = \frac{L}{N} (N - n) E \left[\frac{1}{V} \right].$$

We call the amount of time for which a user is served in a cell, as the service time. The overall service time of a new call (b_e) and that of a handed over call (b_h) on average equal (by unconditioning)⁶:

$$b_e = \sum_{i=-N}^N b_n \pi_n = C_{b,e} L E[1/V] \text{ and } b_h = b_{-N} = C_{b,h} L E[1/V] \text{ where} \quad (4)$$

$$C_{b,e} := \frac{1}{N} \sum_{n=-N}^N \pi_n (N - n) \text{ and } C_{b,h} := 2.$$

Extra handover bytes s_h : So far, in the analysis we did not consider the effect of the extra bytes required for handover s_h , which only impact a handover call. In the following, we will modify the expressions derived so far (only for the ones specific to handover calls), after considering the effect of these s_h handover bytes. The time taken for exchange of handover bytes s_h has to be included in the time of the cell utilized by the user and hence the (handover) service time b_h (which by approximation is the time taken to traverse the entire cell) does not change with s_h . However $P_{h,ho}$, the probability of a handover call not completing the call before reaching the (opposite) boundary, changes to the following

$$\psi_{-N,h} = Prob \left(S + s_h < \frac{L}{NV} \sum_{m=-N}^N r_m \right) \approx (C_{h,ho} P L^{1-\beta} E[1/V] - \mu s_h) = 1 - P_{h,ho}. \quad (5)$$

Before we proceed further, we would like to note down an important result regarding the limit of the speeds that can be supported, which is a counter part of the [5, Theorem 2] for a system supporting finite number (N) of transmission rates. A user entering at $-L$ when moving with speed V , can transfer at maximum

$$g^N(L) := \frac{L}{NV} \sum_{m=-N}^N r_m = C_{h,ho} P L^{1-\beta} / V, \quad (6)$$

bytes of information in a cell, out of which s_h are used for handover purposes. So, useful communication is possible (and $\psi_{-N,h} > 0$) only when

$$g^N(L) = C_{h,ho} P L^{1-\beta} \frac{1}{V} > \mu s_h.$$

With $\beta > 1$ (the practical range of path loss factors), $g^N(L)$ reduces with L and so useful communication is not possible for any cell size if $g^N(Nd_0)$ itself is less than μs_h and hence we have (substituting $C_{h,ho}$ from (3)):

⁶The higher speed users are handed over more frequently than the lower speed ones. So, the distribution of the HO user velocities need not be uniform even if the new users arrive with uniform speeds. However as in [5, Appendix B], one can show that the HO user speed distribution converges to uniform distribution as the cell size reduces to zero.

Theorem 1: When $\beta > 1$, there exists a limit on the maximum possible velocity that can be supported by the system for a given transmission power P :

$$V_{lim}(P) := \frac{1}{s_h} r_0 N^{\beta-1} \sum_{n=-N}^N |n|^{-\beta} P d_0^{1-\beta}. \square$$

Comparison between systems with finite and infinite \mathbb{R} : Theorem 1 is equivalent of the limit result derived for an ideal system supporting infinite number of transmission rates, [5, Theorem 1]. In [5], we considered a system which can change the transmission rate instantaneously (with the distance between the user and the serving BS) to the maximum possible one (capacity). We derived the total number of bytes that can be communicated while the user moving at speed V traverses one entire cell (see expression for $g(L)$ as given by equation (3) in [5] with $\beta > 1$). This quantity determines the system performance to a great extent and is reproduced here ([5, equation (3)]):

$$g(L) = \frac{2Pd_0}{V(\beta-1)} \left(\beta - \left(\frac{L}{2d_0} \right)^{1-\beta} \right).$$

We see that $g(L)$ increases with L , the cell size and finally saturates to $2Pd_0\beta/(V(\beta-1))$. On the other hand, for the system considered in this paper with finite set of transmission rates (\mathbb{R} , with $|\mathbb{R}| = N$), the corresponding number (of bytes transferred in a cell) is given by $g^N(L)$ defined in (6). For this finite system $g^N(L)$ actually reduces with L in contrast to the ideal system of [5]. This is because when the number of possible transmission rates is fixed at N , even with the 'best'⁷ rate allocations given by (2), the serving (transmission) rate at any point in the cell reduces as the cell size L increases. *Thus with limited resources (limited number of feasible transmission rates) the system behavior changes drastically.* Note that one can achieve a system close to the one in [5] if the number of supported rates, N , increases appropriately with L .

Because of these effects resulting from the limitation of resources, one would need the transmit power P to be scaled (with L) by a proportionality factor greater than β (the beta+ power scaling mentioned in the Introduction). The beta+ scaling is discussed at the end of this subsection after deriving the appropriate system metrics.

Non elastic traffic and relevant performance metrics: We now continue with the derivation of the performance metrics that capture the trade off between handovers losses and the better service rates. In this work, we consider non elastic traffic. Non elastic traffic comprises of users demanding immediate

⁷In (2), we divided the cell into equal area regions and assigned each region with the best the possible transmission rate, the 'maximal' rate (capacity) of the farthest user of the region.

service. These users (e.g., voice calls) drop the call if it is not picked up immediately, i.e., if all the K servers are busy. The probability that a call is not picked up immediately is called the Busy probability and the probability that a call that was picked up is ever dropped before completing its service is called the Drop probability. Both these performance metrics depend upon handovers as well the average service times and hence captures the required trade-off. In this paper we work with metrics proportional to P_{Busy} . One can obtain P_{Drop} as in ([5], [1]) and we omit its discussion here. Performance metrics in this section are derived in a way similar to that in [5] and [1]. We briefly explain the derivations and more details can be found in the two papers.

As a first step we obtain the rate, $\lambda_{h,L}$, at which handovers occur. Let λ_L represent the fractions of the calls that arrive in the cell of interest $[-L, L]$ which equals λL for uniform arrivals (for appropriate $\lambda > 0$). A fraction of the new arrivals as well as handover calls get converted (again) to handover calls. The calls that have not finished their service before reaching the boundary are exactly this fraction, whose value is given by $P_{e,ho}$ and $P_{h,ho}$ respectively for new arrivals and the calls that resulted from a handover. By memory less nature of S (the bytes to be transferred) we have a stochastic equivalence between the calls entering and leaving the cell (see [5], [1]). Using this, $\lambda_{h,L}$ satisfies the fixed point equation, $\lambda_{h,L} = \lambda_L P_{e,ho} + \lambda_{h,L} P_{h,ho}$. Hence, from equations (3), (5)

$$\lambda_{h,L} = \lambda_L \frac{P_{e,ho}}{1 - P_{h,ho}} = \lambda_L \frac{1 - PL^{1-\beta} E[1/V] C_{e,ho}}{PL^{1-\beta} E[1/V] C_{h,ho} - \mu s_h}.$$

The overall expected service time (considering handover and new arrivals) equals (see (4)),

$$\bar{b} = \frac{1}{\lambda_L + \lambda_{h,L}} (\lambda_L b_e + \lambda_{h,L} b_h).$$

The product of the average service times and the arrival rate gives the rate at which the overall load is arriving into the system and the ratio of this product with the number of servers is called load factor, which is given by:

$$\rho = \frac{1}{K} (\lambda_L + \lambda_{h,L}) \bar{b}.$$

Simplifying

$$\rho = \frac{1}{K} \lambda_L b_e + \lambda_{h,L} b_h = \frac{1}{K} \lambda L^2 E[1/V] \left(C_{b,e} + C_{b,h} \frac{1 - PL^{1-\beta} E[1/V] C_{e,ho}}{PL^{1-\beta} E[1/V] C_{h,ho} - \mu s_h} \right). \quad (7)$$

Busy probability P_{Busy} and the load factor ρ : A small cell catering to non elastic traffic can be modeled by an M/G/K/K queue (as in [5]). Then using Erlang's loss formula the busy probability can be calculated as,

$$P_{Busy}(L) := \frac{\rho^K / K!}{\sum_{k=0}^K \rho^k / k!}.$$

Busy probability, P_{Busy} , depends upon L only via ρ and both are differentiable in L (see [5] for similar details) and by differentiating twice one can immediately obtain the following:

Lemma 1: The optimizers of ρ and P_{Busy} are same, i.e., $L_\rho^* := \min_L \rho = \min_L P_{Busy}(L) =: L_{P_{Busy}}^*$. ■

Because of Lemma 1, we hence forth work with the load factor ρ .

Beta+ power scaling: For the system with limited resources, from (6) the key factor $g^N(L)$ (the number of bytes transfered while traversing one entire cell) decreases monotonically with L . Performance metrics like drop or busy probability further degrade due to handover losses. This intuition is directly established by the expression (7) for ρ : load factor ρ increases monotonically with L . So, unless one scales the transmit power P with L , the optimal cell size will be the trivial, Nd_0 . With cell size L , there exists approximately D/L cells in the system and if one uses transmit power P_L (now including the dependency on cell size L in the notations) for cell size L , then the total power used in the system is proportional to P_L/L . One can scale $P_L \propto L$, to maintain the total power constant. From (7), the load factor ρ depends upon L via two factors L^2 and $P_L L^{1-\beta}$ and ρ increases with the first factor and decreases with the second factor. With linear power scaling ($P_L L^{1-\beta}$ is then proportional to $L^{2-\beta}$) the second factor decreases at a smaller rate than the first factor (L^2) and hence and again the optimizer for ρ would be the trivial, Nd_0 . It is clear that the second factor $P_L L$ increases at a better rate and then ρ does not increase monotonically with L only if the transmission power P_L scales by a proportionality factor greater than β , i.e., if $P_L \propto L^{\beta+\gamma}$ for some $\gamma > 0$. We call this as *beta+ scaling*.

B. Optimal power law for a given cell size L and power constraint \bar{P} :

As explained in the Introduction, based on the results of [5], we are mainly interested in dividing the users into different categories based on their speeds and then dividing the power across the various user classes. From (7), load factor ρ depends upon the user speed profile only via $E[1/V]$. Thus, we consider I disjoint velocity regimes, each of which is identified with its $E[1/V]$ in one of the I values $\{1/V_1, \dots, 1/V_I\}$. Let p_i represent the probability that $E[1/V]$ equals $1/V_i$ and let P_i represent the transmit power utilized by the users of speed regime i .

The handover rates of the different user classes can be calculated as before and they equal:

$$\lambda_{h,L,i} = \lambda_{L,i} \frac{P_{e,ho,i}}{1 - P_{h,ho,i}} = \lambda L p_i \frac{1 - P_i L^{1-\beta} E[1/V_i] C_{e,ho}}{P_i L^{1-\beta} E[1/V_i] C_{h,ho} - \mu s h}.$$

Similarly the expected service time for different user classes can be calculated and then the over all

service factor ρ simplifies to (with $\delta_i := P_i E[1/V_i] L^{1-\beta}$)

$$\rho(L, \mathbf{P}) = \lambda_L b_e + \sum_i \lambda_{h,L,i} b_{h,i} = \lambda L^2 \left(C_{b,e} \sum_i p_i E[1/V_i] + C_{b,h} \sum_i p_i E[1/V_i] \frac{1 - \delta_i C_{e,ho}}{\delta_i C_{h,ho} - \mu s_h} \right). \quad (8)$$

We are interested power division $\mathbf{P} := \{P_1, \dots, P_I\}$ which optimizes load factor ρ while satisfying the power constraint: $\sum_i p_i P_i \leq \bar{P}$. The average power constraint can be satisfied by substituting $P_I = \frac{1}{p_I} (\bar{P} - \sum_{i<I} p_i P_i)$, and then (with $\mathbf{P}_{-I} := (P_1, \dots, P_{I-1})$)

$$\begin{aligned} \rho(L; \bar{P}) := \rho \left(L, \left(\mathbf{P}_{-I}, \frac{\bar{P} - \sum_{i<I} p_i P_i}{p_I} \right) \right) &= \lambda L^2 \left(C_{b,e} \sum_i p_i E[1/V_i] + C_{b,h} \sum_{i<I} p_i E[1/V_i] \frac{1 - \delta_i C_{e,ho}}{\delta_i C_{h,ho} - \mu s_h} \right) \\ &\quad + \lambda L^2 C_{b,h} p_I E[1/V_I] \frac{p_I - (\bar{P} - \sum_{i<I} p_i P_i) E[1/V_I] L^{1-\beta} C_{e,ho}}{(\bar{P} - \sum_{i<I} p_i P_i) E[1/V_I] L^{1-\beta} C_{h,ho} - p_I \mu s_h}. \end{aligned}$$

We are now interested in the optimal power division:

$$\mathbf{P}_{-I}^*(L; \bar{P}) = \arg \max_{\{\mathbf{P}_{-I} \in \mathbb{P}\}} \rho(L; \bar{P}) \text{ with } \mathbb{P}(\bar{P}, L) := \left\{ \mathbf{P}_{-I} : \min_{i \leq I} \delta_i > \mu s_h / C_{h,ho} \text{ with } P_I := \frac{\bar{P} - \sum_{i<I} p_i P_i}{p_I} \right\}.$$

Clearly, \mathbb{P} is non-empty⁸ and the optimization will a solution if the power constraint \bar{P} is sufficiently large and we assume the same in this paper. Calculating the partial derivatives and equating them to zero, $\partial \rho / \partial P_i = 0$ for all u , the optimal power division $\mathbf{P}^*(L; \bar{P})$, for a given cell size and power constraint \bar{P} equals (for all i):

$$P_i^*(L; \bar{P}) = \bar{P} + \frac{\mu s_h}{L^{1-\beta} C_{h,ho}} \sum_{j<I} p_j \left(\frac{1}{E[1/V_I]} - \frac{1}{E[1/V_j]} \right) - \frac{\mu s_h}{L^{1-\beta} C_{h,ho}} \left(\frac{1}{E[1/V_I]} - \frac{1}{E[1/V_i]} \right). \quad (9)$$

C. Optimal cell size after using the optimal power law (9)

As already mentioned one needs a sufficiently large power constraint for a non empty domain of optimization, \mathbb{P} . Further, by beta+ scaling, we have a L dependent power constraint i.e., $\bar{P} = \tilde{P} L^{\beta+\gamma}$ for some $\tilde{P}, \gamma > 0$ and we assume \tilde{P} is sufficiently large. The final goal of this paper is to obtain the optimal cell size when the users are allocated (optimal) transmission powers based on their speeds as in (9) and when the power constraint is beta+ scaled:

$$L^* = \arg \max_{\{L: \mathbb{P}(\tilde{P} L^{\beta+\gamma}, L) \neq \emptyset\}} \rho \left(L, \mathbf{P}^*(L; \tilde{P} L^{\beta+\gamma}) \right).$$

However increasing the power constraint by $\beta+$ scaling, implies increasing the cost for power and hence we consider the following joint cost to obtain the optimal cell size:

$$L^* = \arg \max g(L) \quad \text{with} \quad g(L) := \rho \left(L, \mathbf{P}^*(L; \tilde{P} L^{\beta+\gamma}) \right) + w_P \tilde{P} L^{\beta+\gamma},$$

⁸The set \mathbb{P} is inverse image of a continuous mapping on a connected set and it grows with increase in \bar{P} .

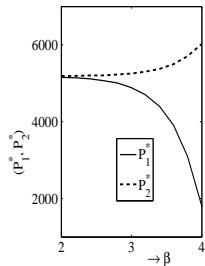


Fig. 3. Optimal power division ($\bar{P} = 0.0001$, $L = 160$)

Common Settings

$$N = 3 \quad d_0 = 5$$

$$r_0 = .4d_0^\beta \quad \gamma = 0.5$$

$$K = 500$$

$$\lambda = 10^{-7} \quad s_h = 0.2$$

$$\mu = 0.000001$$

$$\omega_p = 0.0002$$

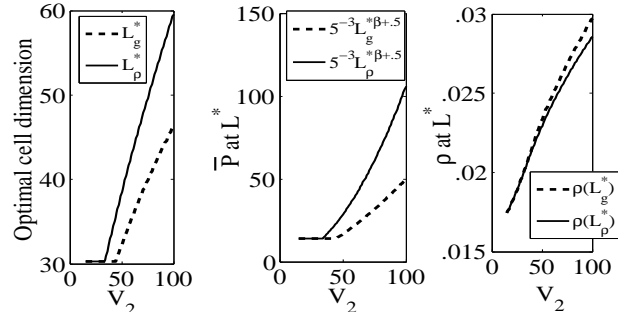


Fig. 4. Optimal cell size, performance and power budget versus V_2

where w_P is the weight given to the power cost. This is a complicated optimization and we obtain the optimal cell size via some numerical examples.

IV. NUMERICAL EXAMPLES

In Figure 3 we plot optimal power division as a function of β with settings as in figure. Additionally, we have two speeds: $E[1/V] \in [.067 \ .011]$ with $(p_1, p_2) = (0.2, 0.8)$. Interestingly we notice: the more the losses (pathloss factor) are, the more diverse the two allocated powers are.

We next consider another example in Figure 4 where we obtain optimal cell size after $\beta+$ scaling and optimal power division. We consider two user speeds ($I = 2$) and fix the lower speed at $E[1/V_1] = 1/15$ while vary the second one. The settings are same in before (except for $\omega_p = 0.005$ and $\bar{P} = 0.0005$) and we plot the optimal cell size for both total cost (L_g^*) as well as ρ (L_p^* when $\omega_p = 0$ in g) as a function of $V_2 := 1/E[1/V_2]$. We also plot the power budget per cell size ($\tilde{P}L^{\beta+\gamma}$), ρ at the optimizers.

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