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A reconstruction of  
Krause's table of factors  
(1804)

Denis Roegel

20 December 2011



# 1 Krause (1781–1832)

Karl Christian Friedrich Krause (1781–1832) was a German philosopher. He studied philosophy and mathematics at the University of Jena. He later went to Berlin, Göttingen and Munich. In Göttingen, he was one of Schopenhauer’s teachers.

## 2 Krause’s table of factors and primes

One of Krause’s first books was his table of factors and primes, published in 1804 [9]. In this book, Krause gave a number of lists in a very compact and unusual way.<sup>1</sup> Here, we will only focus on two of his tables: the table of primes and the table of factors.

In his introduction, Krause mentions the tables of Lambert [10] and Felkel [2, 3, 4, 5], and he understands the objections towards the use of letters for numbers which was favored by Felkel. He nevertheless decided to provide his two short tables, in order to show that letters can be used more conveniently than in Felkel’s table. The main tables in Krause’s book, however, do not use letters.

Krause wrote that his tables were computed (*neuberechnet*), and presumably not copied, although he certainly compared his tables with earlier ones. He also announced an extension of his table of factors to one million (with only the smallest factors), as well as an extension of the table of primes to two millions. These tables have never been published.

### 2.1 Krause’s table of primes

In this table, Krause gave a list of all primes from 1 to 100000. Each prime is encoded by a letter and the whole table fits on two pages. The first five primes are given explicitly, and they are followed by the list “efghkmnprstwyzbdegilo 1abcdflnpquvxabeghmnop zeklmoqrvxacdfhin 3cefgnptuwybegilo ...”

The first sequence gives the primes up to 97. The symbols are to be read in the following table :

01	03	07	09	11	13	17	19	21	23	27	29	31	33	37	39	41	43	47	49	51	53	57	59	61	63	67	69	71	73	77	79	81	83	87	89	91	93	97	99
a	b	c	d	e	f	g	h	i	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	k	l	m	n	o	p

We have therefore the primes 11 (e), 13 (f), 17 (g), etc. The symbols from “a” to “p” can refer to two different numbers, but the sequences above remove any ambiguity. The last symbol of the first sequence, namely “o”, represents 97, and not 33, because the numbers represented by the symbols appear in increasing order.

After 97, we have a new sequence, starting with “1”. This sequence gives all prime numbers between 100 and 200. The first digit is “1” and the two other digits are given by the symbol, using the encoding given above. Each sequence therefore corresponds to an interval of 100 integers. Every ten such sequences, the prefix is given in isolation: 10, 20, etc. Every hundred such sequences, the prefix (or rather the number of thousands) is given in the margin.

One might wonder about the ambiguities which could arise given that some symbols are duplicated in the encoding list. It appears that there is only one ambiguous case in

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<sup>1</sup>One of the lists (see figure 5), in particular, gives the Farey sequence from  $\frac{1}{30}$  to 1, with denominators not exceeding 30. But such lists of fractions were not yet called Farey sequences by then.

the range of the table, namely for the interval 31400 to 31500. In this interval, the first prime is 31469, and the first alphabet is not at all used, causing possible confusion. In that case, Krause wrote the sequence “*4cfhl*” instead of “*4cfhl*”, and he drew the attention of the reader to this special case at the end of his list of primes.

In our reconstruction, we have faithfully followed Krause’s scheme and used exactly the same line and page breaks as he did. During the reconstruction, we have found one error in Krause’s table, namely that he gave 16587 and 16593 as prime, the correct values being 16567 and 16573. This was possibly a typographic error at some point, as the two errors seem to be related. There may be other errors, but at least the values at the line breaks should otherwise be correct.

## 2.2 Krause’s table of factors

In this table, Krause gave the list of all factors for all numbers not divisible by 2, 3, and 5 until 10000. Each number is also encoded by a letter, but the factors are always given explicitly. Moreover, there is now no ambiguity in the symbols, since Krause uses latin and gothical alphabets.

Krause intended to give all factors, and not merely the simple factors. The composite numbers are given with prefixes and almost the same symbols as above. The only difference, apart from the use of a second alphabet, are that the latin “o” is written “*φ*,” presumably in order to avoid a confusion with the digit “0,” although such a confusion normally does not arise.

01 03 07 09 11 13 17 19 21 23 27 29 31 33 37 39 41 43 47 49 51 53 57 59 61 63 67 69 71 73 77 79 81 83 87 89 91 93 97 99  
a b c d e f g h i k l m n *φ* p q r s t u v w x y z a b c d e f g h i f l m n o p

The list of composite numbers starts with

u 7.7 f 7.11 m 7.13 - 1 - h 7.17 i 11.11 *φ* 7.19 s 11.13 z 7.23 c 13.13 f 11.17 - 2 - ...

The first “u” stands for 49, then “f” is for 77, and so on. Hundreds are separated like in the first table, so that the first “h” stands for the number 119.

Each composite number is followed by the list of its simple factors, separated by dots. When a simple factor has a multiplicity greater than 1, it is repeated, as for  $9n = 931 = 7 \times 7 \times 19$ :

... - 9 - a 17.53 f 11.83 g 7.131 k 13.71 n 7.7.19.133.49 s 23.41 ...

This example also shows that the list of simple factors given in increasing order is followed by the list of composite factors given in decreasing order. 931 has two such composite factors (other than itself), namely 133 and 49.

Although Krause intended to give all factors, the lists of composite factors are often incomplete. One of the last numbers of the table is 9947 and Krause omits the factors 203 and 49. His list of factors should have been instead

t 7.7.7.29.1421.343.203.49

Since Krause forgot a number of composite factors, we have decided not to follow his line breaks in our reconstruction. It should nevertheless be useful, first in order to convey Krause’s scheme, and second in order to provide a tool for checking Krause’s table.

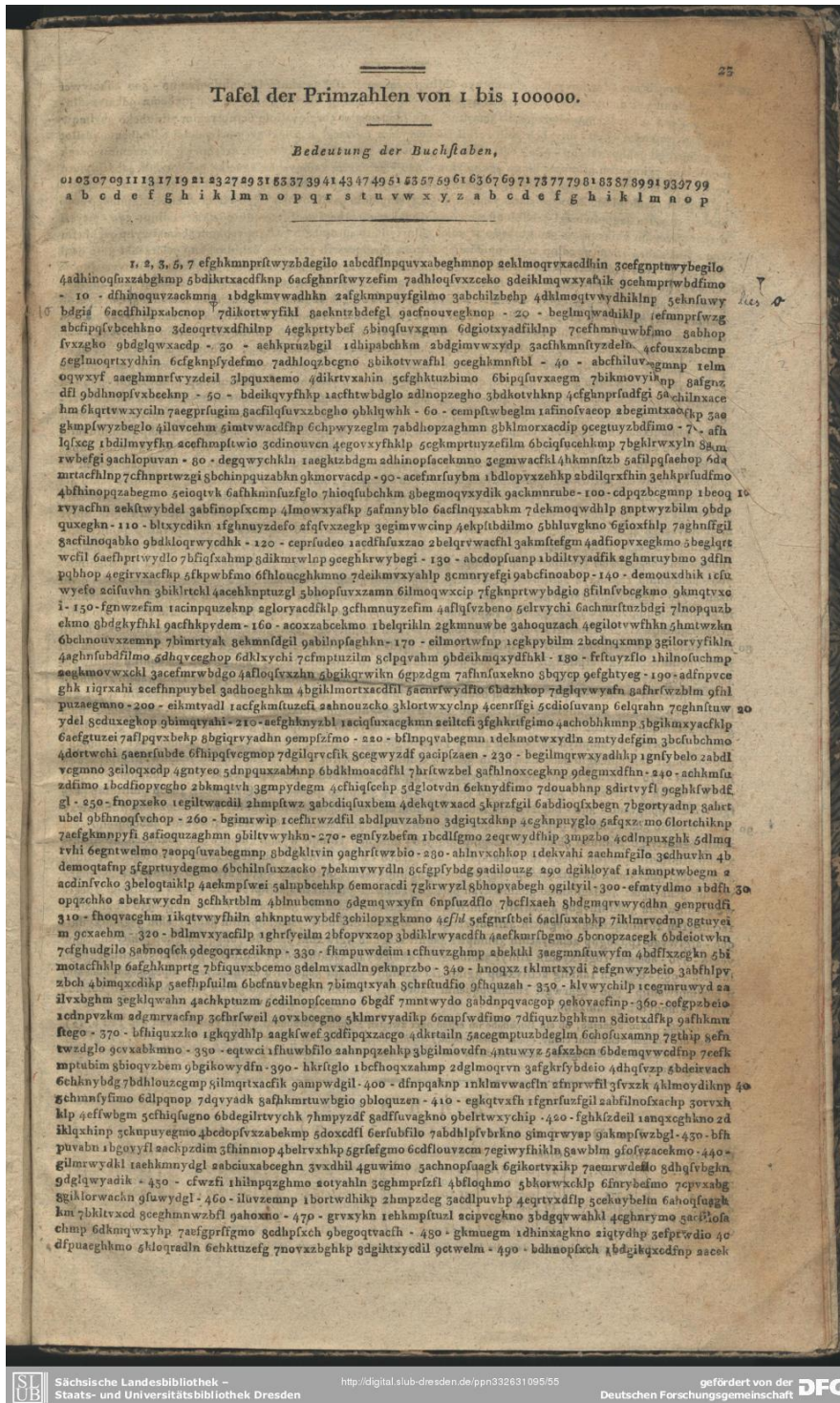


Figure 1: The first page of Krause's table of primes. (source: Staats- und Universitätsbibliothek Dresden)



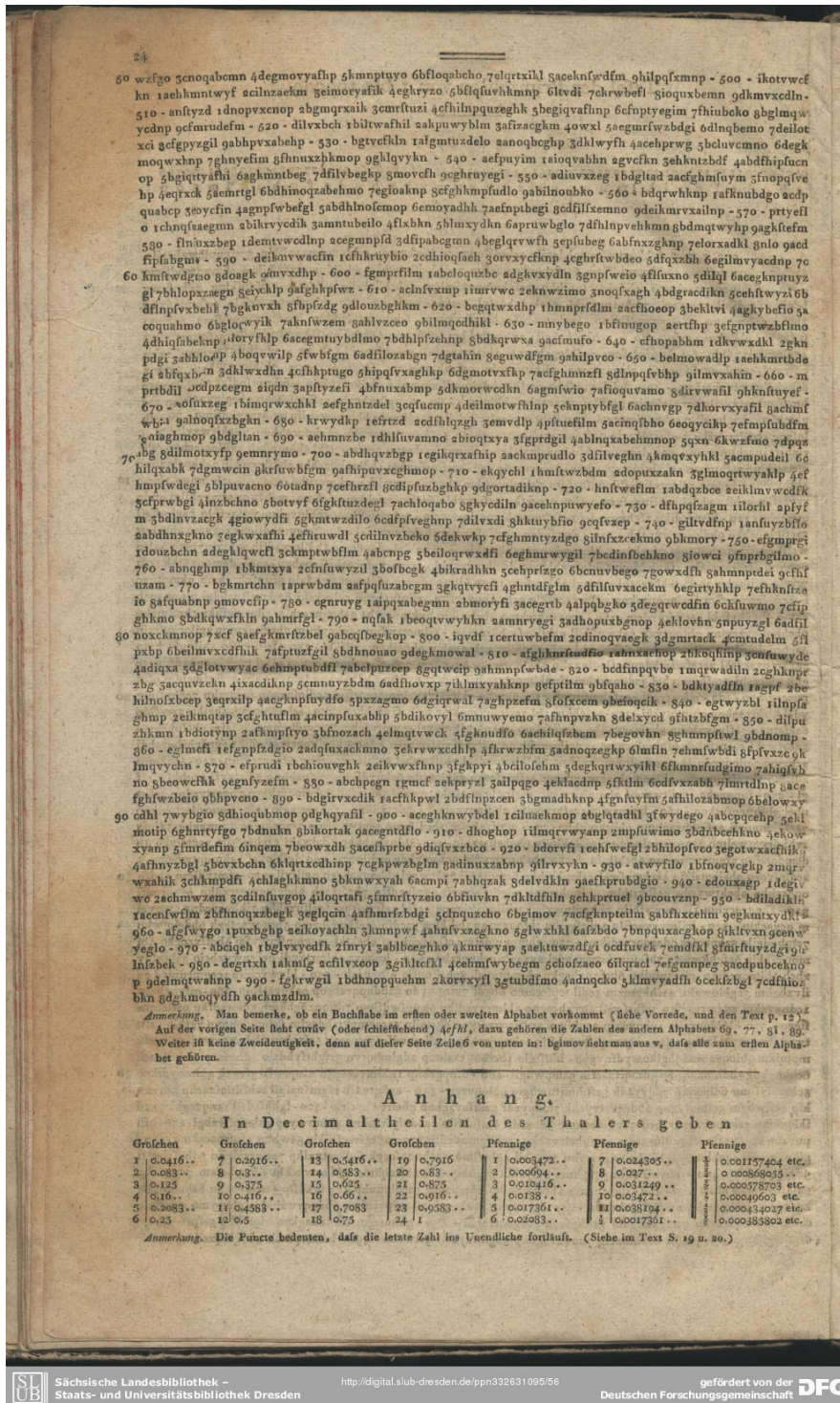


Figure 2: The second page of Krause's table of primes. (source: Staats- und Universitätsbibliothek Dresden)



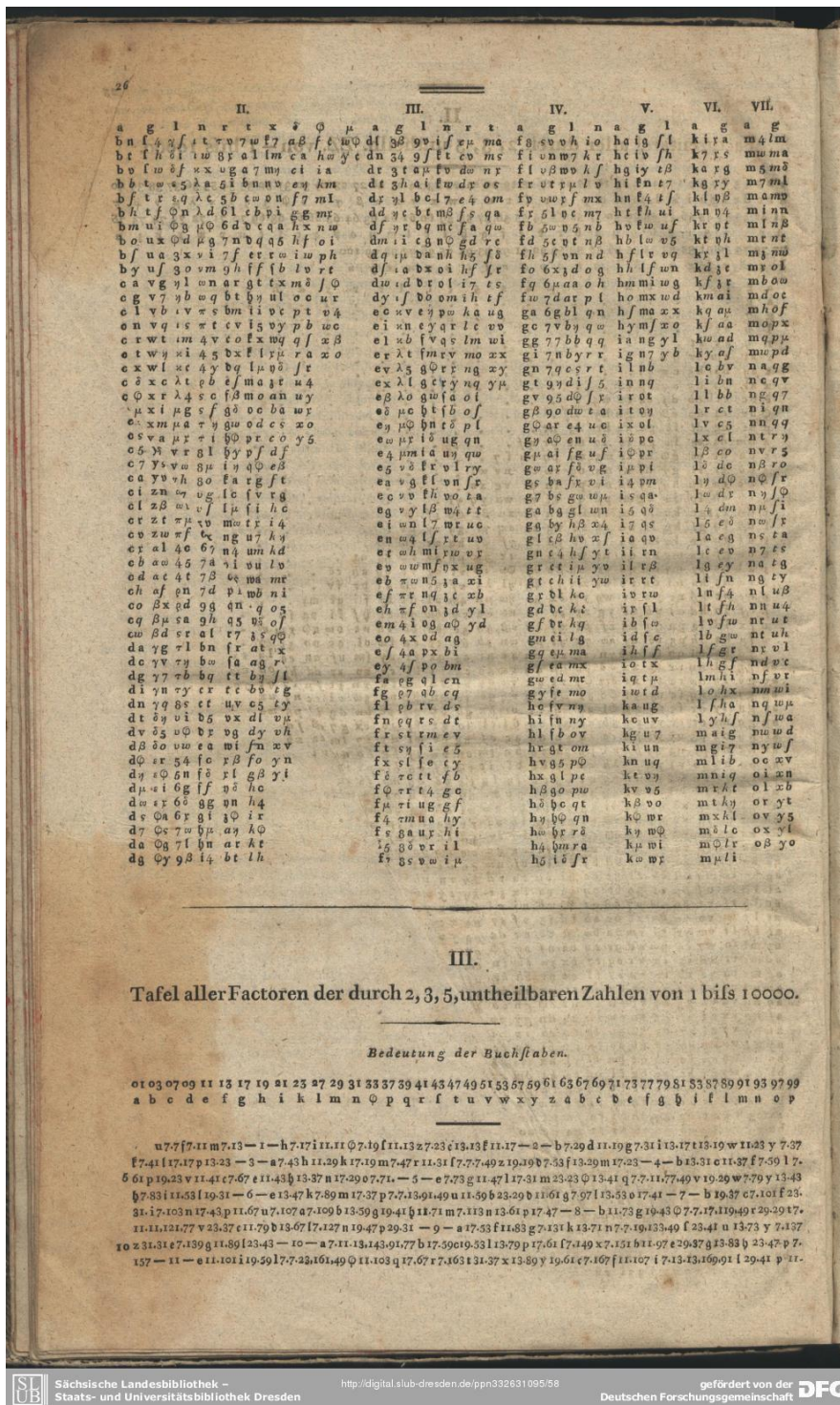


Figure 3: The first page of Krause's table of factors. (source: Staats- und Universitätsbibliothek Dresden)



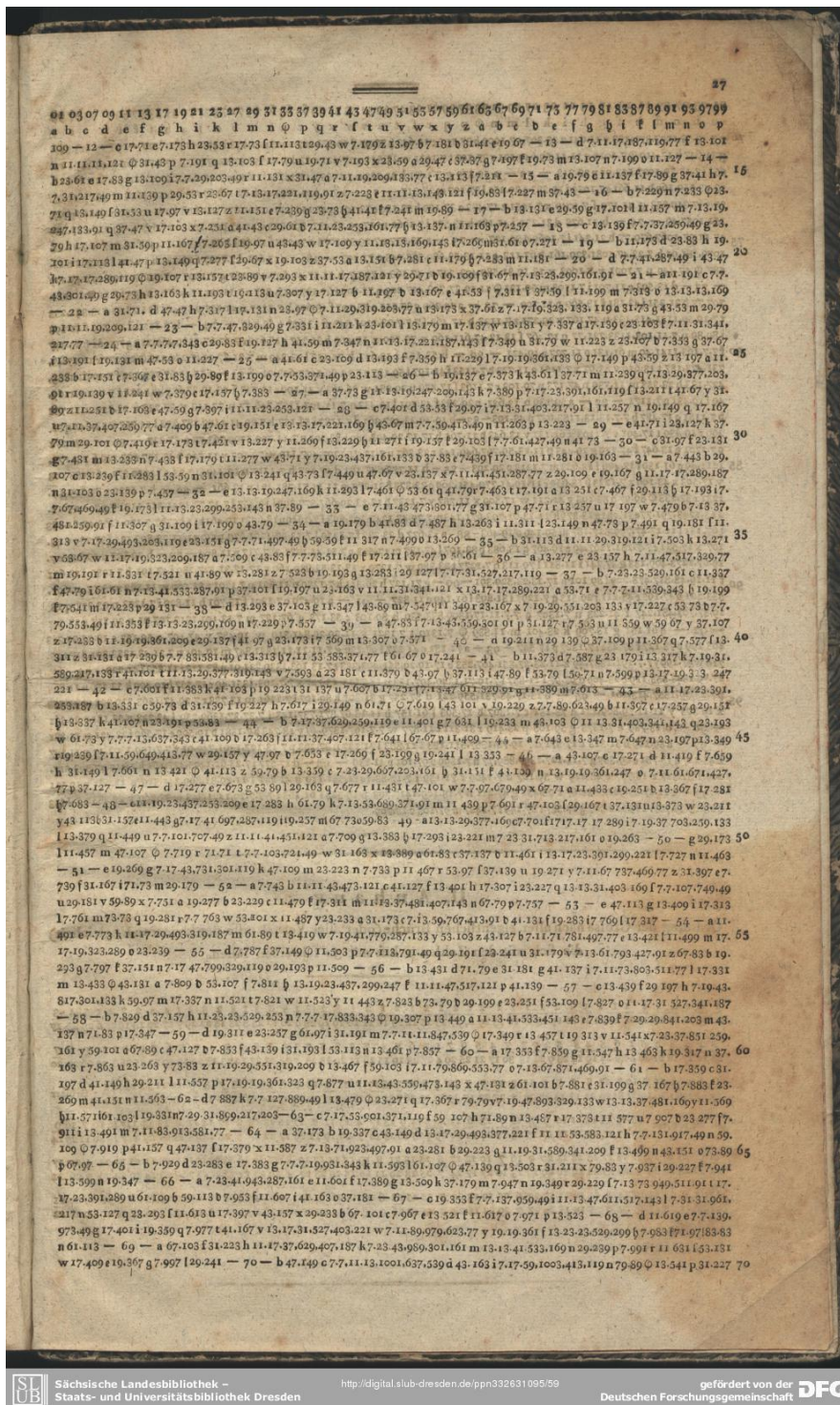


Figure 4: The second page of Krause's table of factors. (source: Staats- und Universitätsbibliothek Dresden)



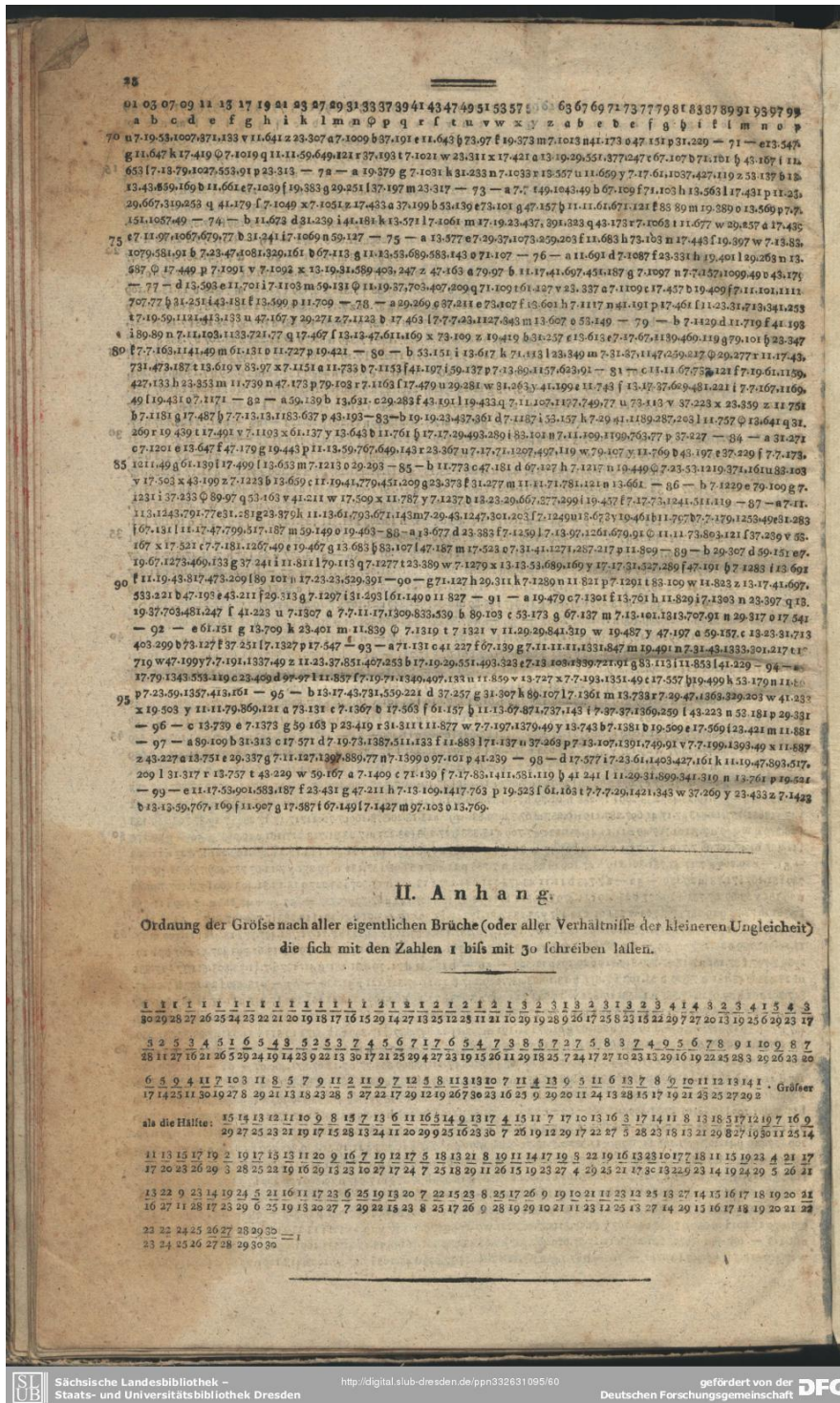


Figure 5: The third page of Krause's table of factors. (source: Staats- und Universitätsbibliothek Dresden)

## References

The following list covers the most important references<sup>2</sup> related to Krause’s table. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.

- [1] Maarten Bullynck. Factor tables 1657–1817, with notes on the birth of number theory. *Revue d’histoire des mathématiques*, 16(2):133–216, 2010.
- [2] Anton Felkel. *Tafel aller einfachen Factoren der durch 2, 3, 5 nicht theilbaren Zahlen von 1 bis 10 000 000. I. Theil. Enthaltend die Factoren von 1 bis 144000*. Wien: von Ehelenschen, 1776. [There is also a Latin edition [3] of this first part.] [reconstructed in [12]]
- [3] Anton Felkel. *Tabula omnium factorum simplicium numerorum per 2, 3, 5 non divisibilium, ab 1 usque 10 000 000. Pars I. Exhibens factores ab 1 usque 144000*. Wien: A. Gheleniana, 1777. [Latin version of [2].] [not seen] [reconstructed in [12]]
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- [6] Christian Ernst Gabler. Literarische Anzeigen. *Intelligenzblatt der Allgemeine Literatur-Zeitung*, 4(206):1687–1688, 1803. [Review of Krause’s book.]
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- [8] James Whitbread Lee Glaisher. Table, mathematical. In Hugh Chisholm, editor, *The Encyclopædia Britannica, 11th edition*, volume 26, pages 325–336. Cambridge, England: at the University Press, 1911.
- [9] Karl Christian Friedrich Krause. *Factoren und Primzahlentafel von 1 bis 100000 Neuberechnet und zweckmäßig eingerichtet nebst einer Gebrauchsanleitung und Abhandlung der Lehre von Factoren und Primzahlen*. Jena: Christian Ernst Gabler, 1804.

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<sup>2</sup>**Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.

- [10] Johann Heinrich Lambert. *Zusätze zu den Logarithmischen und Trigonometrischen Tabellen zur Erleichterung und Abkürzung der bey Anwendung der Mathematik vorfallenden Berechnungen*. Berlin: Haude und Spener, 1770. [the table of factors was reconstructed in [14]; [11] is a Latin translation of this book]
- [11] Johann Heinrich Lambert and Anton Felkel. *Supplementa tabularum logarithmicarum et trigonometricarum*. Lisbon, 1798. [Latin translation of [10]; the table of factors was reconstructed in [13]]
- [12] Denis Roegel. A reconstruction of Felkel’s tables of primes and factors (1776). Technical report, LORIA, 2011. [This is a reconstruction and an extension of Felkel’s tables [2, 3, 4, 5].]
- [13] Denis Roegel. A reconstruction of Lambert and Felkel’s table of factors (1798). Technical report, LORIA, Nancy, 2011. [This is a reconstruction of the table of factors in [11].]
- [14] Denis Roegel. A reconstruction of Lambert’s table of factors (1770). Technical report, LORIA, Nancy, 2011. [This is a reconstruction of the table of factors in [10].]
- [15] Paul Peter Heinrich Seelhoff. Geschichte der Factorentafeln. *Archiv der Mathematik und Physik*, 70:413–426, 1884.
- [16] Carl von Prantl. Krause, Karl Christian Friedrich. In Historische Kommission bei der Bayerischen Akademie der Wissenschaften, editor, *Allgemeine Deutsche Biographie*, volume 17, pages 75–79. Leipzig: Duncker & Humblot, 1883.





Krause's table of factors (1804) (reconstruction, D. Roegel, 2011)

01 03 07 09 11 13 17 19 21 23 27 29 31 33 37 39 41 43 47 49 51 53 57 59 61 63 67 69 71 73 77 79 81 83 87 89 91 93 97 99  
a b c d e f g h i k l m n o p q r s t u v w x y z a b c d e f g h i k l m n o p

1, 2, 3, 5, 7 e f g h k m n p r s t v y z b d e g i l o 1 a b c d f i n p q u v x a b e g h m n o p 2 e k l m o q r v x a c d f h i n 3 c e f g n p t u w y b e g i l o  
4 a d h i n o q s u x z a b g k m p 5 b d i k r t x a c d f k n p 6 a c f g h n r s t w y z e f i m 7 a d h l o q s v x z c e k o 8 d e i k l m q w x y a f h i k 9 c e h m p r t w b d f i m o  
- 10 - d f h i n o q u v z a c k m n o 1 b d g k m n v w a d h k n 2 a f g k m n p u y f g i l m o 3 a b c h i l z b e h p 4 d k l m o q t v w y d h i k l n p 5 e k n s u w y  
b d g i o 6 a c d f h i l p x a b c n o p 7 d i k o r t w y f i k l 8 a e k n t z b d e f g l 9 a c f n o u v e g k n o p - 20 - b e g l m q w a c h i k l p 1 e f m n p r s w z g  
2 b c f i p q s v b c e h k n o 3 d e o q r t v x d f h i l n p 4 e g k p r t y b e f 5 b i n q s u v x g m n 6 d g i o t x y a d f i k l n p 7 c e f h m n r u w b f i m o 8 a b h o p  
s v x z g k o 9 b d g l q w x a c d p - 30 - a e h k p r u z b g i l 1 d h i p a b c h k m 2 b d g i m v w x y d p 3 a c f h k m n s t y z d e l m 4 c f o u z a b c m p  
5 e g l m o q r t x y d h i n 6 c f g k n p s y d e f m o 7 a d h l o q z b c g n o 8 b i k o t v w a f h l 9 c e g h k m n s t b l - 40 - a b c f h i l u v x e g m n p 1 e l m  
o q w x y f 2 a e g h m n r s w y z d e i l o 3 l p q u x a e m o 4 d i k r t v x a h i n 5 c f g h k t u z b i m o 6 b i p q s u v x a e g m 7 b i k m o v y i k l n p 8 a f g n z  
d f l 9 b d h n o p s v x b c e k n p - 50 - b d e i k q v y f h k p 1 a c f h t w b d g l o 2 d l n o p z e g h o 3 b d k o t v h k n p 4 c f g h n p r s u d f g i 5 a b c h i l n x a c e  
h m 6 k q r t v w x y c i l n 7 a e g p r s u g i m 8 a c f i l q s u v x z b c g h o 9 b k l q w h k - 60 - c e m p s t w b e g l m 1 a f i n o s v a e o p 2 b e g i m t x a c d f k p 3 a e  
g k m p s w y z b e g l o 4 i l u v c e h m 5 i m t v w a c d f h p 6 c h p w y z e g l m 7 a b d h o p z a g h m n 8 b k l m o r x a c d i p 9 c e g t u y z b d f i m o - 70 - a f h  
l q s x c g 1 b d i l m v y f k n 2 c e f h m p s t w i o 3 c d i n o u v e n 4 e g o v x y f h k l p 5 c g k m p r t u y z e f i m 6 b c i q s u c e h k m p 7 b g k l r w x y l n 8 g k m  
r w b e f g i 9 a c h l o p u v a n - 80 - d e g q w y c h k l n 1 a e g k t z b d g m 2 d h i n o p s a c e k m n o 3 e g m w a c f k l 4 h k m n s t z b 5 a f i l p q s a e h o p 6 d k l  
m r t a c f h l n p 7 c f h m p r t w z g i 8 b c h i n p q u z a b k n 9 k m o r v a c d p - 90 - a c e f m r s u y b m 1 b d l o p v x z e h k p 2 b d i l q r x f h i n 3 e h k p r s u d f i m o  
4 b f h i n o p q z a b e g m o 5 e i o q t v k 6 a f h k m n s u z f g l o 7 h i o q s u b c h k m 8 b e g m o q v x y d i k 9 a c k m n r u b e - 100 - c d p q z b c g m n p 1 b e o q 10  
r v y a c f h n 2 e k s t w y b d e l 3 a b f i n o p s x c m p 4 l m o w x y a f k p 5 a f m n y b l o 6 a c f n q v x a b k m 7 d e k m o q w d h l p 8 n p t w y z b i l m 9 b d p  
q u x e g k n - 110 - b l t x y c d i k n 1 f g h n u y z d e f o 2 f q s v x z e g k p 3 e g i m v w c i n p 4 e k p s t b d i l m o 5 b h l u v g k n o 6 g i o x f h l p 7 a g h n s f g i l  
8 a c f i l n o q a b k o 9 b d k l o q r w y c d h k - 120 - c e p r s u d e o 1 a c d f h s u x z a o 2 b e l q r v w a c f h l 3 a k m s t e f g m 4 a d f i o p v x e g k m o 5 b e g l q r t  
w e f i l 6 a e f h p r t w y d l o 7 b f i q s x a h m p 8 d i k m r w l n p 9 c e g h k r w y b e g i - 130 - a b c d o p s u a n p 1 b d i l t v y a d f i k 2 g h m r u y b m o 3 d f i n  
p q b h o p 4 e g i r v x a c f k p 5 f k p w b f i m o 6 f h l o u c g h k m n o 7 d e i k m v x y a h l p 8 c m n r y e f g i 9 a b c f i n o a b o p - 140 - d e m o v x d h i k 1 c s u  
w y e f o 2 c i s u v h n 3 b i k l r t c k l 4 a c e h k n p t u z g l 5 b h o p s u v x z a m n 6 i l m o q w x c i p 7 f g k n p r t w y b d g i o 8 f i l n s v b c g k m o 9 k m q t v x c  
i - 150 - f g n w z e f i m 1 a c i n p q u z e k n p 2 g l o r y a c d f k l p 3 c f h m n u y z e f i m 4 a f l q s v z b e n o 5 e l r v y c h i 6 a c h m r s t u z b d g i 7 l n o p q u z b  
e k m o 8 b d g k y f h k l 9 a c f h k p y d e m - 160 - a c o x z a b c e k m o 1 b e l q r i k l n 2 g k m n u w b e 3 a h o q u z a c h 4 e g i l o t v w f h k n 5 h m t w z b e  
6 b c h n o u v x z e m n p 7 b m r t y a k 8 e k m n s d g i l 9 a b i l n p s a g h k n - 170 - e i l m o r t w f n p 1 c g k p y b i l m 2 b c d n q x m n p 3 g i l o r v y f i k l n  
4 a g h n s u b d f i l m o 5 d h q v c e g h o p 6 d k l x y c h i 7 c f m p t u z i l m 8 c l p q v a h m 9 b d e i k m q x y d f h k l - 180 - f r s t u y z f l o 1 h i l n o s u c h m p  
2 e g k m o v w x c k l 3 a c e f m r w b d g o 4 a f l o q s v x z h n 5 b g i k q r w i k n 6 g p z d g m 7 a f h n s u x e k n o 8 b q y c p 9 e f g h t y e g - 190 - a d f n p v c e  
g h k 1 i q r x a h i 2 c e f h n p u y b e l 3 a d h o e g h k m 4 b g i k l m o r t x a c d f i l 5 a c n r s w y d f i o 6 b d z h k o p 7 d g l q v w y a f n 8 a f h r s w z b l m 9 f h l  
p u z a e g m n o - 200 - e i k m t v a d l 1 a c f g k m s t u z e f i 2 a h n o u z c k o 3 k l o r t w x y c l n p 4 c e n r s f g i 5 c d i o s u v a n p 6 e l q r a h n 7 c g h n s t u w 20  
y d e l 8 c d u x e g k o p 9 b i m q t y a h i - 210 - a e f g h k n y z b l 1 a c i q s u x a c g k m n 2 e i l t c f i 3 f g h k r t f g i m o 4 a c h o b h k m n p 5 b g i k m x y a c f k l p  
6 a e f g t u z e i 7 a f l p q v x b e k p 8 b g i q r v y a d h n 9 e m p s z f i m o - 220 - b f n p q v a b e g m n 1 d e k m o t w x y d l n 2 m t y d e f g i m 3 b c s u b c h m o  
4 d o r t w c h i 5 a e n r s u b d e 6 f h i p q s v c e g m o p 7 d g i l q r v c f i k 8 c e g w y z d f 9 a c i p s z a e n - 230 - b e g i l m q r w x y a d h k p 1 g n s y b e l o 2 a b d l  
v e c m n o 3 e i l o q x c d p 4 n t y e o 5 d n p q u x z a b h n p 6 b d k l m o a c d f k l 7 h r s t z w b e l 8 a f h l n o x c e g k n p 9 d e g m x d f h n - 240 - a c h k m s u  
z d f i m o 1 b c d f i o p v c g h o 2 b k m q t v h 3 g m p y d e g m 4 c f h i q s c e h p 5 d g l o t v d n 6 e k n y d f i m o 7 d o u a b h n p 8 d i r t v y f l 9 c g h k s w b d f  
g l - 250 - f n o p x e k o 1 e g i l t w a c d i l 2 h m p s t w z 3 a b c d i q s u x b e m 4 d e k q t w x a c d 5 k p r z f g i l 6 a b d i o q s x b e g n 7 b g o r t y a d n p 8 a h r t  
u b e l 9 b f h n o q s v c h o p - 260 - b g i m r w i p 1 c e f h r w z d f i l 2 b d l p u v z a b n o 3 d g i q t x d k n p 4 c g k n p u y g l o 5 a f q x z e m o 6 l o r t c h i k n p  
7 a e f g k m n p y f i 8 a f i o q u z a g h m n 9 b i l t v w y h k n - 270 - e g n s y z b e f m 1 b c d l s g m o 2 e q r w y d f h i p 3 m p z b o 4 c d l n p u x g h k 5 d l m q  
r v h i 6 e g n t w e l m o 7 a o p q s u v a b e g m n p 8 b d g k l t v i n 9 a g h r s t w z b i o - 280 - a h l n v x c h k o p 1 d e k v a h i 2 a e h m f g i l o 3 c d h u v k n 4 b  
d e m o q t a f n p 5 f g p r t u y d e g m o 6 b c h i l n s u x z a c k o 7 b e k m v w y d l n 8 c f g p s y b d g 9 a d i l o u z g - 290 - d g i k l o y a f 1 a k m n p t w b e g m 2  
a c d i n s v c k o 3 b e l o q t a i k l p 4 a e k m p s w e i 5 a l n p b c e h k p 6 e m o r a c d i 7 g k r w y z l 8 b h o p v a b e g h 9 g i l t y i l - 300 - e f m t y d l m o 1 b d f h 30  
o p q z c h k o 2 b e k r w y c d n 3 c f h k r t b l m 4 b l n u b c m n o 5 d g m q w x y f n 6 n p s u z d f l o 7 b c f l x a e h 8 b d g m q r v w y c d h n 9 e n p r u d f i  
- 310 - f h o q v a c g h m 1 i k q t v w y f h i l n 2 h k n p t u w y b d f 3 c h i l o p x g k m n o 4 c f h l 5 e f g n r s t b e i 6 a c l s u x a b k p 7 i k l m r v c d n p 8 t u y e i  
m 9 c x a e h m - 320 - b d l m v x y a c f i l p 1 g h r s y e i l m 2 b f o p v x z o p 3 b d i k l r w y a c d f h 4 a e f k m r s b g m o 5 b c n o p z a c e g k 6 b d e i o t w k n  
7 c f g h u d g i l o 8 a b n o q s c k 9 d e g o q r x c d i k n p - 330 - f k m p u w d e i m 1 c f h u v z g h m p 2 b e k t k l 3 a e g m n s t u w y f m 4 b d f l x z c g k n 5 b i  
m o t a c f h k l p 6 a f g h k m p r t g 7 b f i q u v x b c e m o 8 d e l m v x a d l n 9 e k n p r z b o - 340 - h n o q x z 1 k l m r t x y d i 2 e f g n w y z b e i o 3 a b f h l p v  
z b c h 4 b i m q x c d i k p 5 a e f h p s u i l m 6 b c f n u v b e g k n 7 b i m q t x y a h 8 c h r s t u d i o 9 f h q u z a h - 350 - k l v w y c h i l p 1 c e g m r u w y d 2 a  
i l v x b g h m 3 e g l q w a h n 4 a c h k p t u z m 5 c d i l n o p s c e m n o 6 b g d f 7 m n t w y d o 8 a b d n p q v a c g o p 9 e k o v a c f n p - 360 - c e f g p z b e i o  
1 c d n p v z k m 2 d g m r v a c f n p 3 c f h r s w e i l 4 o v x b c e g n o 5 k l m r v y a d i k p 6 c m p s w d f i m o 7 d f i q u z b g h k m n 8 d i o t x d f k p 9 a f h k m n  
s t e g o - 370 - b f h i q u x z k o 1 g k y d h l p 2 a g k s w e f 3 c d f i p q x z a c g o 4 d k r t a i l n 5 a c e g m p t u z b d e g l m 6 c h o s u a m n p 7 g t h i p 8 e f n  
t w z d g l o 9 c v x a b k m n o - 380 - e q t w c i 1 f h u w b f i l o 2 a h n p q z e h k p 3 b g i l m o v d f n 4 n t u w y z 5 a s x z b c n 6 b d e m q v w c d n p 7 e e f k  
m p t u b i m 8 b i o q v z b e m 9 g b i k o y d f n - 390 - h k r s t g l o 1 b c f h o q x z a h m p 2 d g l m o q r v n 3 a f g k r s y b d e i o 4 d h q s v z p 5 b d e i r v a c h  
6 c h k n y b d g 7 b d h l o u z c g m p 8 i l m q r t x a c f i k 9 a m p w d g i l - 400 - d f n p q a k n p 1 e k l m v w a c f n 2 f n p r w f i l 3 s v x z k 4 k l m o y d i k n p 40  
5 c h m n s y f i m o 6 d l p q n o p 7 d q v y a d k 8 a f h k m r t u w b g i o 9 b l o q u z e n - 410 - e g k t v x f h 1 f g n r s u z f g i l 2 a b f i l n o s x a c h p 3 o r v x h  
k l p 4 e f s w b g m 5 c f h i q s u g n o 6 b d e g i l r t v y c h k 7 h m p y z d f 8 a d f s u v a g k n o 9 b e l r t w x y c h i p - 420 - f g h k s z d e i l 1 a n q x c g h k n o 2 d  
i k l q x h i n p 3 c k n p u y e g m o 4 b c d o p s v x z a b e k m p 5 d o x c d f l 6 e r s u b f i l o 7 a b d h l p s v b e k n o 8 i m q r w y a p 9 a k m p s w z b g l - 430 - b f h  
p u v a b n 1 b g o v y f l 2 a c k p z d i m 3 f h i n m o p 4 b e l r v x h k p 5 g r s e f g m o 6 c d f l o u v z c m 7 e g i w y f h i k l n 8 a w b l m 9 f o s v z a c e k m o - 440 -  
g i l m r w y d k l 1 a e h k m n y d g l 2 a b c i u x a b c e g h n 3 v x d h i l 4 g u w i m o 5 a c h n o p s u a g k 6 g i k o r t v x i k p 7 a e m r w d e f l o 8 d h q s v b g k n  
9 d g l q w y a d i k - 450 - c f w z f i 1 h i l n p q z g h m o 2 o t y a h l n 3 c g h m p r s z f l 4 b f l o q h m o 5 b k o r w x c k l p 6 f n r y b e f m o 7 c p v x a b g  
8 g i k l o r w a c k n 9 s u w y d g l - 460 - i l u v z e m n p 1 b o r t w d h i k p 2 h m p z d e g 3 a c d l p u v h p 4 e q r t v x d f l p 5 c e k u y b e l m 6 a h o q s u a g h  
k m 7 b k l t v x c d 8 c e g h m n w z b f l 9 a h o x n o - 470 - g r v x y k n 1 e h k m p s t u z l 2 c i p v c g k n o 3 b d g q v w a h k l 4 c g h n r y m o 5 a c f i l o s a  
c h m p 6 d k m q w x y h p 7 a e f g p r s f g m o 8 c d h p s x c h 9 b e g o q t v a c f h - 480 - g k m u e g m 1 d h i n x a g k n o 2 i q t y d h p 3 e f p r w d i o 4 c  
d f p u a e g h k m o 5 k l o q r a d l n 6 e h k t u z e f g 7 n o v x z b g h k p 8 d g i k t x y c d i l 9 c t w e l m - 490 - b d h n o p s x c h 1 b d g i k q x c d f n p 2 a c e k

Krause's table of factors (1804) (reconstruction, D. Roegel, 2011)

50 wzfgo 3cnoqabcmm 4degmovyafhp 5kmmptuyo 6bfloqabcho 7elqrtxikl 8aceknswdfm 9hilpqsxmnp - 500 - ikotvwfc  
kn 1aehkmntwyf 2cilnzaekm 3eimoryafik 4egkryzo 5bflqsuvhkmnp 6ltvdi 7ckrwbeff 8ioquxbemn 9dkmvxcdln  
- 510 - anstyzd 1dnopvxcnop 2bgmqrxaik 3cmrstuzi 4cfhilnpquzeghk 5begiqvafhnp 6cfnptyegim 7fhiubcko 8bglmqw  
ycdnp 9cfmrudefm - 520 - dilvxbch 1biltwafhil 2akpuwyblm 3afizacgkm 4owxl 5aegmrswzbdgi 6dlngbemo 7deilot  
xci 8cfcgpyzgil 9abhpxabehp - 530 - bgtvcfkl 1afgmtuzdelo 2anoqbcghp 3dklwylf 4acehprwg 5beluvcmno 6degk  
moqwxhnp 7ghnyefim 8fhnuxzhkmop 9gklqvykn - 540 - aefpuyim 1aioqvabhn 2gvcfkn 3ehkntzbd 4abdfhipsucn  
op 5bgqrtyafhi 6agkmntbeg 7dfilvbegkp 8movcfh 9cghrueygi - 550 - adiuvxzeg 1bdgltd 2acfhgmsuym 3fnopqsve  
hp 4eqrxck 5aemrtgl 6bdhinoqzabehmo 7egioaknp 8cfghkmepsudlo 9abilnoubko - 560 - bdqrwhknp 1afknubdgo 2cdp  
quabcp 3eoycfm 4agnpswbefgl 5abdhlnozmop 6emoyadhk 7aefnptbegi 8cdfilsxemno 9deikmrvxailnp - 570 - prtyeff  
o 1chnqsuaegmn 2bikrvyedik 3amntubeilo 4flxbkn 5blmxydkn 6apruwbgl 7dfhlnpvehkmn 8bdmqtwyhp 9agkstem  
- 580 - flnsuzzbep 1demtwcdlnp 2cegmnpsd 3dfipabcmn 4beglqrwfh 5epsupeg 6abfnxzgknp 7elorxadkl 8nlo 9acd  
fipsabgmo - 590 - deikmvwacfm 1cfhkruybio 2cdhioqsaeh 3orvxycknp 4cghrstwbdeo 5dfqzxbh 6egilmvyacdnp 7c  
60 kmstwdgmo 8doagk 9imvxdhp - 600 - fgmpfilm 1abcloquzbc 2dgkvxydl 3gnpsweio 4flsuxno 5dilql 6acegkntuyz  
gl 7bhlopzxaegn 8eiycklp 9afghkpswz - 610 - aclsvxmp 1imrvwc 2eknwzimo 3noqsxagh 4bdgracdikn 5cehstwyzi 6b  
dfnpsvxbehk 7bgkmvxh 8fhpszdg 9dloubghkm - 620 - begqtxwdhp 1hmnprsdm 2acfhoeop 3bekltvi 4agkybefio 5a  
coquahmo 6bgloqwyik 7aknszwem 8ahlvzceo 9bilmqcdhikl - 630 - mnybego 1bfnugop 2ertfhp 3efgnptwzbfmo  
4dhiqsabeknp 5iloryfklp 6acegmtuybdm 7bdhlpszehnp 8bdkqrwxa 9acfmufu - 640 - cfhopabhm 1dkvwxdkl 2gkn  
pdgi 3abhloehp 4boqvwilp 5fwbfgm 6adfilozabgn 7dgtahin 8eguwdfgm 9ahilpvco - 650 - belmowadlp 1aehkmrtbde  
gi 2bfqxbckn 3dklwxhdn 4cfhkptugo 5hipqsvxaghk 6dgmotvxfk 7acfhgmznzl 8dlmpqsvbhp 9ilmvxahin - 660 - m  
prtbdil 1bcdpzcem 2iqdn 3apstyzefi 4bfmxabmp 5dkmorwedkn 6agmswio 7afioquvamo 8dirvwafil 9hknstuyef  
- 670 - biosuzzeg 1bimqrwxchkl 2efghntzdel 3cqsucmp 4deilmotwfhlnp 5ekntybfgl 6achnvgp 7dkorvxyafil 8achms  
wbim 9alnopszbgkn - 680 - krwydkp 1eftzld 2cdfhlqzgh 3emvdlp 4pstudefilm 5acinqsbho 6eoqycik 7efmnsbdfm  
8fhiaghmo 9bdgltdan - 690 - aehmznbe 1dhlsuvmno 2bioqtxya 3fgprdigl 4ablnqxabehmnp 5qxn 6kwzfm 7dpqz  
70 abg 8dilmotxyfp 9emnrymo - 700 - abdhqvzbgp 1egikqrxfhip 2ackmprudlo 3dfilveghn 4kmqvxyhkl 5acmpudeil 6c  
hilqxakb 7dgmwcin 8krsuwbfgm 9afhipuvxcghmop - 710 - ekqychl 1hmstwbzdm 2dopuxzakn 3glmoqrtyyaklp 4ef  
hmpswdegi 5blpuvacno 6otadnp 7cefhrzfl 8cdipsuzbghk 9dgortadiknp - 720 - hnstweflm 1abdqbzce 2eiklmvwedfk  
3cfrwvbi 4inzbchn 5botvyf 6fgkstuzdegl 7achloqabo 8gkyediln 9aceknpuwyefo - 730 - dfhpqszagm 1ilorhl 2psyf  
m 3bdlnvzacgk 4giowydfi 5gkmtwzdilo 6cdfpsveghnp 7dilvxd 8hktuybfio 9cqsvezep - 740 - giltvdfnp 1ansuyzbfl  
2abdhnxgkno 3egkwxafhi 4efhruwdl 5cdilnvzbeke 6dekwp 7cfghmntydzgo 8ilnsxzcekmo 9bkmory - 750 - efgmprgi  
1douzbchn 2degklqwcf 3ckmptwbfm 4abcnpg 5beiloqrwxdfi 6eghmrvygil 7bedinsbehkno 8iowci 9fnprbgilmo  
- 760 - abnqghmp 1bkmtxya 2cfnsuwyzil 3bosbcgk 4bikradhkn 5cehprszgo 6bcnuvbego 7gowxdfh 8ahmnpdtei 9cfhs  
uzam - 770 - bgkmrtchn 1aprwbm 2afpqsuzabcm 3gkqtvyefi 4ghntdfglm 5dfilsuvxacekm 6egirtyhklp 7efhkntste  
io 8afquabnp 9movcfip - 780 - cgnruyg 1aipqxabegmn 2bmoryfi 3acegrtb 4alpqbgko 5degqrwcdm 6cksuwmo 7cfip  
ghkmo 8bdkqwxfln 9ahmrflg - 790 - nqsak 1beoqtvwyhkn 2amnruegi 3adhoppuxbgop 4eklovhn 5npuyzgl 6adfil  
80 noxckmnp 7xcf 8aefgkmrstzbel 9abcqsbegekop - 800 - iqvdf 1certuwbefm 2cdinoqvaegk 3dgmrtack 4cmtudelm 5fl  
pxbp 6beilmvxcdfhik 7afptuzfgil 8bdlnouao 9degkmowal - 810 - afgkhrstudfio 1ahnxaehop 2bkoqhnp 3cnsuwyde  
4adiqxa 5dglotvwyac 6ehmptubdf 7abclpuzcep 8gqtwcip 9ahmnpwbde - 820 - bcdinpqvbe 1mqrwadiln 2cghknpr  
zbg 3acquvzekn 4ixacdiknp 5cmnuyzbdm 6adfhovxp 7iklmxyahknp 8efptilm 9bfqaho - 830 - bdktyadfn 1agpf 2bc  
hilnosxbcep 3eqrxilp 4acgknpsuydfo 5pxzagmo 6dgiqrwal 7aghpezem 8fosxcm 9beioqic - 840 - egtwyzbl 1ilnpsa  
ghmp 2eikmqtap 3cghltufm 4acinpsuxabhp 5bdikovy 6mnuwyemo 7afhnvzkn 8delxycd 9fhtzbfm - 850 - dilpu  
zhknn 1bdioty 2afkmpstyo 3bfnozach 4elmqtvwck 5fgknudfo 6achilqszbcm 7begovhn 8ghmnpstwl 9bdnomp  
- 860 - eglmcfi 1efgnpszdgio 2adqsuxackmno 3ekrvwxdhlp 4fkrwzbfm 5adnoqzegk 6lmfln 7ehmswdbi 8fsvxzc 9k  
lmqvychn - 870 - efprudi 1bchiouvghk 2eikvwxfhnp 3fgkpyi 4bcilosehm 5degqrwtwyikl 6fkmrsudgimo 7ahiqsvb  
no 8beowcfhk 9egnzyzefm - 880 - abchpcgn 1gmcf 2ekpryzl 3ailpqgo 4eklacdnp 5fktlm 6cdsvxzabh 7imrtdlnp 8ace  
fghswzbeio 9bhpcvno - 890 - bdgirvxdik 1acfhkpw 2bdfnppcen 3bgmadhkn 4fgnsuyfm 5afhilozabmop 6belowxy  
90 cdhl 7wybgio 8dhioqubmop 9dgkqyafil - 900 - aceghknwybdel 1ciluaekmop 2bglqtadhl 3fwydego 4abcpqcehp 5ekl  
motip 6ghnrtyfgo 7bdnukn 8bikortak 9acegntdflo - 910 - dhoghop 1ilmqrwyainp 2mpsuwimo 3bdnbcchkno 4ekow  
xyanp 5fmrdefim 6inqem 7beowxdh 8acefkprbe 9diqsvxzbc - 920 - bdorvfi 1cehswefgl 2bhilopsvco 3egotwxacfhik  
p 4afhnyzbl 5bcvxbchn 6klqrtxcdhnp 7cgkpwzbgm 8adinuzabnp 9ilrvxykn - 930 - atwyfio 1bfnoqvcgk 2mqr  
wxahik 3chkmpdfi 4chlaghkmo 5bkmwxyah 6acmpi 7abhqzak 8delvdikn 9aefkprubdgo - 940 - cdouxagp 1degi  
vwc 2achmwzem 3cdilnsuvnop 4iloqrtafi 5fmrstyzieo 6bfuivkn 7dkltdfhl 8ehkprtuel 9bcouvznp - 950 - bdiladikn  
1acenswflm 2bfhnoqzxbegk 3eglcqin 4afhmrszbdgi 5clnquzcho 6bgimov 7acfgknpteilm 8abfhxcehm 9egkmtxydkl  
- 960 - afgswygo 1puxbghp 2eikoyachln 3kmpwf 4ahnsvxzegkno 5glwxhkl 6aszbd 7bnpqxacgk 8ikltvxn 9cenw  
yeglo - 970 - abciqeh 1bglvxyedfk 2fnryi 3ablbceghko 4kmrwyap 5aektuwzdfgi 6cdfuvek 7emdfkl 8fmrstuyzdi 9h  
lnszbek - 980 - degtrhx 1akmsg 2cflvxcop 3giklctfkl 4cehmswybegm 5choszaeo 6ilqracl 7efgmnp 8acdpubcekno  
p 9delmqtwahnp - 990 - fgkrwil 1bdhnopquehm 2korvxyfl 3gtubdfmo 4adnqcko 5klmvyadfh 6cekszbl 7cdfhioz  
bkn 8dgmqoydfh 9ackmzdlm.

Krause's table of factors (1804) (reconstruction, D. Roegel, 2011)

01 03 07 09 11 13 17 19 21 23 27 29 31 33 37 39 41 43 47 49 51 53 57 59 61 63 67 69 71 73 77 79 81 83 87 89 91 93 97 99  
a b c d e f g h i k l m n o p q r s t u v w x y z a b c d e f g h i f l m n o p

u 7.7 f 7.11 m 7.13 - 1 - h 7.17 i 11.11 φ 7.19 s 11.13 z 7.23 c 13.13 f 11.17 - 2 - b 7.29 d 11.19 g 7.31 i 13.17 t 13.19  
w 11.23 y 7.37 f 7.41 l 17.17 p 13.23 - 3 - a 7.43 h 11.29 k 17.19 m 7.47 r 11.31 s 7.7.7.49 z 19.19 ð 7.53 f 13.29 m 17.23 - 4 -  
b 13.31 c 11.37 f 7.59 l 7.61 p 19.23 v 11.41 c 7.67 e 11.43 b 13.37 n 17.29 o 7.71 - 5 - e 7.73 g 11.47 l 17.31 m 23.23 φ 13.41 5  
q 7.7.11.77.49 v 19.29 w 7.79 y 13.43 b 7.83 i 11.53 l 19.31 - 6 - e 13.47 k 7.89 m 17.37 p 7.7.13.91.49 u 11.59 b 23.29  
ð 11.61 g 7.97 l 13.53 o 17.41 - 7 - b 19.37 c 7.101 f 23.31 i 7.103 n 17.43 p 11.67 u 7.107 a 7.109 b 13.59 g 19.41 b 11.71  
m 7.113 n 13.61 p 17.47 - 8 - b 11.73 g 19.43 φ 7.7.17.119.49 r 29.29 t 7.11.11.121.77 v 23.37 c 11.79 ð 13.67 l 7.127 n 19.47  
p 29.31 - 9 - a 17.53 f 11.83 g 7.131 k 13.71 n 7.7.19.133.49 s 23.41 u 13.73 y 7.137 z 31.31 e 7.139 g 11.89 l 23.43 - 10 - 10  
a 7.11.13.143.91.77 b 17.59 c 19.53 l 13.79 p 17.61 s 7.149 x 7.151 b 11.97 e 29.37 g 13.83 b 23.47 p 7.157 - 11 - e 11.101  
i 19.59 l 7.7.23.161.49 φ 11.103 q 17.67 r 7.163 t 31.37 x 13.89 y 19.61 c 7.167 f 11.107 i 7.13.13.169.91 l 29.41 p 11.109  
- 12 - c 17.71 e 7.173 h 23.53 r 17.73 s 11.113 t 29.43 w 7.179 z 13.97 b 7.181 ð 31.41 e 19.67 - 13 - d 7.11.17.187.119.77  
f 13.101 n 11.11.11.121 φ 31.43 p 7.191 q 13.103 s 17.79 u 19.71 v 7.193 x 23.59 a 29.47 c 37.37 g 7.197 f 19.73 m 13.107  
n 7.199 o 11.127 - 14 - b 23.61 e 17.83 g 13.109 i 7.7.29.203.49 r 11.131 x 31.47 a 7.11.19.209.133.77 c 13.113 f 7.211 - 15 -  
a 19.79 c 11.137 f 17.89 g 37.41 h 7.7.31.217.49 m 11.139 p 29.53 r 23.67 t 7.13.17.221.119.91 z 7.223 e 11.11.13.143.121 15  
f 19.83 l 7.227 m 37.43 - 16 - b 7.229 n 7.233 φ 23.71 q 11.149 s 31.53 u 17.97 v 13.127 z 11.151 e 7.239 g 23.73 b 41.41  
f 7.241 m 19.89 - 17 - b 13.131 e 29.59 g 17.101 l 11.157 m 7.13.19.247.133.91 q 37.47 v 17.103 x 7.251 a 41.43 c 29.61  
ð 7.11.23.253.161.77 b 13.137 n 11.163 p 7.257 - 18 - c 13.139 f 7.7.37.259.49 g 23.79 h 17.107 m 31.59 p 11.167 r 7.263  
s 19.97 u 43.43 w 17.109 y 11.13.13.169.143 i 7.269 m 31.61 o 7.271 - 19 - b 11.173 d 23.83 h 19.101 i 17.113 l 41.47  
p 13.149 q 7.277 s 29.67 x 19.103 z 37.53 a 13.151 b 7.281 c 11.179 b 7.283 m 11.181 - 20 - d 7.7.41.287.49 i 43.47 20  
k 7.17.17.289.119 φ 19.107 r 13.157 t 23.89 v 7.293 x 11.11.17.187.121 y 29.71 ð 19.109 f 31.67 n 7.13.23.299.161.91  
- 21 - a 11.191 c 7.7.43.301.49 g 29.73 h 13.163 k 11.193 t 19.113 u 7.307 y 17.127 b 11.197 ð 13.167 e 41.53 f 7.311  
i 37.59 l 11.199 m 7.313 o 13.13.13.169 - 22 - a 31.71 d 47.47 h 7.317 l 17.131 n 23.97 φ 7.11.29.319.203.77 u 13.173  
x 37.61 z 7.17.19.323.133.119 a 31.73 g 43.53 m 29.79 p 11.11.19.209.121 - 23 - b 7.7.47.329.49 g 7.331 i 11.211 k 23.101  
l 13.179 m 17.137 w 13.181 y 7.337 a 17.139 c 23.103 f 7.11.31.341.217.77 - 24 - a 7.7.7.343.49 c 29.83 f 19.127 h 41.59  
m 7.347 n 11.13.17.221.187.143 s 7.349 u 31.79 w 11.223 z 23.107 ð 7.353 g 37.67 i 13.191 l 19.131 m 47.53 o 11.227 -  
25 - a 41.61 c 23.109 d 13.193 f 7.359 h 11.229 l 7.19.19.361.133 φ 17.149 p 43.59 z 13.197 a 11.233 b 17.151 c 7.367 25  
e 31.83 b 29.89 f 13.199 o 7.7.53.371.49 p 23.113 - 26 - b 19.137 e 7.373 k 43.61 l 37.71 m 11.239 q 7.13.29.377.203.91  
r 19.139 v 11.241 w 7.379 c 17.157 b 7.383 - 27 - a 37.73 g 11.13.19.247.209.143 k 7.389 p 7.17.23.391.161.119 s 13.211  
t 41.67 y 31.89 z 11.251 ð 17.163 e 47.59 g 7.397 i 11.11.23.253.121 - 28 - c 7.401 d 53.53 f 29.97 i 7.13.31.403.217.91  
l 11.257 n 19.149 q 17.167 u 7.11.37.407.259.77 a 7.409 b 47.61 c 19.151 e 13.13.17.221.169 b 43.67 m 7.7.59.413.49  
n 11.263 p 13.223 - 29 - e 41.71 i 23.127 k 37.79 m 29.101 φ 7.419 r 17.173 t 7.421 v 13.227 y 11.269 f 13.229 b 11.271  
i 19.157 f 29.103 l 7.7.61.427.49 n 41.73 - 30 - c 31.97 f 23.131 g 7.431 m 13.233 n 7.433 s 17.179 t 11.277 w 43.71 30  
y 7.19.23.437.161.133 ð 37.83 e 7.439 f 17.181 m 11.281 o 19.163 - 31 - a 7.443 b 29.107 c 13.239 f 11.283 l 53.59 n 31.101  
φ 13.241 q 43.73 s 7.449 u 47.67 v 23.137 x 7.11.41.451.287.77 z 29.109 e 19.167 g 11.17.17.289.187 n 31.103 o 23.139  
p 7.457 - 32 - e 13.13.19.247.169 k 11.293 l 7.461 φ 53.61 q 41.79 r 7.463 t 17.191 a 13.251 c 7.467 f 29.113 b 17.193  
i 7.7.67.469.49 f 19.173 l 11.13.23.299.253.143 n 37.89 - 33 - e 7.11.43.473.301.77 g 31.107 p 47.71 r 13.257 u 17.197  
w 7.479 b 7.13.37.481.259.91 f 11.307 g 31.109 i 17.199 o 43.79 - 34 - a 19.179 b 41.83 d 7.487 h 13.263 i 11.311 l 23.149  
n 47.73 p 7.491 q 19.181 s 11.313 v 7.17.29.493.203.119 e 23.151 g 7.7.71.497.49 b 59.59 f 11.317 n 7.499 o 13.269 - 35  
- b 31.113 d 11.11.29.319.121 i 7.503 k 13.271 v 53.67 w 11.17.19.323.209.187 a 7.509 c 43.83 f 7.7.73.511.49 f 17.211 35  
l 37.97 p 59.61 - 36 - a 13.277 e 23.157 h 7.11.47.517.329.77 m 19.191 r 11.331 t 7.521 u 41.89 w 13.281 z 7.523 b 19.193  
g 13.283 i 29.127 l 7.17.31.527.217.119 - 37 - b 7.23.23.529.161 c 11.337 f 47.79 i 61.61 n 7.13.41.533.287.91 p 37.101  
s 19.197 u 23.163 v 11.11.31.341.121 x 13.17.17.289.221 a 53.71 e 7.7.7.11.539.343.77.49 b 19.199 f 7.541 m 17.223  
p 29.131 - 38 - d 13.293 e 37.103 g 11.347 l 43.89 m 7.547 q 11.349 r 23.167 x 7.19.29.551.203.133 y 17.227 c 53.73  
ð 7.7.79.553.49 i 11.353 f 13.13.23.299.169 n 17.229 p 7.557 - 39 - a 47.83 f 7.13.43.559.301.91 p 31.127 r 7.563 u 11.359  
w 59.67 y 37.107 z 17.233 ð 11.19.19.361.209 e 29.137 f 41.97 g 23.173 i 7.569 m 13.307 o 7.571 - 40 - d 19.211 n 29.139 40  
φ 37.109 p 11.367 q 7.577 s 13.311 z 31.131 a 17.239 b 7.7.83.581.49 c 13.313 b 7.11.53.583.371.77 f 61.67 o 17.241 - 41 -  
b 11.373 d 7.587 g 23.179 i 13.317 k 7.19.31.589.217.133 r 41.101 t 11.13.29.377.319.143 v 7.593 a 23.181 c 11.379 ð 43.97  
b 37.113 i 47.89 f 53.79 l 59.71 n 7.599 p 13.17.19.323.247.221 - 42 - c 7.601 f 11.383 k 41.103 p 19.223 t 31.137 u 7.607  
b 17.251 f 7.13.47.611.329.91 g 11.389 m 7.613 - 43 - a 11.17.23.391.253.187 b 13.331 c 59.73 d 31.139 f 19.227 h 7.617  
i 29.149 n 61.71 φ 7.619 s 43.101 v 19.229 z 7.7.89.623.49 b 11.397 c 17.257 g 29.151 b 13.337 f 41.107 n 23.191 p 53.83 -  
44 - b 7.17.37.629.259.119 e 11.401 g 7.631 l 19.233 m 43.103 φ 11.13.31.403.341.143 q 23.193 w 61.73 y 7.7.7.13.637.  
343.91.49 c 41.109 ð 17.263 f 11.11.37.407.121 f 7.641 l 67.67 p 11.409 - 45 - a 7.643 e 13.347 m 7.647 n 23.197 p 13.349 45  
r 19.239 s 7.11.59.649.413.77 w 29.157 y 47.97 ð 7.653 e 17.269 f 23.199 g 19.241 l 13.353 - 46 - a 43.107 c 17.271 d 11.419  
f 7.659 h 31.149 l 7.661 n 11.421 φ 41.113 z 59.79 b 13.359 c 7.23.29.667.203.161 b 31.151 f 43.109 n 13.19.19.361.247  
o 7.11.61.671.427.77 p 37.127 - 47 - d 17.277 e 7.673 g 53.89 l 29.163 q 7.677 r 11.431 t 47.101 w 7.7.97.679.49  
x 67.71 a 11.433 c 19.251 ð 13.367 f 17.281 b 7.683 - 48 - c 11.19.23.437.253.209 e 17.283 h 61.79 k 7.13.53.689.371.91  
m 11.439 p 7.691 r 47.103 s 29.167 t 37.131 u 13.373 w 23.211 y 43.113 b 31.157 e 11.443 g 7.17.41.697.287.119 i 19.257  
m 67.73 o 59.83 - 49 - a 13.13.29.377.169 c 7.701 f 17.17.17.289 i 7.19.37.703.259.133 l 13.379 q 11.449 u 7.7.101.707.49  
z 11.11.41.451.121 a 7.709 g 13.383 b 17.293 m 7.23.31.713.217.161 o 19.263 - 50 - g 29.173 l 11.457 m 47.107 φ 7.719 50  
r 71.71 t 7.7.103.721.49 w 31.163 x 13.389 a 61.83 c 37.137 ð 11.461 i 13.17.23.391.299.221 l 7.727 n 11.463 - 51 -  
e 19.269 g 7.17.43.731.301.119 k 47.109 m 23.223 n 7.733 p 11.467 r 53.97 s 37.139 u 19.271 y 7.11.67.737.469.77



Krause's table of factors (1804) (reconstruction, D. Roegel, 2011)

z 13.397 e 7.739 f 31.167 i 71.73 m 29.179 - 52 - a 7.743 b 11.11.43.473.121 c 41.127 f 13.401 h 17.307 i 23.227  
q 13.13.31.403.169 s 7.7.107.749.49 u 29.181 v 59.89 x 7.751 a 19.277 b 23.229 c 11.479 f 17.311 m 11.13.37.481.407.143  
n 67.79 p 7.757 - 53 - e 47.113 g 13.409 i 17.313 l 7.761 m 73.73 q 19.281 r 7.7.109.763.49 w 53.101 x 11.487 y 23.233  
a 31.173 c 7.13.59.767.413.91 d 41.131 f 19.283 i 7.769 l 17.317 - 54 - a 11.491 e 7.773 k 11.17.29.493.319.187 m 61.89  
t 13.419 w 7.19.41.779.287.133 y 53.103 z 43.127 b 7.11.71.781.497.77 e 13.421 f 11.499 m 17.17.19.323.289 o 23.239  
55 - 55 - d 7.787 f 37.149 φ 11.503 p 7.7.113.791.49 q 29.191 s 23.241 u 31.179 v 7.13.61.793.427.91 z 67.83 b 19.293  
g 7.797 f 37.151 n 7.17.47.799.329.119 o 29.193 p 11.509 - 56 - b 13.431 d 71.79 e 31.181 g 41.137 i 7.11.73.803.511.77  
l 17.331 m 13.433 φ 43.131 a 7.809 d 53.107 f 7.811 b 13.19.23.437.299.247 f 11.11.47.517.121 p 41.139 - 57 - c 13.439  
f 29.197 h 7.19.43.817.301.133 k 59.97 m 17.337 n 11.521 t 7.821 w 11.523 y 13.443 z 7.823 b 73.79 d 29.199 e 23.251  
f 53.109 l 7.827 o 11.17.31.527.341.187 - 58 - b 7.829 d 37.157 h 11.23.23.529.253 n 7.7.7.17.833.343.119.49 φ 19.307  
p 13.449 a 11.13.41.533.451.143 e 7.839 f 7.29.29.841.203 m 43.137 n 71.83 p 17.347 - 59 - d 19.311 e 23.257 g 61.97  
i 31.191 m 7.7.11.11.847.539.121.77.49 φ 17.349 r 13.457 t 19.313 v 11.541 x 7.23.37.851.259.161 y 59.101 a 67.89  
60 c 47.127 d 7.853 f 43.139 i 31.193 l 53.113 n 13.461 p 7.857 - 60 - a 17.353 f 7.859 g 11.547 h 13.463 k 19.317  
n 37.163 r 7.863 u 23.263 y 73.83 z 11.19.29.551.319.209 d 13.467 f 59.103 i 7.11.79.869.553.77 o 7.13.67.871.469.91 -  
61 - b 17.359 c 31.197 d 41.149 h 29.211 l 11.557 p 17.19.19.361.323 q 7.877 u 11.13.43.559.473.143 x 47.131 z 61.101  
b 7.881 c 31.199 g 37.167 b 7.883 f 23.269 m 41.151 n 11.563 - 62 - d 7.887 k 7.7.127.889.49 l 13.479 φ 23.271 q 17.367  
r 79.79 v 7.19.47.893.329.133 w 13.13.37.481.169 y 11.569 b 11.571 i 61.103 l 19.331 n 7.29.31.899.217.203 - 63 -  
c 7.17.53.901.371.119 f 59.107 h 71.89 n 13.487 r 17.373 t 11.577 u 7.907 d 23.277 f 7.911 i 13.491 m 7.11.83.913.581.77  
- 64 - a 37.173 b 19.337 c 43.149 d 13.17.29.493.377.221 f 11.11.53.583.121 h 7.7.131.917.49 n 59.109 φ 7.919 p 41.157  
q 47.137 s 17.379 x 11.587 z 7.13.71.923.497.91 a 23.281 b 29.223 g 11.19.31.589.341.209 f 13.499 n 43.151 o 73.89  
65 p 67.97 - 65 - b 7.929 d 23.283 e 17.383 g 7.7.7.19.931.343.133.49 k 11.593 l 61.107 φ 47.139 q 13.503 r 31.211 x 79.83  
y 7.937 i 29.227 f 7.941 l 11.599 n 19.347 - 66 - a 7.23.41.943.287.161 e 11.601 f 17.389 g 13.509 k 37.179 m 7.947  
n 19.349 r 29.229 s 7.13.73.949.511.91 t 17.17.23.391.289 u 61.109 b 59.113 d 7.953 f 11.607 i 41.163 o 37.181 - 67 -  
c 19.353 f 7.7.137.959.49 i 11.13.47.611.517.143 l 7.31.31.961.217 n 53.127 q 23.293 s 11.613 u 17.397 v 43.157 x 29.233  
b 67.101 c 7.967 e 13.521 f 11.617 o 7.971 p 13.523 - 68 - d 11.619 e 7.7.139.973.49 g 17.401 i 19.359 q 7.977 t 41.167  
v 13.17.31.527.403.221 w 7.11.89.979.623.77 y 19.19.19.361 f 13.23.23.529.299 b 7.983 f 71.97 l 83.83 n 61.113 - 69 -  
a 67.103 f 31.223 h 11.17.37.629.407.187 k 7.23.43.989.301.161 m 13.13.41.533.169 n 29.239 p 7.991 r 11.631 s 53.131  
70 w 17.409 e 19.367 g 7.997 l 29.241 - 70 - b 47.149 c 7.7.11.13.1001.637.539.143.91.77.49 d 43.163 i 7.17.59.1003.413.119  
n 79.89 φ 13.541 p 31.227 u 7.19.53.1007.371.133 v 11.641 z 23.307 a 7.1009 b 37.191 e 11.643 b 73.97 f 19.373 m 7.1013  
n 41.173 o 47.151 p 31.229 - 71 - e 13.547 g 11.647 k 17.419 φ 7.1019 q 11.11.59.649.121 r 37.193 t 7.1021 w 23.311  
x 17.421 a 13.19.29.551.377.247 c 67.107 d 71.101 b 43.167 i 11.653 l 7.13.79.1027.553.91 p 23.313 - 72 - a 19.379 g 7.1031  
k 31.233 n 7.1033 r 13.557 u 11.659 y 7.17.61.1037.427.119 z 53.137 b 13.13.43.559.169 d 11.661 e 7.1039 f 19.383 g 29.251  
l 37.197 m 23.317 - 73 - a 7.7.149.1043.49 b 67.109 f 71.103 h 13.563 l 17.431 p 11.23.29.667.319.253 q 41.179 s 7.1049  
x 7.1051 z 17.433 a 37.199 b 53.139 e 73.101 g 47.157 b 11.11.61.671.121 f 83.89 m 19.389 o 13.569 p 7.7.151.1057.49 -  
74 - b 11.673 d 31.239 i 41.181 k 13.571 l 7.1061 m 17.19.23.437.391.323 q 43.173 r 7.1063 t 11.677 w 29.257 a 17.439  
75 c 7.11.97.1067.679.77 d 31.241 i 7.1069 n 59.127 - 75 - a 13.577 e 7.29.37.1073.259.203 f 11.683 h 73.103 n 17.443 s 19.397  
w 7.13.83.1079.581.91 b 7.23.47.1081.329.161 d 67.113 g 11.13.53.689.583.143 o 71.107 - 76 - a 11.691 d 7.1087 f 23.331  
h 19.401 l 29.263 n 13.587 φ 17.449 p 7.1091 v 7.1093 x 13.19.31.589.403.247 z 47.163 a 79.97 b 11.17.41.697.451.187  
g 7.1097 n 7.7.157.1099.49 o 43.179 - 77 - d 13.593 e 11.701 i 7.1103 m 59.131 φ 11.19.37.703.407.209 q 7.109 t 61.127  
v 23.337 a 7.1109 c 17.457 d 19.409 f 7.11.101.1111.707.77 b 31.251 i 43.181 f 13.599 p 11.709 - 78 - a 29.269 c 37.211  
e 73.107 f 13.601 h 7.1117 n 41.191 p 17.461 s 11.23.31.713.341.253 t 7.19.59.1121.413.133 u 47.167 y 29.271 z 7.1123  
d 17.463 l 7.7.7.23.1127.343.161.49 m 13.607 o 53.149 - 79 - b 7.1129 d 11.719 f 41.193 i 89.89 n 7.11.103.1133.721.77  
q 17.467 s 13.13.47.611.169 x 73.109 z 19.419 b 31.257 c 13.613 e 7.17.67.1139.469.119 g 79.101 b 23.347 f 7.7.163.1141.49  
80 m 61.131 o 11.727 p 19.421 - 80 - b 53.151 i 13.617 k 71.113 l 23.349 m 7.31.37.1147.259.217 φ 29.277 r 11.17.43.731.  
473.187 t 13.619 v 83.97 x 7.1151 a 11.733 d 7.1153 f 41.197 i 59.137 p 7.13.89.1157.623.91 - 81 - c 11.11.67.737.121  
f 7.19.61.1159.427.133 h 23.353 m 11.739 n 47.173 p 79.103 r 7.1163 s 17.479 u 29.281 w 31.263 y 41.199 e 11.743  
f 13.17.37.629.481.221 i 7.7.167.1169.49 l 19.431 o 7.1171 - 82 - a 59.139 b 13.631 c 29.283 f 43.191 l 19.433 q 7.11.  
107.1177.749.77 u 73.113 v 37.223 x 23.359 z 11.751 b 7.1181 g 17.487 b 7.7.13.13.1183.637.169.91.49 p 43.193 - 83 -  
b 19.19.23.437.361 d 7.1187 i 53.157 k 7.29.41.1189.287.203 l 11.757 φ 13.641 q 31.269 r 19.439 t 17.491 v 7.1193 x 61.137  
y 13.643 d 11.761 b 17.17.29.493.289 i 83.101 n 7.11.109.1199.763.77 p 37.227 - 84 - a 31.271 c 7.1201 e 13.647 f 47.179  
g 19.443 p 11.13.59.767.649.143 r 23.367 u 7.17.71.1207.497.119 w 79.107 y 11.769 d 43.197 e 37.229 f 7.7.173.1211.49  
85 g 61.139 i 17.499 l 13.653 m 7.1213 o 29.293 - 85 - b 11.773 c 47.181 d 67.127 h 7.1217 n 19.449 φ 7.23.53.1219.371.161  
u 83.103 v 17.503 x 43.199 z 7.1223 b 13.659 c 11.19.41.779.451.209 g 23.373 f 31.277 m 11.11.71.781.121 n 13.661 - 86 -  
b 7.1229 e 79.109 g 7.1231 i 37.233 φ 89.97 q 53.163 v 41.211 w 17.509 x 11.787 y 7.1237 d 13.23.29.667.377.299 i 19.457  
f 7.17.73.1241.511.119 - 87 - a 7.11.113.1243.791.77 e 31.281 g 23.379 k 11.13.61.793.671.143 m 7.29.43.1247.301.203  
s 7.1249 u 13.673 y 19.461 b 11.797 d 7.7.179.1253.49 e 31.283 f 67.131 l 11.17.47.799.517.187 m 59.149 o 19.463 - 88 -  
a 13.677 d 23.383 f 7.1259 l 7.13.97.1261.679.91 φ 11.11.73.803.121 s 37.239 v 53.167 x 17.521 c 7.7.181.1267.49  
e 19.467 g 13.683 b 83.107 m 17.523 o 7.31.41.1271.287.217 p 11.809 - 89 - b 29.307 d 59.151 e 7.19.67.1273.469.133  
g 37.241 i 11.811 l 79.113 q 7.1277 t 23.389 y 7.1279 x 13.13.53.689.169 y 17.17.31.527.289 f 47.191 b 7.1283 i 13.691  
90 f 11.19.43.817.473.209 l 89.101 n 17.23.23.529.391 - 90 - g 7.127 h 29.311 k 7.1289 n 11.821 p 7.1291 t 83.109 w 11.823  
z 13.17.41.697.533.221 d 47.193 e 43.211 f 29.313 g 7.1297 i 31.293 l 61.149 o 11.827 - 91 - a 19.479 c 7.1301 f 13.701  
h 11.829 i 7.1303 n 23.397 q 13.19.37.703.481.247 s 41.223 u 7.1307 a 7.7.11.17.1309.833.539.187.119.77.49 b 89.103  
c 53.173 g 67.137 m 7.13.101.1313.707.91 n 29.317 o 17.541 - 92 - e 61.151 g 13.709 k 23.401 m 11.839 φ 7.1319 t 7.1321

Krause's table of factors (1804) (reconstruction, D. Roegel, 2011)

v 11.29.29.841.319 w 19.487 y 47.197 a 59.157 c 13.23.31.713.403.299 d 73.127 f 37.251 l 7.1327 p 17.547 - 93 - a 71.131  
c 41.227 f 67.139 g 7.11.11.11.1331.847.121.77 m 19.491 n 7.31.43.1333.301.217 t 13.719 w 47.199 y 7.7.191.1337.49 z 11.  
23.37.851.407.253 b 17.19.29.551.493.323 e 7.13.103.1339.721.91 g 83.113 i 11.853 l 41.229 - 94 - a 7.17.79.1343.553.119  
c 23.409 d 97.97 l 11.857 s 7.19.71.1349.497.133 u 11.859 v 13.727 x 7.7.193.1351.49 c 17.557 b 19.499 f 53.179 n 11.863  
p 7.23.59.1357.413.161 - 95 - b 13.17.43.731.559.221 d 37.257 g 31.307 k 89.107 l 7.1361 m 13.733 r 7.29.47.1363.329.203 95  
w 41.233 x 19.503 y 11.11.79.869.121 a 73.131 c 7.1367 d 17.563 f 61.157 b 11.13.67.871.737.143 i 7.37.37.1369.259 l 43.223  
n 53.181 p 29.331 - 96 - c 13.739 e 7.1373 g 59.163 p 23.419 r 31.311 t 11.877 w 7.7.197.1379.49 y 13.743 b 7.1381  
d 19.509 e 17.569 i 23.421 m 11.881 - 97 - a 89.109 b 31.313 c 17.571 d 7.19.73.1387.511.133 f 11.883 l 71.137 n 37.263  
p 7.13.107.1391.749.91 v 7.7.199.1393.49 x 11.887 z 43.227 a 13.751 e 29.337 g 7.11.127.1397.889.77 n 7.1399 o 97.101  
p 41.239 - 98 - d 17.577 i 7.23.61.1403.427.161 k 11.19.47.893.517.209 l 31.317 r 13.757 t 43.229 w 59.167 a 7.1409 c 71.139  
f 7.17.83.1411.581.119 b 41.241 l 11.29.31.899.341.319 n 13.761 p 19.521 - 99 - e 11.17.53.901.583.187 f 23.431 g 47.211  
h 7.13.109.1417.763.91 p 19.523 s 61.163 t 7.7.7.29.1421.343.203.49 w 37.269 y 23.433 z 7.1423 d 13.13.59.767.169  
f 11.907 g 17.587 i 67.149 l 7.1427 m 97.103 o 13.769.