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A Bandwidth Marketplace for Heterogeneous Mobile Networks

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Abstract—In recent years, with the evolution of new and content-rich Internet services, mobile network operators face the challenging task to guarantee ubiquitous and seamless access to their customers, while minimizing the installation and operating costs of their network. On the other hand, in residential areas most of the network capacity that residential users lease from ISP for Internet access remains unexploited.

Mobile network operators can therefore rent the unexploited capacity of residential user access devices (e.g., wireless access points or femtocells) when the traffic demand of their mobile customers exceeds the operator’s network capacity.

To minimize the leasing costs for the use of the capacity made available by residential users and prevent market manipulation, we formulate the allocation mechanism as a combinatorial reverse auction, which guarantees that all users reveal their real price for the utilization of their wireless access points.

Index Terms—Reverse Auction, Network Design, Heterogeneous Wireless Networks.

I. INTRODUCTION

In recent years, the rapid growth of the bandwidth demand required by content-rich Internet services accessed by mobile users through their 3G/3GPP devices has increased the pressure on mobile network operators for the upgrading of their cellular networks. Nevertheless, the increasing installation and management costs might limit the deployment of new base stations to provide the bandwidth required for accessing Internet services. This problem might further worsen with the integration of energy efficient mechanisms [1], which switch off the underused base stations in order to reduce the power consumption, since, as a side effect, they also reduce the overall bandwidth of the mobile network.

Several works investigate the benefit of opportunistically exploiting WiFi access networks to improve the QoS experienced by 3G devices [2], [3], [4]. Consequently, mobile network operators could provide a better service without limiting the maximum traffic of their customers through a wise management of their resources and the opportunistic utilization of other access networks.

In this position paper, we therefore propose a new marketplace which acts as an incentive for exploiting opportunistically the unused Internet connections of residential users. In particular, we envision a marketplace scenario for the unexploited bandwidth of the devices used by residential users for Internet access, which can be shared or even sold to third

party devices through a wireless connection, like in Wireless Community Networks [5]. A mobile network operator rents the capacity made available by residential users through their access devices in order to satisfy the additional traffic demand that mobile customers need for accessing content-rich web services.

As any marketplace, the misbehavior of even few residential users playing strategically might seriously affect the efficiency of the allocation mechanism used by the network operator, thus discouraging the mobile network operator and residential users from participating to the market. To address this issue, we present a *reverse truthful auction* targeted for the heterogeneous network scenario described above, which forces each residential user interested in leasing the unexploited bandwidth of its Internet connection to reveal its real valuation for such capacity.

The temporary agreements that can be stipulated by the mobile network operator and residential users through our marketplace are economically profitable, since the operator can provide a better service to its mobile customers without increasing the installation costs of its network, while residential users can earn money from their unexploited Internet connection.

In an effort to design a mechanism which would boost economically the deployment of heterogeneous networks, our work makes the following contributions:

- We propose and analyze an innovative marketplace for selecting the cheapest available Access Points to serve the additional traffic demand of mobile users.
- We propose a combinatorial truthful auction that minimizes the leasing costs sustained by a mobile network operator, which is resilient against any market manipulation.

The rest of this paper is structured as follows: Section II discusses related work. Section III presents the network and economic models considered in our work. Section IV formulates the combinatorial auction as an optimization model. Numerical results are discussed in Section V. Finally, Section VI concludes this work.

II. RELATED WORK

In recent years, *auction theory* has turned out to be a powerful tool to solve efficiently resource allocation problems in several networking contexts.

In particular, with the upcoming generation of cognitive radio networks, market-based auctions have been extensively studied as an efficient mechanism to dynamically sublease the unexploited licensed spectrum to secondary users and increase the revenue of the spectrum owner [6], [7], [8], [9], [10].

VERITAS [6] pinpoints the limits of conventional auctions and proposes a truthful and flexible mechanism requiring only polynomial complexity for solving the spectrum allocation problem. TRUST [7] further develops this approach to support multi-party spectrum trading through a truthful double spectrum auction based on the well-known McAfee mechanism. The work presented in [9] adopts a similar approach to model and solve a broad class of problems concerning the allocation of spectrum resources to primary and secondary users in cognitive radio networks, while [10] analyzes also the interplay among the spectrum broker, service providers which are interested in leasing bands of spectrum, and end-users. The analysis determines the conditions for the pricing policy that allow both service providers and end users to maximize their utilities. In [8] the authors investigate a spectrum marketplace where the spectrum owner's uncertainty about the private valuations of spectrum bidders is modeled using a Bayesian approach. Under this model, the authors propose an efficient algorithm to maximize the spectrum owner's expected revenue, while exploiting efficiently the spectrum resources.

Auction theory has also been exploited to design innovative traffic engineering techniques and routing protocols, both to enhance the utilization of unused network paths and force the collaboration of intermediate relaying nodes [11], [12], [13], [14], [15].

Finally, recent research has analyzed virtual network scenarios where several service providers compete among each other for using the resources owned and managed by a network operator [16], [17]. In particular, Jain et al. in [16] present a mechanism for per-link bandwidth allocation of end-to-end paths in wired network, whereas Fu et al. in [17] design an auction-based stochastic game for resource allocation of virtual operators in wireless cellular networks.

Unlike recent literature, our work envisions a new marketplace based on reverse auctions, where access devices of community networks (either WiFi Access Points or Femtocells) are exploited by mobile network operators to serve their customers. This marketplace would reduce the installation and management costs for mobile network operators, as well as foster the development of community networks.

III. SYSTEM MODEL

This section presents the communication and network models considered in our work, as well as the definitions and assumptions we adopt in the design of our auction mechanisms.

Let us refer to the Heterogeneous Network (HN) scenario illustrated in Figure 1, which is composed of a Mobile

Network (MN) formed by three Base Stations and a set of wireless access devices (APs) connected to the Internet (like, for example, Wifi access points or cellular femtocells).

The MN is managed by a single operator that provides ubiquitous access to its mobile customers (MCs), while each residential user, which participates to the marketplace, is the owner of a wireless access device (AP).

Residential users lease the unused capacity of their wireless access devices, so that the mobile operator can rent the available APs' bandwidth when the traffic demand of its customers exceeds the capacity provided by its mobile network.

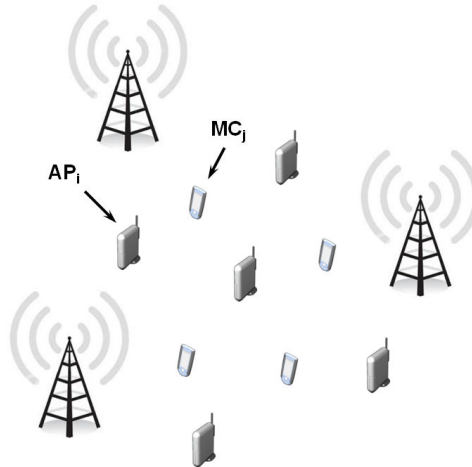


Fig. 1: Network scenario considered in this work. The MN is managed by a single operator that provides access to its customers (e.g., MC_j), while residential users lease the unused capacity of their wireless access devices (e.g., AP_i).

The mechanism we propose selects the residential access devices that are used to satisfy the traffic demand of mobile customers which exceeds the MN capacity. More specifically, the MN operator leases the available bandwidth provided by the access devices, which in turn is used to satisfy its mobile customers.

Each seller ¹ i owns an AP that he is willing to lease for a given price a_i . The MN operator decides which access points (APs) are selected to satisfy the extra-demand of its mobile customers (MCs), and the price paid to the winners p_i for exploiting their devices.

We further assume that APs use orthogonal channels; therefore, the different subsets of MCs assigned to each AP do not interfere with each other.

We observe that the transmission rate and the channel utilization required to satisfy the extra demand depend on the distance between the mobile customer device and the access point to which it is connected; hence, the allocation mechanism influences the number of access points that the MN operator uses to satisfy the extra traffic demand.

¹In this paper we use interchangeably the terms *sellers*, *residential users* and *access points*, while we use the term *buyer* to refer to the *mobile network operator*.

Therefore, the aim of the operator is to minimize its cost by allocating the traffic demand of its mobile customers to the cheapest access points, i.e., to those residential users that are willing to ask the lowest price for the utilization of their devices.

To this end, we design a truthful auction that, in addition to minimizing the overall price paid by the MN operator, prevents market distortion by forcing every residential user to declare and ask its true valuation, $v_i = a_i$.

Each residential user i submits its bid, a_i , representing the price that the seller i asks for leasing its AP to the MN operator. To simplify the discussion, in this paper we consider only one type of residential users. However, our model can be extended straightforwardly by considering different types of residential users (classified, for example, according to their hidden cost, wired connection, or residual bandwidth).

The MN operator turns the extra traffic demand of its mobile customers d_j into a vector of channel utilizations, $\vec{o}_j = [o_{j1} \ o_{j2} \ \dots \ o_{jn} \ \dots \ o_{jn}]$, where each pair (j, i) refers to a possible allocation of MC j to AP i , whereas n represents the number of APs in the network. Channel utilizations are computed as follows:

$$o_{ji} = \frac{d_j}{r_{ji}^{(max)}}, \quad (1)$$

where the element o_{ji} represents the channel utilization perceived by AP i when it satisfies the demand of MC j , and it is computed as the ratio between the required traffic demand d_j and the maximum achievable transmission rate of the wireless link that might connect MC j and AP i , $r_{ji}^{(max)}$. Note that this latter value can be easily obtained from the MAC layer through a scanning of the wireless channels, which is performed periodically by all network devices.

The allocation problem that minimizes the overall cost sustained by the MN operator to use the APs of residential users can be solved designing a *combinatorial single-minded reverse auction*. Specifically, the auction run by the MN operator aims at finding the cheapest APs to serve the traffic demand of the mobile customers. Each seller is interested in handling at least one mobile customer (i.e., a set composed of a mobile customer or any superset of it), since, in this case, he is paid by the MN operator which, in turn, uses its AP to serve the mobile customers. Therefore, the private valuation of residential user i can be defined according to Equation (2):

$$v_i(S_i) = \begin{cases} v_i < 0 & \text{if } |S_i| \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$S_i = \{j \in \mathcal{M} : j \text{ is assigned to } i\}$$

where \mathcal{M} represents the set of mobile customer devices (MCs).

We observe that MCs do not pay any additional price for the services provided through the residential APs (e.g., Internet access). Indeed, the MN operator benefits from using a subset of these APs, since it can satisfy the growing bandwidth

demand of its customers without installing new Base Stations, which have high installation and operational costs.

Let us denote by $p_i \leq 0$ the price paid to residential user i when its AP is used by the operator. Then, the utility of user i is defined as the difference between its private valuation v_i and the price paid to exploit the bandwidth of its AP, p_i , according to the following expression:

$$u_i = \begin{cases} v_i - p_i & \text{if } |S_i| \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Obviously, when the residential user's AP i is not used, its utility is null, since both the paid price and its valuation are null. Note that both the price p_i and the valuation v_i are negative, since the mechanism pays the players (i.e., the residential users) that participate to the marketplace.

We further assume that the MN operator has only a limited and imperfect knowledge about the real valuation residential users are willing to receive for leasing their access points, which we refer to as v_i . We model the operator's uncertainty about the real value of any residential user i as a continuous probability distribution over a finite interval. More specifically, let $f_i : [lb_i, ub_i] \rightarrow \mathbb{R}^+$ represent the probability density function of the real valuation that user i gives to its access point available capacity. We assume that the probability density function, $f_i = \frac{d}{dy} F_i(y)$, is known to the MN operator.

Note that the distribution describing the operator's uncertainty of residential users valuations can be determined by using appropriate prediction methods or exploiting the *a-priori* operator's knowledge of the system. We can confidently assume the use of an automatic system for implementing the bidding strategy of any AP owner, in which the residential user sets the lower and upper bounds of its bids, which are then automatically risen by the system.

According to the Bayesian approach to optimal auction design proposed in [18], the virtual valuation of any residential user i , over which the mechanism seeks to optimize, can be defined as follows:

$$\phi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}. \quad (4)$$

IV. OPTIMAL REVERSE AUCTION FOR AP SELECTION

This section presents the combinatorial auction mechanism we propose to select the wireless access points for satisfying the extra-demand of mobile customers.

We formalize the optimal and truthful auction mechanism in two steps. First, we present an Integer Linear Programming (ILP) model which provides the optimal allocation for the auction. Then, we describe the algorithm that, exploiting the allocation of the ILP model, makes the auction truthful. This algorithm computes the price paid to the residential users in such a way that the optimal strategy for each residential user i is to ask its real valuation v_i for the utilization of its available capacity c_i .

We first describe the meaning of the variables that we use in our model, then we provide the ILP description of the problem.

Let \mathcal{M} denote the set of mobile customer devices (MCs), and \mathcal{A} the set of wireless access points (APs) whose owners (the residential users) participate to the reverse auction of the mobile operator. Let us define $\mathcal{M}_i \subseteq \mathcal{M}, i \in \mathcal{A}$ as the set of MCs that are covered by AP i (i.e., the MCs that are in the radio range of AP i).

We can now introduce the decision variables used in our ILP model. Binary variables $x_i, i \in \mathcal{A}$, indicate which residential users win the auction, i.e., the APs whose available capacity is exploited by the mobile operator to serve the extra-traffic of its MCs ($x_i = 1$ if the available capacity of AP i is used, 0 otherwise).

Binary variables $y_{ji}, i \in \mathcal{A}, j \in \mathcal{M}$, provide the assignment of MCs to APs ($y_{ji} = 1$ if MC j is assigned to AP i , 0 otherwise).

Given the above definitions and notation, the reverse combinatorial auction problem can be stated as follows:

$$\max \sum_{i \in \mathcal{N}} \phi_i(a_i) \cdot x_i \quad (5)$$

s.t.

$$y_{ji} \leq x_i \quad \forall i \in \mathcal{A}, \forall j \in \mathcal{M}_i \quad (6)$$

$$\sum_{i \in \mathcal{A}} y_{ji} = 1 \quad \forall j \in \mathcal{M} \quad (7)$$

$$\sum_{j \in \mathcal{M}_i} y_{ji} o_{ji} \leq 1 \quad \forall i \in \mathcal{A} \quad (8)$$

$$\sum_{j \in \mathcal{M}_i} y_{ji} d_j \leq x_i c_i \quad \forall i \in \mathcal{A} \quad (9)$$

$$y_{ji} = 0 \quad \forall i \in \mathcal{A}, \forall j \notin \mathcal{M}_i \quad (10)$$

$$x_i, y_{ji} \in \{0, 1\} \quad \forall i \in \mathcal{A}, \forall j \in \mathcal{M} \quad (11)$$

The objective function (5) maximizes the social welfare, which coincides with the expected cost sustained by the MN operator to exploit the APs provided by the residential users (recall that both a_i and $\phi_i(a_i)$ are negative, since user i gets money from the mechanism).

Constraints (6) are coherence constraints ensuring that only the access points that win the auction can be used to serve mobile customer devices.

The set of constraints (7) provide full coverage of all those mobile customer devices that are served by the MN operator through the leased residential access points.

Constraints (8) and (9) prevent the allocation of an overall traffic demand that cannot be satisfied by an access point, due to the maximum achievable transmission rate of the wireless channel and the limited capacity of the Internet connection made available by the residential user, while constraints (10) avoid the assignment of MCs to APs that are not in the reciprocal radio range. Note that the channel assignment of a network composed of several access points can be optimized in order to avoid interference effects caused by the utilization of the same wireless channel.

Finally, constraints (11) ensure the integrality of the binary decision variables.

Having defined the ILP model representing the optimal auction, we now illustrate the algorithm that forces residential users to ask their real valuation for the utilization of the capacity that they make available through their access points.

Algorithm 1 describes the steps performed by the MN operator to select the cheapest APs. The algorithm receives

Algorithm 1: Optimal and Truthful Reverse Auction

Input : $\mathcal{M}, \mathcal{A}, c_i, a_i, d_{ji}$

Output: x_i, p_i, y_{ji}

1 Compute channel utilizations o_{ji} ;

2 Compute virtual bids $\phi_i(a_i)$;

3 $x_i \leftarrow$ Solve the ILP model (5)-(11);

4 **foreach** $i \in \mathcal{A}$ **do**

if $x_i = 1$ **then**

$\tilde{p}_i \leftarrow$

$\max_{x_{-i}} \sum_{h \neq i} x_h \phi_h(a_h) - \max_x \sum_{h \neq i} x_h \cdot \phi_h(a_h)$;

$p_i \leftarrow \max \{ \phi^{-1}(0), \tilde{p}_i \}$;

else

$p_i \leftarrow 0$;

end

end

as input the parameters which describe the network topology and all residential users' offers; these latter are composed of the capacity c_i made available through the APs and its cost a_i . It produces as output the allocation of mobile customer devices to access points, y_{ji} , as well as the price p_i paid to each winning residential user to exploit the available capacity of its access point ($\exists p_i \leq 0, \forall i \in \mathcal{A} : x_i = 1$).

The algorithm proceeds in 4 steps. In steps 1 and 2, the traffic demands of mobile customers are transformed into equivalent channel utilizations, and the virtual valuations of residential users are computed using both their offered bids and the valuation distribution function $F(x)$. Step 3 consists in solving the ILP model to find the allocation that maximizes the social welfare, or equivalently, the expected cost paid by the operator. Finally, in step 4, the operator computes the prices paid to the winners according to the Myerson's mechanism, which guarantees a truthful auction.

V. NUMERICAL RESULTS

In this section, we illustrate the numerical results obtained solving the reverse auction with the algorithm detailed in the previous section. We first describe the experimental methodology employed in our simulations, and then we measure the performance of our proposed solutions.

In our simulations, we consider network topologies composed of 10 Access Points and 100 Mobile Customers randomly scattered over an area of $800 \times 800m^2$. Both the extra traffic demands of MCs and the bids offered by residential users for their APs are drawn from uniform distributions; in particular, we vary the average values of both the MCs traffic demands and APs bids in the sets $\{1, 2, 3\}$ Mbps and $\{15, 25, 35\}$ monetary units (e.g., US dollars), respectively.

The channel capacity of both access and backbone links is defined according to the reception sensitivity of the Wistron

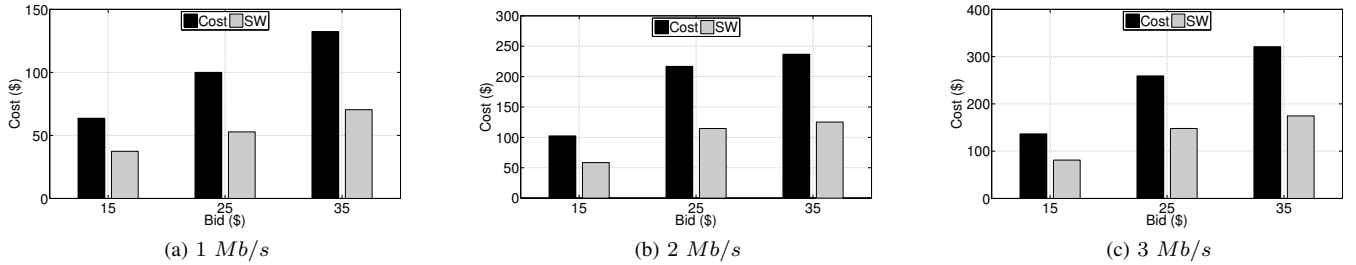


Fig. 2: Total Cost paid by the operator and Social Welfare, measured as a function of the average bid of mobile customers.

CM9 commercial wireless cards (based on Atheros chipset)². The path loss, which is necessary to evaluate the sensitivity of the receiving node, is computed according to the Friis propagation model. We observe that all the above assumptions do not affect the proposed algorithms, which can be used to solve any network scenario.

In order to analyze and compare the performance of the proposed algorithm, we consider the following metrics:

- *Total Cost*: defined as the sum of the prices paid by the operator to all winners, $\sum_{i \in \mathcal{A}} p_i \cdot x_i$.
- *Social Welfare*: defined as the sum of the winner virtual bids, $\sum_{i \in \mathcal{A}} \phi_i(a_i) \cdot x_i$. This is the function maximized by our mechanisms, and represents the ideal bound for the *Total Cost* paid by the MN operator.

Note that, for the sake of clarity, we have considered the absolute value of both the *Total Cost* and *Social Welfare*. For each network scenario, we performed 10 independent measurements, computing very narrow 95% confidence intervals.

Figure 2 shows the *Total Cost* and *Social Welfare* paid by the MN operator for three different mean values of the customers data traffic measured in the network topologies described above, as a function of the average bid of the residential users.

It can be observed that the overall cost paid by the operator using our allocation scheme well approaches the *Social Welfare* in all network scenarios, thus confirming the validity of the proposed approach.

VI. CONCLUSION

This paper proposed a new marketplace where mobile network operators can rent the available bandwidth of Internet connections sold by residential users through their wireless access devices (either WiFi Access Points or femtocells).

The allocation problem faced by the mobile network operator is formulated as a combinatorial auction in order to minimize the cost paid for exploiting the residential user devices and prevent market manipulation, which may decrease the effectiveness of the proposed mechanism, forcing all residential users to reveal their real valuation. Numerical results demonstrate the effectiveness of the proposed approach.

Future research issues include the design and analysis of an efficient algorithm to solve the auction in large scale network scenarios

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²Available on-line <http://www.diswire.com/SpecsCM9.pdf>