Automatic characterization of emergent phenomena in complex systems
Thomas Moncion, Patrick Amar, Guillaume Hutzler

To cite this version:

HAL Id: hal-00644627
https://inria.hal.science/hal-00644627
Submitted on 25 Nov 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Automatic characterization of emergent phenomena in complex systems

Thomas Moncion\textsuperscript{1,2}, Patrick Amar\textsuperscript{1}, Guillaume Hutzler\textsuperscript{2}

\textsuperscript{1} LRI, UMR 8623, CNRS/Paris-Sud 11 University, Bât 490 Université Paris-Sud 11 91405 Orsay Cedex, France
\textsuperscript{2} IBISC, EA 4526, Evry University, 523 Place des Terrasses, 91000 EVRY, France

corresponding author, e-mail: guillaume.hutzler@ibisc.univ-evry.fr

Abstract

One of the main characteristics of complex systems is that the interrelations between the entities composing the system are not permanently established but evolve along time. As opposed to complicated systems, the structure of complex systems also evolve in a dynamic organizational process. When studying complex systems, self-organization and emergent phenomena must therefore be taken into account and studied carefully. In this paper, we propose to provide tools in order to automatically detect and characterize the emergent phenomena occurring in agent-based simulations. To this end, we consider the interactions between the entities at the lower level as the main organizational forces that shape the structure of the system at a higher level. These interactions are detected during the simulation and represented as dynamic graphs. Measures can then be made on various properties of the graph so as to detect the occurrence of structuring processes. Groups detection and tracking techniques are then introduced so as to characterize more precisely the exact nature of these processes.

1 Introduction

Complicated systems on the one hand, like horological movements, may be constituted by a great number of components that interact with one another. They may also be dynamical, meaning that their parts can move. Their structure however is static, the interrelations between the various parts of the system being fixed. Complex systems on the other hand are characterized by the fact that the interactions between the entities comprising the system may be created and destroyed dynamically. This is because the entities are loosely coupled inside the system, as opposed to complicated systems in which they are tightly coupled inside a rigid structure. This does not mean, however, that complex systems don’t and couldn’t have any structure at all. But the structure, instead of being engineered beforehand and imposing its constraints to the system, is the emergent result of the local interactions between the entities of the system. A particular class is constituted by systems whose structure evolves along time, which corresponds to the notion of \textit{Dynamical Systems with Dynamical Structure (\((DS)^2\))} introduced by Giavitto in [1].

It is therefore essential, when studying complex systems, to elucidate the mechanisms of this dynamic and emergent structuring of the system. \textit{Self-organization} and \textit{emergence} are most commonly invoked as the main structuring processes in complex systems, although they can not easily be defined. Self-organization is often referred to as an increase in order (measured as a decrease of entropy), without the structuration process being guided by external inputs or controls. We will rely on the definition given by De Wolf and Holvoet [2], which states that \textit{self-organization is a dynamical and adaptive process where systems acquire and maintain structure themselves, without external control}. As far as emergence is concerned, it is classical, paraphrasing Aristotle\textsuperscript{3},

\textsuperscript{3}In the case of all things which have several parts and in which the totality is not, as it were, a mere heap, but
to say that the whole is more than the sum of its parts, meaning that the interactions between the local entities give rise to a global behaviour, which can not be deduced from the study of these individual entities. Again, we will refer to the definition given in [2], which states that a system exhibits emergence when there are coherent emergents at the macro-level that dynamically arise from the interactions between the parts at the micro-level. Such emergents are novel with respect to the individual parts of the system.

Although in many systems, self-organization and emergence occur together, they are not strictly synonymous. Indeed, one can find systems with emergent properties but no self-organization and vice-versa. The pressure of a gas is typically an emergent property of the interaction between the particles that compose the gas but their is no self-organization. Reciprocally, a system of agents may organize autonomously through the interactions between agents, without necessarily the emergence of any new global property. In both emergence and self-organization however, the appearance of stable and robust new properties, functions or structures is the result of the local interaction between the individual entities of the system, and between the entities and their environment.

We will therefore be interested in the appearance of emergent structures in complex systems, and to this end, will focus on the local interactions between their parts, and between the parts and the environment. All these interactions constitute a dense and dynamic network of interrelations that evolve along with the system itself. The basic hypothesis underlying our work is that the topology of this network can be seen as the fingerprint of the structure of the system at a given time, and that the evolution of the network can be seen as the fingerprint of the processes dynamically structuring the system. By analyzing the topology of this interaction network, it may thus be possible to detect the emergence of structures and to characterize them.

In the remaining of the paper, we will first review some related works in section 2 before presenting our own approach. In section 3, we explain how to build the interaction network. In section 4, we present measures and algorithms to analyze the structure of the graph. In section 5, we finally develop the application of the techniques presented in the paper on a simple example.

2 Related works

In order to be able to automatically detect emergent phenomena, one has first to define precisely what can be considered as emergent. The trouble is that the notion of emergence is itself subject to a heated debate for a very long time, with no real consensus. First, emergence is a general term, which covers very different phenomena [5, 6]. In addition, emergent phenomena are often defined relatively to an external observer, who qualifies the phenomenon as emergent [3, 7, 8], emergent being in some cases synonymous with surprising [9]. Worse, in its strong interpretation [10,11], emergence is even sometimes considered as something mysterious or magical since the global phenomena cannot be deduced from the local activity of the entities composing the system. Some authors thus question the possibility to simulate emergence at all using Turing-equivalent machines and suggest the use of alternative computing devices such as analogue computers [12]. All of this results in relatively few works on the subject of computationally detecting emergence in complex systems until recently.

As a conclusion of their review on emergent phenomena [3], Bonabeau et al. insist however that “a framework for characterizing emergence is needed, especially if one wants to go further than the simple awareness that things “emerge” in the world.” In the framework that they propose [7], they rely on levels of organization, which are “defined by a set of elements and relations R between them”. They propose to use detectors, that could be based on different formal measures of complexity, which are more or less related to entropy or information theory. Since the notion of entropy has long been related to that of order and disorder, although it is now preferably seen as a measure of how close a system is to equilibrium, it has indeed been proposed by some authors to determine whether a system exhibits or not emergent phenomena [13–16]. These measures however have themselves different interpretations, in physics, information science, economics, social sciences and it is therefore difficult to use it as a universal measure of organization (or disorganization) in complex systems. Furthermore, they are global measures. In the best case, it can only be used to sign the fact...
that there is emergence or not, but not to characterize how this emergence is occurring. Furthermore, it sometimes requires to reformulate the model using a given formalism [17, 18], which lacks generality and raises the question of extending the approach to more complex models.

Although they constitute interesting contributions to the understanding of the concept of emergence, theoretical works such as [17] or [19] are not so pertinent when it comes to analyze the dynamics of simulated complex systems such as agent-based models. In that particular field, some preliminary works have been done in specific application domains (see for example [20, 21] for the detection of emergent hydrological structures), or for specific simulators and emergent phenomena [22, 23]. If we stick to emergence in a large interpretation, we stick to general measures with limited applicability. But since organization may be seen as the result of an interaction dynamics between lower-level entities, and between these entities and their environment, one can think of studying the construction of these organizations by studying the dynamics of the underlying interaction networks. In social sciences, there is a long tradition of studying human communities, using interaction networks as a tool of choice [24–28]. This enables the detection of the emergence of groups based on the density of connections between the individuals. Bertelle et al. have developed on this idea so as to propose tools to study the dynamic organization of agent-based simulations in order to allow their parallelization [29–32]. This approach is but limited to a narrow view of the notion of interactions between agents. Chen et al. for their part propose a richer interaction framework to analyze emergent behaviours with the notion of complex events, corresponding to combinations of events that are related with one another in space and/or time [33–35].

Our proposal is to develop a generic mixed approach that may enable to analyze agent-based simulation by observing events occurring in the simulation. To this end, we wish to remain independent from any particular simulation framework by only relying on events sent by the platform to describe the evolving state of the simulated system. This mean that the modelled behaviour of the entities is a priori unknown. The proposal relies on:

- a wide and rich definition of interaction events;
- the use of dynamic graphs to represent the interaction networks;
- the use of tools from complex networks and information theory so as to characterize the complexity of the network;
- the use of tools from graph theory so as to detect and to analyze groups emerging in the simulations.

3 Building the interaction network

In order to observe an agent-based simulation and to characterize the emergent phenomena, the first step is to build the interaction network by observing the events occurring in the simulation. It has first to be defined what can be considered as an interaction or an event, then to see how the observation of these interactions can be used to build a dynamic graph representing the interaction network.

3.1 Interaction signs

Interaction is usually defined as a kind of action that occurs as two or more objects have an effect upon each other (two agents, or an agent and the environment). But since the modelled behaviour of the entities is unknown, it is rarely possible to say for sure, based on the events occurring in the simulation, that a given entity has achieved an action upon another one. What we will look for are rather traces, signs, indications that an interaction has occurred or may occur in a near future. But in fact, ultimately, what we will look are indications that two agents belong to the same group. We call these indications interaction signs. These interaction signs can be multiple, capturing all the different influences that we can think of between agents and with the environment.

Since the locality of the interactions is of fundamental importance when speaking about emergence in complex systems, the first criterion that we chose to take into consideration is the distance between two agents. If we think about the flocking model of C. Reynolds [36], it relies on three simple steering behaviours which describe how an individual boid maneuvers based on the positions and velocities of flockmates within a certain small neighbourhood around itself. In its original version, this neighbourhood was only defined as “a spherical zone of sensitivity centered at the boid’s local origin”. Hence, if we take $r$
to be the radius of this sphere, any boid within distance \( r \) from boid \( b \) may influence its behaviour. We say in that case that being at a distance less than a certain distance \( r \) from another boid corresponds to the distance interaction sign. We will thus say that there is an interaction between boids \( b_1 \) and \( b_2 \) if they are in each other’s neighbourhood or, said differently, if they match the distance interaction sign. In later versions, the neighbourhood was “characterized by a distance (measured from the center of the boid) and an angle, measured from the boid’s direction of flight”. In this version, the neighbourhood is not symmetric anymore, so that we will now consider interaction between boids \( b_1 \) and \( b_2 \) if \( b_1 \) is in the neighbourhood of \( b_2 \) or if \( b_2 \) is in the neighbourhood of \( b_1 \). Taking the new definition of neighbourhood into account only requires to modify the definition of the distance interaction sign or to define a new one. One can see that in that particular interpretation, the notion of interaction is not oriented.

Another important aspect of Reynold’s model is that boids try to align with their local flockmates by steering towards the average heading amongst them (velocity in the initial model). We can thus enrich our catalog of interaction signs by defining one relative to the boids’ heading (two boids match the heading interaction sign if the difference between their headings is less than a given threshold) and another one relative to their velocities. More generally, all the characteristics pertaining to the movement of agents in a 3D space may be used as interaction signs (position, heading, velocity, acceleration, angular speed, etc.). We can generalize still further by adopting a different point of view and noticing that boids try to modify their inner state so as to resemble their local flockmates. If we define the state of an agent as a point in a \( d \)-dimensional space \((x_1, x_2, ..., x_d)\), we can add a state interaction sign corresponding to the distance between the states of two agents, and \( i \)-state interaction signs corresponding to the difference of value between the \( x_i \).

The number of interaction signs can be multiplied at will as to capture the emergent processes at work in the system as precisely as desired. In particular, one can think of agentifying the environment so as to take into account the interactions between the agents and their environment. One can also add temporal criteria so as to consider phase shifts between agents or to address the case where an agent \( a_1 \) deposits some kind of pheromone in the environment and is followed at a distance by another agent \( a_2 \) that smells the pheromones. Finally, one can also consider using complex events as proposed by Chen et al. [35] as interaction signs so as detect more complex interaction situations.

### 3.2 The interaction network

As we just saw, there can be many different interaction signs defined to analyze a single simulation. It is therefore possible for two agents \( a_1 \) and \( a_2 \) to be considered as being in relation simultaneously according to several interaction signs. In a flock for example, two boids may be both in their mutual neighbourhood (distance interaction sign) and flying in the same direction and at the same speed (direction and velocity interaction signs). The design of the interaction network has thus to reflect this diversity of relationships between the agents. Another important aspect is that a simulation is a dynamic process in which the interrelations between the agents are continuously changing. Two options may therefore be chosen: either generating the interaction network in at different timesteps \( t \) and studying the time series \( i(t) \), or updating a single interaction network as the simulation runs. The latter option allows to take into consideration cases in which two agents are not related at time \( t \) but remain connected in the interaction network because they had been related for a long period of time before.

In its simplest formulation, the interaction network is a graph in which the nodes correspond to the agents in the simulation and the edges correspond to an interrelation between two agents. Since this relation can be associated to several interaction signs, the edges are labeled with the corresponding sign. Formally, we have thus a labeled multigraph, with potentially several labeled edges between any two nodes. To take into account the dynamic evolution of the graph along time, we add weights on the edges so as to characterize the strength of the relation between two agents. The higher the weight, the longer the relation between the two agents. Formally, we have thus a weighted labeled multigraph. When a new relation is detected between agents \( a_1 \) and \( a_2 \) with respect to the interaction sign \( I_i \), a new edge is created with the label \( I_i \) and an initial weight of \( w_{inc} \). If that same relation (same agents, same interaction sign) is preserved at the next timestep, it is incremented by the factor \( \delta w_{inc} \), else it is decremented by the factor \( \delta w_{dec} \). If the weight of the edge becomes less or equal than zero, the edge is destroyed.

4
4 Analyzing the interaction network

Now that we have built an interaction network, it is necessary to analyze it. The aim of the analysis is twofold. First, it is to detect phase transitions during periods when the system self-organizes. Second, it is to detect the emergence of groups in the system, and track them as the simulation advances.

4.1 Analyzing the global properties of the graph

To simplify the analysis, we consider the graph, not as a multigraph but as a set of simple weighted graphs each associated to a single interaction sign. We can then compute, for each of these graphs, some classical properties from the fields of complex networks or graph theory [37]. By tracing the evolution of these properties along time, the aim is to identify phase transitions in the dynamics of the system.

Let’s study $G(V, E)$, an undirected graph where $V$ is the set of vertices and $E$ the set of edges. A first simple measure is the sum of the weights of all the edges. Since the weights are indicators of the strength of the relations between agents, this global measure is an indication of the autonomy of the agents or, on the contrary their implication in structured organizations. This value is given by:

$$W_{sum}(G) = \sum_{i \neq j} w_{ij}$$

where $w_{ij}$ is the weight of edge $e_{ij}$ connecting vertices $v_i$ and $v_j$.

Another simple measure is the mean degree of the nodes. The degree of a node, denoted by $deg(v)$ is the number of connections it has to other nodes. A node with a high degree corresponds to an agent that interacts with a lot of other agents, thus has a central role in the system, whereas a node with a low degree corresponds to a more autonomous agent. It is expected that the mean degree of the graph will increase as the system organizes. This value is given by:

$$Deg_{mean}(G) = \frac{1}{n} \sum_{v_i \in V} deg(v_i)$$

where $n$ is the number of nodes.

Another parameter that determines the connectivity of the system is related to the distance between the nodes of the graph, that is the number of edges in a shortest path connecting them. Since some of the nodes are disconnected, the distance to them is infinite. We thus cannot directly compute the mean distance between the nodes and use the global efficiency instead. This measure assess the connectivity of the system as a whole and is defined as follows:

$$E_{glob}(G) = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{d(v_i, v_j)}$$

where $d(v_i, v_j)$ is the distance between nodes $v_i$ and $v_j$.

To assess the presence of communities inside the graph, we can use the measure of local efficiency, which is derived from the global efficiency and is defined as follows:

$$E_{loc}(G) = \frac{1}{n} \sum_{i=1}^{n} E_{glob}(G_i)$$

where $G_i$ is the subgraph of the neighbours of node $v_i$.

4.2 Detecting and tracking groups of agents

Detecting groups of agents inside the graph corresponds to identifying clusters of nodes that are more densely or more strongly interconnected. A lot of clustering algorithms however have to make hypotheses about the number of groups that are searched for in the graph. This is ill-suited to our case since we cannot know beforehand how many groups will emerge in the simulation, if any. In addition, most algorithms are very time-consuming, which isn’t adapted either since we have to analyze the graph at every timestep. We thus developed our own algorithm, simpler but very fast and satisfactory in most cases. This algorithm is based on the idea that members of groups share more and stronger interrelations than the mean of the population. The same algorithm can thus be instantiated using of two criteria.

The first criterion is based on the degree of the nodes. Let’s consider a graph $G(V, E)$. The mean degree of the graph is given by $deg_{mean} = 2 * \frac{|E|}{|V|}$. We define a group in this graph, as a connected
subgraph $C(V_C, E_C)$ where, for any node $N_i$ of $V_C$, $\text{deg}(N_i) > \text{deg}_{\text{mean}}$. The second criterion is based on the weights of the edges. A group is then defined as a connected subgraph $C(V_C, E_C)$ where, for any edge $E_i$ of $E_C$, $W(E_i) > W_{\text{mean}}$, where $W_{\text{mean}}$ is the mean weight of the edges and $W(E_i)$ is the weight of edge $E_i$.

![Group Detection Example](image)

Figure 1: Example of group detection based on the degree of the nodes ($\text{deg}_{\text{mean}} = 2 \times (16/11) = 2.9$)

Whatever the criterion (degree of the nodes or weight of the edges), the algorithm examines every node in turn and determines whether it belongs or not to a group (figure 1 shows an example based on the degree of the nodes). The complexity is in $O(m + n)$, where $m$ is the number of nodes and $n$ is the number of edges. After determining the nodes that belong to a group, their distribution amongst the groups is done iteratively by selecting a node that hasn’t been assigned yet to a particular group, and adding to its group all the nodes that are connected to it. Once detected, the groups are labelled using a unique identifier. The trouble is that groups detected at time $t$ are labelled independently of the groups detected at time $t-1$. The same group may thus be labelled differently from one timestep to the other. It is then necessary to relabel the groups so as to be able to track them as the simulation advances. This is done, for each group at time $t$, by searching for the group, at time $t-1$, which shares the greatest number of common nodes. The latter gives its label to the former. If a group has divided into several smaller groups, all of these groups should inherit the same label. In that case, only the biggest one inherits the label, the others being labelled with other names.

5 Case study

5.1 Methodology

To illustrate our approach, we will now examine the results that may be obtained on the flocking model of Reynolds [36]. In this model, creatures called *boids* move according to three steering behaviour: *separation* (steer to avoid crowding local flockmates), *alignment* (steer towards the average heading of local flockmates) and *cohesion* (steer to move toward the average position of local flockmates). As a result, boids self-organize so as to form *flocks*, which can be seen as groups of boids exhibiting a coherent movement behaviour.

In order to study this model (the same holds for any model), one has to decide first the interaction signs that will be used for the creation of the interaction network, and second the properties of the graphs that will be measured. As the simulation runs, the simulator has to export the relevant data with a chosen periodicity. These data are used to update the interaction graph and the corresponding measures. After the simulation has stopped, the history of the interaction network and the time series of the chosen properties are provided to the modeller for analysis.

For the flocking model, since we know that the behaviour of the boids is influenced both by locality (boids only interact with local flockmates) and directionality (boids align with flockmates heading), it seems reasonable to use both the distance interaction sign and the direction interaction sign. In that case, it is necessary to define a composite interaction sign associating the two.

5.2 Results

Figure 2 shows snapshots of a flocking simulation at different timesteps and the corresponding interaction networks. In that simulation, boids have first aggregated in small groups (a), which then began to adopt similar headings. At a given time, a single group aggregated almost all the agents of the simulation (b). Several smaller groups then developed, travelling with approximately the same direction. We can see in that figure that the interaction network exactly reflects this evolution.

Figure 3 shows the evolution of the four global properties presented in section 4.1. The increase of the global efficiency is the sign that the agents of
the simulation are getting more and more connected with one another for the chosen interaction sign, that is moving both close to each other and in the same direction. The maximum is obtained when almost all the agents are aggregated in the same group. The value however is not very high (less than 0.4, to be compared to the global efficiency of a random network, which is 0.28), meaning that the agents are not connected with all the other agents but only with the closest ones. This corresponds to the fact that agents are spread inside the groups because of the separation rule.

We can see that the evolution of the local efficiency is the inverse of the global efficiency. It is first at the maximum of 1, meaning that the agents are completely connected inside the groups. This corresponds to the fact that the groups that are first created are small with only two or three agents. As the simulation advances, groups are getting bigger and bigger until all the agents are included in a single group, which results in a lower local efficiency, because of the spreading of the group. It then stabilizes at a slightly higher value as several smaller groups arise.

The mean degree of the graph increases as groups are formed corresponding to the fact that the agents get connected with each other. It then stabilizes (with stochastic variations), once again because of the spreading of the groups, which limits the number of connections. If the agents were spread along a hexagonal lattice for example, the degree of the agents would be exactly 6 and couldn’t be more than that. In our case, the mean degree oscillates around 8 because agents can be more densely packed and because of the inertia introduced by the weights. Indeed, we can observe that the sum of the weights increases with a steeper slope than the mean degree, corresponding to the fact that the connections between the agents are rather stable, which results in the increase of the weight of the connections.

Figure 4 shows different measures made on one of the groups that have been detected during the simulation. As we can see with the number of agents, groups don’t evolve smoothly by gaining or losing agents one at a time, but by splitting into distinct groups or merging with other groups. The mean length of the shortest path evolves accordingly. Because of the spreading of groups, the bigger the group, the longer the shortest path. As for the clustering coefficient, it appears to slowly decrease during the “life” of the group (with variations), indicating that the agents are less and less densely connected, which may provide hints on why the group disappeared.

6 Conclusion

We developed in this paper a generic, modular and extensible method to study emergent structures in agent-based simulations. This approach is based on the definition of interaction signs which constitute indicators that agents are in interaction or that have interacted in the past. Interaction signs can be tailored at will to fit the modeller’s view of what should be considered as an interaction. And they enable to reconstitute the interaction network between the agents, which we represent as a weighted labelled multigraph. Different measures can be computed so as to characterize the graph and the groups of agents can be detected and followed during the simulation.

We have shown in section 5.2 that these measures can give useful informations about the dynamics of the system as a whole and about the structures that arise during the simulation. We are but conscious that a lot of information still needs to be provided by the modeller and that it orients the structures that are detected by the system. What will be explored in the future is the automatization of the analysis process in order to get rid of the modeller’s subjectivity. As a first thing, it is necessary to detect the interaction sign or combination of interaction signs for which the phase transition is the clearest. It is
then necessary to automatize the interpretation of the
time series of the properties characterizing the
graph.

In addition, the analysis of the groups should be
made finer so as to characterize such parameters as
their speed or heading. Ultimately, this should en-
able to reify groups as agents. By introducing such
new abstractions inside the simulation, the aim is
not so much to obtain computational gains as to de-
telop a new hybrid simulation framework that would
constitute a crucial first step towards truly multilevel
simulations.

References

tions, transformations and their application to
the modeling and the simulation of dynamical

or-organisation: Different concepts but promising
when combined, Engineering Self-Organising

A. , Characterizing Emergent Phenomena (1):
A critical review, Revue Internationale de

about the emergence issue: A survey of emer-

[5] Dessalles J.-L. , Müller J.-P. , Phan D. , Emer-
gence in multi-agent systems: conceptual and
methodological issues, Agent-based modelling
and simulation in the social and human sciences
(Amblard F. Phan D. , eds.), The Bardwell-

gence in agent based computational social sci-
ence: conceptual, formal and diagrammatic
analysis, Intelligent complex adaptive systems
(Shyan Y. Yang A. , eds.), IGI Global, 2008,

Characterizing Emergent Phenomena (2): A
conceptual framework, Revue Internationale de

[8] Bonabeau E. Dessalles J.-L. , Detection and

[9] Ronald E. , Sipper M. , Caparrère M. , De-
sign, observation, surprise! a test of emergence,
Artificial Life 5 (1999), no. 3, 225–239.

[10] Bedau M. A. , Weak emergence, Philosophi-
cal Perspectives: Mind, Causation, and World
(Tomberlin J. , ed.), vol. 11, Blackwell Publishers,

[11] Chalmers D. J. , Strong and weak emergence,
The Re-Emergence of Emergence (Davies
P. Clayton P. , eds.), Oxford University Press,
2006.

[12] Boschetti F. Gray R. , Emergence and com-
putability, Emergence: Complexity and Orga-

[13] Langton C. , Computation at the edge of chaos:
phase transitions and emergent computation,
Physica D 42 (1990), 12–37.

[14] Standish R. K. , On complexity and emergence,

[15] Boschetti F. , Prokopenko M. , Macreadie I. ,
Grisogono A.-M. , Defining and detecting emer-
gence in complex networks, KES (4), 2005,
pp. 573–580.

[16] Prokopenko M. , Boschetti F. , Ryan A. J. ,
An information-theoretic primer on complex-
ity, self-organization, and emergence, Complex-

[17] Yamins D. , Towards a theory of “local to
global” in distributed multi-agent systems (I),

[18] ______, Towards a theory of “local to global” in
distributed multi-agent systems (II), AAMAS,

[19] Kubik A. , Toward a formalization of emer-

[20] Servat D. , Perrier E. , Treuil J.-P. , Drogoul A. ,
When agents emerge from agents: Introduc-
ing multi-scale viewpoints in multi-agent sim-


Figure 3: Evolution of the global properties of the graph during the simulation

Figure 4: Evolution of the properties of a group detected during the simulation