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# Market Share Game with adversarial Access providers : A Neutral and a Non-neutral Network Analysis

(Invited Paper)

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**Abstract**—Internet pricing schemes have not been concerned in the network-neutrality research. In this paper, we study the implication of non-neutrality on the competition between Internet Access Providers. We interpret non-neutral network when a service provider privileges a Content Provider (CP) in order to propose a high quality of service for this content like time-sensitive applications (voice over Internet protocol, live video streaming, online gaming). We present a competitive model that describes the interaction between several competing telecommunications Access Providers (APs), their subscribers, and a network owner. Competition between the access providers is assumed to take place in their pricing decisions as well as in terms of the Quality of Service (QoS) they offer. Furthermore, our work focuses on the analysis of games between APs under two cases : case of neutral network and case of non-neutral network. After having discussed the existence and uniqueness of equilibrium, we analyzed the impact of non-neutrality versus neutrality. In particular, we showed that non-neutrality has a significant effect on the charged tariffs and the perceived quality of services in case where exist restrictions on the available bandwidth at the AP-CP link.

**Index Terms** - Pricing, Network neutrality, Nash equilibrium, Market share, Quality of Service.

## I. INTRODUCTION

Network Neutrality is a concept that while not clearly defined, has generated great debate in many different venues such as economics, academia, law, the Internet industry and Congresses. Network Neutrality, or the limitation of traffic discrimination over Internet, has become an issue that exceeds academia or basic theory, making it an issue of debate in the halls of congresses. The term Network Neutrality was introduced as a result of the common carrier concept. The two potential behaviors most often cited are the network providers ability to control access to and the pricing of broadband facilities, and the incentive to favor network owned content, thereby placing unaffiliated content providers at a competitive disadvantage [6]. In this paper, we focus on the impact of the second behavior on the competition between access providers. Several works addressed the problem of the network neutrality

from different aspects [9], [1], [5]. [11] provides a recent and comprehensive overview in the context of the net neutrality debate. The contributions that are most related to ours are [4], [5].

In this paper, we consider that several access providers share a market and the competition between access providers takes places in pricing as well as in terms of the quality of service they offer. We assume that the demand for the service of a given access provider is a function of the vector of prices and QoS of all access providers. We establish the existence and uniqueness of Nash equilibrium for both schemes of neutrality or non-neutrality. We analyze the effects of net neutrality on the charged price and offered QoS as behaviours of APs.

The rest of the paper is organized as follows. In section II, we describe the contention model between service providers and their subscribers, also, we present definition of the Nash equilibrium. In sections III and IV we provide theorems for existence and uniqueness of equilibria respectively under net neutrality case and non-neutrality case. Section V presents numerical study to validate our claims and Section VI concludes the paper.

## II. PROBLEM MODELING

We consider a system in which there are  $N$  Access Providers (APs) competing over a set of customers. Figure 1 shows a typical example of systems with a single Content Provider (CP), several access providers where each one serves a set of End Users (EUs). Here, the content provider provides multimedia contents (e.g., Music, Streaming, VoD, ...). This multimedia content is transported and made available to EUs over the physical infrastructure of  $AP_i$ . Under neutral network configuration, EUs and the content provider pay only for their direct access. On one hand and depending on own demand rate of  $AP_j$ , the CP decides to invest and lease (allocate) a bandwidth  $\Phi_c^j$  to that latter. This can be seen as a privileged contract made between  $AP_i$  and the single content provider. Yet, as the demand of  $AP_i$  increases as it becomes more

beneficial to CP to invest more and more. On the other each  $AP_i$  charges its registered users an amount  $p_i$  of Euros per unit of traffic. Moreover, each  $AP_i$  allocates a bandwidth  $\Phi_i$  and advertises to its registered users a quality of service denoted by  $\tilde{q}_i$ . In the remaining, we consider the following notations :  $\mathbf{p} = (p_1, \dots, p_j, \dots)$  and  $\tilde{\mathbf{q}} = (\tilde{q}_1, \dots, \tilde{q}_j, \dots)$  for the charged prices vector and the promised QoS respectively.

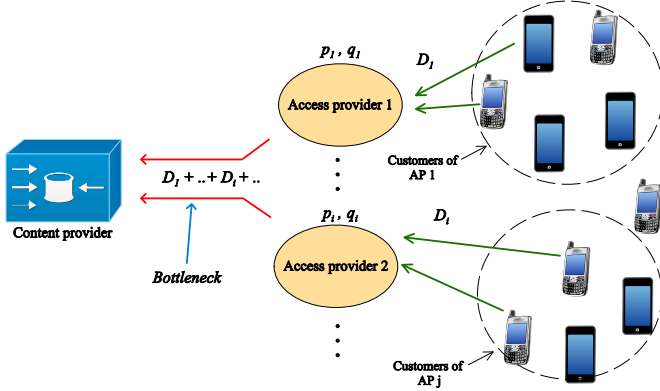


Fig. 1. Single content provider and several access provides.

### A. End-to-end Quality of Service (QoS)

Quality of service is the ability to convey in good conditions a given type of traffic, in terms of availability, throughput, transmission delay, jitter, packet loss rate... Quality of service guarantees are important if the network capacity is very limited, especially for real-time streaming multimedia applications such as voice over IP, online games and IP-TV. This latter reason encourages -in addition to the price constraint- users to subscribe to an AP promising a good QoS. Clearly, the end-to-end (e2e) QoS depends on investments of both APs and CP. We claim that it implicitly depends on the demand  $D_i(\cdot)$  for services of  $AP_i$ , amount of bandwidth  $\Phi_i$  reserved by  $AP_i$  to end users and the amount of bandwidth  $\Phi_c^i$  reserved by the content provider to this AP. In other words, the e2e QoS depends on the qualities perceived on both links User-AP and AP-CP.

Definitely, the end-to-end quality of service can express the transmission e2e delay, e2e throughput, e2e loss probability or any combination of several QoS indicators. Later, we will restrict to some special cases where end-to-end QoS is simple enough (e.g., e2e delay is the sum of delays experienced on both links User-AP and AP-CP).

### B. Demand Model

We consider that the demand function  $D_i$  for services of the tagged  $AP_i$  is linear with respect to price  $p_i$  fixed by  $AP_i$  and its promised e2e QoS, see [7]. This demand depends also on prices  $p_{-i}$  and QoS of competitors, namely the demand

function depends on  $\mathbf{p}$  and  $\tilde{\mathbf{q}}$ . Ultimately,  $D_i$  is decreasing w.r.t.  $p_i$  and increasing w.r.t.  $p_{-i}$ . Whereas it is increasing w.r.t.  $\tilde{q}_i$  and decreasing w.r.t.  $\tilde{q}_{-i}$ . Then, the demand functions w.r.t services of  $AP_i$  can be written as follows :

$$D_i(\mathbf{p}, \tilde{\mathbf{q}}) = a_i - \alpha_i^i p_i + \beta_i^i \tilde{q}_i + \sum_{j, j \neq i} \left[ \alpha_i^j p_j - \beta_i^j \tilde{q}_j \right], \quad (1)$$

where  $a_i$  is a positive constant used to insure non-negative demands over the feasible region. Whereas  $\alpha_i^j$  and  $\beta_i^j$  are positive and normalized constants representing sensitivity of  $AP_i$  to other access providers parameters, i.e. :

$$\sum_{j=1}^N \alpha_i^j = 1 \text{ and } \sum_{j=1}^N \beta_i^j = 1, \quad i = 1, \dots, N.$$

*Assumption 1:* For any price profile, the price mutual sensitivities satisfy :

$$\alpha_i^i \geq \sum_{j, j \neq i} \alpha_i^j, \quad \forall i, j = 1, \dots, N. \quad (2)$$

Assumption 1 is realistic and does not impact applicability domain of this work. Indeed, it simply considers that the interplay of price/tariff charged by a tagged  $AP_i$  has more weight compared to the interplay of the tariffs charged by the competitors APs. This condition could then take into account the presence of customer loyalties and/or partial knowledge of competitors' prices. Moreover, this assumption is a reasonable condition that will ensure the uniqueness of the Nash equilibrium.

*Assumption 2:* For any fixed price vector of the competitors, the total demand  $D(\mathbf{p}, \tilde{\mathbf{q}}) = \sum_{i=1}^N D_i(\mathbf{p}, \tilde{\mathbf{q}})$  is a non-increasing function w.r.t. individual price  $p_i$  of  $AP_i$ .

Considering the customers willingness-to-pay, it becomes plausible to consider that the total demand is decreasing with the individual price set by tagged  $AP_i$ . When  $AP_i$  decides to increase its tariff, the attached customers would either migrate and subscribe with its competitors or alternatively decide to unsubscribe. In other words, this assumption says that the interplay of self price is still higher than the perceived aggregate influence coming from the competitors. An important feature is that the variation of the total demand, with respect to individual price, is very small since small amount of customers would decide to completely unsubscribe.

*Remark 1:* From Assumption 2, we deduce that the first derivative of the total demand w.r.t  $p_i$  is negative, then, the price mutual sensitivities satisfy :

$$\alpha_i^i \geq \sum_{k, k \neq i} \alpha_i^k, \quad \forall i, k = 1, \dots, N. \quad (3)$$

### C. Utility Model

We turn now to derive the utility function of each APs and the single CP. Let  $q_i$  (end-to-end QoS) represents the real QoS perceived by users of  $AP_i$ , (in this case  $\tilde{q}_i$  means the QoS promised/advertised by the  $AP_i$  to end users). The net revenue of  $AP_i$  is exactly the difference between its total income minus its total outcome. This later corresponds to the sum of cost to

cover the fee of bandwidth  $\Phi_i$  and a certain penalty if he did not meet the quality  $\tilde{q}_i$  promised. So, the net revenue is given by the following :

$$U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}}) = p_i D_i(\mathbf{p}, \tilde{\mathbf{q}}) - \vartheta_i \phi_i + \theta_i (q_i - \tilde{q}_i), \quad (4)$$

where  $\vartheta_i$  and  $\theta_i$  are positive constants.

#### D. Game Analysis

In the rest of this paper we are going to analyze the game that arises both under neutrality and non-neutrality setting. Players are the APs who have to set their strategies regarding the price  $p_i$  and promised quality of service ( $\tilde{q}_i$ ). The concept of neutrality is taken into account by assuming that the CP distributes its bandwidth according to the processor sharing principle. In another hand, non-neutrality scenario is taken into account by assuming that there is specific AP (namely  $AP_1$ ) that has signed a contract with CP, in order to reserve him an amount of bandwidth to guarantee the promised QoS  $\tilde{q}_1$ . In this context, competitors of  $AP_1$ , share fairly the remaining bandwidth, as if we have neutrality by considering the topology when we do not have the  $AP_1$ .

#### Definition of Nash Equilibrium :

We consider an N-player strategic-form game  $\Gamma = \{P_1, \dots, P_N, U_1^{AP}, \dots, U_N^{AP}\}$ , where  $P_i$  is player  $i$ 's finite set of price strategies and  $U_i^{AP}$  is player  $i$ 's utility function.

*Definition 1: Nash equilibrium* specifies a strategy  $p_i^* \in P_i$  for each player  $i$  (where  $i = 1, \dots, N$ ) such that :

$$U_i^{AP}(\mathbf{p}^*, \tilde{\mathbf{q}}) = \max_{p_i \in P_i} U_i(p_1^*, \dots, p_{i-1}^*, p_i, p_{i+1}^*, \dots, p_N^*, \tilde{\mathbf{q}}). \quad (5)$$

Where the vector of QoS parameters,  $\tilde{\mathbf{q}}$ , of all the providers is fixed at some predetermined point.

In words, a Nash equilibrium specifies a strategy for each player, in such a way that each player's strategy yields the player at least as high a payoff as any other strategy of the player, given the strategies of the other players and vice versa.

Hereafter, we analyze the competitive pricing of N APs who aim to maximize their utilities. To do so, we show the existence and uniqueness of the equilibrium of games between N APs, after we calculate the equilibrium point. In order to analyze game potential equilibrium we have to find properties on the utility function which require detailed expression in both cases.

### III. NEUTRAL NETWORKS

Let us remind that for APs Sharing capacity under the Process Sharing (PS) principles,  $\Phi_i$  is given by combining both expression of delay in the links between  $AP_i$  and EUs

$$Delay_i^u = \frac{1}{\Phi_i - D_i},$$

and expression of the QoS promised by the  $AP_i$  to end users

$$\tilde{q}_i = \frac{1}{Delay_i^u + c_i},$$

thus :

$$\Phi_i = D_i(\mathbf{p}, \tilde{\mathbf{q}}) + \frac{\tilde{q}_i}{1 - \tilde{q}_i c_i}, \quad (6)$$

where  $c_i$  is the expected delay in the link between  $AP_i$  and CP.

The end to end delay (denoted  $Delay_c^u$ ) experienced by end users of  $AP_i$ , i.e, accumulative delay on both links EU- $AP_i$  and  $AP_i$ -CP, depends on : the total demand for services of CP,  $D(\mathbf{p}, \tilde{\mathbf{q}})$ , the demand for services transported by  $AP_i$ ,  $D_i(\mathbf{p}, \tilde{\mathbf{q}})$ , the bandwidth allocated by CP to the  $AP_i$ ,  $\Phi_c$ , and the bandwidth  $\Phi_i$  offered by  $AP_i$  to its end users:

$$Delay_c^u = \frac{1}{\Phi_i - D_i(\mathbf{p}, \tilde{\mathbf{q}})} + \frac{1}{\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})}.$$

The real QoS perceived by end users of  $AP_i$  is the inverse of the experienced end to end delay,

$$q_i = \frac{1}{Delay_c^u} = \frac{\tilde{q}_i (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}}))}{\tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}}))}. \quad (7)$$

By replacing  $\Phi_i$  and  $q_i$  by their expressions in equation (4), the utility function of  $AP_i$  can be written as :

$$U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}}) = (p_i - \vartheta_i) D_i(\mathbf{p}, \tilde{\mathbf{q}}) - \frac{\vartheta_i \tilde{q}_i}{1 - \tilde{q}_i c_i} + \theta_i \tilde{q}_i \left( \frac{\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})}{\tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}}))} - 1 \right). \quad (8)$$

The question is then, under general assumptions, when can we ensure the existence and uniqueness of the resulting equilibrium across APs.

We consider the single parameter price game (5) which arises when the QoS level of all APs is fixed. So, under assumption 1 and according to remark 1, one has the following general result on the uniqueness of price based Nash equilibrium among the N APs.

*Lemma 1 (Existence):* Consider the price game which arises when the vector of QoS levels is fixed for all APs, there exists in less one price based Nash equilibrium  $\mathbf{p}^*$  of the APs's game.

*Proof:*

Equations (9) and (10) represent respectively the derivative and second derivative of utility function (8) with respect to the price :

$$\frac{\partial U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i} = D_i(\mathbf{p}, \tilde{\mathbf{q}}) - \alpha_i^i (p_i - \vartheta_i) + \frac{\theta_i \tilde{q}_i^2 \left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right)}{(\tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})))^2}, \quad (9)$$

Thus

$$\frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i^2} = -2\alpha_i^i - 2\theta_i \tilde{q}_i^2 \frac{\left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right)^2 (1 - \tilde{q}_i c_i)}{(\tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})))^3} \leq 0 \quad (10)$$

The second derivative of the utility function is negative, then the utility function is thus concave and the existence follows, [10].

*Lemma 2 (Uniqueness):* Under Assumption 1 and according to Remark 1, the equilibrium point  $\mathbf{p}^*$ , of the APs's game, which arises when the vector of QoS levels is fixed for all APs, is unique.

*Proof:* Uniqueness of the equilibrium point is guaranteed if the utility function satisfies Rosen's conditions [10]. Moulin, [8], derived the dominance solvability condition, which is another alternative to satisfy Rosen's conditions : The Nash equilibrium point is unique if

$$-\frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i^2} - \sum_{j, j \neq i} \left| \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} \right| \geq 0, \quad (11)$$

The mixed partial is written as :

$$\begin{aligned} \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} &= \alpha_i^j - 2\theta_i \tilde{q}_i^2 (1 - \tilde{q}_i c_i) \\ &\quad \frac{\left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right) \left( \alpha_j^j - \sum_{k, k \neq j} \alpha_k^j \right)}{\left( \tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})) \right)^3}, j \neq i. \end{aligned} \quad (12)$$

*Worst case (Maximum demand) :* Nowadays, the rate at which customers unsubscribe or new customers subscribe to mobile services tends to be very weak. The total demand is almost saturated/constant or may vary slightly<sup>1</sup>. Naturally, a maximum demand  $D_{max}$  is obtained when the tariff profile set by all APs is minimum. Namely

$$\begin{aligned} \lim_{\mathbf{p} \rightarrow \mathbf{p}_{min}} \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} &= \alpha_i^j - 2\theta_i \tilde{q}_i^2 (1 - \tilde{q}_i c_i) \\ &\quad \frac{\left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right) \left( \alpha_j^j - \sum_{k, k \neq j} \alpha_k^j \right)}{\left( \tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D_{max}) \right)^3} \\ &\leq \alpha_i^j - 2\theta_i \tilde{q}_i^2 (1 - \tilde{q}_i c_i) \\ &\quad \frac{\left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right) \left( \alpha_j^j - \sum_{k, k \neq j} \alpha_k^j \right)}{\left( \tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})) \right)^3}, \forall j \neq i. \end{aligned} \quad (13)$$

The second term of the right-hand side of equation (13) is very small and can be neglected as well. Yet, this term is exactly equal to the product of the derivative of total demand w.r.t to  $p_i$  and  $p_j$ , and a constant weak term. Hence

$$\lim_{\mathbf{p} \rightarrow \mathbf{p}_{min}} \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} \geq 0.$$

It follows that :

$$\frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} \geq 0,$$

so :

$$\left| \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} \right| = \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j}.$$

<sup>1</sup>For example, the penetration rate of mobile services has reached 100% in France (source : Autorité de Régulation des Communications Électroniques et des Postes, <http://www.arcep.fr/index.php?id=10865>).

From this later, and after substitution of (10) and (12) in (11), we have :

$$\begin{aligned} -\frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i^2} - \sum_{j, j \neq i} \left| \frac{\partial^2 U_i^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_i \partial p_j} \right| &= 2\alpha_i^i - \sum_{j, j \neq i} \alpha_i^j + 2\theta_i \tilde{q}_i^2 \\ &\quad \frac{\left[ \left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right)^2 + \left( \alpha_i^i - \sum_{k, k \neq i} \alpha_k^i \right) \sum_{j, j \neq i} \left( \alpha_j^j - \sum_{k, k \neq j} \alpha_k^j \right) \right]}{(1 - \tilde{q}_i c_i)} \\ &\quad \frac{1}{\left( \tilde{q}_i + (1 - \tilde{q}_i c_i) (\Phi_c - D(\mathbf{p}, \tilde{\mathbf{q}})) \right)^3}, \end{aligned}$$

Regarding Assumption 1 and according to Remark 1 yields positivity of the last expression. This means that the Moulin's dominance solvability condition is fulfilled, then this game satisfies Rosen's conditions and the uniqueness of the equilibrium follows [8].

Under lemmas (1, 2), we deduce the following theorem :

*Theorem 1:* Consider the price game (5) which arises when the vector of quality of service levels is fixed for all APs, under the Assumption 1 and according to the Remark 1, there exists one price based Nash equilibrium  $\mathbf{p}^*$  of the APs's game.

#### IV. NON-NEUTRAL NETWORKS

As we mentioned above (subsection II-D), in non-neutrality networks we assume that there is specific AP (namely  $AP_1$ ) that has a privileged contract with CP, in order to reserve to him an amount of bandwidth to guarantee the promised QoS  $\tilde{q}_1$ . Under non neutral scheme,  $AP_1$  is insured to get the reserved capacity, then the expression of utility of  $AP_1$  becomes simpler since  $q_i = \tilde{q}_i$  i.e no penalty appears in the utility function, namely

$$U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}}) = (p_1 - \vartheta_1) D_1(\mathbf{p}, \tilde{\mathbf{q}}) - \frac{\vartheta_1 \tilde{q}_1 (\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}}))}{\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}}) - \tilde{q}_1}. \quad (14)$$

In this context, competitors of  $AP_1$ , share fairly the remaining bandwidth. In absence of  $AP_1$ , the network behaves as a neutral network where other APs compete over the common bandwidth. The utility function of other APs ( $AP_1$ 's competitors) is then given by equation (8).

*Theorem 2:* [Existence and Uniqueness] Consider the price game (5) which arises without net neutrality, under Assumption 1 and according to Remark 1, there exists one price based Nash equilibrium, Therefore, the utility function of the APs, satisfies the properties of concavity and dominance solvency.

*Proof:* It is clear that the expression of utility function of APs who have no privileged contract with the CP, is the same as that of APs in net neutrality case, i.e., equation (8). Henceforth, the utility function of these APs, satisfies the properties of concavity and dominance solvability condition, equation (11). Therefore, to prove existence and uniqueness of an equilibrium under non-neutrality, it suffices to show that the utility function of  $AP_1$  (equation (14)), checks the properties of concavity (existence) and dominance solvability condition, equation (11), (Uniqueness).

*Existence :*

Equation (15) represents the second derivative of utility function of  $AP_1$  :

$$\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1^2} = -2\alpha_1^1 - 2 \frac{\vartheta_1 (\alpha_1^1)^2 \tilde{q}_1^2}{(\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}}) - q_1)^3}, \quad (15)$$

Since we have  $\tilde{q}_1 = \frac{1}{\text{Delay}_u^1 + c_1}$ , then  $1 - \tilde{q}_1 c_1 \geq 0$ , and as,  $c_1 = \frac{1}{\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}})}$ , we find that,  $\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}}) - \tilde{q}_1 \geq 0$ . Next, it is easy to note (from Equation (15)) that  $\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1^2} \leq 0$ . So, the utility function is concave, thereafter, existence of Nash equilibrium follows from [10].

*Uniqueness :*

Similarly, To show the uniqueness of equilibrium point, we should verify if  $U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})$  in equation (14) satisfies the dominance solvability condition, i.e., equation (11).

However, we have :

$$\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1 \partial p_l} = \alpha_1^l + \frac{2\vartheta_1 \tilde{q}_1^2 \alpha_1^1 \alpha_1^l}{(\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}}) - \tilde{q}_1)^3} \geq 0, \quad (16)$$

so,

$$-\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1^2} - \sum_{l, l \neq 1} \left| \frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1 \partial p_l} \right| = -\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1^2} - \sum_{l, l \neq 1} \frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1 \partial p_l} \quad (17)$$

From this later, and after replacing (15) and (16) in (17), we have :

$$-\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1^2} - \sum_{l, l \neq 1} \left| \frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1 \partial p_l} \right| = 2\alpha_1^1 - \sum_{l, l \neq 1} \alpha_1^l + 2\vartheta_1 \tilde{q}_1^2 \alpha_1^1 \frac{\left( \alpha_1^1 - \sum_{l, l \neq 1} \alpha_1^l \right)}{(\Phi_c - D_1(\mathbf{p}, \tilde{\mathbf{q}}) - \tilde{q}_1)^3},$$

Following assumption (1), we have  $\alpha_1^1 > \sum_{l, l \neq 1} \alpha_1^l$ , so :

$$-\frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1^2} - \sum_{l, l \neq 1} \left| \frac{\partial^2 U_1^{AP}(\mathbf{p}, \tilde{\mathbf{q}})}{\partial p_1 \partial p_l} \right| \geq 0, \quad (18)$$

This last result shows that for the utility function of  $AP_1$  the Moulin's dominance solvability condition is satisfied and the result follows. ■

## V. NUMERICAL RESULTS

We turn now to discuss how to take benefit from our analytical findings. We suggest to study numerically the market share game while considering the previous expressions of demand and utility functions of APs. For illustrative purpose, we consider two homogeneous APs seeking to maximize their respective payoffs. Until we contraindicate, the parameter values are summarized in Table I.

**System parametrization :** We start by pointing out the impact of different parameters on the system behaviour. The design paradigm of network neutrality argues, in essence, that no bit of information should be prioritized over another. Due to lack of space, this discussion on network parametrization is omitted

$\alpha_1^1 = \alpha_2^2$	$\alpha_2^1 = \alpha_1^2$	$\beta_1^1 = \beta_2^2$	$\beta_2^1 = \beta_1^2$	$a_1 = a_2$	$\vartheta_1 = \vartheta_2$
0.55	0.45	0.7	0.3	200	40
$c_1 = c_2$	$\theta_1 = \theta_2$	$p_1^{min} = p_2^{min}$	$p_1^{max} = p_2^{max}$	$\tilde{q}_1 = \tilde{q}_2$	$\phi_c$
0,001	5000	0	1000	200	1000

TABLE I  
PARAMETERS SETTING USED FOR NUMERICAL EXAMPLES.

but is available in our technical report [3]. In particular we have addressed the influence of bandwidth cost  $\vartheta$ , QoS penalty factor  $\theta$  and the expected delay  $c$  on the AP-CP link when neutrality is upheld.

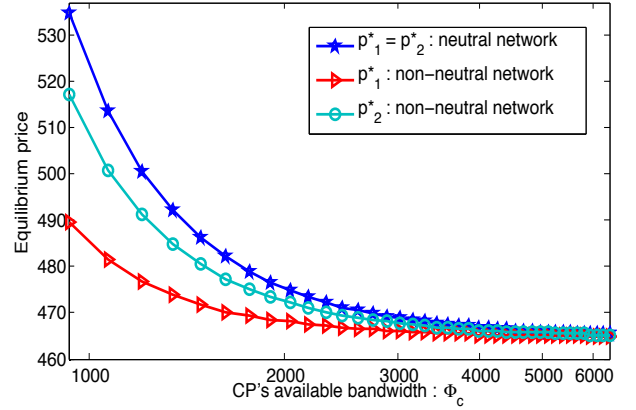


Fig. 2. Impact of available bandwidth  $\Phi_c$  at CP on the equilibrium price.

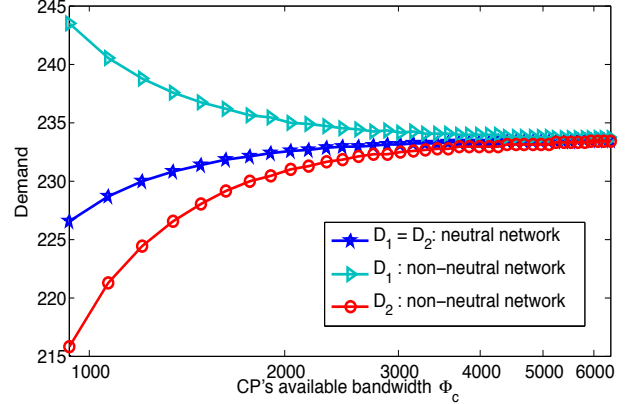


Fig. 3. Impact of available bandwidth  $\Phi_c$  at CP on the demand partitioning.

**Neutral setup VS. non-neutral setup :** In order to prioritize its content over that of AP2, we consider that AP1 has signed a privileged contract with the content provider. This contract could be seen as roadblocks or shortcuts set by the CP to discerns the contents of AP1. Figures 2, 3 and 4 show respectively the variation of prices, demands and real QoS at Nash equilibrium while varying the amount of bandwidth  $\Phi_c$  offered by CP. We note that the equilibrium prices for all APs in neutral and in non-neutral network are decreasing with  $\Phi_c$ . A special feature is that equilibrium prices under non-neutrality are less than prices under neutral network. This

encourages customers to subscribe services, which explains why the total demand is higher under non-neutrality (see Fig. 3). This can be explained as follows : AP1 has more power and becomes the master of the market, this gives it the ability to offer services with cheaper price and better QoS. This has the consequence of attracting more subscriber, which decrease AP2's demand and its revenue although it sell its services at higher price. When  $\Phi_c$  is relatively small and is not sufficient to handle the total demand, the AP with no privileged contract earns less than its competitor, this comes from lack of bandwidth that was consumed by AP1. Whereas, we note that when content provider handles a huge bandwidth, i.e., enough to handle the total demand, the neutrality metrics meet the non-neutrality metrics. Thus AP1 would have no reason to invest in signing a privileged contract since both settings (neutrality and non-neutrality) will yield same result.

**Non-neutrality sustains monopolistic and unfair competition :** Fig. 5 depicts the net revenue of both access providers for neutral and non-neutral settings. Because of its attractive prices and no penalty concerning QoS (since announced QoS is met under non-neutrality), AP1 attracts a higher market share. It is clear that this situation where AP1 has these advantages over AP2 is quite unfair. Indeed, this situation induces some kind of monopole position among contended APs. Yet, this monopoly situation implicitly prohibits competitors from entering the market by using unfair competitive practices derived from its market influence as a privileged AP.

## VI. CONCLUSION

We have presented in this paper a non-cooperative game for market share game. Each access provider advertises some benchmark QoS that it pretends to guarantee to its subscribers. Then, each access provider, taking into account the generated demand, determines the best price maximizing its own net revenue. Furthermore, both neutral (no discrimination over data traveling the network) and non-neutral where some specific AP has signed a special contract with the content provider in order to prioritize its content. Based on Rosen's condition, we have proved existence and uniqueness of a Nash equilibrium for both schemes. We have shown numerically that non-neutral is beneficial for end users since it implies cheaper tariffs and improved QoS. However, it may sustain monopolistic and unfair competition among access providers.

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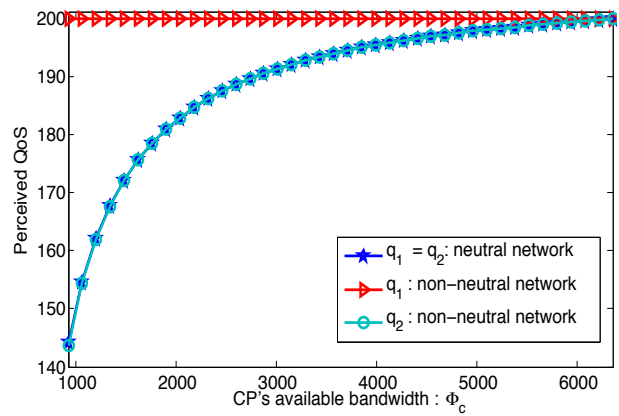


Fig. 4. Real/perceived e2e QoS versus available bandwidth  $\Phi_c$  at CP.

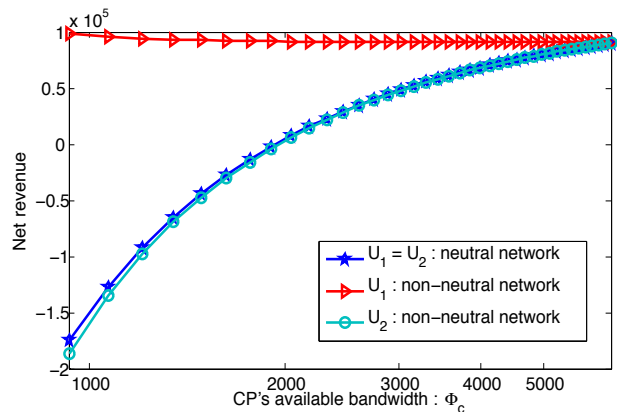


Fig. 5. The net revenues versus available bandwidth  $\Phi_c$  at CP.

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