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**Abstract:** Many distributed applications require transactions. However, transactional protocols that require strong synchronization are costly in large scale environments. Two properties help with scalability of a transactional system: genuine partial replication (GPR), which leverages the intrinsic parallelism of a workload, and snapshot isolation (SI), which decreases the need for synchronization. We show that, under standard assumptions (data store accesses are not known in advance, and transactions may access arbitrary objects in the data store), it is impossible to have both SI and GPR. To circumvent this impossibility, we propose a weaker consistency criterion, called Non-Monotonic Snapshot Isolation (NMSI). NMSI retains the most important properties of SI, i.e., read-only transactions always commit, and two write-conflicting updates do not both commit. We present a GPR protocol that ensures NMSI, and has lower message cost (i.e., it contacts fewer replicas and/or commits faster) than previous approaches.

**Key-words:** distributed systems; transactional systems; replication; concurrency control; transactions; database

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# Non-Monotonic Snapshot Isolation

**Résumé :** Cet article étudie deux propriétés favorisant le passage à l'échelle des systèmes répartis transactionnels: la réplication partielle authentique (GPR), et le critère de cohérence Snapshot Isolation (SI). GPR spécifie que pour valider une transaction  $T$ , seules les répliques des données accédées par  $T$  effectuent des pas de calcul. SI définit que toute transaction doit lire une vue cohérente du système, et que deux transactions concurrentes ne peuvent écrire la même donnée. Nous montrons que SI et GPR sont deux propriétés incompatibles. Afin de contourner cette limitation, nous proposons un nouveau critère de cohérence: Non-Monotonic Snapshot Isolation (NMSI). NMSI est proche de SI et néanmoins compatible avec GPR. Afin de justifier ce dernier point, nous présentons un protocole authentique implémentant de manière efficace NMSI. Au regard des travaux précédents sur le contrôle de concurrence dans les systèmes répartis transactionnels, notre protocole est le plus performant en latence et/ou en nombre de messages échangés.

**Mots-clés :** systèmes répartis, systèmes transactionnels, contrôle de concurrence, transaction, base de données

## 1 Introduction

Large scale transactional systems have conflicting requirements. On the one hand, strong transactional guarantees are fundamental to many applications. On the other, remote communication and synchronization is costly and should be avoided.<sup>1</sup>

To maintain strong consistency guarantees while alleviating the high cost of synchronization, Snapshot Isolation (SI) is a popular approach in both distributed database replications [1–3], and software transactional memories [4, 5]. Under SI, a transaction accesses its own *consistent snapshot* of the data, which is unaffected by concurrent updates. A read-only transaction always commits unilaterally and without synchronization. An update transaction synchronizes on commit to ensure that no concurrent conflicting transaction has committed before it.

Our first contribution is to prove that SI is equivalent to the conjunction of the following properties: *(i)* no cascading aborts, *(ii)* strictly consistent snapshots, i.e., a transaction observes a snapshot that coincides with some point in (linear) time, *(iii)* two concurrent write-conflicting update transactions never both commit, and *(iv)* snapshots observed by transactions are monotonically ordered. Previous definitions [6, 7] of SI extend histories with abstract snapshot points. Our decomposition shows that SI can be expressed on plain histories like serializability [8].

Modern data stores replicate data for both performance and availability. Full replication does not scale, as every process must perform all updates. *Partial replication* (PR) aims to address this problem, by replicating only a subset of the data at each process. Thus, if transactions would communicate only over the minimal number of replicas, synchronisation and computation overhead would be reduced. However, in the general case, the overlap of transactions cannot be predicted; therefore, many PR protocols perform system-wide global consensus [1, 2] or communication [9]. This negates the potential advantages of PR; hence, we require *genuine* partial replication [10] (GPR), in which a transaction communicates only with those processes that replicate some object accessed in the transaction. With GPR, independent transactions do not interfere with each other, and the intrinsic parallelism of a workload can be exploited. Our second contribution is to show that SI and GPR are incompatible. More precisely, we prove that an asynchronous message-passing system supporting GPR cannot compute monotonically ordered snapshots, nor strictly consistent ones, even if it is failure-free.

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<sup>1</sup>We address general-purpose transactions, i.e., we assume that a transaction may access any object in the system, and that its read- and write-sets are not known in advance.

The good news is our third contribution: a consistency criterion, called *Non-Monotonic Snapshot Isolation* (NMSI) that overcomes this impossibility. NMSI is very similar to SI, as every transaction observes a consistent snapshot, and two concurrent write-conflicting updates never both commit. However, under NMSI, snapshots are neither strictly consistent nor monotonically ordered.

Our final contribution is a GPR protocol ensuring NMSI, called Jessy. Jessy uses a novel variant of version vectors, called *dependence vectors*, to compute consistent partial snapshots asynchronously. To commit an update transaction, Jessy uses a single atomic multicast. Compared to previous protocols, Jessy commits transactions faster and/or contacts fewer replicas.

This paper proceeds as follows. We introduce our system model in Section 2. Section 3 presents our decomposition of SI. Section 4 shows that GPR and SI are mutually incompatible. We introduce NMSI in Section 6. Section 7 describes Jessy, our NMSI protocol. We compare with related work in Section 8, and conclude in Section 9.

## 2 Model

This section defines the elements in our model and formalizes SI and GPR .

### 2.1 Objects & transactions

Let *Objects* be a set of objects, and  $\mathcal{T}$  be a set of transaction identifiers. Given an object  $x$  and an identifier  $i$ ,  $x_i$  denotes *version*  $i$  of  $x$ . A *transaction*  $T_{i \in \mathcal{T}}$  is a finite permutation of read and write operations followed by a *terminating* operation, commit ( $c_i$ ) or abort ( $a_i$ ). We write  $w_i(x_i)$  to denote transaction  $T_i$  writing version  $i$  of object  $x$ , and  $r_i(x_j)$  to mean that  $T_i$  reads version  $j$  of object  $x$ . In a transaction, every write is preceded by a read on the same object, and every object is read or written at most once.<sup>2</sup> We note  $ws(T_i)$  the write set of  $T$ , i.e., the set of objects written by transaction  $T_i$ . Similarly,  $rs(T_i)$  denotes the read set of transaction  $T_i$ . The *snapshot* of  $T_i$  is the set of versions read by  $T_i$ . Two transactions *conflict* when they access the same object and one of them modifies it; they *write-conflict* when they both write to the same object.

### 2.2 Histories

A *complete history*  $h$  is a partially ordered set of operations such that (1) for every operation  $o_i$  appearing in  $h$ , transaction  $T_i$  terminates in  $h$ , (2) for every two operations  $o_i$  and  $o'_i$  appearing in  $h$ , if  $o_i$  precedes  $o'_i$  in  $T_i$ , then  $o_i <_h o'_i$ , (3) for every read  $r_i(x_j)$  in  $h$ , there exists a write operation  $w_j(x_j)$  such that  $w_j(x_j) <_h r_i(x_j)$ , and (4) any two write operations over the same objects are ordered by  $<_h$ . A *history* is a prefix of a complete history. For some history  $h$ , order  $<_h$  is the *real-time order* induced by  $h$ . Transaction  $T_i$  is *pending* in history  $h$  if  $T_i$  does not commit, nor abort in  $h$ . We note  $\ll_h$  the version order induced by  $h$  between different versions of an object, i.e., for every object  $x$ , and every pair of transactions  $(T_i, T_j)$ ,  $x_i \ll_h x_j \Leftrightarrow w_i(x_i) <_h w_j(x_j)$ . Following Bernstein et al. [11], we depict a history as a graph. We illustrate this with history  $h_1$  below in which transaction  $T_a$  reads the initial versions of objects  $x$  and  $y$ , while transaction  $T_1$  (respectively  $T_2$ ) updates  $x$  (resp.  $y$ ).<sup>3</sup>

When order  $<_h$  is total, we shall write a history as a permutation of operations, e.g.,  $h_2 = r_1(x_0).r_2(y_0).w_2(y_2).c_1.c_2$ .

<sup>2</sup>This restriction can be easily overcome by reading shared object in private variables and computing with them.

<sup>3</sup>Throughout the paper, read-only transactions are specified with an alphabet subscript, and update transactions are shown with numeric subscript.



$$\begin{array}{l}
 h_1 = r_a(x_0) \longrightarrow r_1(x_0).w_1(x_1).c_1 \\
 \searrow \\
 r_a(y_0).c_a \longrightarrow r_2(y_0).w_2(y_2).c_2
 \end{array}$$

## 2.3 Snapshot Isolation

Snapshot isolation (SI) was introduced by Berenson et al. [8], then later generalized under the name GSI by Elnikety et al. [7]. In this paper, we make no distinction between SI and GSI.

Let us consider a function  $\mathcal{S}$  which takes as input a history  $h$ , and returns an extended history  $h_s$  by adding a *snapshot point* to  $h$  for each transaction in  $h$ . Given a transaction  $T_i$ , the snapshot point of  $T_i$  in  $h_s$ , denoted  $s_i$ , precedes every operation of transaction  $T_i$  in  $h_s$ . A history  $h$  is in SI if and only if there exists a function  $\mathcal{S}$  such that  $h_s = \mathcal{S}(h)$  and  $h_s$  satisfies the following rules:

### D1 (Read Rule)

$$\begin{array}{l}
 \forall r_i(x_{j \neq i}), w_{k \neq j}(x_k), c_k \in h_s : \\
 c_j \in h_s \quad (D1.1) \\
 \wedge c_j <_{h_s} s_i \quad (D1.2) \\
 \wedge c_k <_{h_s} c_j \vee s_i <_{h_s} c_k \quad (D1.3)
 \end{array}$$

### D2 (Write Rule)

$$\begin{array}{l}
 \forall c_i, c_j \in h_s : \\
 ws(T_i) \cap ws(T_j) \neq \emptyset \\
 \Rightarrow (c_i <_{h_s} s_j \vee c_j <_{h_s} s_i)
 \end{array}$$

## 2.4 System

We consider a message-passing system of  $n$  processes  $\Pi = \{p_1, \dots, p_n\}$ . Links are quasi-reliable. We shall define our synchrony assumptions later. Following Fischer et al. [12], an execution is a sequence of steps made by one or more processes. During an execution, processes may fail by crashing. A process that does not crash is said *correct*; otherwise it is *faulty*. We note  $\mathfrak{F}$  the refinement mapping [13] from executions to histories, i.e., if  $\rho$  is an execution of the system, then  $\mathfrak{F}(\rho)$  is the history produced by  $\rho$ . A history  $h$  is *acceptable* if there exists an execution  $\rho$  such that  $h = \mathfrak{F}(\rho)$ . We consider that given two sequences of steps  $U$  and  $V$ , if  $U$  precedes  $V$  in some execution  $\rho$ , then the operations implemented by  $U$  precedes (in the sense of  $<_h$ ) the operations implemented by  $V$  in the history  $\mathfrak{F}(\rho)$ .<sup>4</sup>

## 2.5 Partial Replication

A data store  $\mathcal{D}$  is a finite set of tuples  $(x, v, i)$  where  $x$  is an object (data item),  $v$  a value, and  $i \in \mathcal{T}$  a version. Each process in  $\Pi$  holds a data store

<sup>4</sup>Notice that since steps to implement operations may interleave,  $<_h$  is not necessarily a total order.

such that initially every object  $x$  has version  $x_0$ . For an object  $x$ ,  $replicas(x)$  denotes the set of processes, or *replicas*, that hold a copy of  $x$ . By extension for some set of objects  $X$ ,  $replicas(X)$  denotes the replicas of  $X$ ; given a transaction  $T_i$ ,  $replicas(T_i)$  equals  $replicas(rs(T_i) \cup ws(T_i))$ .

We make no assumption about how objects are replicated. The coordinator of  $T_i$ , denoted  $coord(T_i)$ , is in charge of executing  $T_i$  on behalf of some client (not modeled). The coordinator does not know in advance the read set or the write set of  $T_i$ . To model this, we consider that every prefix of a transaction (followed by a terminating operation) is a transaction with the same id.

Genuine Partial Replication (GPR) aims to ensure that, when the workload is parallel, throughput scales linearly with the number of nodes [10]:

- **GPR.** For any transaction  $T_i$ , only processes that replicate objects accessed by  $T_i$  make steps to execute  $T_i$ .

## 2.6 Progress

The read rule of SI does not define what is the snapshot to be read. According to Adya [6], “transaction  $T_i$ ’s snapshot point needs not be chosen after the most recent commit when  $T_i$  started, but can be selected to be some (convenient) earlier point.” To avoid that read-only transactions always observe outdated data, we need a progress rule.

Below, we introduce the notion of non-triviality. It is defined with relation to a consistent criteria  $\mathcal{C}$ . When a system ensures non-triviality with respect to SI, we shall say that it satisfies non-trivial SI.

- **Non-triviality.** Consider an acceptable history  $h$  and a transaction  $T_i$  pending in  $h$  such that the next operation invoked by  $T_i$  is a read on some object  $x$ . Note  $x_j$  the latest committed version of  $x$  prior to the first operation of  $T_i$  in  $h$ . Let  $\rho$  be an execution satisfying  $\mathfrak{F}(\rho) = h$ . If  $h.r_i(x_j)$  belongs to  $\mathcal{C}$  then *there exists* an execution  $\rho'$  extending  $\rho$  such that in history  $\mathfrak{F}(\rho')$ , transaction  $T_i$  reads at least (in the sense of  $\ll_h$ ) version  $x_j$  of  $x$ .

In addition, we consider that the system supports obstruction-free updates (OFU) and wait-free queries (WFQ).

- **Obstruction-free Updates.** For every update transaction  $T_i$ , if  $coord(T_i)$  is correct then  $T_i$  eventually terminates. Moreover, if  $T_i$  does not conflict with some concurrent transaction, then  $T_i$  eventually commits.

- **Wait-free Queries.** A read-only transaction  $T_i$  never waits for another transaction and eventually commits.

OFU is the weakest progress property a system of practical interest must support. WFQ is a property that ensures the system scales when the number of queries increases. Because a system that supports SI at core ensures WFQ, historically, it has been sometimes wrongly considered as a property of SI itself.

### 3 Decomposing SI

This section defines four properties, whose conjunction is necessary and sufficient to attain SI. We later use these properties in Section 4 to derive our impossibility result.

#### 3.1 Cascading Aborts

Intuitively, a read-only transaction must abort if it observes the effects of an uncommitted transaction that later aborts. By guaranteeing that every version read by a transaction is committed, rules D1.1 and D1.2 of SI prevent such a situation to occur. In other words, these rules *avoid cascading aborts*. We formalize this property below:

**Definition 1** (Avoiding Cascading aborts). *History  $h$  avoids cascading aborts, if for every read  $r_i(x_j)$  in  $h$ ,  $c_j$  precedes  $r_i(x_j)$  in  $h$ . ACA denotes the set of histories that avoid cascading aborts.*

#### 3.2 Consistent and Strictly Consistent Snapshots

Consistent and strictly consistent snapshots are defined by refining causality into a dependency relation as follows:

**Definition 2** (Dependency). *Consider a history  $h$  and two transactions  $T_i$  and  $T_j$ . We note  $T_i \triangleright T_j$  when  $r_i(x_j)$  is in  $h$ . Transaction  $T_i$  depends on transaction  $T_j$  when  $T_i \triangleright^* T_j$  holds.<sup>5</sup> Transaction  $T_i$  and  $T_j$  are independent if neither  $T_i \triangleright^* T_j$ , nor  $T_j \triangleright^* T_i$  hold.*

This means that a transaction  $T_i$  depends on a transaction  $T_j$  if  $T_i$  reads an object modified by  $T_j$ , or such a relation holds by transitive closure. To illustrate this definition, consider history  $h_3 = r_1(x_0).w_1(x_1).c_1.r_a(x_1).c_a.r_b(y_0).c_b$ . In  $h_3$ , transaction  $T_a$  depends on  $T_1$ . Notice that, even if  $T_1$  causally precedes  $T_b$ ,  $T_b$  does not depend on  $T_1$  in  $h_3$ .

We now define consistent snapshots with the above dependency relation. A transaction sees a consistent snapshot iff it observes the effects of all transactions it depends on [14]. For example, consider the history  $h_4 = r_1(x_0).w_1(x_1).c_1.r_2(x_1).r_2(y_0).w_2(y_2).c_2.r_a(y_2).r_a(x_0).c_a$ . In this history, transaction  $T_a$  does not see a consistent snapshot:  $T_a$  depends on  $T_2$ , and  $T_2$  also depends on  $T_1$ , but  $T_a$  does not observe the effect of  $T_1$  (i.e.,  $x_1$ ). Formally, consistent snapshots are defined as follows:

<sup>5</sup>We note  $\mathcal{R}^*$  the transitive closure of some binary relation  $\mathcal{R}$ .

**Definition 3** (Consistent snapshot). *A transaction  $T_i$  in a history  $h$  observes a consistent snapshot iff, for every object  $x$ , if (i)  $T_i$  reads version  $x_j$ , (ii)  $T_k$  writes version  $x_k$ , and (iii)  $T_i$  depends on  $T_k$ , then version  $x_k$  is followed by version  $x_j$  in the version order induced by  $h$  ( $x_k \ll_h x_j$ ). We write  $h \in \text{CONS}$  when all transactions in  $h$  observe a consistent snapshot.*

SI requires that a transaction observes the committed state of the data at some *point* in the past. This requirement is stronger than consistent snapshot. For some transaction  $T_i$ , it implies that (i) there exists a snapshot point for  $T_i$  (SCONSa), and (ii) if transaction  $T_i$  observes the effects of transaction  $T_j$ , it must also observe the effects of all transactions that precede  $T_j$  in time (SCONSb). A history is called strictly consistent if both SCONSa and SCONSb hold. For instance, consider the following history:  $h_5 = r_1(x_0).w_1(x_1).c_1.r_a(x_1).r_2(y_0).w_2(y_2).c_2.r_a(y_2).c_a$ . Because  $r_a(x_1)$  precedes  $c_2$  in  $h_5$ ,  $y_2$  cannot be observed when  $T_a$  takes its snapshot. As a consequence, the snapshot of transaction  $T_a$  is not strictly consistent. This issue is disallowed by SCONSa. Now, consider history  $h_6 = r_1(x_0).w_1(x_1).c_1.r_2(y_0).w_2(y_2).c_2.r_a(x_0).r_a(y_2).c_a$ . Since  $c_1$  precedes  $c_2$  in  $h_6$  and transaction  $T_a$  observes the effect of  $T_2$  (i.e.,  $y_2$ ), it should also observe the effect of  $T_1$  (i.e.,  $x_1$ ). SCONSb prevents history  $h_6$  to occur.

**Definition 4** (Strictly consistent snapshot). *Snapshots in history  $h$  are strictly consistent, when for any committed transactions  $T_i, T_j, T_{k \neq j}$  and  $T_l$ , the following two properties hold:*

$$- \forall r_i(x_j), r_i(y_l) \in h : r_i(x_j) \not\prec_h c_l \quad (\text{SCONSa})$$

$$- \forall r_i(x_j), r_i(y_l), w_k(x_k) \in h : \\ c_k <_h c_l \Rightarrow c_k <_h c_j \quad (\text{SCONSb})$$

We note  $\text{SCONS}$  the set of strictly consistent histories.

### 3.3 Snapshot Monotonicity

In addition, SI requires what we call monotonic snapshots. For instance, although history  $h_7$  below satisfies SCONS, this history does not belong to SI: since  $T_a$  reads  $\{x_0, y_2\}$ , and  $T_b$  reads  $\{x_1, y_0\}$ , there is no extended history that would guarantee the read rule of SI.

$$h_7 = r_a(x_0) \xrightarrow{\quad} r_1(x_0).w_1(x_1).c_1 \xrightarrow{\quad} r_b(x_1).c_b \\ r_b(y_0) \xrightarrow{\quad} r_2(y_0).w_2(y_2).c_2 \xrightarrow{\quad} r_a(y_2).c_a$$

SI requires monotonic snapshots. However, the underlying reason is intricate enough that some previous works [4, for instance] do not ensure this

property, while claiming to be SI. Below, we introduce an ordering relation between snapshots to formalize snapshot monotonicity.

**Definition 5** (Snapshot precedence). *Consider a history  $h$  and two distinct transactions  $T_i$  and  $T_j$ . The snapshot read by  $T_i$  precedes the snapshot read by  $T_j$  in history  $h$ , written  $T_i \rightarrow T_j$ , when  $r_i(x_k)$  and  $r_j(y_l)$  belong to  $h$  and either (i)  $r_i(x_k) <_h c_l$  holds, or (ii) transaction  $T_l$  writes  $x$  and  $c_k <_h c_l$  holds.*

For more illustration, consider  $h_8 = r_1(x_0).w_1(x_1).c_1.r_2(y_0).w_2(y_2).r_a(x_1).c_2.r_b(y_2).c_a.c_b$  and  $h_9 = r_1(x_0).w_1(x_1).c_1.r_a(x_1).c_a.r_2(x_1).r_2(y_0).w_2(x_2).w_2(y_2).c_2.r_b(y_2).c_b$ . In history  $h_8$ ,  $T_a \rightarrow T_b$  holds because  $r_a(x_1)$  precedes  $c_2$  and  $T_b$  reads  $y_2$ . In  $h_9$ ,  $c_1$  precedes  $c_2$  and both  $T_1$  and  $T_2$  modify object  $x$ . Thus,  $T_a \rightarrow T_b$  also holds. We define snapshot monotonicity using snapshot precedence as follows:

**Definition 6** (Snapshot monotonicity). *Given some history  $h$ , if the relation  $\rightarrow^*$  induced by  $h$  is a partial order, the snapshots in  $h$  are monotonic. We note *MON* the set of histories that satisfy this property.*

According to this definition, since both  $T_a \rightarrow T_b$  and  $T_b \rightarrow T_a$  hold in history  $h_7$ , this history does not belong to *MON*.

Non-monotonic snapshots are observed under update serializability [15], that is when queries observe consistent state, but only updates are serializable.

### 3.4 Write-Conflict Freedom

Rule D2 of SI forbids two concurrent write-conflicting transactions from both committing. Since in our model we assume that every write is preceded by a corresponding read on the same object, every update transaction depends on a previous update transaction (or on the initial transaction  $T_0$ ). Therefore, under SI, concurrent conflicting transactions must be independent:

**Definition 7** (Write-Conflict Freedom). *A history  $h$  is write-conflict free if two independent transactions never write to the same object. We denote by *WCF* the histories that satisfy this property.*

### 3.5 The decomposition

Theorem 1 below establishes that a history  $h$  is in SI iff (1) every transaction in  $h$  sees a committed state, (2) every transaction in  $h$  observes a strictly consistent snapshot, (3) snapshots are monotonic, and (4)  $h$  is write-conflict free.

**Lemma 1.** *Consider a history  $h \in SI$  and two versions  $x_i$  and  $x_j$  of some object  $x$ . If  $x_i \ll_h x_j$  holds then  $T_j \triangleright^* T_i$  is true.*

*Proof.* Assume some history  $h \in SI$  such that  $x_i \ll_h x_j$  holds. Let  $h_s$  be an extended history for  $h$  that satisfies rules D1 and D2. According to the model, transaction  $T_j$  first reads some version  $x_k$ , then writes version  $x_j$ .

First, assume that there is no write to  $x$  between  $w_i(x_i)$  and  $w_j(x_j)$ . Since  $x$  belongs to  $ws(T_i) \cap ws(T_j)$ , rule D2 tells us that either  $c_i <_{h_s} s_j$ , or  $c_j <_{h_s} s_i$  holds. We observe that because  $x_i \ll_h x_j$  holds, it must be true that  $c_i <_{h_s} s_j$ . Since there is no write to  $x$  between  $w_i(x_i)$  and  $w_j(x_j)$ ,  $x_k \ll x_i$  holds, or  $k = i$ . Observe that in the former case rule D1.3 is violated. Thus, transaction  $T_j$  reads version  $x_i$ . To obtain the general case, we apply inductively the previous reasoning.  $\square$

**Lemma 2.** *Let  $h \in SI$  be a history, and  $\mathcal{S}$  be a function such that  $h_s = \mathcal{S}(h)$  satisfies D1 and D2. Consider  $T_i, T_j \in h$ . If  $T_i \rightarrow T_j$  holds then  $s_i <_{h_s} s_j$ .*

*Proof.* Consider two transactions  $T_i$  and  $T_j$  such that the snapshot of  $T_i$  precedes the snapshot of  $T_j$ . By definition of the snapshot precedence relation, there exist  $T_k, T_l \in h$  such that  $r_i(x_k), r_j(y_l) \in h$  and either (i)  $r_i(x_k) <_h c_l$ , or (ii)  $w_l(x_l) \in h$  and  $c_k <_h c_l$ . Let us distinguish each case:

(Case  $r_i(x_k) <_h c_l$ ) By definition of function  $\mathcal{S}$ ,  $s_i$  precedes  $r_i(x_k)$  in  $h_s$ . From  $r_j(y_l) \in h$  and rule D1.2,  $c_l <_{h_s} s_j$  holds. Hence,  $s_i <_{h_s} s_j$  holds.

(Case  $c_k <_h c_l$ ) From (i)  $r_i(x_k), w_l(x_l) \in h$ , (ii)  $c_k <_h c_l$  and (iii) rule D1.3, we obtain  $s_i <_{h_s} c_l$ . From  $r_j(y_l) \in h$  and rule D1.2,  $c_l <_{h_s} s_j$  holds. It follows that  $s_i <_{h_s} s_j$  holds.  $\square$

**Lemma 3.** *Consider a history  $h \in ACA \cap CONS \cap WCF$ , and two versions  $x_i$  and  $x_j$  of some object  $x$ . If  $x_i \ll_h x_j$  holds then  $c_i <_h c_j$ .*

*Proof.* Since both  $T_i$  and  $T_j$  write to  $x$  and  $h$  belongs to WCF either  $T_j \triangleright^* T_i$  or  $T_i \triangleright^* T_j$  holds. We distinguish the two cases below:

(Case  $T_j \triangleright^* T_i$ ) First, assume that  $T_j \triangleright T_i$  holds. Note  $y$  an object such that  $r_j(y_i)$  is in  $h$ . Since  $h$  belongs to ACA,  $c_i <_h r_j(y_i)$  holds. Because  $h$  is an history,  $r_j(y_i) <_h c_j$  must hold. Hence we obtain  $c_i <_h c_j$ . By a short induction, we obtain the general case.

(Case  $T_i \triangleright^* T_j$ ) Let us note  $x_k$  the version of  $x$  read by transaction  $T_i$ . From the definition of an history and since  $h$  belongs to ACA, we know that  $w_k(x_k) <_h c_k <_h r_i(x_k) <_h w_i(x_i)$  holds. As a consequence,  $x_k \ll_h x_i$  is true. Since (i)  $h$  belongs to CONS, (ii)  $T_i \triangleright^* T_j$ , and (iii)  $T_j$  writes to  $x$ , it must be the case that  $x_j \ll_h x_k$ . We deduce that  $x_j \ll_h x_i$  holds; a contradiction.

□

Using these lemmata, we successively prove each inclusion.

**Proposition 1.**  $SI \subseteq ACA \cap SCONS \cap WCF \cap MON$

*Proof.* Choose  $h$  in SI. Note  $\mathcal{S}$  a function such that history  $h_s = \mathcal{S}(h)$  satisfies rules D1 and D2.

( $h \in ACA$ ) It is immediate from rules D1.1 and D1.2.

( $h \in WCF$ ) Consider two independent transactions  $T_i$  and  $T_j$  modifying the same object  $x$ . By the definition of a history,  $x_i \ll_h x_j$ , or  $x_j \ll_h x_i$  holds. Applying Lemma 1, we conclude that in the former case  $T_j$  depends on  $T_i$ , and that the converse holds in the later.

( $h \in SCONSa$ ) By contradiction. Assume three transactions  $T_i$ ,  $T_j$  and  $T_l$  such that  $r_i(x_j), r_i(y_l) \in h$  and  $r_i(x_j) <_h c_l$  are true. In  $h_s$ , the snapshot point  $s_i$  of transaction  $T_i$  is placed prior to every operation of  $T_i$  in  $h_s$ . Hence,  $s_i$  precedes  $r_i(x_j)$  in  $h_s$ . This implies that  $s_i <_{h_s} c_l \wedge r_i(y_l) \in h_s$  holds. A contradiction to rule D1.2.

( $h \in SCONSb$ ) Assume for the sake of contradiction four transactions  $T_i, T_j, T_{k \neq j}$  and  $T_l$  such that:  $r_i(x_j), r_i(y_l), w_k(x_k) \in h$ ,  $c_k <_h c_l$  and  $c_k \not<_h c_j$  are all true. Since transaction  $T_j$  and  $T_k$  both write  $x$ , by rule D2, we know that  $c_j <_{h_s} c_k$  holds. Thus,  $c_j <_{h_s} c_k <_{h_s} c_l$  holds. According to rule D1.2, since  $r_i(y_l)$  is in  $h$ ,  $c_l <_{h_s} s_i$  is true. We consequently obtain that  $c_j <_{h_s} c_k < s_i$  holds. A contradiction to rule D1.3.

( $h \in MON$ ) If  $\rightarrow^*$  is not a partial order, there exist transactions  $T_1, \dots, T_{n \geq 1}$  such that:  $T_1 \rightarrow \dots \rightarrow T_n \rightarrow T_1$ . Applying Lemma 2, we obtain that the relation  $s_1 <_{h_s} s_1$  is true. A contradiction.

□

**Proposition 2.**  $ACA \cap SCONS \cap WCF \cap MON \subseteq SI$

*Proof.* Consider some history  $h$  in  $ACA \cap SCONS \cap WCF \cap MON$ . If history  $h$  belongs to SI then there must exist a function  $\mathcal{S}$  such that  $h' = \mathcal{S}(h)$  satisfies rules D1 and D2. In what follows, we build such an extended history  $h'$ , then we prove its correctness.

[Construction] Initially  $h'$  equals  $h$ . For every transaction  $T_i$  in  $h'$  we add a snapshot point  $s_i$  in  $h'$ , and for every operation  $o_i$  in  $h'$ , we execute the following steps:

**S1.** We add the order  $(s_i, o_i)$  to  $h'$ .

**S2.** If  $o_i$  equals  $r_i(x_j)$  for some object  $x$  then

**S2a.** we add the order  $(c_j, s_i)$  to  $h'$ ,

**S2b.** and, for every committed transaction  $T_k$  such that  $w_k(x_k)$  is in  $h$ , if  $c_k <_h c_j$  does not hold then we add the order  $(s_i, c_k)$  to  $h'$ .



[Correctness] We now prove that  $h'$  is an extended history that satisfies rules D1 and D2.

- $h'$  is an extended history.

Observe that for every transaction  $T_i$  in  $h'$ , there exists a snapshot point  $s_i$ , and that according to step S1,  $s_i$  is before all operations of transaction  $T_i$ . It remains to show that order  $<_{h'}$  is acyclic. We proceed by contradiction.

Since  $h$  is a history, it follows that any cycle formed by relation  $<_{h'}$  contains a snapshot point  $s_i$ . Furthermore, according to steps S1 and S2 above, we know that for some operation  $c_{j \neq i}$ , relation  $c_j <_{h'} s_i <_{h'}^* c_j$  holds.

By developing relation  $s_i <_{h'}^* c_j$ , we obtain the following three relations. The first two relations are terminal, while the last is recursive.

- Relation  $s_i <_{h'} c_j$  holds. This relation has to be produced by step S2b. Hence, there exist operations  $r_i(x_k), w_j(x_j)$  in  $h'$  such that  $c_j <_h c_k$  does not hold. Observe that since  $h$  belongs to  $ACA \cap CONS \cap WCF$ , by Lemma 3, it must be the case that  $c_k <_h c_j$  holds.
- Relation  $s_i <_{h'} o_i <_h^* c_j$  holds for some read operation  $o_i$  in  $T_i$ . (If  $o_i <_h^* c_j$  with  $o_i$  a write or a terminating operation, we may consider a preceding read that satisfies the same relation.)
- Relation  $s_i <_{h'} o_i <_{h'}^* c_j$  holds for some read operation  $o_i$  in  $T_i$ , and  $o_i <_{h'}^* c_j$  does not imply  $o_i <_h^* c_j$ . (Again if  $o_i$  is a write or a terminating operation, we may consider a preceding read that satisfies this relation.) Relation  $o_i <_{h'}^* c_j$  cannot be produced by steps S1 and S2. Hence, there must exist a commit operation  $c_k$  and a snapshot point  $s_l$  such that  $s_i <_{h'} o_i <_h c_k <_{h'} s_l <_{h'}^* c_j$  holds.

From the result above, we deduce that there exist snapshot points  $s_1, \dots, s_{n \geq 1}$  and commit points  $c_{k_1} \dots c_{k_n}$  such that:

$$s_1 \prec c_{k_1} <_{h'} s_2 \prec c_{k_2} \dots s_n \prec c_{k_n} <_{h'} s_1 \quad (1)$$

where  $s_i \prec c_{k_i}$  is a shorthand for either (i)  $s_i <_{h'} c_{k_i}$  with  $r_i(x_j), w_{k_i}(x_{k_i}) \in h$  and  $c_j <_h c_{k_i}$ , or (ii)  $s_i <_{h'} o_i <_h c_{k_i}$  with  $o_i$  is some read operation.

We now prove that for every  $i$ ,  $T_i \rightarrow T_{i+1}$  holds. Consider some  $i$ . First of all, observe that a relation  $c_{k_{i-1}} < s_i$  is always produced by step S2a. Then, since relation  $s_i \prec c_{k_i} <_{h'} s_{i+1}$  holds we may consider the two following cases:

- Relation  $s_i <_{h'} c_{k_i} <_{h'} s_{i+1}$  holds with  $r_i(x_j), w_{k_i}(x_{k_i}) \in h$  and  $c_j <_h c_{k_i}$ . From  $c_{k_i} <_{h'} s_{i+1}$  and step S2a, there exists an object  $y$  such that  $r_{i+1}(y_{k_i})$ . Thus, by definition of the snapshot precedence relation,  $T_i \rightarrow T_{i+1}$  holds.
- Relation  $s_i < c_{k_i}$  equals  $s_i <_{h'} o_i <_h c_{k_i}$  where  $o_i$  is some read operation of  $T_i$ . Since  $c_{k_i} <_{h'} s_{i+1}$  is produced by step S2a, we know that for some object  $y$ ,  $r_{i+1}(y_{k_i})$  belongs to  $h$ . According to the definition of the snapshot precedence,  $T_i \rightarrow T_{i+1}$  holds.

Applying the result above to Equation 1, we obtain:  $T_1 \rightarrow T_2 \dots \rightarrow T_n \rightarrow T_1$ . History  $h$  violates MON, a contradiction.

- $h'$  satisfies rules D1 and D2.
  - ( $h'$  satisfies D1.1) Follows from  $h \in \text{ACA}$ ,
  - ( $h'$  satisfies D1.2) Immediate from step S1.
  - ( $h'$  satisfies D1.3) Consider three transactions  $T_i, T_j$  and  $T_k$  such that operations  $r_i(x_j), w_j(x_j)$  and  $w_k(x_k)$  are in  $h$ . The definition of a history tells us that either  $x_k \ll_h x_j$  or the converse holds. We consider the following two cases:
    - (Case  $x_k \ll_h x_j$ ) Since  $h$  belongs to  $\text{ACA} \cap \text{CONS} \cap \text{WCF}$ , Lemma 3 tells us that  $c_k <_h c_j$  holds. Hence,  $c_k <_{h'} c_j$  holds.
    - (Case  $x_j \ll_h x_k$ ) Applying again Lemma 3, we obtain that  $c_j <_h c_k$  holds. Since  $<_h$  is a partial order, then  $c_j <_h c_k$  does not hold. By step S2b, the order  $(s_i, c_k)$  is in  $h'$ .
  - ( $h'$  satisfies D2) Consider two conflicting transaction  $(T_i, T_j)$  in  $h'$ . Since  $h$  belongs to WCF, one of the following two cases occurs:
    - (Case  $T_i \triangleright^* T_j$ ) At first glance, assume that  $T_i \triangleright^* T_j$  holds. By step S2a,  $s_i$  is in  $h'$  after every operation  $c_j$  such that  $r_i(x_j)$  is in  $h'$ , and by step S1,  $s_i$  precedes the first operation of  $T_i$ . Thus  $c_j <_{h'} s_i$  holds, and  $h'$  satisfies D2 in this case. To obtain the general case, we applying inductively the previous reasoning.
    - (Case  $T_j \triangleright^* T_i$ ) The proof is symmetrical to the case above, and thus omitted.

□

From the conjunction of Proposition 1 and Proposition 2, we deduce our decomposition theorem.

**Theorem 1.**  $SI = \text{ACA} \cap \text{SCONS} \cap \text{MON} \cap \text{WCF}$

Notice that this decomposition is well-formed in the sense that the four properties SCONS, MON, WCF and ACA are all distinct and that no strict subset of  $\{\text{SCONS}, \text{MON}, \text{WCF}, \text{ACA}\}$  attains SI.

**Proposition 3.** *For every  $S \subsetneq \{SCONS, MON, WCF, ACA\}$ , it is true that  $\cap_{X \in S} X \neq SI$ .*

*Proof.* For every set  $S \subsetneq \{SCONS, MON, WCF, ACA\}$  containing three of the four properties, we exhibit below a history in  $\cap_{X \in S} X \setminus SI$ . Trivially, the result then holds for every  $S$ .

- $SCONS \cap ACA \cap WCF$ : History  $h_7$  in Section 3.2.
- $MON \cap ACA \cap WCF$ : History  $h_6$  in Section 3.2.
- $SCONS \cap MON \cap WCF$ : History  $r_1(x_0).w_1(x_1).r_a(x_0).c_1.c_a$ .
- $SCONS \cap MON \cap ACA$ : History  $r_1(x_0).r_2(x_0).w_1(x_1).w_2(x_2).c_1.c_2$ .

□

To the best of our knowledge, this result is the first to prove that SI can be split into simpler properties. Theorem 1 also establishes that SI is definable on plain histories. This has two interesting consequences: (i) a transactional system does not have to explicitly implement snapshots to support SI, and (ii) one can compare SI to other consistency criterion without relying on a phenomena based characterization.<sup>6</sup>

<sup>6</sup>Contrary to, e.g., the work of Adya [6].

## 4 The impossibility of SI with GPR

This section leverages our previous decomposition result to show that SI is inherently non-scalable. In more details, we prove that none of MON, SCONS<sub>a</sub> or SCONS<sub>b</sub> is attainable in some asynchronous failure-free GPR system  $\Pi$  when updates are obstruction-free and queries are wait-free. To prove these results, we first characterize below histories acceptable by  $\Pi$ .

**Lemma 4.** *Let  $h = \mathfrak{F}(\rho)$  be an acceptable history by  $\Pi$  such that a transaction  $T_i$  is pending in  $h$ . Note  $X$  the set of objects accessed by  $T_i$  in  $h$ . Only processes in  $\text{replicas}(X)$  make steps to execute  $T_i$  in  $\rho$ .*

*Proof.* (By contradiction.) Consider that a process  $p \notin \text{replicas}(X)$  makes steps to execute  $T_i$  in  $\rho$ . Since the prefix of a transaction is a transaction with the same id, we can consider an extension  $\rho'$  of  $\rho$  such that  $T_i$  does not execute any additional operation in  $\rho'$  and  $\text{coord}(T_i)$  is correct in  $\rho'$ . The progress requirements satisfied by  $\Pi$  imply that  $T_i$  terminates in  $\rho'$ . However, process  $p \notin \text{replicas}(X)$  makes steps to execute  $T_i$  in  $\rho'$ . A contradiction to the fact that  $\Pi$  is GPR.  $\square$

We now state that monotonic snapshots are not constructable in  $\Pi$ . Our proof holds because objects accessed by a transaction are not known in advance.

**Theorem 2.** *No asynchronous failure-free GPR system implements MON*

*Proof.* (By contradiction.) Let us consider (i) four objects  $x, y, z$  and  $u$  such that for any two objects in  $\{x, y, z, u\}$ , their replica sets do not intersect; (ii) four queries  $T_a, T_b, T_c$  and  $T_d$  accessing respectively  $\{x, y\}$ ,  $\{y, z\}$ ,  $\{z, u\}$  and  $\{u, x\}$ ; and (iii) four updates  $T_1, T_2, T_3$  and  $T_4$  modifying respectively  $x, y, z$  and  $u$ .

Obviously, history  $r_b(y_0)$  is acceptable, and since updates are obstruction-free,  $r_b(y_0).r_2(y_0).w_2(y_2).c_2$  is also acceptable. Applying that  $\Pi$  satisfies non-trivial SI, we obtain that history  $r_b(y_0).r_2(y_0).w_2(y_2).c_2.r_a(x_0).r_a(y_2)$  is acceptable. Since  $T_a$  is wait-free,  $h = r_b(y_0).r_2(y_0).w_2(y_2).c_2.r_a(x_0).r_a(y_2).c_a$  is acceptable as well. Using a similar reasoning,  $h' = r_d(u_0).r_4(u_0).w_4(u_4).c_4.r_c(z_0).r_c(u_4).c_c$  is also acceptable. We note  $\rho$  and  $\rho'$  respectively two sequences of steps such that  $\mathfrak{F}(\rho) = h$  and  $\mathfrak{F}(\rho') = h'$ .

System  $\Pi$  is GPR. As a consequence, Lemma 4 tells us that only processes in  $\text{replicas}(x, y)$  make steps in  $\rho$ . Similarly, only processes in  $\text{replicas}(u, z)$  make steps in  $\rho'$ . By hypothesis,  $\text{replicas}(x, y)$  and  $\text{replicas}(u, z)$  are disjoint. Applying a classical indistinguishability argument [12, Lemma 1], both  $\rho'.\rho$  and

$\rho.\rho'$  are admissible by  $\Pi$ . Thus, histories  $h'.h = \mathfrak{F}(\rho'.\rho)$  and  $h.h' = \mathfrak{F}(\rho.\rho')$  are acceptable.

Since updates are obstruction-free, history  $h'.h.r_3(z_0).w_3(z_3).c_3$  is acceptable. Note  $U$  the sequence of steps following  $\rho'.\rho$  with  $\mathfrak{F}(U) = r_3(z_0).w_3(z_3).c_3$ . Observe that by Lemma 4  $\rho'.\rho.U$  is indistinguishable from  $\rho'.U.\rho$ . Then consider history  $\mathfrak{F}(\rho'.U.\rho)$ . In this history,  $T_b$  is pending and the latest version of object  $z$  is  $z_3$ . As a consequence, because  $\Pi$  satisfies non-trivial SI, there exists an extension of  $\rho'.U.\rho$  in which transaction  $T_b$  reads  $z_3$ . From the fact that queries are wait-free and since  $\rho'.\rho.U$  is indistinguishable from  $\rho'.U.\rho$ , we obtain that history  $h_1 = h'.h.r_3(z_0).w_3(z_3).c_3.r_b(z_3).c_b$  is acceptable. We note  $U_1$  the sequence of steps following  $\rho'.\rho$  such that  $\mathfrak{F}(U_1)$  equals  $r_3(z_0).w_3(z_3).c_3.r_b(z_3).c_b$ .

With a similar reasoning, history  $h_2 = h'.h.r_1(x_0).w_1(x_1).c_1.r_d(x_1).c_d$  is acceptable. Note  $U_2$  the sequence satisfying  $\mathfrak{F}(U_2) = r_1(x_0).w_1(x_1).c_1.r_d(x_1).c_d$ .

Executions  $\rho'.\rho.U_1$  and  $\rho'.\rho.U_2$  are both admissible. Because  $\Pi$  is GPR, only processes in  $\text{replicas}(y, z)$  (resp.  $\text{replicas}(x, u)$ ) make steps in  $U_1$  (resp.  $U_2$ ). By hypothesis, these two replica sets are disjoint. Applying again an indistinguishability argument,  $\rho'.\rho.U_1.U_2$  is an execution of  $\Pi$ . Therefore, the history  $\hat{h} = \mathfrak{F}(\rho'.\rho.U_1.U_2)$  is acceptable. In this history, relation  $T_a \rightarrow T_b \rightarrow T_c \rightarrow T_d \rightarrow T_a$  holds. Thus,  $\hat{h}$  does not belong to MON. Contradiction.  $\square$

Our next theorem states that SCONSb is not attainable. Similarly to Attiya et al. [16], our proof builds an infinite execution in which a query  $T_a$  on two objects never terminates. We first define a finite execution during which we interleave between any two consecutive steps to execute  $T_a$ , a transaction updating one of the objects read by  $T_a$ . We show that during such an execution, transaction  $T_a$  does not terminate successfully. Then, we prove that asynchrony allows us to continuously extend such an execution, contradicting the fact that queries are wait-free.

**Definition 8** (Flippable execution). *Consider two distinct objects  $x$  and  $y$ , a query  $T_a$  over both objects, and a set of updates  $T_{j \in [1, m]}$  accessing  $x$  if  $j$  is odd, and  $y$  otherwise. An execution  $\rho = U_1 V_2 U_2 \dots V_m U_m$  where,*

- *transaction  $T_a$  reads in history  $h = \mathfrak{F}(\rho)$  at least version  $x_1$  of  $x$ ,*
- *for any  $j$  in  $[1, m]$ ,  $U_j$  is the execution of transaction  $T_j$  by processes  $Q_j$ ,*
- *for any  $j$  in  $[2, m]$ ,  $V_j$  are steps to execute  $T_a$  by processes  $P_j$ , and*
- *both  $(Q_j \cap P_j = \emptyset) \oplus (P_j \cap Q_{j+1} = \emptyset)$  and  $Q_j \cap Q_{j+1} = \emptyset$  hold,*

is called *flippable*.

**Lemma 5.** *Let  $\rho$  be an execution admissible by  $\Pi$ . If  $\rho$  is flippable and histories accepted by  $\Pi$  satisfy SCONSb, query  $T_a$  does not terminate.*

*Proof.* Let  $h$  be the history  $\mathfrak{F}(\rho)$ . In history  $h$  transaction  $T_j$  precedes transaction  $T_{j+1}$ , it follows that  $h$  is of the form  $h = w_1(x_1).c_1.*.w_2(y_2).c_2.*.\dots$ , where each symbol  $*$  corresponds to either no operation, or to some read operation by  $T_a$  on object  $x$  or  $y$ .

Because  $\rho$  is flippable, transaction  $T_a$  reads at least version  $x_1$  of object  $x$  in  $h$ . For some odd natural  $j \geq 1$ , let  $x_j$  denote the version of object  $x$  read by  $T_a$ . Similarly, for some even natural  $l$ , let  $y_l$  be the version of  $y$  read by  $T_a$ . Assume that  $j < l$  holds. Therefore,  $h$  is of the form  $h = \dots w_j(x_j) \dots w_l(y_l) \dots$ .

Note  $k$  the value  $l + 1$ , and consider the sequence of steps  $V_k$  made by  $P_k$  right after  $U_l$  to execute  $T_a$ . Applying the definition of a flippable execution, we know that (F1)  $(Q_l \cap P_k = \emptyset) \oplus (P_k \cap Q_k = \emptyset)$ , and (F2)  $Q_l \cap Q_k = \emptyset$ . Consider now the following cases:

(Case  $Q_l \cap P_k = \emptyset$ .) It follows that  $\rho$  is indistinguishable from the execution  $\rho'' = \dots U_j \dots V_k U_l U_k \dots$ . Then from fact F2,  $\rho$  is indistinguishable from execution  $\rho' = \dots U_j \dots V_k U_k U_l \dots$ .

(Case  $P_k \cap Q_k = \emptyset$ .) With a similar reasoning, we obtain that  $\rho$  is indistinguishable from  $\rho' = \dots U_j \dots U_k U_l V_k \dots$ .

(Case  $P_k \cap (Q_l \cup Q_k) = \emptyset$ .) This case reduces to any of the two above cases. Note  $h'$  the history  $\mathfrak{F}(\rho')$ . Observe that since  $\rho'$  is indistinguishable from  $\rho$ , history  $h'$  is acceptable. In history  $h'$ ,  $c_k <_{h'} c_l$  holds. Moreover,  $c_j <_{h'} c_k$  holds by the assumption  $j < l$  and the fact that  $k$  equals  $l + 1$ . Besides, operations  $r_i(x_j)$ ,  $r_i(y_l)$  and  $w_k(x_k)$  all belong to  $h'$ . According to the definition of SCONSb, transaction  $T_a$  does not commit in  $h'$ . (The case  $j > l$  follows a symmetrical reasoning to the case  $l > j$  we considered previously.)  $\square$

**Theorem 3.** *No asynchronous failure-free GPR system implements SCONSb.*

*Proof.* (By contradiction.) Consider two objects  $x$  and  $y$  such that  $\text{replicas}(x)$  and  $\text{replicas}(y)$  are disjoint. Assume a read-only transaction  $T_a$  that reads successively  $x$  then  $y$ . Below, we exhibit an execution admissible by  $\Pi$  during which transaction  $T_a$  never terminates. We build this execution as follows:

[Construction.] Consider some empty execution  $\rho$ . Repeat for all  $i \geq 1$ : Let  $T_i$  be an update of  $x$ , if  $i$  is odd, and  $y$  otherwise. Start the execution of transaction  $T_i$ . Since no concurrent transaction is write-conflicting with  $T_i$  in  $\rho$  and updates are obstruction-free, there must exist an extension  $\rho.U_i$  of  $\rho$  during which  $T_i$  commits. Assign to  $\rho$  the value of  $\rho.U_i$ . Execution  $\rho$  is

flippable. Hence, Lemma 5 tells us that transaction  $T_a$  does not terminate in this execution. Consider the two following cases: (Case  $i = 1$ ) Because  $\Pi$  satisfies non-trivial SI, there exists an extension  $\rho'$  of  $\rho$  in which transaction  $T_a$  reads at least version  $x_1$  of object  $x$ . Notice that execution  $\rho'$  is of the form  $U_1.V_2.s\dots$  where (i) all steps in  $V_2$  are made by processes in  $\text{replicas}(x)$ , and (ii)  $s$  is the first step such that  $\mathfrak{F}(U_1.V_2.s) = r_1(x_0).w_1(x_1).c_1.r_a(x_1)$ . Assign  $U_1.V_2$  to  $\rho$ . (Case  $i > 2$ ) Consider any step  $V_{i+1}$  to terminate  $T_a$  and append it to  $\rho$ .

Execution  $\rho$  is admissible by  $\Pi$ . Hence  $\mathfrak{F}(\rho)$  is acceptable. However, in this history transaction  $T_a$  does not terminate. This contradicts the fact that queries are wait-free.  $\square$

SCONSa disallows some real time orderings between operations accessing different objects. Our last theorem shows that this property cannot be maintained under GPR.

**Theorem 4.** *No asynchronous failure-free GPR system implements SCONSa.*

*Proof.* (By contradiction.) Consider two distinct objects  $x$  and  $y$  such that  $\text{replicas}(x)$  and  $\text{replicas}(y)$  are disjoint. Let  $T_1$  be an update accessing  $y$ , and  $T_a$  be a query reading both objects.

Obviously, history  $h = r_a(x_0)$  is acceptable. Note  $U_a$  a sequence of steps satisfying  $U = \mathfrak{F}(r_a(x_0))$ . Because  $\Pi$  supports obstruction-free updates, we know the existence of an extension  $U_a.U_1$  of  $U_a$  such that  $\mathfrak{F}(U_1) = r_1(y_0).w_1(y_1).c_1$ . By Lemma 4, we observe that  $U_a.U_1$  is indistinguishable from  $U_1.U_a$ . Then, since  $\Pi$  satisfies non-trivial SI and read-only transactions are wait-free, there must exist an extension  $U_1.U_a.V_a$  of  $U_1.U_a$  admissible by  $\Pi$  and such that  $\mathfrak{F}(V_a) = r_a(y_1).c_a$ . Finally, since  $U_a.U_1$  is indistinguishable from  $U_1.U_a$  and  $U_1.U_a.V_a$  is admissible,  $U_a.U_1.V_a$  is admissible too. The history  $\mathfrak{F}(U_a.U_1.V_a)$  is not in SCONSa. Contradiction.  $\square$

As a consequence of the above, no asynchronous system, even if it is failure-free, can support both GPR and SI. In particular, even if the system is augmented with failure detectors [17], a common approach to model partial synchrony, SI cannot be implemented under GPR. This fact strongly hinders the usage of SI at large scale. In the following section, we further discuss implications of this result.

## 5 Discussion

In this section, we discuss the consequences of our impossibility results on other consistency criteria than SI.

### 5.1 Strict Serializability and Opacity

A straightforward corollary of any of the theorems we proved in Section 4 is that neither strict serializability [18], nor opacity [19] is attainable under GPR. In the case of opacity, this answers negatively to a problem posed by Peluso et al. [20].

### 5.2 Serializability (SER)

**Permissiveness** A transactional system  $\Pi$  is *permissive* with respect to a consistency criterion  $\mathcal{C}$  when every history  $h \in \mathcal{C}$  is acceptable by  $\Pi$ . Permissiveness [21] measures the optimal amount of concurrency a system allows. If we consider again histories  $h_1$  and  $h_2$  in the proof of Theorem 2, we observe that both histories are serializable. Hence, every system permissive with respect to SER accepts both histories. By relying on the very same argument as the one we exhibit to close the proof of Theorem 2, we conclude that no transactional system is both GPR and permissive with respect to SER. For instance, P-Store [10], a GPR protocol that ensures SER, does not accept history  $h_{10} = r_1(x_0).w_1(x_1).c_1.r_2(x_0).r_2(y_0).w_2(y_2).c_2$ .

**Wait-free Queries** As a corollary of Theorem 2, no SER system can ensure both WFQ and GPR. P-Store [10] favors GPR over WFQ. On the contrary, the protocol of Sciascia et al. [22] ensures GPR but not WFQ.

Peluso et al. [23] recently claim a GPR protocol guaranteeing SER while providing WFQ. Unfortunately, because of Theorem 2, this algorithm is unable to ensure MON, hence is not SER. For example, consider history  $h_{11}$  below. History  $h_{11}$  is non-serializable, but acceptable by the protocol of Peluso

$$\begin{array}{c}
 h_{11} = r_1(u_0).w_1(u_1).c_1.r_a(u_1).r_a(x_0) \implies r_3(x_0).w_3(x_3).c_3 \implies r_b(x_3).c_b \\
 \phantom{h_{11} = } \phantom{r_1(u_0).w_1(u_1).c_1.r_a(u_1).r_a(x_0)} \phantom{\implies} \phantom{r_3(x_0).w_3(x_3).c_3} \phantom{\implies} \phantom{r_b(x_3).c_b} \\
 r_2(z_0).w_2(z_2).c_2.r_b(z_2).r_b(y_0) \implies r_4(y_0).w_4(y_4).c_4 \implies r_a(y_4).c_a
 \end{array}$$

et al. [23], as we explain next: The system is composed of four processes  $p_1, p_2, p_3$  and  $p_4$  that replicate objects  $u, x, y$  and  $z$  respectively. Queries  $T_a$  and  $T_b$  are initiated on  $p_1$  and  $p_4$  respectively. They first read objects  $u$  and  $z$  locally, thus setting their *sid* to 1 (Peluso et al. [23, Algorithm 1, Upon Read]). Then they try to read objects  $x$  and  $y$  from remote processes.  $T_a$  first tries to read object  $x$  and  $T_b$  tries to read object  $y$ . Since both  $x$  and



$y$  are exclusively unlocked, according to Peluso et al. [23, Algorithm 1, function `doRead`], they read versions  $x_0$  and  $y_0$  respectively. After the first read,  $T_a$  tries to read object  $y$  and  $T_b$  tries to read object  $x$ . In the meantime, transactions  $T_3$  and  $T_4$  update objects  $x$  and  $y$  respectively. Upon receiving a read request for object  $y$  of  $T_a$ , and since the *commitid* of  $p_3$  is greater or equal to *sid* of  $T_a$ , and because object  $y$  is exclusively unlocked, it returns  $y_4$ . Likewise,  $p_2$  returns  $x_3$  upon receiving a read request from transaction  $T_b$ . Because the two queries are wait-free, they commit both.

History  $h_{11}$  is not in SER, still it belongs to update serializability. Update serializability (US) was introduced by Garcia-Molina and Wiederhold [24], then later extended (EUS) for abort transactions by Hansdah and Patnaik [25]. EUS provides the same guarantees as SER for update transactions, i.e., update transactions are serialized. In addition, EUS ensures queries see a serializable histories. However, two queries may not agree on the order in which non-conflicting transactions occur. One can show that the protocol of Peluso et al. [23] supports in fact EUS.

### 5.3 Declaring the Read-set in Advance

When a transaction declares objects it accesses *in advance*, a GPR system can install a strictly consistent and monotonic snapshot just after the start of the transaction. As a consequence, such an assumption sidesteps our impossibility result. The protocol of Armendáriz-Iñigo et al. [2] makes such an assumption to ensure SI, but it still uses atomic broadcast to install a snapshot. It is nevertheless possible to replace atomic broadcast by a genuine atomic multicast. In such a case, we obtain a GPR system that supports SI.

### 5.4 Parallel Snapshot Isolation (PSI)

Recently, Sovran et al. [9] introduce a weaker consistency criterion called PSI. PSI allow snapshots to be non-monotonic, but still require them to ensure SCONSa. Sovran et al. justify the use of PSI in Walter by the fact that SI is too expensive in a geographically distributed environment [9, page 4]. Our impossibility result establishes that, in order to scale, a transactional system needs supporting both non-monotonic *and* non-strictly consistent snapshots. Thus, while being more scalable than SI, it yet cannot be implemented in a GPR system.

## 6 Non-Monotonic Snapshot Isolation

We just showed that the SI requirements of strictly consistent (SCONS) and monotonic (MON) snapshots hurt scalability, as they are impossible with GPR. To overcome the impossibility, this section presents a slightly weaker criterion, called Non-Monotonic Snapshot Isolation (NMSI).

NMSI retains the most important properties of SI, namely snapshots are consistent, a read-only transaction can commit locally without synchronization, and two concurrent conflicting updates do not both commit. However, NMSI allows non-strict, non-monotonic snapshots. For instance, history  $h_7$  in Section 3.3, which is not in SI, is allowed by NMSI. Formally, we define NMSI as follows:

**Definition** (Non-Monotonic Snapshot Isolation). *A history  $h$  is in NMSI iff  $h$  belongs to  $ACA \cap CONS \cap WCF$ .*

To clarify our understanding of NMSI, Table 1 compares it to well-known approaches, based on the anomalies an application might observe. In addition to the classical anomalies [6, 8] (dirty reads, non-repeatable reads, read skew, dirty writes, lost updates, and write skew), we also consider the following: (Non-Monotonic Snapshots) snapshots taken by transactions are not monotonically ordered, and (Real-Time Causality Violation) a transaction  $T_2$  observes the effect of some transaction  $T_1$ , but does not observe the effect of all the transactions that precede (in real time)  $T_1$ .

	Strict Serializ- ability [18]	Serializability [8]	Update Serializ- ability [15]	Snapshot Isola- tion	NMSI
Dirty Reads	x	x	x	x	x
Non-repeatable Reads	x	x	x	x	x
Read Skew	x	x	x	x	x
Dirty Writes	x	x	x	x	x
Lost Updates	x	x	x	x	x
Write Skew	x	x	x	-	-
Non-Monotonic Snapshots	x	x	-	x	-
Real-time Causality Violation	x	-	-	x	-

Table 1: Comparing consistency criterion by their anomalies (x: disallowed)

Write Skew, the classical anomaly of SI, is observable under NMSI. (Cahill et al. [26] show how an application can easily avoid it.) Because NMSI does not ensure SCONSb, it suffers the Real-Time Causality Violation anomaly.

Note that it is not new, as it occurs with serializability as well; this argues that it is not considered a problem in practice. Non-Monotonic Snapshots occur both under NMSI and update serializability. Following Garcia-Molina and Wiederhold [15], we believe that this is a small price to pay for improved performance.

## 7 Protocol

We now describe Jessy, a scalable transactional system that implements NMSI with GPR. Because distributed locking policies do not scale [27, 28], Jessy employs deferred update replication: transactions are executed optimistically, then certified by a termination protocol. Jessy uses a novel clock mechanism to ensure that snapshots are both fresh and consistent, while preserving wait-freedom of queries and genuineness. We describe it in the next section.

### 7.1 Building Consistent Snapshots

To compute consistent snapshots, Jessy makes use of a novel data type called *dependence vectors*. Each version of each object is assigned its own dependence vector. The dependence vector of some version  $x_i$  reflects all the versions read by  $T_i$ , or read by transactions that precede  $T_i$ , as well as the writes of  $T_i$  itself:

**Definition** (Dependence Vector). *A dependence vector is a function  $V$  that maps every read (or write) operation  $o(x)$  in a history  $h$  to a vector  $V(o(x)) \in \mathbb{N}^{|\text{Objects}|}$  such that:*

$$\begin{aligned} V(r_i(x_0)) &= \mathbf{0}^{|\text{Objects}|} \\ V(r_i(x_j)) &= V(w_j(x_j)) \\ V(w_i(x_i)) &= \max \{ V(r_i(y_j)) : y_j \in rs(T_i) \} \\ &\quad + \sum_{z_i \in ws(T_i)} \mathbf{1}_z \end{aligned}$$

where  $\max \mathcal{V}$  is the vector containing for each dimension  $z$ , the maximal  $z$  component in the set of vectors  $\mathcal{V}$ , and  $\mathbf{1}_z$  is the vector that equals 1 on dimension  $z$  and 0 elsewhere.

To illustrate this definition, consider history  $h_{10}$  below. In this history, transactions  $T_1$  and  $T_2$  update objects  $x$  and  $y$  respectively, while transaction  $T_3$  reads  $x$ , then updates  $y$ . The dependence vector of  $x_1$  equals  $\langle 1, 0 \rangle$ , and

$$\begin{array}{ccc} h_{10} = r_1(x_0).w_1(x_1).c_1 & \searrow & r_3(x_1).r_3(y_2).w_3(y_3).c_3 \\ & & \nearrow \\ & r_2(y_0).w_2(y_2).c_2 & \end{array}$$

of  $y_1$  equals  $\langle 0, 1 \rangle$ . Since transaction  $T_3$  reads  $x$  then updates  $y$ , this implies that dependence vector of  $y_3$  equals  $\langle 1, 2 \rangle$ .

**Definition** (Compatibility Relation). *Consider a transaction  $T_i$  and two versions  $x_j$  and  $y_l$  read by  $T_i$ . We shall say that  $x_j$  and  $y_l$  are compatible for  $T_i$ , written  $\text{compat}(T_i, x_j, y_l)$ , when both  $V(r_i(x_j))[x] \geq V(r_i(y_l))[x]$  and  $V(r_i(y_l))[y] \geq V(r_i(x_j))[y]$  hold.*

Using the compatibility relation, we can prove that dependence vectors fully characterize consistent snapshots. First of all, we show in Lemma 6 that if transaction  $T_i$  depends on transaction  $T_j$  then the dependence vector of any object written by  $T_i$  is greater than the dependence vector of any object written by  $T_j$ .

**Lemma 6.** *Consider a history  $h$  in NMSI, and two transactions  $T_i$  and  $T_j$  in  $h$ . Then,*

$$T_i \triangleright^* T_j \Leftrightarrow \forall x, y \in \text{Objects} : \forall w(x), w(y) \in T_i \times T_j : V(w_i(x_i)) > V(w_j(y_j))$$

*Proof.* The proof goes as follows:

- ( $\Rightarrow$ ) First consider that  $T_i \triangleright T_j$  holds. By definition of relation  $\triangleright$ , we know that for some object  $z$ , operations  $r_i(z_j)$  and  $w_j(z_j)$  are in  $h$ . According to definition of function  $V$  we have:  $V(w_i(x_i)) \geq V(r_i(z_j)) + 1_x$ . Besides, always according to the definition of  $V$ , it is true that the following equalities hold:  $V(r_i(z_j)) = V(w_j(z_j)) = V(w_j(y_j))$ . Thus, we have:  $V(w_i(x_i)) > V(w_j(y_j))$ . The general case  $T_i \triangleright^* T_j$  is obtained by applying inductively the previous reasoning.
- ( $\Leftarrow$ ) From the definition of function  $V$ , it must be the case that  $r_i(y_{j'})$  is in  $h$  with  $j' \neq 0$ . We then consider the following two cases: (Case  $j' = j$ ) By definition of relation  $\triangleright$ ,  $T_i \triangleright T_j$  holds. (Case  $j' \neq j$ ) By construction, we have that:  $T_i \triangleright T_{j'}$ . By definition of function  $V$ , we have that  $V(r_{j'}(y_{j'})) = V(w_{j'}(y_{j'}))$ . Since  $V(w_i(x_i)) > V(w_j(y_j))$  holds,  $V(w_{j'}(y_{j'}))[y] \geq V(w_j(y_j))[y]$  is true. Both transactions  $T_j$  and  $T_{j'}$  write  $y$ . Since  $h$  belongs to NMSI, it must be the case that either  $T_j \triangleright^* T_{j'}$  or that  $T_{j'} \triangleright^* T_j$  holds. If  $T_j \triangleright^* T_{j'}$  holds, then we just proved that  $V(w_j(y_j)) > V(w_{j'}(y_{j'}))$  is true. A contradiction. Hence necessarily  $T_{j'} \triangleright^* T_j$  holds. From which we conclude that  $T_i \triangleright^* T_j$  is true. □

The following theorem shows that dependence vectors enable taking consistent snapshots.

**Theorem 5.** *Consider a history  $h$  in NMSI and a transaction  $T_i$  in  $h$ . Transaction  $T_i$  sees a consistent snapshot in  $h$  if, and only if, every pair of versions  $x_j$  and  $y_l$  read by  $T_i$  is compatible.*

*Proof.* The proof goes as follows:

- ( $\Rightarrow$ ) By contradiction. Assume the existence of two versions  $x_l$  and  $y_j$  in the snapshot of  $T_i$  such that  $V(r_i(x_l))[x] < V(r_i(y_j))[x]$  holds. By definition of function  $V$ , we have  $V(r_i(x_l)) = V(w_l(x_l))$  and  $V(r_i(y_j)) =$

$V(w_j(y_j))$ . Hence,  $V(w_l(x_l))[x] < V(w_j(y_j))[x]$  holds. Again from the definition of function  $V$ , there exists a transaction  $T_{k \neq 0}$  writing on  $x$  such that (i)  $V(w_j(y_j)) \geq V(w_k(x_k))$  and (ii)  $V(w_j(y_j))[x] = V(w_k(x_k))[x]$ . Applying Lemma 6 to (i), we obtain  $T_j \triangleright^* T_k$ . From which we deduce that  $T_i \triangleright^* T_k$ . Now since both transactions  $T_l$  and  $T_k$  write  $x$  and  $h$  belongs to NMSI,  $T_l \triangleright^* T_k$  or  $T_k \triangleright^* T_l$  holds. From (ii) and  $V(w_l(x_l))[x] < V(w_j(y_j))[x]$ , we deduce that  $V(w_l(x_l))[x] < V(w_k(x_k))[x]$ . As a consequence of Lemma 6,  $T_k \triangleright^* T_l$  holds. Hence  $x_l \ll_h x_k$ . But  $T_i \triangleright^* T_k$  and  $r_i(x_l)$  is in  $h$ . It follows that  $T_i$  does not read a consistent snapshot. Contradiction.

- ( $\Leftarrow$ ) By contradiction. Assume that there exists an object  $x$  and a transaction  $T_k$  on which  $T_i$  depends such that  $T_i$  reads version  $x_j$ ,  $T_k$  writes version  $x_k$ , and  $x_j \ll_h x_k$ . First of all, since  $h$  is in NMSI, one can easily show that  $T_k \triangleright^* T_j$ . Since  $T_k \triangleright^* T_j$ , Lemma 6 tells us that  $V(w_k(x_k)) > V(w_j(x_j))$  holds. Since  $T_i \triangleright^* T_k$  holds, a short induction on the definition of function  $V$  tells us that  $V(r_i(x_j))[x] \geq V(w_k(x_k))[x]$  is true. From which we obtain that:  $V(r_i(x_j))[x] \geq V(w_k(x_k))[x] > V(w_j(x_j))[x] = V(r_i(x_j))[x]$ . Contradiction.  $\square$

Despite that in the common case dependence vectors are sparse, they might be large for certain workloads. For instance, if transactions execute random accesses, the size of each vector tends asymptotically to the number of objects in the system. To address the above problem, Jessy employs a mechanism to approximate dependencies safely, by coarsening the granularity, grouping objects into disjoint partitions and serializing updates in a group as if it was a single larger object. We cover this mechanism in what follows.

## 7.2 Partitioned Dependence Vector

Consider some partition  $\mathcal{P}$  of *Objects*. For some object  $x$ , note  $P(x)$  the partition  $x$  belongs to, and by extension, for some  $S \subseteq \text{Objects}$ , note  $P(S)$  the set  $\{P(x) \mid x \in S\}$ . A partition is *proper* when updates inside the same partition are serialized, that is, for every  $X \in \mathcal{P}$  and every two writes  $w_i(x_i)$ ,  $w_j(y_j)$  with  $P(x) = P(y)$ , either  $w_i(x_i) <_h w_j(y_j)$  or  $w_j(y_j) <_h w_i(x_i)$  holds.

Now, consider some history  $h$ , and for every object  $x$  replace every operation  $o_i(x)$  in  $h$  by  $o_i(P(x))$ . We obtain a history that we note  $h^{\mathcal{P}}$ . The following result linked the consistency of  $h$  to the consistency of  $h^{\mathcal{P}}$ :

**Proposition 4.** *Consider some history  $h$ . If  $\mathcal{P}$  is a proper partition of *Objects* for  $h$  and history  $h^{\mathcal{P}}$  belongs to *CONS*, then  $h$  is in *CONS*.*

*Proof.* First of all we observe that for any two transactions  $T_i$  and  $T_j$ :

- If  $T_i \triangleright^* T_j$  holds in  $h$  then  $T_i \triangleright^* T_j$  holds in  $h^{\mathcal{P}}$ .

*Proof.* If  $T_i \triangleright T_j$  holds in  $h$ , then  $r_i(x_j)$  is in  $h$ . Thus  $r_i(\mathcal{P}(x_j))$  is in  $h^{\mathcal{P}}$ . It follows that  $T_i \triangleright T_j$  holds in  $h^{\mathcal{P}}$ . If  $T_i \triangleright^* T_j$  in  $h$  then there exist a set of transactions  $\{T_1, \dots, T_m\}$  such that:  $T_i \triangleright T_1 \dots \triangleright T_m \triangleright T_j$  hold in  $h$ . From the result above, we deduce that  $T_i \triangleright T_1 \dots \triangleright T_m \triangleright T_j$  hold in  $h^{\mathcal{P}}$ . Hence,  $T_i \triangleright^* T_j$  holds in  $h^{\mathcal{P}}$ .  $\square$

- If  $x_i \ll x_j$  holds in  $h$  then  $\mathcal{P}(x_i) \ll \mathcal{P}(x_j)$  holds in  $h^{\mathcal{P}(x)}$ .

*Proof.* If  $x_i \ll x_j$  holds in  $h$  then  $\mathcal{P}(x_i) \ll \mathcal{P}(x_j)$  holds in  $h$ .  $\square$

For the sake of contradiction, assume that  $h^{\mathcal{P}}$  is in CONS while  $h$  is not in CONS. It follows that there exist a transaction  $T_i$ , some object  $x$  and a transaction  $T_k$  on which  $T_i$  depends such that in  $h$ ,  $T_i$  reads version  $x_j$ ,  $T_k$  writes version  $x_k$ , and  $x_j \ll_h x_k$ . From the two observations above, we obtain that  $T_i \triangleright T_j$ ,  $T_i \triangleright^* T_k$  and  $\mathcal{P}(x_j) \ll_h \mathcal{P}(x_k)$  hold in  $h^{\mathcal{P}}$ . Hence,  $h^{\mathcal{P}}$  is not consistent. Contradiction.  $\square$

Given two operations  $o_i(x_j)$  and  $o_k(y_l)$ , let us introduce relation  $o_i(x_j) \leq_h^{\mathcal{P}} o_k(y_l)$  when  $o_i(x_j) = o_k(y_l)$ , or  $o_i(x_j) <_h o_k(y_l) \wedge \mathcal{P}(x) = \mathcal{P}(y)$  is true. Based on Proposition 4, we define below a function that approximates dependencies safely:

**Definition 9** (Partitioned Dependence Vector). *A partitioned dependence vector is a function  $PV$  that maps every read (or write) operation  $o(x)$  in a history  $h$  to a vector  $PV(o(x)) \in \mathbb{N}^{|\mathcal{P}|}$  such that:*

$$\begin{aligned} PV(r_i(x_0)) &= 0^{|\mathcal{P}|} \\ PV(r_i(x_j)) &= \max \{ PV(w_l(y_l)) \mid w_l(y_l) \leq_h^{\mathcal{P}} r_i(x_j) \\ &\quad \wedge (\forall k : x_j \ll_h x_k \Rightarrow w_l(y_l) \leq_h^{\mathcal{P}} w_k(x_k)) \} \\ PV(w_i(x_i)) &= \max \{ PV(r_i(y_j)) \mid y_j \in rs(T_i) \} \cup \\ &\quad \{ PV(w_k(z_k)) : w_k(z_k) \leq_h^{\mathcal{P}} w_i(x_i) \} \\ &\quad + \sum_{X \in \mathcal{P}(ws(T_i))} 1_X \end{aligned}$$

The first two rules of function  $PV$  are identical to the ones that would give us function  $V$  on history  $h^{\mathcal{P}}$ . The second part of the third rule serializes objects in the same partition

We now prove that partitioned dependence vectors properly capture consistent snapshots. Consider the following definition of  $compat(T_i, x_j, y_l)$  for a proper partition  $\mathcal{P}$ :

**Case  $\mathcal{P}(x) \neq \mathcal{P}(y)$ .** This case is identical to the definition we gave for function  $V$ . In other words, both  $PV(r_i(x_j))[\mathcal{P}(x)] \geq PV(r_i(y_l))[\mathcal{P}(x)]$  and  $PV(r_i(y_l))[\mathcal{P}(y)] \geq PV(r_i(x_j))[\mathcal{P}(y)]$  must hold.

**Case  $\mathcal{P}(x) = \mathcal{P}(y)$ .** This case deals with the fact that inside a partition writes are serialized. We have (i) if  $PV(r_i(x_j))[\mathcal{P}(y)] > PV(r_i(y_l))[\mathcal{P}(y)]$  holds then  $y_l = \max \{y_k \mid w_k(y_k) \leq_h^{\mathcal{P}} w_j(x_j)\}$ , or symmetrically (ii) if  $PV(r_i(y_l))[\mathcal{P}(x)] > PV(r_i(x_j))[\mathcal{P}(x)]$  holds then  $x_j = \max \{x_k \mid w_k(x_k) \leq_h^{\mathcal{P}} w_l(y_l)\}$ , or otherwise (iii) the predicate equals *true*.

We prove next that the “if” part of Theorem 5 holds for the above definition of compatibility:

**Proposition 5.** *Consider a history  $h$  in NMSI and a transaction  $T_i$  in  $h$ . If every pair of versions  $x_j$  and  $y_l$  read by  $T_i$  is compatible, then transaction  $T_i$  sees a consistent snapshot in  $h$*

*Proof.* Using a reasoning identical to the one we depicted in the proof of Theorem 5, we can prove that  $h^{\mathcal{P}}$  belongs to CONS. Then, from Proposition 4, we know that if  $h^{\mathcal{P}}$  belongs to CONS, then  $h$  belong to CONS.  $\square$

As discussed in [29], we notice here the existence of a trade-off between the size of the vectors and the freshness of the snapshots. For instance, if  $x$  and  $y$  belong to the same partition and transaction  $T_i$  reads a version  $x_j$ ,  $T_i$  cannot read a version  $y_l$  that committed after a version  $x_k$  posterior to  $x_j$ .

### 7.3 Transaction Lifetime in Jessy

Jessy is a distributed system of processes which communicate by message passing. When a client (not modeled) executes a transaction  $T_i$  with Jessy,  $T_i$  is handled by a coordinator. The coordinator of a transaction can be any process in the system. A transaction  $T_i$  can be in one of the following four states at some process:

- *Executing:* Each non-termination operation  $o_i(x)$  in  $T_i$  is executed optimistically (i.e., without synchronization with other replicas) at the transaction coordinator  $coord(T_i)$ . If  $o_i(x)$  is a read,  $coord(T_i)$  returns the corresponding value, fetched either from the local replica or a remote one. If  $o_i(x)$  is a write,  $coord(T_i)$  stores the corresponding update value in a local buffer, enabling (i) subsequent reads to observe the modification, and (ii) a subsequent commit to send the write-set to remote replicas.



**Algorithm 1** Execution Protocol of Jessy

---

```

1: Variables:
2:   db, submitted, committed, aborted
3:
4: remoteRead(x, Ti)
5:   pre: received ⟨REQUEST, Ti, x⟩ from q
6:          $\exists(x, v, j) \in db : \forall y_l \in rs(T_i) : compat(T_i, x_j, y_l)$ 
7:   eff: send ⟨REPLY, Ti, x, v⟩ to q
8:
9: execute(WRITE, x, v, Ti)
10:  eff:  $up(T_i) \leftarrow up(T_i) \cup \{(x, v, i)\}$ 
11:
12: execute(READ, x, Ti)
13:  eff: if  $\exists(x, v, i) \in up(T_i)$  then return v
14:        else
15:          send ⟨REQUEST, Ti, x⟩ to replicas(x)
16:          wait until received ⟨REPLY, Ti, x, v⟩
17:          return v
18:
19: execute(TERM, Ti)
20:  eff:  $submitted \leftarrow submitted \cup \{T_i\}$ 
21:        wait until  $T_i \in decided$ 
22:        if  $T_i \in committed$  then return COMMIT
23:        return ABORT
24:

```

---

- *Submitted*: Once all the read and write operations of  $T_i$  have executed,  $T_i$  terminates, and the coordinator submits it to the termination protocol. The protocol applies a certification test on  $T_i$  to enforce NMSI. This test ensures that if two concurrent conflicting update transactions terminate, one of them aborts.
- *Committed/Aborted*: When  $T_i$  enters the *Committed* state at  $r \in replicas(T_i)$ , its updates (if any) are applied to the local data store. If  $T_i$  aborts,  $T_i$  enters the *Aborted* state.

## 7.4 Execution Protocol

Algorithm 1 describes the execution protocol in pseudocode. Logically, it can be divided into two parts: action *remoteRead()*, executed at some process, reads an object replicated at that process in a consistent snapshot; and the coordinator *coord(T<sub>i</sub>)* performs actions *execute()* to execute  $T_i$  and to buffer the updates in  $up(T_i)$ .

The variables of the execution protocol are: *db*, the local data store; *submitted* contains locally-submitted transactions; and *committed* (respec-

tively *aborted*) stores committed (respectively aborted) transactions. We use the shorthand *decided* for *committed*  $\cup$  *aborted*.

Upon a read request for  $x$ ,  $coord(T_i)$  checks against  $up(T_i)$  if  $x$  has been previously updated by the same transaction; if so, it returns the corresponding value (line 13). Otherwise,  $coord(T_i)$  sends an (asynchronous) read request to the processes that replicate  $x$  (lines 15 to 17). When a process receives a read request for object  $x$  that it replicates, it returns a version of  $x$  which complies with Theorem 5 (lines 5 to 7).

Upon a write request of  $T_i$ , the process buffers the update value in  $up(T_i)$  (line 10). During commitment, the updates of  $T_i$  will be sent to all replicas holding an object that is modified by  $T_i$ .

When transaction  $T_i$  terminates, it is submitted to the termination protocol (line 20). The execution protocol then waits until  $T_i$  either commits or aborts, and returns the outcome.

## 7.5 Termination Protocol

Algorithm 2 depicts the termination protocol of Jessy. It accesses the same four variables  $db$ ,  $submitted$  and  $committed$ , along with a FIFO queue named  $Q$ .

In order to satisfy GPR, the termination protocol uses a genuine atomic multicast primitive [30]. In our model, this requires that either (i) we form non-intersecting groups of replicas, and an eventual leader oracle is available in each group, or (ii) that a system-wide *reliable* failure detector is available. The latter setting allows Jessy to tolerate a disaster [31].

To terminate an update transaction  $T_i$ ,  $coord(T_i)$  atomic-multicasts it to every process that holds an object written by  $T_i$ . Every such process  $p$  certifies  $T_i$  by calling function  $certify(T_i)$  (line 16). This function returns *true* at process  $p$ , iff for every transaction  $T_j$  committed prior to  $T_i$  at  $p$ , if  $T_j$  write-conflicts with  $T_i$ , then  $T_i$  depends on  $T_j$ . Formally:

$$certify(T_i) \triangleq \forall T_j \in committed : ws(T_i) \cap ws(T_j) \neq \emptyset \Rightarrow T_i \triangleright^* T_j$$

Under partial replication, a process  $p$  might store only a subset of the objects written by  $T_i$ , in which case  $p$  does not have enough information to decide on the outcome of  $T_i$ . Therefore, we introduce a voting phase where replicas of the objects written by  $T_i$  send the result of their certification test in a VOTE message to every process in  $replicas(ws(T_i)) \cup \{coord(T_i)\}$  (lines 17 to 18).

A process can safely decide on the outcome of  $T_i$  when it has received votes from a *voting quorum* for  $T_i$ . A voting quorum  $Q$  for  $T_i$  is a set of

**Algorithm 2** Termination Protocol of Jessy

---

```

1: Variables:
2:   db, submitted, committed, aborted,  $\mathcal{Q}$ 
3:
4: submit( $T_i$ )
5:   pre:  $T_i \in submitted$ 
6:          $ws(T_i) \neq \emptyset$ 
7:   eff: AM-Cast( $T_i$ ) to replicas( $ws(T_i)$ )
8:
9: deliver( $T_i$ )
10:  pre:  $T_i = \text{AM-Deliver}()$ 
11:  eff:  $\mathcal{Q} \leftarrow \mathcal{Q} \circ \langle T_i \rangle$ 
12:
13: vote( $T_i$ )
14:  pre:  $T_i \in \mathcal{Q} \setminus decided$ 
15:         $\forall T_j \in \mathcal{Q}, T_j <_{\mathcal{Q}} T_i \Rightarrow T_j \in decided$ 
16:  eff:  $v \leftarrow certify(T_i)$ 
17:        send  $\langle \text{VOTE}, T_i, v \rangle$  to replicas( $ws(T_i)$ )
18:           $\cup \{coord(T_i)\}$ 
19:
20: commit( $T_i$ )
21:  pre: outcome( $T_i$ )
22:  eff: foreach  $(x, v, i)$  in up( $T_i$ ) do
23:        if  $x \in db$  then  $db \leftarrow db \cup \{(x, v, i)\}$ 
24:         $committed \leftarrow committed \cup \{T_i\}$ 
25:
26: abort( $T_i$ )
27:  pre:  $\neg outcome(T_i)$ 
28:  eff:  $aborted \leftarrow aborted \cup \{T_i\}$ 
29:

```

---

replicas such that for every object  $x \in cert(T_i)$ , the set  $Q$  contains at least one of the processes replicating  $x$ . Formally, a set of processes is a voting quorum for  $T_i$  iff it belongs to  $vquorum(T_i)$ , defined as follows:

$$vquorum(T_i) \triangleq \{Q \subseteq \Pi \mid \forall x \in cert(T_i) : \exists j \in Q \cap replicas(x)\}$$

A process  $p$  makes use of the following (three-values) predicate *outcome*( $T_i$ ) to determine whether some transaction  $T_i$  commits, or not:

$$outcome(T_i) \triangleq$$

```

if  $cert(T_i) = \emptyset$ 
  then true
else if  $\forall Q \in vquorum(T_i), \exists q \in Q,$ 
   $\neg received \langle VOTE, T, \_ \rangle$  from  $q$ 
  then  $\perp$ 
else if  $\exists Q \in vquorum(T_i), \forall q \in Q,$ 
   $received \langle VOTE, T, true \rangle$  from  $q$ 
  then true
else false

```

To commit transaction  $T_i$ , process  $p$  first applies  $T_i$ 's updates to its local data store, then  $p$  adds  $T_i$  to variable *committed* (lines 21 to 24). If instead  $T_i$  aborts,  $p$  adds  $T_i$  to *aborted* (lines 27 to 28).

## 7.6 Correctness of Jessy

We now sketch a correctness proof of Jessy: Proposition 7 establishes that Jessy generates histories in NMSI. Proposition 8 shows that read-only transactions are wait-free. Propositions 9 and 10, respectively, prove that Jessy satisfies obstruction-freedom for updates and non-triviality for NMSI.

### 7.6.1 Safety

**Proposition 6.** *If a transaction  $T_i$  commits (respectively aborts) at some process in  $replicas(ws(T_i)) \cup coord(T_i)$ , it commits (resp. aborts) at every correct process in  $replicas(ws(T_i)) \cup coord(T_i)$ .*

*Proof.* This proposition follows from the properties of atomic multicast, the fact that the queue  $\mathcal{Q}$  is FIFO, the preconditions at lines 14 to 15 in Algorithm 2, and the definitions of *vote()* and *outcome()*.  $\square$

**Proposition 7.** *Every history admissible by Jessy belongs to NMSI.*

*Proof.* We first observe that transactions in Jessy always read committed versions of the objects (line 6 in Algorithm 1). Moreover, we know by Theorem 5 that reads are consistent when Jessy uses dependence vectors, and that this property also holds in case Jessy employs partitioned dependence vectors (Proposition 5). It thus remains to show that histories generated by Jessy are write-conflict free (WCF).

To prove that WCF holds, we consider two independent write-conflicting transactions  $T_i$  and  $T_j$ , and we assume for the sake of contradiction that they

both commit. We note  $p_i$  (resp.  $p_j$ ) the coordinator of  $T_i$  (resp.  $T_j$ ). Since  $T_i$  and  $T_j$  write-conflict, there exists some object  $x$  in  $ws(T_i) \cap ws(T_j)$ . One can show that the following claim holds:

- (C1) For any two replicas  $p$  and  $q$  of  $x$ , denoting  $committed_p$  (resp.  $committed_q$ ) the set  $\{T_j \in committed : x \in ws(T_j)\}$ , at the time  $p$  (resp.  $q$ ) decides  $T_i$ , it is true that  $committed_p$  equals  $committed_q$ .

According to line 21 of Algorithm 2 and the definition of function  $outcome()$ ,  $p_i$  (respectively  $p_j$ ) received a positive VOTE message from some process  $q_i$  (resp.  $q_j$ ) replicating  $x$ . Observe that  $T_i$  (resp.  $T_j$ ) is in variable  $\mathcal{Q}$  at process  $q_i$  (resp.  $q_j$ ) before this process sends its VOTE message. It follows from claim C1 that either (1) at the time  $q_i$  sends its VOTE message,  $T_j <_{\mathcal{Q}} T_i$  holds, or (2) at the time  $q_j$  sends its VOTE message,  $T_i <_{\mathcal{Q}} T_j$  holds. Assume that case (1) holds (the reasoning for case (2) is symmetrical). From the precondition at line 15 in Algorithm 2, we know that process  $q_i$  must wait that  $T_j$  is decided before casting a vote for  $T_i$ . From Proposition 6, we deduce that  $T_j$  is committed at process  $q_i$ . Hence,  $certify(T_i)$  returns *false* at process  $q_i$ ; a contradiction.  $\square$

## 7.6.2 Progress

**Lemma 7.** *For every transaction  $T_i$ , if  $coord(T_i)$  is correct, eventually  $T_i$  is submitted to the termination protocol at  $coord(T_i)$ .*

*Proof.* Transaction  $T_i$  executes all its write operations locally at its coordinator. Now, upon executing a read request on some object  $x$ , if  $x$  was modified previously by  $T_i$ , the corresponding value is returned. Otherwise,  $coord(T_i)$  sends a read request to  $replicas(x)$ . To prove this lemma, we have to show that eventually one of the replica replies to the coordinator.

According to our model, there exists one correct process replica of  $x$ . In what follows, we name it  $p$ . Observe that since links are quasi-reliable,  $p$  eventually receives the read request from  $coord(T_i)$ . Upon receiving this request, process  $p$  tries returning a version of  $x$  compatible with all versions previously read by  $T_i$ .

Consider that Jessy uses dependence vectors (the reasoning for partitioned dependence vectors is similar), and assume, by contradiction, that  $p$  never finds such a compatible version. From the definition of  $compat(T_i, x_j, y_l)$ , this means that the following predicate is always true:

$$\forall (x, v, l) \in db : V(w_l(x_l))[x] < V(r_i(y_j))[x] \\ \vee V(w_l(x_l))[y] > V(r_i(y_j))[y]$$

This means that there exists a version  $x_k$  upon which transaction  $T_i$  depends, and such that  $V(w_k(x_k))[x] = V(r_i(y_j))[x]$ . Transaction  $T_k$  committed at some site. As a consequence, Proposition 6 tells us that eventually  $T_k$  commits at process  $p$ . We conclude by observing that since Jessy satisfies both CONS and WCF,  $V(w_k(x_k))[y] > V(r_i(y_j))[y]$  cannot hold.  $\square$

**Lemma 8.** *For every transaction  $T_i$ , if  $T_i$  is submitted at  $\text{coord}(T_i)$  and  $\text{coord}(T_i)$  is correct, every correct process in  $\text{replicas}(ws(T_i)) \cup \text{coord}(T_i)$  eventually decides  $T_i$ .*

*Proof.* According to Lemma 7 and the properties of atomic multicast, transaction  $T_i$  is delivered at every correct process in  $\text{replicas}(ws(T_i)) \cup \text{coord}(T_i)$ . It is then enqueued in variable  $\mathcal{Q}$  (lines 10 to 11 in Algorithm 2).

Because  $\mathcal{Q}$  is FIFO, processes dequeue transactions in the order they deliver them (lines 14 to 15). The uniform prefix order and acyclicity properties of genuine atomic multicast ensure that no two processes in the system wait for a vote from each other. It follows that every correct replica in  $\text{replicas}(ws(T_i))$  eventually dequeues  $T_i$ , and sends the outcome of function  $\text{certify}(T_i)$  to  $\text{replicas}(ws(T_i)) \cup \text{coord}(T_i)$  (lines 16 to 18).

Since there exists at least one correct replica for each object modified by  $T_i$  eventually every correct process in  $\text{replicas}(ws(T_i)) \cup \text{coord}(T_i)$  collects enough votes to decide upon the outcome of  $T_i$ .  $\square$

**Proposition 8.** *Jessy satisfies WFQ.*

*Proof.* Consider some read-only transaction  $T_i$  and assume that  $\text{coord}(T_i)$  is correct, Lemma 7 tells us that  $T_i$  is eventually submitted at  $\text{coord}(T_i)$ . According to the definition of predicate  $\text{outcome}$ ,  $\text{outcome}(T_i)$  always equals true. Hence, the precondition at line 21 in Algorithm 2 is always true, whereas precondition at line 27 is always false. It follows that  $T_i$  eventually commits.  $\square$

To prove that Jessy satisfies obstruction-freedom for updates (OFU), we make the following two modifications to the protocol:<sup>7</sup>

- (M1) When a process resolves a remote read over some object  $x$ , it always returns the greatest version of  $x$  stored in variable  $db$  that satisfies the precondition at line 6.<sup>8</sup>
- (M2) When a transaction  $T_i$  aborts at the coordinator, a retry mechanism re-executes it. (The client is oblivious of this mechanism.)

<sup>7</sup>Those modifications were omitted in our description of Jessy to ease presentation.

<sup>8</sup>Greatest version has to be understood in the sense of functions  $V$  and  $PV$ .

**Proposition 9.** *Jessy satisfies OFU.*

*Proof.* Consider some update transaction  $T_i$  such that  $coord(T_i)$  is correct. From Lemma 8,  $coord(T_i)$  eventually decides transaction  $T_i$ .

Then, assume that at the time  $T_i$  starts its execution, every write-conflicting transaction has terminated.

By modifications M1 and M2,  $T_i$  eventually depends on every write-conflicting transactions. Thus, the outcome of  $certify(T_i)$  eventually always equals true. Hence, transaction  $T_i$  eventually commits.  $\square$

We now prove that Jessy satisfies non-triviality for NMSI. This results is stated in the case where Jessy employs (non-partitioned) dependence vectors.

**Proposition 10.** *Jessy ensures non-trivial NMSI.*

*Proof.* Consider a replica  $p$  of  $x$  storing version  $x_j$ , and assume an extension of the execution in which  $p$  answers first to a remote read request from  $coord(T_i)$  over  $x$ . Since history  $h.r_i(x_j)$  is in NMSI, it belongs to CONS. Because Jessy use dependence vectors, Theorem 5 tells us that:  $V(r_i(x_j))[x] \geq V(r_i(y_k))[x]$  and  $V(r_i(x_j))[y] \leq V(r_i(y_k))[y]$  hold. According to the preconditions of operation  $remoteRead(x, T_i)$  and modification M1, process  $p$  returns version  $x_j$  to  $coord(T_i)$ .  $\square$

## 8 Related Work

Table 2 compares different partial replication protocols, in terms of time and message complexity (from the coordinator’s perspective), when executing a transaction with  $r_r$  remote reads and  $w_r$  remote writes. A transaction can be of the following three types: a read-only transaction, a local update transaction (the coordinator replicates all the objects accessed by the transaction), or a global update transaction (some object is not available at the coordinator).

Several protocols solve particular instances of the partial replication problem. Some assume that a correct replica holds all the data accessed by a transaction [32, 33]. Others consider that data can be partitioned into conflict sets [34], or that always aborting concurrent conflicting transactions [35] is reasonable. Hereafter, we review in details algorithms that do not make such an assumption.

P-Store [10] is a genuine partial replication algorithm that ensures SER by leveraging genuine atomic multicast. Like in Jessy, read operations are performed optimistically at some replicas and update operations are applied at commit time. However, unlike Jessy, P-Store certifies read-only transactions as well.

A few algorithms [1, 2] offer partial replication with SI semantics. At the start of a transaction  $T_i$ , the algorithm of Armendáriz-Iñigo et al. [2] atomically broadcasts  $T_i$  to all processes. This message defines the consistent snapshot of  $T_i$ . If  $T_i$  is an update transaction,  $T_i$ ’s write set is atomic broadcast to all processes at commit time and each process independently certifies it. The algorithm of Serrano et al. [1] executes a dummy transaction after each commit. As the commit of a transaction is known by all processes, a dummy transaction identifies a snapshot point. This avoids the cost of the start message. As a consequence of the impossibility result depicted in Section 4, none of these algorithms is genuine.

Walter is a transactional key-value store proposed by Sovran et al. [9] that supports Parallel Snapshot Isolation (PSI). PSI is somewhat similar to NMSI; in particular, PSI snapshots are non-monotonic. However, PSI is stronger than NMSI, as it enforces SCONSa: NMSI allows reading versions of objects that have committed after the start of the transaction, as long as it is consistent. On the contrary in PSI, an operation has to read the most recent versions at the time the transaction starts. Enforcing SCONSa does not preclude any anomaly, and it increases the probability that a write skew, or a conflict between concurrent writes occurs. To ensure PSI, Walter relies on a single master replication schema per object and 2PC. After the transaction commits, it is propagated to all processes in the system in the



Algorithm	Cons.	Gen- uine?	Multi- Master?	Message Complexity	Time complexity		
					Read-only	Global Update	Local Update
P-Store [10]	SER	yes	yes	$O(n^2)$	$(r_r \times 2\Delta) + 4\Delta$	$(r_r \times 2\Delta) + 5\Delta$	$4\Delta$
GMU [36]	US	yes	yes	$O(n^2)$	$r_r \times 2\Delta$	$(r_r \times 2\Delta) + 2\Delta$	$2\Delta$
SIPRe[2]	SI	no	yes	$O(N^2)$	$(r_r \times 2\Delta) + 3\Delta$	$(r_r + w_r) \times 2\Delta + 6\Delta$	$6\Delta$
Serrano[1]	SI	no	yes	$O(N^2)$	$r_r \times 2\Delta$	$(r_r + w_r) \times 2\Delta + 3\Delta$	$3\Delta$
Walter [9]	PSI	no	no	$O(N)$	$r_r \times 2\Delta$	$(r_r \times 2\Delta) + 2\Delta$	$2\Delta \mid 0$
Jessy	NMSI	yes	yes	$O(w_r^2)$	$r_r \times 2\Delta$	$(r_r \times 2\Delta) + 5\Delta$	$4\Delta$

*Message complexity: number of messages sent on behalf of transaction. Time complexity: delay for executing a transaction.  $N$ : number of replicas;  $n$ : number of replicas involved in transaction;  $\Delta$ : message latency between replicas;  $r_r$ : number of remote reads;  $w_r$ : number of remote writes. The latency of atomic broadcast (resp. atomic multicast) is considered  $3\Delta$  (resp  $4\Delta$ ) during solo step execution [31].*

Table 2: Comparison of partial replication protocols

background before it becomes visible.

More recently, Peluso et al. [36] proposed GMU, an algorithm that supports an extended form of update serializability. GMU relies on vector clocks to read consistent snapshots. At commit time, both GMU and Walter use locks to commit transactions. Because locks are not ordered before voting (contrary to P-Store and Jessy), these algorithms are subjected to the occurrence of distributed deadlocks, and scalability problems leading to poor performance for global update transactions [37, 38].

## 9 Conclusion

Partial replication and genuineness are two key factors of scalability in replicated systems. This paper shows that ensuring snapshot isolation (SI) in a genuine partial replication (GPR) system is impossible. To state this impossibility result, we introduce four properties whose conjunction is equivalent to SI. We show that two of them, namely snapshot monotonicity and strictly consistent snapshots cannot be ensured.

To side step the incompatibility of SI with GPR, we propose a novel consistency criterion named NMSI. NMSI prunes most anomalies disallowed by SI, while providing guarantees close to SI: transactions under NMSI always observe consistent snapshots and two write-conflicting concurrent updates never both commit.

The last contribution of this paper is Jessy, a genuine partial replication protocol that supports NMSI. To read consistent partial snapshots of the system, Jessy uses a novel variation of version vectors called dependence vectors. An analytical comparison between Jessy and previous partial replication protocol shows that Jessy contacts fewer replicas, and that, in addition, it may commit faster.

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