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Forwarders vs. centralized server: an evaluation of two approaches for locating mobile agents

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Abstract

This paper evaluates and compares the performance of two approaches for locating an agent in a mobile agent environment. The first approach dynamically creates a chain of forwarders to locate a moving agent whereas the second one relies on a centralized server to perform this task. Based on a Markov chain analysis, we compute the performance of each scheme (time to reach an agent, number of forwarders) and compare them first with simulations and second with experimental results obtained by using ProActive, a Java library. Depending on the system parameters we identify the best scheme and observe that over a LAN the server yields the best performance whereas the forwarders yield the best performance over a MAN.

Key words: Mobile code, migration, centralized server, forwarders, Markov chain.

1 Introduction

The Internet has allowed the creation of huge amounts of data located on many different machines. Performing complex operations on some data requires that the data be transferred first to the machine on which the operations are to be executed. This transfer may require a non-negligible amount of bandwidth and may seriously limit performance if it is the bottleneck. However, instead of moving the data to the code, it is possible to move the code to the data, and perform all the operations locally. This simple idea has led to a new paradigm called code-mobility [17]. In this paradigm, a mobile object – sometimes called

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an agent – is given a list of destinations and a series of operations to perform on each one of them. The mobile object will visit all of the destinations, perform the requested operations and possibly pass the result on to another object. Any machine willing to receive an agent must provide an agent-platform which is a placeholder where the agent is executed.

Code mobility has recently received a lot of attention because of its wide application to fields ranging from e-commerce (e.g. looking for the lowest fare on many different sites) to data mining [6]. Mobility can be implemented as a service provided by an operating system [14]; however this severely limits its usefulness in a heterogeneous environment such as the Internet. Another solution is to use a library which provides an application with all of the necessary features [1,3,8,9,11,16,18].

Any mobility mechanism must first provide a way to migrate code from one host to another. It must also ensure that any communication posterior to a migration will not be impaired by it, namely that two objects should still be able to communicate even if one of them has migrated. Such a mechanism is referred to as a routing mechanism or even as a location mechanism since it often relies on the knowledge of the location of the objects to ensure communications. Two location mechanisms are widely used: the first one uses a centralized (location) server which keeps track of the location of mobile objects, whereas the second one relies on special objects – called forwarders – whose role is to forward a message to the mobile object. A more careful description of these approaches will be given later on.

Mobility raises several concerns, among them security [5] (of both the mobile agent and the host sheltering it) and performance issues [4,7]. In [7] the complexity of using forwarders is extensively studied whereas [4] addresses fault-tolerance properties in forwarder-based mechanisms. In this paper we will only focus on performance issues and, more specifically, on the cost of communication in presence of migration. To the best of our knowledge, this is the first time that such an analysis is performed. In [11] the authors only give intuitive criteria on how to select the proper location scheme under certain circumstances. Our analysis can be used to select the best scheme based on its response time. It also allows us to compute the average number of forwarders, which is useful to study the fault-tolerance of forwarding schemes [4].

In this work we develop Markovian models of the forwarders and of the location server as implemented in ProActive [16], a Java library that provides all the necessary primitives for code mobility. Closed-form expressions for various performance measures are derived, including the time needed to reach an agent and the mean number of forwarders. These expressions are in turn used to evaluate the cost of location under various network conditions and for different applications. For the purpose of validation, we have developed
for each mechanism both an event-driven simulator and a benchmark that uses ProActive. Simulations and experiments conducted over a LAN and a MAN have validated both models and have shown that no scheme performs uniformly better than the other, thereby justifying the present research.

The paper is organized as follows: preliminary definitions are introduced in Section 2; the forwarding scheme is presented and evaluated in Section 3 and the centralized server scheme is investigated in Section 4. Simulated and experimental results are reported in Section 5 as well as a theoretical comparison between both approaches. Concluding remarks are given in Section 6.

2 Definitions and notation

In this section we introduce several random variables (rvs) that we will use to construct our models. Throughout the paper a mobile object will indifferently be called a mobile agent or simply an object or an agent.

The $i$-th message is sent by the source to the agent at time $a_i$ and the communication is over at time $d_i := a_i + \tau_i$. The rv $\tau_i$ – referred to as the ($i$-th) communication time – is a scheme-dependent quantity that will be defined later on for each mechanism (forwarders and centralized server). In the time-interval $(d_i, a_{i+1})$ no message is generated by the source. Let $w_{i+1} := a_{i+1} - d_i$ be the length of this time-interval and assume that $w_1 := a_1$ and that no message is generated in $[0, a_1)$.

The $j$-th migration of the mobile occurs at time $m_j > 0$ and it requires the mobile $p_j$ units of times to reach its new location. During a migration period the agent is unreachable. The mobile then spends $u_{j+1}$ units of time at its $j$th location, time during which it can be reached by a message, and then initiates a new migration. We set $u_1 := m_1$ and assume that the mobile does not migrate in $[0, m_1)$ (see Figure 1).

The following assumptions will be enforced throughout the paper:

**A1** The *input* sequences $\{w_i, i \geq 1\}$, $\{p_j, j \geq 1\}$ and $\{u_k, k \geq 1\}$ are assumed to be mutually independent *renewal* sequences of rvs such that $w_i$, $p_j$ and $u_k$
are exponentially distributed rvs with parameters $\lambda > 0$, $\delta > 0$ and $\nu > 0$, respectively.

3 The forwarders

3.1 Description

Forwarding techniques were first introduced in distributed operating systems like DEMOS/MP [15] for finding mobile processes. The mechanism is straightforward: on leaving a host, a process leaves a special reference, called a forwarding reference which points toward its next location. As the system runs, chains of forwarders are built. A consequence of this mechanism is that a caller does not usually know the location of the callee. A special built-in mechanism called short-cutting allows the update of the address as soon as a communication takes place. When a forwarded message reaches a mobile object, the latter communicates its new location to the caller. As a result, all subsequent requests issued by this caller will not go through the existing forwarders – which are shortcut.

An illustration of the short-cutting feature is given in Figure 2: a message is sent by the source to the last known location of the agent (Host B). Since the agent is no longer at this location, the message is then forwarded to the host that was next visited by the agent (Host C). Again, the agent has already moved when the message reaches Host C and the message is forwarded to the next visited host (Host D) where the agent is finally located. A location message is then sent by the agent (located at host D) to the source and the next message will be sent by the source to Host D.

Fig. 2. The short-cutting feature in the forwarding mechanism.

In order to keep the same semantic as with a static program (i.e. with no mobile object) one has to introduce constraints. Mainly, communications through a
chain of forwarders should be synchronous, i.e. the caller stays blocked during the communication time. With these assumptions we can now describe the protocol in use:

- Upon migration, a mobile object leaves a forwarder on the current site;
- This forwarder is linked to the mobile object on the remote host;
- No communication can occur when the object is migrating;
- When receiving a message, the forwarder sends it to the next hop (possibly the mobile object);
- Any successful communication places the mobile object one hop away of the caller.

The above protocol is implemented in various Java libraries (MOA [11], ProActive [16] and Voyager [18]) except for the short-cutting feature that is triggered by the agent at the end of the message processing.

3.2 A Markovian analysis of the forwarders

In this section we will evaluate the performance (number of forwarders in Section 3.2.1 and response time 3.2.2) of the mechanism introduced in Section 3.1 through a Markovian analysis. We will assume that a mobile object does not return to a previously visited site where there is still an active forwarder, i.e. it does not migrate twice (or more) to a particular site between two consecutive epochs $d_i$ (see Section 2). Hence, there can be no loops within a chain. It is clear that under these conditions the length of a chain can extend to infinity if the number of hosts is infinite.

A forwarding mechanism is well represented by the chains of forwarders that it produces. In an application a chain of forwarders connects a single source to a single agent and its dynamic is not affected by other objects; there will be as many chains as there are pairs source-agent. It suffices then to study the behavior of one chain to evaluate the performance of the forwarders approach. To study the dynamics of a chain, one should take into account the state of a chain and the states of the source and the agent at its endpoints. Notice that this doesn’t place any assumption on the number of objects (source or agent) in the application.

From the description given in Section 3.1, it can be seen that at any time the system is in one of the following states (see Figure 3):

- States $(i, 0, 0)$, $i \geq 1$, indicate that the agent is available (i.e. not migrating) and located $i$ hops away from the source, and that no message is traveling;
- State $(1, 0, 1^*)$ indicates that the agent is available and located one hop away from a message, and the latter has never been through any forwarder;
State \((1, 0, 1)\) indicates that the agent is available and located one hop away from a message, the latter having already gone through at least one forwarder; 

States \((i, 0, 1), \ i \geq 2\) indicate that a message is traveling, the agent is available and located \(i\) hops away from the message; 

States \((i, 1, k), \ i \geq 1\), indicate that the agent is migrating and that before the initiation of its migration it was located \(i\) hops away from the source if no message is traveling \((k = 0)\) or it was located \(i\) hops away from the message if a message is traveling \((k = 1)\); 

State \((0, 1, 1)\) indicates that a message that has gone through at least one forwarder and the agent are at the same location but that the agent is migrating (i.e. the agent has initiated a migration just before the arrival of the message); 

State \((0, 0, 1)\) indicates that the message has reached the agent after having traveled through at least one forwarder and that the agent is currently communicating its new position to the source.

State \((1, 0, 1^*)\) takes into account the fact that if a new message reaches the agent after exactly one hop and that the agent has not initiated a migration before the arrival of the message then the cycle is over and the source can transmit a new message; otherwise, the agent will have to communicate its location to the source once the message will have reached it.

Under the enforced assumption

**A2** The traveling time of a message from one host (possibly the source) to the next one (possibly the agent) is an exponential rv with parameter \(\gamma > 0\). The successive traveling times are assumed to be mutually independent and independent of the input sequences \(\{w_i\}_i, \ \{p_i\}_i\) and \(\{u_i\}_i\) introduced in Section 2,

and assumption **A1**\(^4\), it is easily seen that the sojourn time in each state is exponentially distributed and that any state can be reached from any other state in a finite number of steps. In other words, the process defined above is an irreducible Markov process on \(\mathcal{E} := \{(0, 0, 1), (0, 1, 1), (1, 0, 1^*), (i, j, k), \ i \geq 1, \ j, k = 0, 1\}\).

The transition rates are indicated in Figure 3: a transition with rate \(\lambda\) indicates that a new message has been generated; a transition with rate \(\nu\) indicates that the agent has initiated a migration; a transition with rate \(\delta\) indicates the end of a migration; a transition with rate \(\gamma\) indicates that the message has reached the next host on its route (possibly the agent).

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\(^4\) Assumptions **A1** and **A2** are mainly made for sake of mathematical tractability. We have however observed in our experiments that our models are fairly robust to deviations from these assumptions – see Section 5.
Let $p_{i,j,k}$ be the stationary probability that the process is in state $(i, j, k) \in \mathcal{E}$. If the Markov process is ergodic then the stationary probabilities are the unique strictly positive solution of the Chapman-Kolmogorov (C-K) equations [10]

\[
\begin{align*}
(\lambda + \nu) p_{1,0,0} &= \gamma (p_{0,0,1} + p_{1,0,1}) \\
(\lambda + \nu) p_{i,0,0} &= \delta p_{i-1,1,0} & i &= 2, 3, \ldots \\
(\lambda + \delta) p_{i,1,0} &= \nu p_{i,0,0} & i &= 1, 2, \ldots \\
\delta p_{0,1,1} &= \gamma p_{1,1,1} \\
(\delta + \gamma) p_{1,1,1} &= \nu (p_{1,0,1} + p_{1,1,1}) + \lambda p_{1,1,0} + \gamma p_{2,1,1} \\
(\delta + \gamma) p_{i,1,1} &= \nu p_{i,0,1} + \lambda p_{i,1,0} + \gamma p_{i+1,1,1} & i &= 2, 3, \ldots \\
(\nu + \gamma) p_{1,0,1} &= \lambda p_{1,0,0} \\
&= p_{1,0,1} \\
(\nu + \gamma) p_{i,0,1} &= \delta p_{i-1,1,1} + \gamma p_{i+1,0,1} & i &= 2, 3, \ldots 
\end{align*}
\]

such that $\sum_{(i,j,k) \in \mathcal{E}} p_{i,j,k} = 1$. We will not try to solve equations (1)-(10) explicitly. Instead, we will use the standard $z$-transform approach to characterize the ergodicity condition and the invariant measure of the Markov process. To this end, define for $|z| \leq 1$

\[
f(z) := \sum_{i=1}^{\infty} z^i p_{i,0,0}; \quad g(z) := \sum_{i=1}^{\infty} z^i p_{i,1,0}; \quad h(z) := \sum_{i=0}^{\infty} z^i p_{i,0,1}; \quad k(z) := \sum_{i=0}^{\infty} z^i p_{i,1,1}
\]

the $z$-transform of the stationary probabilities $\{p_{i,0,1}\}_{i \geq 0}$, $\{p_{i,1,0}\}_{i \geq 1}$, $\{p_{i,1,1}\}_{i \geq 0}$ and $\{p_{i,0,0}\}_{i \geq 1}$, respectively. Last, introduce $F(z) := p_{1,0,1} + f(z) + g(z) + h(z) + k(z)$ ($|z| \leq 1$) the $z$-transform of the stationary probabilities $p_{i,j,k}$, $(i, j, k) \in \mathcal{E}$. $\hat{F}(z)$ is determined in the following proposition (proof in Appendix A):

**Proposition 3.1 (z-transform of the Markov process)**

The Markov process depicted in Figure 3 is ergodic if and only if $1/\gamma < 1/\nu + 1/\delta$. In steady-state, the elements of $\hat{F}(z)$ are given by
\[ f(z) = \frac{z \gamma(\lambda + \delta)(\lambda + \nu)(\gamma + \nu)}{\nu(\nu + \lambda + \gamma) a(z)} p_{1,0,1} \quad g(z) = \frac{z \gamma(\lambda + \nu)(\gamma + \nu)}{(\nu + \lambda + \gamma) a(z)} p_{1,0,1} \]

\[ h(z) = -\frac{z^2 \delta \gamma}{b(z)} p_{0,1,1} + \left[ \frac{\delta(\nu - \gamma) z^2 - \gamma(\nu + \delta) z + \gamma^2}{b(z)} \right] p_{1,0,1} \]

\[ k(z) = \frac{\gamma(\gamma - z(\gamma + \nu))}{b(z)} p_{0,1,1} - \frac{\gamma(\lambda + \nu)(\gamma + \nu)(\lambda + \delta)(\lambda + \nu)}{(\nu + \lambda + \gamma) a(z) b(z)} z^2 p_{1,0,1} \]

\[ p_{1,0,1} = \frac{\lambda \gamma}{\nu(\nu + \lambda + \gamma)} p_{1,0,1} \]

\[ p_{0,1,1} = \frac{1 + \left[ \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right) + \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right)^2 \left( \frac{(\gamma - z_0(\gamma + \nu))(\nu + \lambda + \gamma) a(z_0)}{(\lambda + \nu)(\gamma + \nu)(\lambda + \delta)(\lambda + \nu)) z_0^2 \right) \right]}{1 + \left[ \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right) + \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right)^2 \left( \frac{(\gamma - z_0(\gamma + \nu))(\nu + \lambda + \gamma) a(z_0)}{(\lambda + \nu)(\gamma + \nu)(\lambda + \delta)(\lambda + \nu)) z_0^2 \right) \right]} \]

\[ p_{1,0,1} = \frac{1 + \left[ \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right) + \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right)^2 \left( \frac{(\gamma - z_0(\gamma + \nu))(\nu + \lambda + \gamma) a(z_0)}{(\lambda + \nu)(\gamma + \nu)(\lambda + \delta)(\lambda + \nu)) z_0^2 \right) \right]}{1 + \left[ \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right) + \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right)^2 \left( \frac{(\gamma - z_0(\gamma + \nu))(\nu + \lambda + \gamma) a(z_0)}{(\lambda + \nu)(\gamma + \nu)(\lambda + \delta)(\lambda + \nu)) z_0^2 \right) \right]} \]

with \(a(z) := -\delta \nu z + (\lambda + \delta)(\lambda + \nu)\), \(b(z) := \delta \nu z^2 - \gamma(\gamma + \nu + \delta) z + \gamma^2\) and \(z_0 := \gamma \left( \gamma + \nu + \delta - \sqrt{(\gamma + \nu + \delta)^2 - 4 \nu \delta} \right) / (2 \nu \delta)\).

### 3.2.1 The expected number of forwarders

In this section we compute the expected number of forwarders in steady-state between the agent and the message (resp. the source if there is no message). Let \(q(i)\) be the probability that the agent is located \(i \geq 1\) hops away from a message (resp. from the source if there is no message), which corresponds to a situation where there are exactly \(i - 1\) forwarders in the system. Clearly,

\[ q(i) = p_{i,0,0} + p_{i,1,0} + p_{i,0,1} + p_{i,1,1} + 1(i = 1) p_{1,0,1} \quad i = 1, 2, \ldots \]

and the mean number \(N\) of forwarders is given by

\[ N = \sum_{i=1}^{\infty} (i-1) q(i) = f'(1) + g'(1) + h'(1) + k'(1) + p_{1,0,1} - (1 - p_{0,0,1} - p_{0,1,1}) \]

where \(\phi'(1)\) denotes the derivative of \(\phi(z)\) at point \(z = 1\). From Proposition 3.1 and (8) we find that

\[ N = \frac{\delta^2(\gamma^2 - \gamma \nu + \nu^2)(1 - p_{1,0,1})}{\gamma(\delta + \nu)(\gamma(\delta + \nu) - \delta \nu)} + \frac{\nu \gamma(\gamma + \lambda)(\gamma + \nu)}{\lambda^2(\nu + \lambda + \gamma)} - \frac{\gamma^2 - \nu^2}{\nu} \frac{\delta p_{1,0,1}}{\gamma(\delta + \nu) - \delta \nu} \]

where \(p_{1,0,1}\) is given in (16). The average number of forwarders is expected to grow when \(\delta\) or \(\nu\) increases and it should back off when \(\lambda\) or \(\gamma\) increases which...
is illustrated in Figure 4. The crosses in each graph correspond to the same values of the model parameters: $\lambda = 1$, $\nu = 10$, $\delta = 20$, $\gamma = 50$. In Figure 4, the plots for $\delta = 40 s^{-1}$ (lower graph at the left) and $\gamma = 9 s^{-1}$ (lower graph at the right) depict the behavior of our model when the ergodicity condition is close to being violated. In that situation, the expected number of forwarders grows to infinity. As shown in [4] $N$ can be used to evaluate the fault-tolerance of the protocol. This issue is not addressed here.

![Graphs showing expected number of forwarders](image)

*Fig. 4. The expected number of forwarders.*

### 3.2.2 The expected communication time

In this section we determine the expected communication time. The communication time is the time needed to a message to reach the mobile object, to which we must add the time it takes to the mobile object to send its new position to the source in case the message has to go through at least one forwarder. If the message reaches the mobile object after exactly one hop then there is no need to the mobile object to send its position to the source since the source knows it; in this case, the communication terminates once the message has reached the object, thereby justifying the definition of the communication time given above.

At the end of a communication the agent is not migrating, the source idles and it is one hop away from the agent. This corresponds to state $(1, 0, 0)$. At this time, the source stays idle for an exponentially distributed duration with mean
After this idling period a new communication is initiated and the system can be in any one of the following states: \((1, 0, 1^*)\), \((i, 0, 1)\) for \(i \geq 2\), or \((i, 1, 1)\) for \(i \geq 1\). Define \(T_{i,j,k}\) with \(i \geq 1\), \(j = 0, 1\), \(k = 1\) or with \((i, j, k) = (1, 0, 1^*)\), as the expected communication time given that a message was generated when the system was in state \((i, j, k)\) just after the generation of the message.

The expected communication time \(T_F\) (subscript \(F\) refers to “Forwarders”) is given by

\[
T_F = q_F(1, 0, 1^*) T_{(1, 0, 1^*)} + \sum_{i=2}^{\infty} q_F(i, 0, 1) T_{(i, 0, 1)} + \sum_{i=1}^{\infty} q_F(i, 1, 1) T_{(i, 1, 1)}
\]

where \(q_F(i, j, k)\) denotes the probability of reaching state \((i, j, k)\) given that the process initiated in state \((1, 0, 0)\). It actually represents the probability that a communication starts when the system moves from state \((i, j, 0)\) to state \((i, j, k)\). With the help of Figure 3 we find that (with \(r := \frac{\delta}{(\lambda+\delta)(\lambda+\nu)} < 1\))

\[
q_F(1, 0, 1^*) = \frac{\lambda}{\lambda+\nu}; \quad q_F(1, 0, 1) = \frac{\lambda(\lambda+\delta)}{\delta\nu} r^i; \quad q_F(1, 1, 1) = \frac{\lambda}{\delta} r^j
\]

with \(i \geq 2\) and \(j \geq 1\), so that

\[
T_F = \frac{\lambda}{\lambda+\nu} T_{1,0,1^*} + \frac{\lambda(\lambda+\delta)}{\delta\nu} (G(r) - r T_{1,0,1} - T_{0,0,1}) + \frac{\lambda}{\delta} (H(r) - T_{0,1,1})
\]

with \(G(z) := \sum_{i=0}^{\infty} z^i T_{i,0,1}\) and \(H(z) := \sum_{i=0}^{\infty} z^i T_{i,1,1}\). It remains to determine the generating functions \(G(z)\) and \(H(z)\) at \(z = r\). To this end we will use the following recursive equations that follow from the Markovian description of the protocol displayed in Figure 3:

\[
T_{1,0,1^*} = \frac{1}{\nu+\gamma} + \frac{\nu}{\nu+\gamma} T_{1,1,1}; \quad T_{0,0,1} = \frac{1}{\nu}; \quad T_{0,1,1} = \frac{1}{\gamma} + T_{1,0,1}
\]

\[
T_{i,0,1} = \frac{1}{\nu+\gamma} + \frac{\nu}{\nu+\gamma} T_{i,1,1} + \frac{\gamma}{\nu+\gamma} T_{i-1,0,1} \quad i = 1, 2, \ldots
\]

\[
T_{i,1,1} = \frac{1}{\delta+\gamma} + \frac{\delta}{\delta+\gamma} T_{i+1,0,1} + \frac{\gamma}{\delta+\gamma} T_{i-1,1,1} \quad i = 1, 2, \ldots
\]

which yields

\[
(\nu + \gamma (1-z)) G(z) - \nu H(z) = \frac{1}{1-z} + \frac{\nu}{\gamma} - \nu T_{0,0,1}
\]

\[-\delta G(z) + (\delta + \gamma (1-z)) z H(z) = \frac{z}{1-z} - \frac{\delta}{\gamma} + \gamma z T_{0,1,1}.
\]

Solving for \(G(z)\) and \(H(z)\) gives
\[ G(z) = \frac{1}{D(z)} \left( \frac{(\delta + \nu)}{1 - z} + \frac{\nu}{\gamma} (1 - z)(\gamma z - \delta) + \gamma z + \nu(\gamma z - \delta) z T_{0,1,1} \right) \]

\[ H(z) = \frac{1}{D(z)} \left( \frac{(\delta + \nu)}{1 - z} + (\gamma + \delta) z - (\gamma^2 z^2 - \gamma(\gamma + \nu) z + \delta \nu) T_{0,1,1} \right) \]

with \( D(z) := (z - 1)(\gamma^2 z^2 - \gamma(\gamma + \nu + \delta) z + \delta \nu) \). Finally,

\[ T_F = \frac{1}{\alpha(\lambda)} \left[ ((\lambda + \delta)(\lambda + \nu) - \gamma \nu) T_{0,1,1} - \frac{(\lambda + \delta)(\lambda + \nu + \delta)}{\delta} \right. \]

\[
- \left. \frac{(\lambda + \delta)(\lambda + \nu)(\lambda(\lambda + \nu) + \nu(\gamma + \nu)) + \delta \gamma \nu \lambda}{\lambda(\lambda + \nu)(\gamma + \nu)} \right] \]

with \( \alpha(\lambda) := (\lambda + \delta)(\lambda + \nu) - \gamma(\lambda + \nu + \delta) \). It remains to identify the constant \( T_{0,1,1} \) in (18). This can be done by noticing that \( \alpha(\lambda) \) has a single non-negative zero \( \lambda_0 := (\gamma - \nu - \delta + \sqrt{(\gamma + \nu + \delta)^2 - 4\delta \nu}) / 2 \). In order for \( T_F \) to be well-defined for all non-negative values of \( \lambda \), the coefficient of \( 1/\alpha(\lambda) \) in (18) must vanish when \( \lambda = \lambda_0 \), which gives us an extra relation from which we can determine \( T_{0,1,1} \). We obtain \( T_{0,1,1} = (\gamma(\lambda_0 + \nu + \delta) + \delta \lambda_0) / (\gamma \delta \lambda_0) \). A routine application of l’Hôpital’s rule gives \( T_F \) when \( \lambda = \lambda_0 \).

4 Centralized server

4.1 Description

An alternative to the forwarders approach for locating a mobile object is to use a location server. Such a server keeps track of the location of mobile objects in a database. Servers like this are widely used in the Internet. For instance, the Domain Name Server [12] uses a hierarchically organized servers to associate location (IP address) to symbolic name. For sake of simplicity, we will consider here a single centralized server, although many different schemes can be conceived to improve speed and reliability. We will also assume that each object (source or mobile agent) knows the location of this server.

The idea behind location server is simple: each time a mobile object migrates, it informs the server of its new location. Whenever the source wants to reach the mobile, it sends a message to the last known location of the mobile; if this communication fails, then the source sends a location request to the server. We now give a careful description of the protocol used by the source and the mobile object to communicate with the server:

- The Mobile Object
Step 1: Performs the migration;
Step 2: Sends its new location to the server.

• The Source
  Step 1: Issues a message to the mobile object with the recorded location.
  Upon failure goes to Step 2;
  Step 2: Queries the server to have the current location of the mobile object;
  Step 3: Issues a message to the mobile object with the location provided
          by the server. Upon failure, returns to Step 2.

The above protocol is implemented in various Java libraries (MOA [11] and
ProActive [16]). It is rather straightforward, the only issue being when the
agent is migrating while the source receives a reply from the server. This
situation is illustrated in Figure 5: the server does not give the correct location
of the agent to the source since the agent has initiated a migration in the mean-
time. As a result, the source will have to send a second location request
          to the server (event no. 9) before finally being able to reach the agent (event no.

Regarding the service policy, there exist several different schemes that can be
implemented. In ProActive [16], the performance of the server has been opti-
mized so that it does not act as a bottleneck. The buffer at the server is com-
pletely partitioned in the sense that each pair source-agent possesses its own
queue. The server polls these different queues and attends the queue having
the oldest query. Within each queue, there is a priority scheduling mechanism
that gives priority to update requests over location requests. If a queue
contains several update requests then only the newest is processed and the
others are destroyed. The service policy is non-preemptive since ProActive is
a Java library (the execution of a Java method cannot be stopped). As a conse-
quence an update request under processing cannot be preempted by a more
recent one, which may harm the performance of the protocol. (Notice that

Fig. 5. One possible scenario in the centralized approach from the source point of
view.

Step 1: Wrong location in Reply
Step 2: Correct location in Reply

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Step 1 {1} fail, step 2 {3, 5}, step 3 {7} fail, step 2 {9, 10}, successful step 3 {11}
only the queue that is attended by the server may have two update location requests, all other ones having at most one single pending update request.) Fortunately, it has been observed that this event is very rare in practice. Another consequence of this scheme is that upon serving a location request, the server has to check whether the corresponding queue contains an update request coming from the object in which case the location request is sent back to the queue; otherwise, it sends the position of this object (as found in the database) to the source. Finally, communications in ProActive between the server and the different sources are synchronous. Therefore, a source that has sent a location request cannot perform any other task before it gets a reply from the server, which implies there is at most one pending location request per source at the server.

4.2 A Markovian analysis of the server

An exact modeling of the centralized approach would consist in modeling the system as a polling server with vacations. In this section, we develop an approximate model that considers a single queue – thereby a single source communicating with a single agent – where the processing speed of the server is reduced to account for the contention due to the potential presence of several agents/sources. This queue may have at most a single location request and two update requests. By arguing that it is highly unlikely that a coming update request finds an update request (from the same agent) being processed, we will assume that there can be at most one pending update request at the queue or, in other words, that an update request under processing is preempted by a more recent one (cf. Section 4.1). The latter restriction can easily been removed at the expanse of enlarging the state-space (which would yield a 29.6% increase in the number of states).

We assume that the set of assumptions A1 holds (cf. Section 2). Note, however, that in this context \( p_j \) will represent the sum of the \( j \)-th traveling time of the agent to its new host and the travel time of the associated update request to the server site. We further assume that the traveling times between the source and the presumed location of the mobile object (resp. between the source and the location server) are i.i.d. exponentially distributed rvs with parameter \( \gamma_1 > 0 \) (resp. \( \gamma_2 > 0 \)). Finally, we assume that the service times – regardless of the query type – are i.i.d. exponentially distributed rvs with parameter \( \mu > 0 \) (notice that if there are only one source and one agent in an application \( \mu \) would be the server processing speed). All these rvs are assumed to be mutually independent.

We model the system behavior by a finite-state Markov process whose transition diagram and rates are given in Figure 6. A state has the representation
(i, j, k) with i \in \{A, B, \ldots, G\}, j \in \{0, 0^*, 1, 1^*\} and k \in \{0, 1, 1^*, 2\}, where the 2-dimensional vectors A, B, \ldots, G are defined in Figure 6.

More precisely, the vector \( i = (i_1, i_2) \) represents the state of a queue at the server, namely, the type of the messages in the queue, with \( i_l \in \{0, \lambda, \nu\} \) the type of the message that occupies the \( l \)-th position in the queue (\( l = 1, 2 \)). By convention, \( i_l = 0 \) (resp. \( i_l = \lambda, i_l = \nu \)) indicates that the \( l \)-th position is not occupied (resp. the \( l \)-th position is occupied by a location request, the \( l \)-th position is occupied by an update request). Recall that update requests have non-preemptive priority over a location request and that an arriving update request that finds one update request will destroy it at once.

The component \( j \in \{0, 0^*, 1, 1^*\} \) in the state description \((i, j, k)\) represents the state of the object: \( j = 1 \) (resp. \( j = 1^* \)) indicates that the object is migrating and that the source knows (resp. does not know) the location of the host that the object is leaving; similarly, \( j = 0 \) (resp. \( j = 0^* \)) indicates that the object is not migrating and that the source knows (resp. does not know) its location.

Finally, the component \( k \in \{0, 1, 1^*, 2\} \) in the state description \((i, j, k)\) represents the state of the source: \( k = 0 \) if the source has no activity, \( k = 1 \) if it has sent a message to the object, \( k = 1^* \) if the message sent by the source has reached the host that the object is leaving, and \( k = 2 \) if the source has sent a location request to the server. The latter only occurs if the presumed location of the active object is no longer valid.

The system enters state \((A, 0, 0)\) just after the end of a communication (defined as the instant when a message has reached the agent). It remains in that state for an exponentially distributed duration with mean \( 1/\lambda \), and then a new message is generated by the source. The time that elapses between the generation of a new message by the source and the next visit to state \((A, 0, 0)\) is the communication time (i.e. quantities \( \tau_i \)'s introduced in Section 2). In other words, the successive communication times \( \{\tau_i\} \) are initialized when \( k \) goes from 0 to 1, and are stopped when \( k \) goes from 1 to 0. Each time a communication fails, a request is issued to the server and \( k \) switches from 1 to 2. As soon as the server replies, \( k \) switches back to 1 and the message is re-issued by the source to the location of the object returned by the server.

Under the above description/assumptions, the process depicted in Figure 6 is an irreducible finite-state Markov process on the state-space \( \mathcal{F} \), where \( \mathcal{F} \) is the set of all states indicated in Figure 6 (\( \mathcal{F} \) contains 27 elements). Let \( p = \{ p_{i,j,k}, (i, j, k) \in \mathcal{F} \} \) be the stationary probability of this Markov process (\( p \) exists since an irreducible finite-state Markov process is ergodic). If \( Q \) denotes the infinitesimal generator of this Markov process (whose elements can easily be identified from Figure 6), then we know (see e.g. [10]) that \( p \) is the unique solution of the system of linear equations \( p.Q = 0, p.1 = 1 \), which
where $q$ can be solved by using a routine numerical procedure.

4.2.1 The expected communication time

We are interested in finding the expected communication time, denoted as $T_S$ (subscript $S$ refers to “Server”). Recall that a communication begins when the source – after an idling period with mean $1/\lambda$ – sends a message to the active object and terminates when the object is reached. It is seen from Figure 6 that a message may only be generated when the system is in 6 distinct states – all states where $k = 0$. As soon as $k = 1$ a message is generated. Hence, a communication may start only when the system is in one of the following states: $(A, 0, 1), (A, 1, 1), (A, 0^*, 1), (A, 1^*, 1), (B, 0^*, 1)$ or $(B, 1^*, 1)$.

Let $T_{i,j,k}$ denote the expected time to hit state $(A, 0, 0)$ starting from state $(i,j,k)$. The expected response time $T_S$ of the system is given by

$$T_S = \sum_{j=0,1} \left( q_S(A, j, 1) T_{A,j,1} + q_S(A, j^*, 1) T_{A,j^*,1} + q_S(B, j^*, 1) T_{B,j^*,1} \right) \quad (19)$$

where $q_S(i,j,1)$ denotes the probability that the communication is initiated when the system is in state $(i,j,1)$. With Figure 6 it is seen that [2]

$$q_S(A, 0, 1) = \frac{\lambda}{\lambda + \nu}; \quad q_S(A, 1^*, 1) = \frac{\nu \mu (2\lambda + \delta + \mu + \nu) q_S(B, 0^*, 1)}{(\lambda + \delta)(\lambda + \nu)(\mu + \lambda + \delta)}$$

$$q_S(A, 0^*, 1) = \frac{\mu q_S(B, 0^*, 1)}{\lambda + \nu}; \quad q_S(B, 0^*, 1) = \frac{\lambda + \nu + \mu (\lambda + \delta + \nu)}{\nu q_S(B, 0^*, 1)}$$

$$q_S(A, 1, 1) = \frac{\nu \lambda}{(\lambda + \delta)(\lambda + \nu)}; \quad q_S(B, 1^*, 1) = \frac{\mu + \lambda + \delta}{\mu + \lambda + \delta}.$$  

(20)
It remains to compute the hitting times $T_{i,j,k}$. They are easily identified from the infinitesimal generator matrix $Q$ as follows (see pp. 113-114 in [13])

$$T_{A,0,0} = 0 \quad \text{and} \quad \sum_{i,j,k} q(Y',k'),(i,j,k) T_{i,j,k} = -1 \quad \text{for } (Y',k',k') \neq (A,0,0) \quad (21)$$

where $q(Y',k'),(i,j,k)$ are the elements of the generator matrix $Q$. In order to simplify (21), let us introduce $M_{A,0,0}$ as the minor of the matrix $Q$ obtained by removing the row and the column corresponding to state $(A,0,0)$. Let $T = \{T_{i,j,k}, (i,j,k) \in F - \{(A,0,0)\}\}$ be the vector of hitting times, except $T_{A,0,0}$ (which is equal to 0). Equation (21) then reads

$$M_{A,0,0} \cdot T = -1. \quad (22)$$

Solving for (22) and using the group of equations (20) yields $T_S$.

5 Validation and comparison

5.1 Validation through simulations

The theory developed in Sections 3 and 4 relies on several assumptions. Mainly, idle times for a source and for an agent, migration durations, communications latencies and service times (centralized approach only) were all assumed to be exponential rvs and independent from each other. In this section we undertake validation of the Markovian models presented in Sections 3 and 4 against results obtained from event-driven simulators of both schemes. We have run each simulator several times having one assumption violated at a time, and one time having all assumptions violated. This way we can observe the robustness of the corresponding model and the impact of each assumption on its performance. In the simulations, we have considered a single agent and a single source.

In order to test realistic distributions for the rvs in hand we have collected measurements on a LAN and a MAN and fitted the resulting data to well-known distributions. Except for the service times which are approximately constant, all other (network-dependent) parameters are well represented by a Weibull distribution. The distributions of the communication rate $\lambda$ (inverse of the average idle times for the source) and the migration rate $\nu$ (inverse of the average idle times for the agent) depend only on the application. To test the robustness of the models against each assumption, we have run simulations where all random variables are exponential except one whose distribution is either deterministic (case of idle times and service times) or Weibull (case of migration durations and travel times). At last, we have run the simulators
Table 1 reports the sample mean and the percentiles of the relative error (expressed in percent) between simulated results and theoretical expected response times as predicted by both models. Our observations are summarized below:

(1) Both models are very robust against deterministic idle times for the agent (see lines 2 and 7 in Table 1) and Weibull travel times (see lines 4, 9 and 10).

(2) The model of the location server is very robust against deterministic service times (largest error observed is 3.7%). See line 11.

(3) The models are more sensitive to the distribution of the idle times for source (see lines 1 and 6) and the migration durations (see lines 3 and 8). Still, the largest relative error stays under 17.2%.

(4) The performance of the models is fair when all the assumptions concerning the distribution of the rvs are violated. In half of the simulations the relative error on the response time is less than 10.5% and its mean is...
The robustness of the models against correlated processes will be addressed in the next section.

5.2 Validation through experiments

In order to further validate the models, we have conducted extensive experiments over a LAN and a MAN. All benchmarks were written using the ProActive library [16] which provided us with all the necessary mobile primitives. The network was composed of various Pentium II and Pentium III machines running Linux (2.2.18) and Sun SPARC running SunOS interconnected with a 100Mb/s switched LAN or a 7Mb/s MAN (four machines were used overall). We used Java 1.2.2 Green threads in all experiments.

For each approach, the testbed was composed of a single mobile object and a single source. Idle times for the source (resp. the agent) are exponentially distributed with rate $\lambda$ (resp. rate $\nu$). Parameters $\lambda$ and $\nu$ can be modified from one session to another session. All the other model parameters ($\delta, \gamma, \gamma_1, \gamma_2, \mu$) are system-dependent and cannot be changed. Hence, among all the assumptions that we made to construct the models, only 2 assumptions are not violated (the ones concerning the input sequences $\{w_i, i \geq 1\}$ and $\{u_k, k \geq 1\}$ (see Section 2)). Indeed, migration durations, communications latencies and service times were found to have Weibull distributions both on a LAN and a MAN. Further, assumptions on the independence of these processes are not likely to be valid under real conditions. Migration durations and travel times are particularly correlated in real life.

Figure 7 reports both the experimental and theoretical expected response times obtained for a LAN (4 upper graphs) and for a MAN (4 lower graphs). Graphs on the left (resp. on the right) display the expected response time as a function of the communication rate $\lambda$ (resp. of the migration rate $\nu$) for $\lambda$ (resp. $\nu$) ranging from $1\text{s}^{-1}$ to $10\text{s}^{-1}$, for 3 different values of the migration rate $\nu$ (resp. communication rate $\lambda$) (1, 5 and 10). For each value of the pair $(\lambda, \nu)$, the empirical values of $\delta$ and $\gamma$ (resp. $\delta, \gamma_1, \gamma_2$ and $\mu$) were plugged into the expression of $T_F$ (resp. $T_S$) given in (18) (resp. (19)) in case the forwarding mechanism (resp. the centralized mechanism) was used. Observe that analytical and experimental response times are very close to each other over almost all experiments.

For each network configuration (LAN or MAN), the performance of the models are collected in Table 2 in means of order statistics and the sample mean of the relative error between analytical results and experimental values. Results in Table 2 indicate that the theoretical models behave fairly well. Their overall
Fig. 7. Validation with experiments over a LAN and a MAN.

performance are very close to each other (see lines 3 and 6 in Table 2). However, both models are more robust when applied on a LAN (see lines 1 and 4 in Table 2) than when applied on a MAN (see lines 2 and 5 in Table 2).

The model of the server has been validated for one source-agent pair. We still need to perform experiments including multiple sources and/or agents in order to complete the validation of the model. Beside validation, we can use the experimental results to compare the performance of the location mechanisms. Looking at the 4 upper graphs in Figure 7 (experiments on a LAN), it appears
that the location server scheme has a lower response time if compared to the forwarders scheme. Unexpectedly, we observe the opposite in the 4 lower graphs in Figure 7 (experiments on a MAN), where the forwarders scheme achieves the smallest communication time. It looks like the location server scheme performs the best only over high speed networks. When communication latencies increases (and subsequently the migration durations), the forwarding technique surpasses the centralized technique in performance.

5.3 A theoretical comparison of both approaches

As already mentioned, many parameters are network-dependent so that an experimental comparison of both approaches is necessarily limited to a few scenarios. No such limitations occur when comparing them by using the theoretical results obtained in Sections 3 and 4. We will focus on the expected response time given by each approach and, more precisely, on their difference $\Delta T$, namely, $\Delta T = T_F - T_S$ where $T_F$ and $T_S$ are given in (18) and (19), respectively. Four cases have been investigated corresponding to different values of the model parameters. Unless otherwise mentioned, the parameters have the values listed in Table 3. Each entry in Table 3 is the average value obtained over all experiments reported in Section 5.2. Notice that the ergodicity condition was always verified in our experiments. This can easily be explained by the fact that the migration of an agent over the network is achieved by sending several messages from the old site to the new one. Thus we always have $\frac{1}{\delta} > \frac{1}{\gamma}$. In each case we have identified regions where $\Delta T = 0$, which corresponds to situations where both schemes yield the same expected response (communication) time. When $\Delta T > 0$ (resp. $\Delta T < 0$) the location server (resp. forwarding) scheme gives the best performance. Figure 8 displays the sign of $\Delta T$ in all four cases that we have investigated.

Figure 8(a) shows that under LAN conditions (resp. MAN conditions) (refer to 1st line (resp. line 2) in Table 3 for self-contained information), the cen-

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
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<tbody>
<tr>
<td>Sample mean and percentiles of the relative error in both mechanisms.</td>
</tr>
<tr>
<td>Experiment</td>
</tr>
<tr>
<td>Forwarding mechanism 100Mb/s LAN</td>
</tr>
<tr>
<td>mechanism 7Mb/s MAN</td>
</tr>
<tr>
<td>Centralized approach 100Mb/s LAN</td>
</tr>
<tr>
<td>approach 7Mb/s MAN</td>
</tr>
</tbody>
</table>
Fig. 8. Sign of the difference between response times $\Delta T = T_F - T_S$.

In Figure 8(c) we observe that for extremely low service rate ($\mu$ less than $16 \text{s}^{-1}$), the forwarders technique becomes better than the centralized technique. Notice that there should be around 80 active queues at the server ($\mu = 2325$ in our LAN experiments) in order to have service rates per queue as low as 30. Surprisingly enough, increasing the communication rate $\lambda$ while keeping unchanged the migration rate $\nu$ does not have the same effect on the frontier $\Delta T = 0$: for $\nu = 1$, an increase in $\lambda$ from 1 to 5 shifts the frontier to the left (lower service rate) whereas the same increase in $\lambda$ but for $\nu = 5$

Table 3

<table>
<thead>
<tr>
<th>Network</th>
<th>$\delta$ $(\text{s}^{-1})$</th>
<th>$\gamma$ $(\text{s}^{-1})$</th>
<th>$\gamma_1$ $(\text{s}^{-1})$</th>
<th>$\gamma_2$ $(\text{s}^{-1})$</th>
<th>$\mu$ $(\text{s}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAN</td>
<td>10.9</td>
<td>45.6</td>
<td>115.6</td>
<td>76.3</td>
<td>2325</td>
</tr>
<tr>
<td>MAN</td>
<td>1.6</td>
<td>12.3</td>
<td>36.7</td>
<td>12.1</td>
<td>938</td>
</tr>
</tbody>
</table>
shifts the frontier to the right (higher service rate). (A shift to the right (resp. to the left) enlarges the region where forwarders are better (resp. the server is better).)

Figure 8(d) displays the frontiers $\Delta T = 0$ for several values of the migration rate $\nu$ and for $\mu \in [10, 2500]$ and $\lambda \in [1, 50]$; the travel times and the migration durations correspond to a LAN (values in Table 3). When $\nu$ increases the line $\Delta T = 0$ shifts to the right (higher service rate) enlarging the region where forwarders are better because as the migration rate increases the performance of the server degrades more rapidly than the performance of the forwarders.

To conclude this section, we would like to stress the fact that selecting the best location scheme is not as intuitive as it could look in the first place. Our study of the sign of $\Delta T$ has pointed out several unexpected effects when the value of some parameter is changed.

6 Concluding remarks

We have proposed simple Markovian analytical models for evaluating the performance of two approaches for locating mobile objects. One approach uses forwarders to enable communication between a source and a mobile object; in the other approach communications are ensured by a centralized server. Our models have been validated through simulations and extensive experiments over a LAN and a MAN. In all the experiments that we have conducted, we have observed that the server yields the best performance over a LAN and the forwarders are more efficient over a MAN. Using the theoretical expected response times of both schemes, we have identified the best location scheme under a wide variety of conditions. At last, note that our contribution in the field of code mobility is twofold. First, we have identified the best location scheme depending on the network conditions. Our conclusions are not that intuitive and they were made possible thanks to our rigorous approach. Second, we have shown that modeling such mechanisms is possible using rather simple techniques, thereby leaving the door open to the performance analysis of similar schemes.

Acknowledgements: The authors would like to thank D. Towsley for constructive comments on a preliminary version of this paper.
A  Proof of Proposition 3.1

We first derive (11). From (2) and (3) we obtain

\[ p_{i,0,0} = \left( \frac{\delta \nu}{(\lambda + \delta)(\lambda + \nu)} \right)^{i-1} p_{1,0,0}; \quad p_{i,1,0} = \left( \frac{\delta \nu}{(\lambda + \delta)(\lambda + \nu)} \right)^{i-1} \frac{\nu}{\lambda + \delta} p_{1,0,0} \]

for \( i \geq 1 \), so that

\[ f(z) = \left[ \frac{z}{(\lambda + \delta)(\lambda + \nu) - z \delta \nu} \right] p_{1,0,0}; \quad g(z) = \left[ \frac{z \nu(\lambda + \nu)}{(\lambda + \delta)(\lambda + \nu) - z \delta \nu} \right] p_{1,0,0}. \]

(A.1)

From (1) and (7)-(8) we find

\[ p_{1,0,0} = \frac{\gamma(\gamma + \nu)}{\nu(\nu + \lambda + \gamma)} p_{1,0,1}. \]

(A.2)

Introducing (A.2) into (A.1) gives (11). On the other hand, we find (14) from (A.2) and (7). We now address the computation of \( h(z) \) and \( k(z) \). From (4)-(10) we find

\[ [z(\gamma + \nu) - \gamma] h(z) = z \nu[p_{1,0,1} - z p_{1,0,1\ast}] - \gamma[p_{1,0,1} + z^2 p_{1,0,1\ast}] + z \lambda f(z) + z^2 \delta k(z) \]
\[ [z(\gamma + \delta) - \gamma] k(z) = \gamma(z - 1)p_{0,1,1} + z \lambda g(z) + z \nu h(z) - z \nu[p_{1,0,1} - z p_{1,0,1\ast}]. \]

Solving for this system of linear equations in the unknowns \( h(z) \) and \( k(z) \) yields (12) and (13). The constants \( p_{0,1,1} \) and \( p_{1,0,1} \) involved in (11)-(13) are determined as follows: \( h(z) \) and \( k(z) \) are well-defined in the unit disk as long as \( b(z) \) does not vanish for \( |z| \leq 1 \) (since \( a(z) \neq 0 \) for \( |z| \leq 1 \)). Since \( b(z) \) as a unique zero \( z = z_0 \) in the unit disk (given in Proposition 3.1) we conclude that the coefficient of \( 1/b(z) \) in \( h(z) \) and in \( k(z) \) must vanish when \( z = z_0 \). This gives rise to two linear relations between \( p_{0,1,1} \) and \( p_{1,0,1} \) that are actually identical. We therefore need another relation between \( p_{0,1,1} \) and \( p_{1,0,1} \) which is nothing but the normalizing condition \( F(1) = 1 \). We can then solve for \( p_{0,1,1} \) and \( p_{1,0,1} \).

It remains to establish the stability condition. The normalizing condition will be satisfied iff

\[ p_{0,1,1} + \frac{(\lambda + \nu)(\gamma + \nu)(\gamma + \lambda)}{\lambda \nu(\nu + \lambda + \gamma)} p_{1,0,1} = \frac{\delta \nu}{\delta + \nu} \left( \frac{1}{\nu} + \frac{1}{\delta} - \frac{1}{\gamma} \right) (1 - p_{1,0,1}) \]

which implies that \( 1/\gamma < 1/\nu + 1/\delta \) is a necessary condition for stability. It is also a sufficient condition since, when it holds, one can find a unique strictly positive and normalized solution to the C-K equations, that are given by the coefficients of the \( z \)-transform \( F(z) \).
References


