M/G/1 queue with repeated inhomogeneous vacations applied to IEEE 802.16e power saving
Sara Alouf, Eitan Altman, Amar Prakash Azad

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1 Introduction and Model

We shall analyze and optimize the parameters of the following two types of power saving of IEEE 802.16e in presence of downlink traffic:

(i) Type I classes: Under the sleep mode operation, sleep and listen windows are interleaved as long as there is no downlink traffic destined to the node. During listen windows, the node checks with the base station whether there is any buffered downlink traffic destined to it in which case it leaves the sleep mode. Each sleep window is twice the size of the previous one but it is not greater than a specified final value. The initial sleep window is \(T_{\text{min}}\), the multiplicative factor is \(a\) \((a = 2\) in the standard) and the final value \(bT_{\text{min}}\) depends on the exponent \(l\).

(ii) Type II classes: All sleep windows are of the same size as the initial window (i.e. \(a = 1\)). Sleep and listen windows are interleaved as in type I classes.

To model these classes, we analyze an \(M/G/1\) queue in which the server begins a vacation of random length each time that the system becomes empty. If the server returns from a vacation to find an empty queue, a new random vacation initiates; otherwise, the server works until the system empties (exhaustive service regime). Request arrivals are assumed to form a Poisson regime. Request arrivals are assumed to form a Poisson process with parameter \(\lambda\). Let \(\sigma\) denote a generic random variable having the same (general) distribution as the queue service times. The queue regenerates each time it empties and the cycles are i.i.d. Each regeneration cycle consists of (see Fig. 1):

(i) an idle period; let \(I\) denote a generic random variable having the same distribution as the queue idle periods, a generic idle period \(I\) consists of \(I\) vacation periods denoted \(V_1, \ldots, V_i\);

(ii) a warm-up period; it is a fixed duration denoted \(T_w\) during which the server is warming up to start serving requests;

(iii) a busy period; let \(B\) denote a generic random variable having the same distribution as the queue busy periods.

Let \(X(t)\) denote the queue size at time \(t\). Define (i) \(\tilde{V}_i\): the end of the \(i\)th vacation period, \(i = 1, \ldots, \zeta\); the idle period ends at \(\tilde{V}_i\): we have \(\tilde{V}_i = \sum_{j=1}^{i} V_j\) and \(I = \tilde{V}_\zeta = \sum_{i=1}^\zeta V_i\);

(ii) \(T_Z\): the start of the busy period \(B\); let \(Z := X(T_Z)\), the queue size at the beginning of a busy period;

(iii) \(T_I\): the first time the queue size decreases to the value \(i\) (i.e. \(X(T_I) = i\)) for \(i = Z - 1, \ldots, 0\); the cycle ends at \(T_0\).

The times \(\{T_i\}_{i=Z, Z-1, \ldots, 0}\) delimit \(Z\) subperiods in \(B\). We can write \(B = \sum_{i=1}^Z B_i\) where \(B_i = T_i - T_{i-1}\).

\(Z\) is in fact the number of arrivals from \(t = 0\) until time \(T_Z\). Introduce \(Z_I\) as the number of requests that have arrived up to time \(\tilde{V}_\zeta\) (i.e. during period \(I\)) and \(Z_w\) as the number of arrivals during the warm-up period \(T_w\). Hence \(Z = Z_I + Z_w\). Observe that \(X(I) = Z_I\).
From Fig. 1 and using (1) we can write (note that $B_1 = B_{M/G/1}$)

$$E[B] = E[B_1]E[Z] = \frac{\rho}{1 - \rho} (E[I] + T_w).$$

The Queue Size and Sojourn Time. Let

$$A := \int_{V_\cdot}^{V_\cdot} X(t)dt, \quad A_w := \int_{V_{\cdot}}^{T_w} X(t)dt, \quad Q_Z := \int_{T_w}^{T_w} X(t)dt,$$

be defined as the total area under the curve $X(t)$ for the idle, warm-up and busy periods respectively. Then

$$E[X] = \frac{E[A] + E[A_w] + E[Q_Z]}{E[I] + T_w + E[B]}, \quad T = \frac{E[X]}{\lambda}.$$  \hspace{1cm} (2)

Let $N(t), t \geq 0$ be a Poisson process with rate $\lambda$. Let $\tau_i$ be the $i$th arrival epoch. Define $A(t) := E[\alpha(t)|N(t) \geq 1]$ with

$$\alpha(t) := \int_0^t N(s)ds = \sum_{i=1}^{N(t)} (t - \tau_i) = tN(t) - \sum_{i=1}^{N(t)} \tau_i.$$  \hspace{1cm}

We thus need to compute $E[A] = E[A(V_\cdot)]$. We have $E[\alpha(t)] = \lambda t^2/2$. Therefore,

$$A(t) = \frac{\lambda t^2}{2} \sum_{k=0}^{\infty} \exp(-k\lambda t)$$

$$E[A] = \frac{\lambda}{2} \sum_{i=1}^{\infty} \left( \prod_{k=1}^{i-1} L_k(\lambda) \right) \left( 1 - L_i(\lambda) \right) \sum_{k=0}^{\infty} \frac{d^2 L_i(s)}{ds^2} \bigg|_{s=k\lambda}.$$  \hspace{1cm}

From Fig. 1 we can write

$$E[A_w] = E[Z]T_w + \int_0^{T_w} \lambda tdt = \lambda T_w (E[I] + T_w/2).$$

The following recursive equation holds

$$Q_Z = ((Z-1)B_z + Q_1) + Q_{Z-1}.$$  \hspace{1cm}

Hence $E[Q_z] = E[Z]E[Q_1] + (E[Z^2] - E[Z])E[B_1]/2$ where

$$E[Q_1] = E[Q_{M/G/1}] = \frac{E[\sigma]}{1 - \rho} + \frac{\lambda E[\sigma^2]}{2(1 - \rho)^2}$$

which completes the computation of the response time $T$; see (2).

3 Application to Power Saving

Let $S_i$ be a generic random variable that gives the actual time a node is sleeping during the $i$th vacation period. We then have $V_1 = S_1$ and $V_i = T_i + S_i$ for $i = 2, \ldots, \zeta$. The listen window $T_1$ accounts for the time needed by a node to check for messages at the base station. We assume $T_i = T_w$ and consider either (i) $S_i = a^{\min(i-1,\ell)}T_{\min}, \ell = 1, 2, \ldots, (\text{deterministic case})$ (ii) $E[S_i] = a^{\min(i-1,\ell)}T_{\min}, \ell = 1, 2, \ldots, (\text{exponential $S_i$})$.

Define the economy of energy $G := (E_{\text{no sleep}} - E_{\text{sleep}})/E_{\text{no sleep}}$.

The following nonlinear program $P$

$$\text{maximize } G \text{ subject to } T \leq T_{\text{QoS}}$$

is solved for the three protocol parameters: the exponent $l$, the multiplicative factor $a$ and the initial expected sleep window $T_{\text{min}}$.

The system parameters have been selected as follows: $E[\sigma] = 1, E[\sigma^2] = 2, T_1 = T_w = 1, T_{\text{QoS}} = 50, 100$. The ratio between the energy consumptions of a node in idle and active states is set to 0.2. Numerical results for the sojourn time $T$ in the deterministic case have been derived and are displayed graphically in Fig. 2. The sojourn time $T$ appears to be fairly insensitive to parameters $l$ and $a$ except for very small values of $\lambda$. Observe that the load $\rho = \lambda E[\sigma]$ gives the probability of finding the queue busy. Hence, with probability $1 - \lambda E[\sigma]$, an arriving request finds the server on vacation. This explains the high response time for low input rates. For very large input rates, the system response time is also high, but then it is mainly due to queueing delays. We have solved the nonlinear program $P$ for $(l, a, T_{\text{min}})$ with $T_{\text{QoS}} = 50$ for deterministic sleep windows and $T_{\text{QoS}} = 100$ for exponential sleep windows. The values of the optimal protocol parameters $(l, a, T_{\text{min}})$ are in Table 1.

Table 1: Optimal protocol parameters obtained from $P$

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<th>$\lambda$</th>
<th>$l$</th>
<th>$a$</th>
<th>$T_{\text{min}}$</th>
<th>$G$</th>
<th>$\lambda$</th>
<th>$l$</th>
<th>$a$</th>
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References