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# IMPULSIVE INTERFERENCE MITIGATION IN *AD HOC* NETWORKS BASED ON ALPHA-STABLE MODELING AND PARTICLE FILTERING

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## ABSTRACT

In this paper, we tackle the problem of interference mitigation in *ad hoc* networks. In such context, the multiple access interference (MAI) is known to be of an impulsive nature. Therefore, the conventional Gaussian assumption can not be considered to model this type of interference. Contrariwise, it can be accurately modeled by stable distributions. Here, this issue is addressed within an Orthogonal Frequency Division Multiplexing (OFDM) transmission link assuming a symmetric  $\alpha$ -stable model for the signal distortion due to MAI. For this purpose, we propose a method for the joint estimation of the transmitted multicarrier signal and the noise parameters. Based on sequential Monte Carlo (SMC) methods, the proposed scheme allows the online estimation using a Rao-blackwellized particle filter.

**Index Terms**— *ad hoc* networks, multiple access interference, sequential Monte Carlo methods,  $\alpha$ -stable distribution

## 1. INTRODUCTION

Sensor networks need to reduce the energy to sense and transmit data. Signal processing has a key role to play in order to minimize the transmission power necessary to ensure the required link quality and reliability. Interference in low power communications could however be a strong limitation. In *ad hoc* networks, capacities of relay and interference channels are not analytically known.

If recent works from Bresler *et al.* [1] have approximated such a capacity, the way to cope with interference is still an open question. The authors suggest some interference alignment scheme whereas Dabora *et al.* [2] propose to use relays to forward interference in order to properly remove it at the receiver and obtain an interference free link. However, in sensor networks, the hard constraints on consumption and cost make it difficult to implement those complex proposals.

We are then interested in exploiting the non Gaussian behavior of the interference in order to limit its effects. Indeed, the *ad hoc* configuration gives to the multiple access interference (MAI) an impulsive nature: the interfering pulses amplitude may importantly vary and the MAI is conditioned by

the presence of strong interferers. Several works [3, 4] have proposed to use stable distributions to model it and a general mathematical framework was recently proposed by Win *et al.* in [5]. They show the theoretical validity of the stable approach for different situations in network interference, including *ad hoc* and cognitive radio. We also suggest that the *ad hoc* configuration gives the shape of the MAI distribution which is then accurately represented by symmetric  $\alpha$ -stable distributions [6].

In this paper, this modeling of interference is used in an OFDM transmission link and we try to reduce the signal distortion due to MAI. For this purpose, we propose a method for the joint estimation of the multicarrier signal and the noise parameters. The proposed scheme is based on Bayesian estimation using SMC methods. We propose a sequential approach although it is not necessarily justified in OFDM based transmission but we are here interested in the possibility to remove interference without adding delays in the signal processing and therefore in the transmission.

The paper is organized as follows: in section 2, we present some preliminary definitions regarding the  $\alpha$ -stable distribution and its properties. In section 3, we introduce the OFDM system's model and its state space representation. Section 4 is devoted to the description of the proposed particle filter (PF) to estimate both multicarrier signal and noise parameters. Simulation results are shown in section 5 and conclusions are drawn through section 6.

## 2. STABLE DISTRIBUTIONS

Stable distributions can be seen as a generalization of the Gaussian distribution in that sense that they are the only distributions to be stable in convolution, meaning that the sum of two  $\alpha$ -stable distributions is an  $\alpha$ -stable distribution. One difficulty is that they have no closed-form expressions for their probability density function (pdf) and cumulative distribution. They can be most conveniently described by their characteristic functions as follows [7]:

$$\varphi(t) = \exp(i\mu t - \gamma |t|^\alpha [1 + i\beta \text{sign}(t)\omega(t, \alpha)])$$

where

$$\omega(t, \alpha) = \begin{cases} \tan \frac{\alpha\pi}{2}, & \text{if } \alpha \neq 1 \\ \frac{2}{\pi} \log|t|, & \text{if } \alpha = 1 \end{cases}$$

Thus, the stable distribution is completely determined by four parameters  $\alpha, \beta, \gamma$  and  $\mu$  where:

- $\alpha \in ]0, 2]$  is the characteristic exponent. It measures the tails heaviness of distribution. A small value of  $\alpha$  will imply a considerable probability mass in the tails of distribution, while a value of  $\alpha$  close to 2 indicates more Gaussian type behavior.
- $\mu \in \mathbb{R}$  is the location parameter.
- $\gamma > 0$  is the scale parameter. It determines the spread of density around the location parameter  $\mu$ .
- $\beta \in [-1, 1]$  is the symmetry parameter. When  $\beta = 0$ , the distribution is symmetric about  $\mu$ .

To denote an  $\alpha$ -stable distribution with parameters  $\alpha, \beta, \gamma$  and  $\mu$ , we will use the following notation  $\mathcal{S}_\alpha(\beta, \gamma, \mu)$ . An  $\alpha$ -stable distribution is said to be standard when  $\gamma = 1$  and  $\mu = 0$ . For standardized stable distribution we will use the notation  $\mathcal{S}_\alpha(\beta)$ .

$\alpha$ -stable distribution pdf can be obtained analytically only for a few particular cases:  $\alpha = 2$ , corresponding to the Gaussian distribution,  $\alpha = 1$  and  $\beta = 0$ , yielding the Cauchy distribution, and finally  $\alpha = \frac{1}{2}$  and  $\beta = 1$  for the Levy distribution.

### 3. MODEL

In this paper, we consider a dynamic state space model, widely used in communication, to represent OFDM systems [8].

The considered OFDM system is based on  $N$  subcarriers. The transmitted signal, composed of  $N$  samples  $s_t$  with  $t = 0 \dots N - 1$ , is generated through an inverse FFT. Before transmission, a cyclic prefix of length  $N_{cp}$  is introduced in order to avoid intersymbol interference. The resulting signal is transmitted in a time varying frequency selective multipath channel with  $L$  paths.

We define the transmitted multicarrier signal vector as:

$$S_t = [s_t \quad \dots \quad s_{-L+1} \quad \mathbf{0}_{1 \times (N+N_{cp}-t-1)}]^T$$

where  $s_k = 0$  if  $k < 0$ .

The state equation of  $S_t$  can be written in the matrix form as follows:

$$S_t = A_t S_{t-1} + B_t$$

where the transition matrix  $A_t$  is defined as:

$$\begin{bmatrix} \xi_t^T & \\ \mathbf{I}_{(N+N_{cp}+L-2)} & \mathbf{0}_{(N+N_{cp}+L-2) \times 1} \end{bmatrix}$$

with

$$\xi_t^T = \begin{cases} [\mathbf{0}_{1 \times (N+N_{cp}+L-1)}]^T, & \text{if } 0 \leq t \leq N-1 \\ [\mathbf{0}_{1 \times (N-1)} \quad 1 \quad \mathbf{0}_{1 \times (N_{cp}+L-1)}]^T, & \text{if } N \leq t \leq N+N_{cp}-1 \end{cases}$$

and  $B_t$  is a circular Zero-mean Gaussian noise vector with the covariance matrix

$$E[B_t B_t^H] = \begin{bmatrix} \sigma_{b_t}^2 & 0 & \dots & 0 \\ 0 & 0 & & \\ \vdots & & \ddots & \vdots \\ 0 & \dots & & 0 \end{bmatrix}$$

where

$$\sigma_{b_t}^2 = \begin{cases} 1, & \text{if } 0 \leq t \leq N-1 \\ 0, & \text{if } N \leq t \leq N+N_{cp}-1 \end{cases}$$

The received multicarrier signal  $r_t$  can be written as:

$$r_t = H^T S_t + \rho_t u_t$$

for  $t = 0 \dots N + N_{cp} - 1$ , where  $H$  is the channel impulsive response

$$H = [h_0 \quad \dots \quad h_{L-1} \quad \mathbf{0}_{1 \times (N+N_{cp}-1)}]^T$$

Contrary to [8], we assume that  $u_t$  is a symmetric  $\alpha$ -stable noise

$$u_t \sim \mathcal{S}_\alpha(0)$$

Exploiting the mixture of normal representation of a stable distribution,  $u_t$  can be written as follows [9]:

$$u_t = \sqrt{\lambda_t} w_t \quad \text{where} \quad \lambda_t \sim \mathcal{S}_{\frac{\alpha}{2}}(1) \quad \text{and} \quad w_t \sim \mathcal{N}(0, 1)$$

Therefore, the observation equation becomes

$$r_t = H^T S_t + D_t w_t \quad \text{with} \quad D_t = [\rho_t \sqrt{\lambda_t}]$$

### 4. PARTICLE FILTER FOR INTERFERENCE MITIGATION

In this paper, our main target is to jointly estimate the transmitted multicarrier signal vector  $S_t$  and the parameters of the model  $\lambda_t, \alpha$  and  $\rho_t$ . In the context of Bayesian estimation, we need to compute the joint posterior pdf  $p(S_t, \lambda_t, \alpha, \rho_t | r_{0:t})$ . However, this pdf is analytically intractable. Therefore, we propose to use SMC methods in order to find an estimate of  $p(S_t, \lambda_t, \alpha, \rho_t | r_{0:t})$ .

Following [10], we denote  $\theta_t$  the time varying parameter vector given by:

$$\theta_t = [\alpha, \phi_{\rho_t}, \lambda_t] \quad \text{where} \quad \phi_{\rho_t} = \ln(\rho_t^2)$$

The joint posterior pdf can be written as follows:

$$p(S_t, \theta_t | r_{0:t}) = p(S_t | \theta_t, r_{0:t}) p(\theta_t | r_{0:t}) \quad (1)$$

In the previous expression, the first probability density can be computed analytically using the Kalman filter (KF). Since, conditionally on  $\theta_t$ , our model is linear and Gaussian. The probability  $p(S_t|\theta_t, r_{0:t})$  can be written as follows:

$$p(S_t|\theta_t, r_{0:t}) = \mathcal{N}(m_{t|t}, P_{t|t}) \quad (2)$$

where  $m_{t|t}$  and  $P_{t|t}$  are computed using a KF.

The second probability density is intractable and is approximated by a PF [11]:

$$p(\theta_t|r_{0:t}) \approx \sum_{i=1}^M \omega_t^{(i)} \delta_{\theta_t^{(i)}}(\theta_t) \quad (3)$$

where  $\omega_t^{(i)}$  is the normalized weight associated to the  $i$ th particle.

Thus, we obtain the following estimate of the joint posterior pdf:

$$p(S_t, \theta_t|r_{0:t}) \approx \sum_{i=1}^M p(S_t|\theta_t^{(i)}, r_{0:t}) \omega_t^{(i)} \delta_{\theta_t^{(i)}}(\theta_t)$$

Like in [10], the importance function is then chosen as:

$$p(\theta_t|\theta_{t-1}) = p(\alpha_t|\alpha_{t-1})p(\phi_{\rho_t}|\phi_{\rho_{t-1}})p(\lambda_t) \quad (4)$$

where

$$p(\phi_{\rho_t}|\phi_{\rho_{t-1}}) \sim \mathcal{N}(\phi_{\rho_{t-1}}, \delta_{\phi_{\rho}}^2) \quad p(\phi_{\rho_0}) \sim \mathcal{N}(0, \delta_{\phi_{\rho_0}}^2) \\ p(\lambda_t) \sim \mathcal{S}_{\frac{\alpha}{2}}(1)$$

Our PF has to deal with the estimation of the static parameter  $\alpha$ . However, this intensifies the degeneracy problem. Considering this lack, we adopt the approach of artificial parameter evolution described in [11, 10]. Nevertheless, the main problem of this approach is the information loss. In [11], the authors propose a method, to remediate to this gap, based on sampling the updated values of the static parameter from a kernel smoothed density. Using this method helped to define the conditional density evolution of  $\alpha$  as:

$$p(\alpha_{t+1}|\alpha_t) \sim \mathcal{N}(\alpha_{t+1}|d\alpha_t + (1-d)\bar{\alpha}_t, h^2\sigma_t^2)$$

with

$$h^2 = 1 - d^2 = 1 - \left( \frac{3\delta - 1}{2\delta} \right)^2$$

where  $\bar{\alpha}_t$  and  $\sigma_t^2$  are respectively the mean and the variance matrix of the Monte Carlo approximation of  $p(\alpha|r_{0:t})$  and  $\delta$  is a discount factor in  $]0, 1]$ .

Weights are then updated according to:

$$\tilde{\omega}_t^{(i)} \propto \tilde{\omega}_{t-1}^{(i)} p(r_t|\theta_t^{(i)}, r_{t-1}) \quad (5)$$

with

$$p(r_t|\theta_t^{(i)}, r_{t-1}) = \mathcal{N}(r_t, H^T m_{t|t}^{(i)}, G_t^{(i)})$$

where  $G_t^{(i)}$  is the innovation covariance matrix from the KF.

Finally, the proposed PF for interference mitigation is summarized in Algorithm 1:

```

initialization for  $i \leftarrow 1$  to  $M$  do
     $\omega_0^{(i)} = \frac{1}{M}$ ;
end
for  $t \leftarrow 2$  to  $N$  do
    for  $i \leftarrow 1$  to  $M$  do
        Sample  $\theta_t^{(i)}$  using (4);
        Update  $m_{t|t}^{(i)}$  and  $P_{t|t}^{(i)}$  using KF;
        Evaluate the weights using (5);
    end
    Normalize the weights;
    if  $N_{eff} < \frac{M}{3}$  then Resampling step;
    Evaluate  $\hat{S}_t$ ,  $\hat{\rho}_t$  and  $\hat{\alpha}_t$ ;
end

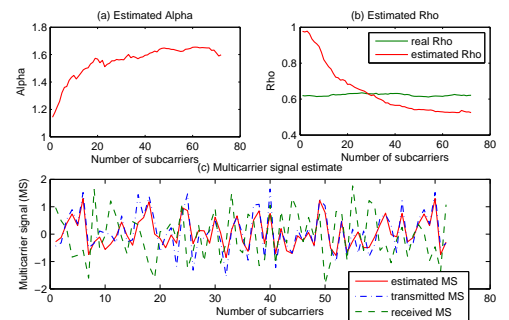
```

**Algorithm 1:** PF for interference mitigation algorithm

## 5. SIMULATIONS

In order to validate our approach, various experiments have been carried out. We have considered the following system parameters: an OFDM system with  $N = 64$  subcarriers, a cyclic prefix of  $N_{cp} = 8$  samples and a multipath channel with  $L = 4$  paths. Both real and imaginary part of the transmitted multicarrier signal have been generated from a Gaussian distribution  $\mathcal{N}(0, \frac{1}{2})$  [8]. The proposed PF has been implemented with 100 particles. Parameters  $\delta_{\phi_{\rho}}^2$  and  $\delta_{\phi_{\rho_0}}^2$  are respectively set to 0.0005 and 0.5. We fixed the discount factor  $\delta$  to 0.95 [11].

At first, the algorithm performance is studied with a multicarrier signal corrupted by a symmetric  $\alpha$ -stable noise with  $SNR = 10$ . The related parameters are set to:  $\alpha = 1.6$ ,  $\mu = 0$  and  $\gamma = 1$ . Results are illustrated in the different plots of Fig. 1. The evolution of the estimated  $\alpha$  and  $\rho_t$  are respectively depicted in Fig. 1.(a) and Fig. 1.(b). We remark that, after some iterations, the estimated parameters start displaying the same trajectory as their actual counterparts. Curves in

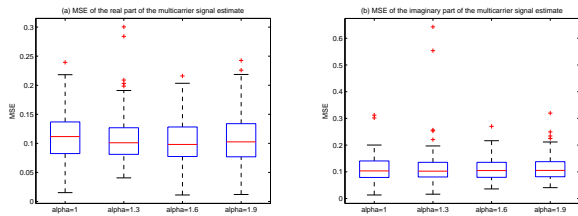


**Fig. 1.** The estimation of the model parameters and the multicarrier signal for  $\alpha = 1.6$  and  $SNR = 10$ .

Fig. 1.(c) show the estimated multicarrier signal as well as the

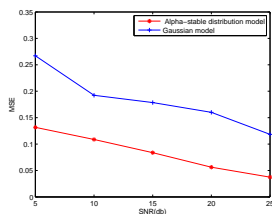
transmitted and the received ones. We can easily observe that the estimated multicarrier signal tends to be very close to the transmitted one.

Secondly, we study the performance of our method for different values of  $\alpha$ . For each one of them, we perform the experiment 100 times. The performances are shown in term of mean square error (MSE) of the multicarrier signal estimate. The average of the obtained MSEs are shown in Fig. 2.



**Fig. 2.** Box and whiskers plots of the MSE of the multicarrier signal estimate for different values of  $\alpha$ .

In order to show the improvement brought by the  $\alpha$ -stable modelling, we have compared the previous estimation scheme to the following : the noise on the observations is  $\alpha$ -stable but the estimation process is carried out by supposing that the observation noise is simply gaussian. As it can be seen in Fig. 3, it is clear that including this impulsive nature in the estimation process brings a real improvement.



**Fig. 3.** MSE of the multicarrier signal estimate versus SNR.

Thus, simulations demonstrate the efficiency of the proposed method for the joint estimation of the multicarrier signal and noise parameters.

## 6. CONCLUSIONS

In this paper, we deal with the problem of impulsive interference mitigation in *ad hoc* networks. In this context, we consider more particularly an OFDM transmission link. In the proposed model, we assumed that the signal distortion due to MAI is represented by a symmetric  $\alpha$ -stable distribution. We propose an approach based on Bayesian estimation using particle filtering, to jointly estimate the multicarrier signal and the noise parameters. Based on the performed results, it can be firstly concluded that  $\alpha$ -stable distributions are well suited to model the noise that exhibit impulsive nature. Furthermore,

the efficiency and the robustness of the proposed algorithm for interference mitigation have been demonstrated.

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