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Networked Control using GPS Synchronization

Alexandre Seuret, Fabien Michaut, Jean-Pierre Richard and Thierry Divoux

***Abstract*— This work concerns the control, the observation and then, the implementation principles of a remote system (Master and Slave parts) through the *Internet* network. This communication link introduces variable delays that have to be taken into account in the control-observation loop. The data-sampling effects will also be considered, even in the aperiodic case. The Slave part is considered to be a linear system. But, since its computation power is supposed to be limited, the control complexity (which, here, is an observer-based state feedback) has to stay in the Master part. The global system must ensure speed performance whatever the delay variation. Such a performance is obtained by showing the robust, exponential stability property, which is proven by using adequate Lyapunov-Krasovskii functionals. This makes possible to compute the controller and observer gains by using LMI optimization. The technical solution we propose is based on a GPS system, which guarantees the Master and Slave clocks to be synchronized. Then, the control and measurement packets are sent together with time-stamps that allow for reconstructing a non-symmetric delay information. It means that Master-to-Slave and Slave-to-Master delays are separately reconstructed by the system (and not only the global RTT, round-trip-time). The last part of the paper provides an example where the Slave is a second-order system.**

I. Introduction

Remote control is a good way to perform tasks in dangerous or hard-to-reach areas, or to broaden the number of users on a localized test-bed. The web technology on the *Internet* now appears as a natural, cheap way to ensure the communication link in such remote controlled systems. Nowadays, the Quality of Service provided by the internet is often good enough for that kind of applications.

However, a communication link unavoidably introduces time delays that have to be taken into account in the control/observation loop. The references [13], [17] provide overviews on control techniques for time-delay systems, which great influence on stability was already mentioned in the 60's [6]. Several works on teleoperation introduced the question of transmission delays, first, in the constant case [2], [5], [15].

In networked control situations, the delays are basically variable (*jitter* phenomenon) and unknown, which is a source of problem when one intend to apply the classical, predictor-based control laws (as, for instance, FSA: finite spectrum assignment). Such techniques generally need the delay to be constant, *i.e.* $h(t) = h$, which assumption does correspond to actual situation.

In the case of variable delays, some researches have used independent-of-delay conditions. Such i.o.d. conditions are generally conservative, but in [4], the results appear to be well-fitted for particular cases such as constant delays, or *symmetric delays*. What we mean here refers to the case where Master-to-Slave $h_1(t)$ and Slave-to-Master $h_2(t)$ transmission delays are equal, *i.e.* $h_1(t) = h_2(t) = R(t)/2$, where $R(t)$ denotes the round trip time (RTT). Another reference [10] considered non-symmetric delays, but only in the case of constant delays, *i.e.* $h_1(t) = h_1 \neq h_2(t) = h_2$.

Another interesting approach was recently given in [21], who generalized the predictor techniques to the case of variable delays. In this case, a maximal upper-bound of the delay is assumed to be known (h_m such that $0 \leq h(t) \leq h_m$), which is not that constraining. The main assumption is that a dynamical ODE model (Ordinary Differential Equation) of the delay is supposed to be available, which is possible in the case of a single-owner *Ethernet* network.

Another possible solution [11], [12], [16] consists in introducing an input buffer that makes the receiver wait until the maximum value of the delay is reached and, then, deliver the information to the control. In this case, from the receiver's point of view, the delay becomes constant and equal to the maximum value h_m. However, it is obvious that this situation maximizes the delay up to its worst (largest) value and, consequently, may decrease the speed performance of the global, remote system. Then, the speed performance has to be figured out within the design phase, which problem was not explicitly considered in [11], [12]. Moreover, these studies were considering *symmetric delays*.

The present study aims at using the *Internet* as a communication media linking a Master system and a Slave one. In such a situation, the generated delays are not only time-varying, non-symmetric, but also unknown (no dynamical model of the delays is available see, [14]). They just can be assumed to have known maxima h_{im}, so that $0 \leq h_i(t) \leq h_{im}$ holds. In the network framework, this assumption means that if a packet is lost, it is not re-emitted because we use UDP (User Datagram Protocol) . One also can assume that the delay variation satisfies $\dot{h}_i(t) \leq 1$, which means that all the packets are re-organized in their chronological emission order.

Now, for the discrete-time implementation, the data-sampling effect has to be taken into account. Following the lines of [8], [22], we consider it produces an additional, variable delay $t - t_k$, where t_k is the kth sampling instant. Generally, due to the computer architecture and operating system, the sampling may be aperiodic, *i.e.* there is no

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exact period T such that $t_k = kT$). So, we just assume a maximum sampling interval T is known, so that $0 \leq t_{k+1} - t_k \leq T$ holds. The global delays resulting from the communication-plus-sampling phenomena will be denoted by $\delta_i(t_k) = h_i(t) + t - t_k$, and one can see that the limit case $\dot{\delta}_i = 1$ can occur.

Of course, additional information are needed for achieving the global performance. The technical solution we propose is based on a GPS system. Both the Slave and the Master are equipped with a GPS antenna, which allows the Master and Slave clocks to be synchronized. Then, the control and measurements packets are sent together with "time-stamps" that permit to reconstruct the *non-symmetric* delay information. By this way, both Master-to-Slave $h_1(t)$ and Slave-to-Master $h_2(t)$ delays are separately reconstructed by the system, and not only the RTT.

The exchanged data correspond to the control (sent by the Master to the Slave) and to the output of the remote system (sent by the Slave to the Master). Since the Slave is not supposed to have a large computation power, the control and observation complexity has to be concentrated in the Master. Our purpose is to guarantee the robustness and speed performances of the global Master-Slave system. In particular, the global system must ensure the closed-loop stability and a guaranteed speed rate whatever the delay variation.

Stabilizing a system in such conditions is not that easy. The Master receives the information he needs for the control computation after it has crossed the communication zone. The GPS-based estimation of the transmission delay, joined to the observer, allows to know what was the Slave state at the instant the information was sent to the Master. Similarly, the control computed by the Master will be applied some time after it is sent to the Slave, and this dead-time is not known in advance.

For simplicity, the Slave is considered to be a linear system. The global performance is obtained by showing an exponential stability property ($\alpha-$stability), robust w.r.t. the delay, and which is proven by using adequate Lyapunov-Krasovskii functionals. This makes possible to compute the controller and observer gains by using LMI optimization.

The last part of the paper gives simulation results for a second order system.

II. Features of the remote system

Figure 1 presents the overall structure of the Master-Slave remote system.

The system has the following features:

- The Slave is supposed to have a limited computation power. Then it can not build its own control. The Master computes and forwards the control to the Slave. The forwarding cannot be instantaneous, because the communication lines induces a delay $h_1(t)$, as well as sampling effects, which create the variable delay $\tau_1(t)$.
- The Slave is driven by a linear, controllable and observable, known model (A,B,C), influenced by an input delay:

$$\begin{cases} \dot{x}(t) = & Ax(t) + Bu(t - \delta_1(t)), \\ y(t) = & Cx(t), \end{cases} \tag{1}$$

where $\delta_1(t)$ is a delay to be defined later on (subsection II-D).

- The Slave measures its sampled-data output variables $y(t)$, that the Master receives after a delay $h_2(t)$. An other delay $\tau_2(t)$ due to the sampling is added, Which means that the Master only can access $y(t - \delta_2(t))$, where δ_2 corresponds to the resulting delay. The Master includes an observer which aims at providing an estimation $\hat{x}$ of the complete Slave state x at the present time. From this estimation, the Master elaborates the control law.
- The sampling instants t_k may not be periodical (*i.e.*, $t_k \neq kT$), but it is supposed there is a known T such that, for any k:

$$t_{k+1} - t_k \leq T. \tag{2}$$

- The two generated delays have a known maximum $\delta_i^m = h_i^m + T$, so that $0 < \delta_i(t) \leq \delta_i^m$ holds, and the delay variation satisfies $\dot{\delta}_i(t) \leq 1$ (the interpretation in the network and sampled-time framework is given in the Introduction part).
- Each part of the Master-Slave system has a GPS card, which gives a shared clock. Thus, the internal clock of Master and Slave are synchronized. Each data packet includes an added time-stamp (the time the packet was sent). By this way, the receiver can calculate the transfer delays, $h_i(t)$ as soon as it receives the packet.

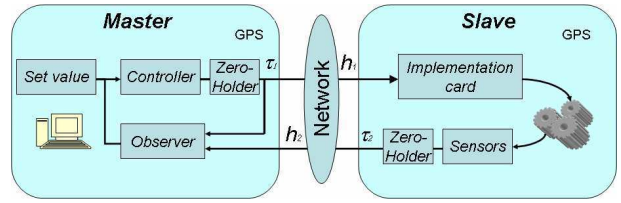

Fig. 1. Features of the remote system

The next subsections present the main features and notations, this is: (1) The modeling the sampling effects, considered as additional, variable delays $\tau_i(t)$; (2) The controller, which is a static, linear state feedback; (3) The different communication delays $h_i(t)$; (4) The observer, which is a linear, Luenberger-type one, but with delays.

A. The sampling delays

From a practical point of view, the global system (including the controller, the observer, the network, the process) cannot be considered as a continuous-time one. If the Slave has fast dynamics, exchanging the packets between Slave and Master in continuous time would mean the network can support a very high data flow. Then, the packets only

give discrete-time information. The corresponding sampling effect represents a possible disturbance to the stabilization of the remote system and must be taken into account in the observer and controller design. Instead of turning into discrete-time, recurrent equations, recent works [8], [19], [22] have considered such sampling effects as continuous-time phenomena with variable time delays. Indeed, the sample $g(t_k)$ of a function $g(t)$ at time t_k can be written as: $g(t_k) = g(t - [t - t_k]) = g(t - \tau(t))$, which notation replaces the sample-and-hold with an additional delay $\tau_k(t) = t - t_k,\ t \in [t_k, t_{k+1}[$. By this way, an aperiodic sampling is modeled as unknown delay with the upper-bound T (defined by (2)). This change allows one to use continuous-time techniques, as Lyapunov-Krasovskii functionals, for the stability study of sampled systems. In our case, we can define a delay $\delta(t)$ which represents the combination of such a sampling delay $\tau_k(t)$ with the delay $h(t_k)$ the transmission line subjects to the packet containing the kth sample. For any signal $g(t)$, this delay will be of the form:

$$\begin{aligned} g(t_k - h(t_k)) \quad &= g(t - h(t_k) - (t - t_k)), \\ &= g(t - \delta(t)), \\ t_k \le t < t_{k+1}, \quad &\delta(t) \triangleq h(t_k) + t - t_k. \end{aligned} \tag{3}$$

B. The control law

The controller computes a control law which takes into account some set value to be reached by the Slave. The state feedback control $u(t)$ is defined from the state estimate $\hat{x}$ given by the observer, as follows:

$$u(t) = K\hat{x}(t). \tag{4}$$

The main difficulty is to determine the linear gain K of the state feedback control so to guarantee the stability of the Slave motion despite the value of the time-varying delay $\delta_1(t)$. This delay is not known by the Master when its control data is sent.

C. Transmission of the control u

The kth data sent by the Master to the Slave includes the control $u(t_{1,k})$ it has just designed, together with the time $t_{1,k}$ when the packet was sent. This packet goes across the network. The Slave receives this information at time $t^r_{1,k}$. Thanks to the GPS clock synchronization, this time has the same meaning for the Slave as for the Master. Then the term $t^r_{1,k} - t_{1,k}$, corresponding to the transmission delay, is known by the Slave once the packet has reached it.

D. Receipt and processing of the control data

The control, sent by the Master at time $t_{1,k}$, is received by the Slave at time $t^r_{1,k} > t_{1,k}$. It will be injected in the Slave input only at the pre-defined "target time" $t^{target}_{1,k} = t_{1,k} + h^m_1$. The corresponding waiting time h^m_1 is depicted on Figure 2. This is realistic because the transmission delay is bounded by a known value (in general, one can choose $h^m_1 = \delta^m_1$. By this way, at any present time, the Master also knows the time $t_{1,k}$ when this control $u(t_{1,k})$ will be injected at the Slave input.

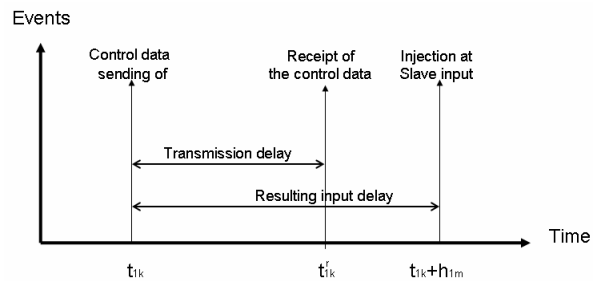

Fig. 2. Control data processing

E. Transmission of the measured output information

The Slave accesses its output y at discrete instants. A sent packet contains the output $y(t_{2,k'})$ together with its measurement instant $t_{2,k'}$ which is the k'^{th} one. The Master receives at time $t^r_{2,k'}$ the output data. Once the packet has reached the Master, the delay $t^r_{2,k'} - t_{2,k'}$ is known thanks to the GPS synchronization.

F. Observation of the process

For a given k and for any $t \in [t_{1,k} + h_{1m}\ ,\ t_{1,k+1} + h_{1m}[$, there exists a k' such that the proposed observer is of the form:

$$\begin{cases} \dot{\hat{x}}(t) = & A\hat{x}(t) + Bu(t_{1,k}) - L(y(t_{2,k'}) - \hat{y}(t_{2,k'})), \\ \hat{y}(t) = & C\hat{x}(t). \end{cases} \tag{5}$$

The index k' corresponds to the most recent output information the Master has received. Note that the Master knows the time $t_{1,k}$ and the control $u(t_{1,k})$ (see Section II-D), which makes this observer realizable.

Using the delay re-writing proposed in (3), one obtains:

$$\begin{cases} \dot{\hat{x}}(t) = & A\hat{x}(t) + Bu(t - \delta_1(t)) \\ & -L(y(t - \delta_2(t)) - \hat{y}(t - \delta_2(t))), \\ \hat{y}(t) = & C\hat{x}(t), \end{cases} \tag{6}$$

with $\delta_1(t) \triangleq t - t_{1,k}$ and $\delta_2(t) \triangleq t - t_{2,k'}$.

In other words, the observer is realizable because the times $t_{1,k}$ and $t_{2,k'}$ defining the observer delays are known, thanks to the common GPS clock. The system features lead to $\delta_1(t) \le h^m_1 + T$ and $\delta_2(t) \le h^m_2 + T$.

III. Design of the controller and observer gains

A. A preliminary result on exponential stabilization

The controller and observer gains will have to be computed so to guarantee the optimal speed rate α despite the presence of delays (communication plus sampling). This subsection gives a result on exponential $\alpha-$stabilization of systems with variable delays. The theorem is an adaptation from [18], [19] and is presented with a sketch of a proof.

Consider the following linear system with bounded variable delays:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t-\delta_1(t)) + Bu(t-\delta_2(t))), \\ x(t) = \phi(t),\ t \in [-h, 0], \end{cases} \tag{7}$$

where the delays $\delta_i(t)$ satisfy, for $i = 1,2$:

$$\delta_i(t) = \delta_i + \eta_i(t), \quad with \quad |\eta_i(t)| \le \mu_i \quad and \quad \dot{\eta}_i(t) \le 1. \tag{8}$$

Note that the delays have a non zero lower bound ("non small delays" [9]). The following theorem uses a polytopic formulation of the variable delays and this leads to the definition of the following extrema, to be involved in the stability conditions:

$$\begin{array}{ll} \beta_{11} = e^{\alpha(\delta_1-\mu_1)}, & \beta_{12} = e^{\alpha(\delta_1+\mu_1)}, \\ \beta_{21} = e^{\alpha(\delta_2-\mu_2)}, & \beta_{22} = e^{\alpha(\delta_2+\mu_2)}. \end{array} \tag{9}$$

Theorem 1 (Exponential stability): Given a gain matrix K, the system (7) is $\alpha-$stable if there exists $n \times n$ matrices $0 < P_1$, P_2, P_3, S_k, Y_{k1}, Y_{k2}, Z_{k1}, Z_{k2}, Z_{k3}, R_k et R_{ka}, for $k = 1,2$ satisfying the LMI conditions:

$$\begin{bmatrix} \Psi_1 & P^T \begin{bmatrix} 0 \\ \beta_{1i}A_1 \end{bmatrix} - Y_1^T & P^T \begin{bmatrix} 0 \\ \beta_{2j}BK \end{bmatrix} - Y_2^T & \mu_1 P^T \begin{bmatrix} 0 \\ \beta_{1i}A_1 \end{bmatrix} & \mu_2 P^T \begin{bmatrix} 0 \\ \beta_{2j}BK \end{bmatrix} \\ * & -S_1 & 0 & 0 & 0 \\ * & * & -S_2 & 0 & 0 \\ * & * & * & -\mu_1 R_{1a} & 0 \\ * & * & * & * & -\mu_2 R_{2a} \end{bmatrix} < 0, \tag{10}$$

$$\forall (i,j) \in \{1,2\}^2$$

and

$$\begin{bmatrix} R_k & Y_k \\ * & Z_k \end{bmatrix} \ge 0, \quad k = 1,2, \tag{11}$$

where

$$\begin{array}{l} P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad Z_k = \begin{bmatrix} Z_{k1} & Z_{k2} \\ Z_{k2}^T & Z_{k3} \end{bmatrix}, \\ Y_k = \begin{bmatrix} Y_{k1} & Y_{k2} \end{bmatrix}, \end{array} \tag{12}$$

$$\begin{aligned} \Psi_1 = \quad & P^T \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix}^T P \\ & + \textstyle\sum_{k=1}^{2} \left\{ \begin{bmatrix} Y_k \\ 0 \end{bmatrix} + \begin{bmatrix} Y_k \\ 0 \end{bmatrix}^T + \delta_k Z_k \right. \\ & \left. + \begin{bmatrix} S_k & 0 \\ 0 & \delta_k R_k + 2\mu_k R_{ka} \end{bmatrix} \right\}. \end{aligned} \tag{13}$$

These LMI conditions will be used in the computation of the gains K of the feedback control and L of the observer. *Proof:* The proof is based on Lyapunov-Krasovskii technics with descriptor representation. Consider the Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) = \quad & \bar{x}_\alpha^T(t) E P \bar{x}_\alpha(t) + \textstyle\sum_{i=1}^{2} \int_{-\delta_i}^{0} \int_{t+\theta}^{t} \dot{x}_\alpha^T(s) R_i \dot{x}_\alpha(s) ds d\theta. \\ & \textstyle\sum_{i=1}^{2} \int_{-\mu_i}^{\mu_i} \int_{t+\theta-\delta_i}^{t} \dot{x}_\alpha^T(s) R_i \dot{x}_\alpha(s) ds d\theta, \end{aligned} \tag{14}$$

where$\bar{x}_\alpha(t) = col\{x_\alpha(t), \dot{x}_\alpha(t)\}$, $x_\alpha(t) = x(t)e^{\alpha t}$ and $E = diag\{I, 0_{(2\times 2)}\}$. Differentiating this functional along the trajectory of (7) and using LMI techniques leads to the LMI conditions of theorem 1. Note that the first integral part of the functional takes in account the constant delays δ_i and the last one the time-varying disturbing delays $\eta_i(t)$ which is norm bounded by μ_i. ■

B. Observer design

Since the pair (A,C) is observable, it is possible to determine a linear gain L such that the observer exponentially converges to the real system in the non-delayed case. The next theorem allows one to design another L so that the observer state $\hat{x}(t)$ converges sufficiently fast (then, with exponential rate α) to the real system state $x(t)$ despite a variable delay $\delta_2(t)$ on the Slave output. The error vector is defined as $e(t) = x(t) - \hat{x}(t)$. From 1 and 6, this error is ruled by:

$$\dot{e}(t) = \quad Ae(t) - LCe(t-\delta_2(t)), \tag{15}$$

Theorem 2: Suppose that, for some positive scalars α and ε, there exists $n \times n$ matrices $0 < P_1$, P, S, Y_1, Y_2, Z_1, Z_2, Z_3, R, R_a and a matrix W with appropriate dimensions such that the following LMI conditions are satisfied for $j = 1,2$:

$$\begin{bmatrix} \Psi_2 & \begin{bmatrix} \beta_{2j}WC - Y_1 \\ \varepsilon\beta_{2j}WC - Y_2 \end{bmatrix} & \mu_2\beta_{2j} \begin{bmatrix} WC \\ \varepsilon WC \end{bmatrix} \\ * & -S & 0 \\ * & * & -\mu_2 R_a \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} R & Y \\ * & Z \end{bmatrix} \ge 0, \tag{17}$$

where β_{2j} are defined by (9) for $j = 1,2$ and the matrices Y, Z and Ψ_2 are given by:

$$Y = [Y_1\ Y_2], \quad Z = \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}, \tag{18}$$

$$\begin{array}{l} \Psi_2^{11} = P^T(A_0 + \alpha I) + (A_0 + \alpha I)^T P + S + \delta_2 Z_1 + Y_1 + Y_1^T, \\ \Psi_2^{12} = P_1 - P + \varepsilon P^T (A_0 + \alpha I)^T + \delta_2 \bar{Z}_2 + \bar{Y}_2, \\ \Psi_2^{22} = -\varepsilon(P + P^T) + \delta_2 \bar{Z}_3 + 2\mu_2 R_a, \end{array}$$

Then, the gain:

$$L = (P^T)^{-1} W, \tag{19}$$

makes the error (15) of observer (6) exponentially converge to the solution $e(t) = 0$, with a decay rate $\alpha > 0$.

Proof: The proof comes from Theorem 1 with a single delay and:

$$P = P_2 = \varepsilon P_3, \quad W = P_2 L. \tag{20}$$

■

Remark 1: In the previous theorem, the delay $\delta_2(t)$ and then, δ_2 and μ_2, are imposed by the quality of the network and maximum the sampling period. The greater α, the faster the stabilization. Thus, the objective is to tune ε to maximize α.

C. Control design

In this part, the interest will be focussed on the design of an ideal controller $u = Kx$, which means a perfect observer ($e(t) = 0, x(t) = \hat{x}(t)$. The influence of the observation dynamics ($e(t) \neq 0$) on the global system will be considered in the next subsection . Then one considers:

$$\dot{x}(t) = \quad Ax(t) + BKx(t - \delta_1(t)), \tag{21}$$

Theorem 3: [19] Suppose that, for some positive numbers α and ε, there exists a positive definite matrix $\bar{P}_1$, matrices of size $n \times n$: $\bar{P}$, $\bar{U}$, $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$, $\bar{Y}_1$, $\bar{Y}_2$ similarly to (18) and a $n \times m$ matrix W, such that the following LMI conditions hold:

$$\Gamma_{3i} = \begin{bmatrix} \Psi_3 & \begin{bmatrix} \beta_i BW - \bar{Y}_1^T \\ \varepsilon\beta_{1i} BW - Y_2^T \end{bmatrix} & \mu_1 \begin{bmatrix} \beta_{1i} BW \\ \varepsilon\beta_{1i} BW \end{bmatrix} \\ * & -\bar{S} & 0 \\ * & * & -\mu_1 \bar{R}_a \end{bmatrix} < 0, \tag{22}$$

$$\forall i = 1, 2,$$

$$\begin{bmatrix} \bar{R} & \bar{Y}_1 & \bar{Y}_2 \\ * & \bar{Z}_1 & \bar{Z}_2 \\ * & * & \bar{Z}_3 \end{bmatrix} \geq 0, \tag{23}$$

where β_{1i}, for $i = 1, 2$, are defined by (9) and

$$\bar{\Psi}_3^{11} = (A_0 + \alpha I)\bar{P} + \bar{P}^T (A_0 + \alpha I)^T + \bar{S} + \delta_1 \bar{Z}_1 + \bar{Y}_1 + \bar{Y}_1^T,$$
$$\bar{\Psi}_3^{12} = \bar{P}_1 - \bar{P} + \varepsilon \bar{P}^T (A_0 + \alpha I)^T + \delta_1 \bar{Z}_2 + \bar{Y}_2,$$
$$\bar{\Psi}_3^{22} = -\varepsilon(\bar{P} + \bar{P}^T) + \delta_1 \bar{Z}_3 + 2\mu_1 \bar{R}_a.$$

Then, the gain:

$$K = W\bar{P}^{-1}, \tag{24}$$

exponentially stabilizes the system (21) with the decay rate α for all delay $\delta_1(t)$ satisfying (8).

Proof: We apply Theorem 1 with: $P_3 = \varepsilon P_2$, where ε is a tuning scalar parameter. Note that P_2 is nonsingular since the only matrix which can be negative definite in the second block on the diagonal of (10) is $-\varepsilon(P_2 + P_2^T)$. We also define:

$$\bar{P} = P_2^{-1}. \tag{25}$$

For any matrix $V \in \{P_1, Y_{ij}, S_i, U, R_i\ R_{ia}, Z_{ik}\}$ for all $i = 1, 2$, $j = 1, 2$, $k = 1, 2, 3$, we define another matrix $\bar{V}$ by $\bar{V} \triangleq \bar{P}^T V \bar{P}$. The proof is achieved by multiplying (10), from the right and the left sides respectively, by $\mathscr{P}_7 = diag\{\bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}\}$ and its transpose $\mathscr{P}_7^T$, and multiplying (11) by $\mathscr{P}_3 = diag\{\bar{P}, \bar{P}, \bar{P}\}$ and its transpose $\mathscr{P}_3^T$, from the right and the left sides respectively. ■

D. Global stability of the remote system

The gains K and L have to be computed in such a way they exponentially stabilize the global Master-Slave-Observer system despite the variable delays $\delta_1(t)$ and $\delta_2(t)$. This global system is:

$$\begin{cases} \dot{x}(t) = & Ax(t) + BK\hat{x}(t - \delta_1(t)), \\ \dot{e}(t) = & Ae(t) - LCe(t - \delta_2(t)), \end{cases} \tag{26}$$

which leads to:

$$\begin{cases} \dot{x}(t) = & Ax(t) + BKx(t - \delta_1(t)) - BKe(t - \delta_1(t)), \\ \dot{e}(t) = & Ae(t) - LCe(t - \delta_2(t)), \end{cases} \tag{27}$$

Introducing the variable $\bar{e}(t) = col\{x(t), e(t)\}$, (27) becomes:

$$\dot{\bar{e}}(t) = \bar{A}_0 \bar{e}(t) + \bar{A}_1 \bar{e}(t - \delta_1(t)) + \bar{A}_2 \bar{e}(t - \delta_2(t)), \tag{28}$$

with

$$\bar{A}_0 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \quad \bar{A}_1 = \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 0 & 0 \\ 0 & LC \end{bmatrix}. \tag{29}$$

Then, the exponential stability of the global system is proven by using Theorem 1.

E. Operating overview

We can summarize here the way one operates the remote system design. The first step consists in the determination of the delay upper-bounds h_1^m, h_2^m, and T. Then, the observer gain L is computed by applying Theorem 2 to system (15), and the controller gain K by applying Theorem 3 to system (21). Once these gains are found, the stability of the closed-loop system is checked by applying Theorem 1 to (28). An additional study of the robustness w.r.t. delay mismatches is possible, even if it not developed here.

IV. Application to a mobile robot

This study is illustrated on the model of a mobile robot (Slave) which can move in 1 direction. The identification phase gives the following dynamics:

$$\begin{cases} \dot{x}(t) = & \begin{bmatrix} 0 & 1 \\ 0 & -11,32 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 11,32 \end{bmatrix} u(t - \delta_1(t)), \\ y(t) = & \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{cases} \tag{30}$$

(15), respectively theorem 3 to system . Once these gains are found, the stability of the closed-loop system is proven by using theorem 1 applied to (28).

The characteristics of transmission delays combined with the sampling and with the computation effect lead to the values (see (8)) $\delta_1 = \delta_2 = 0.37sec.$, and $\mu_1 = \mu_2 = 0.11sec.$ Theorem 2 applied to (15) guarantees that the error dynamics converge exponentially to the solution $e(t) = 0$ with $\alpha = 1.01$ (obtained for $\varepsilon = 3.00$) if the gain L is chosen as:

$$L = \begin{bmatrix} -0.9119 \\ -0.0726 \end{bmatrix}. \tag{31}$$

Theorem 3 applied to (21) ensures the control law will exponentially stabilize the reduced system with $\alpha = 1.01$, obtained for $\varepsilon = 3.43$ and:

$$K = \begin{bmatrix} -0.9125 & -0.0801 \end{bmatrix} \tag{32}$$

With these values, the global stability of the remote system (28) is also ensured by Theorem 1.

Figure 3 shows a simulation result that was obtained for a delay variation law, $\delta_i(t) = \delta_i + \mu_i/2sin(w_i t) + b_i(t)$, depicted on Figure 4. w_i represents the frequency of the time-varying part of the delay and where $b_i(t)$ is a piecewise continuous function which corresponds to the sampling effects and which satisfies $\|b_i(t)\| \leq \mu_i/2$. On Figure 3, the continuous model of the observer $\hat{x}$ corresponds to the blue and red curves, while the sampling instants correspond to the blue and red dots. The blue output is driven to its set value (dark-blue steps).

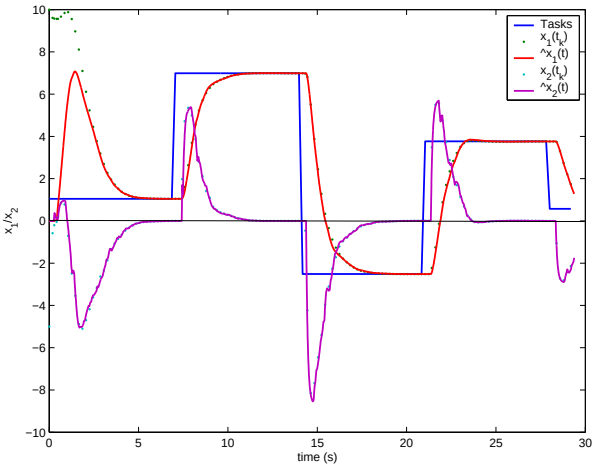

Fig. 3. Simulation results

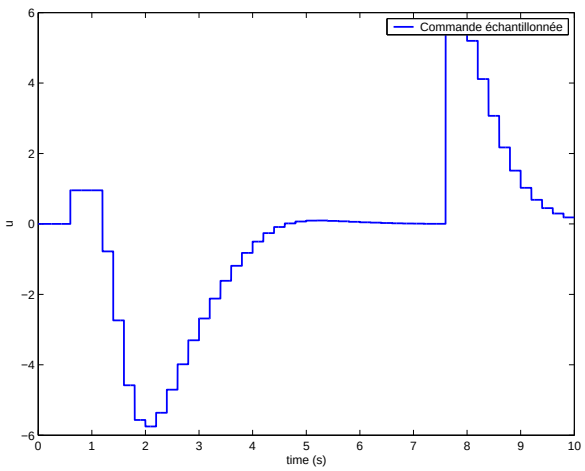

Fig. 4. The corresponding delayed control

V. Aknowledgements

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VI. Concluding remarks

A main feature of our control strategy is that the Master works in continuous time, whereas the Slave works in discrete time. By this way, the observer always works and provides an estimation of the Slave state even if the Slave information is not sent continuously.

Another characteristic of this approach is to consider non-small delays (*i.e.* delays which lower bound is non zero) with few assumptions (non symmetric, unknown, time-varying). The global system will be implemented soon.

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