

Motion planning

Florent Lamiraux

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Motion planning

Florent Lamiraux

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Motion planning

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Motion planning

Introduction

Definitions

Random Sampling

Collision testing

Software

Motion planning

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Context

industrial robot



aerial vehicle



autonomous vehicle



Autonomous mobile systems

- moving in an environment cluttered by obstacles
- possibly subject to kinematic or dynamic constraints

Motion planning : automatically compute a feasible collision-free path between two given configurations.

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Robot

Set of rigib bodies $\mathcal{B}_0, \cdots \mathcal{B}_m$, linked to one another by *joints*.



Joint : mobility of a body in the reference frame of its parent, parameterized by one or several numbers.



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Robot configuration

The configuration \mathbf{q} of a robot is represented by the concatenation of the parameters of each joint.





Motion planning

Forward kinematics

Computation of the position of each joint in world frame.

$$M_i(\mathbf{q}) = M_{parent(i)}(\mathbf{q}) \ M_{i/parent} \ T_i(\mathbf{q})$$





Motion planning

- \blacktriangleright Workspace : $\mathcal{W}=\mathbb{R}^2$ or \mathbb{R}^3 : space in which the robot moves
- Workspace obstacle : compact subset of \mathcal{W} , denoted by \mathcal{O} .
- ► Configuration space : C.
- ▶ Position in configuration **q** of a point $M \in B_i$: $\mathbf{x}_i(M, \mathbf{q})$.
- Configuration space obstacle :

 $\mathcal{C}_{obst} = \{ \mathbf{q} \in \mathcal{C}, \exists i \in \{1, \cdots, m\}, \exists M \in \mathcal{B}_i, \mathbf{x}_i(M, \mathbf{q}) \in \mathcal{O} \text{ ou} \\ \exists i, j \in \{1, \cdots, m\}, \exists M_i \in \mathcal{B}_i, \exists M_j \in \mathcal{B}_j, \\ \mathbf{x}_i(M_i, \mathbf{q}) = x_j(M_j, \mathbf{q}) \}$

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Motion

Motion :

- continuous mapping from [0,1] into C.
- Collision free motion :
 - continuous mapping from [0,1] into C_{free} .

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Motion planning problem



initial configuration

goal configuration

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$$\mathcal{C} = [-2\pi, 2\pi]^6$$

History

- before the 1990's : mainly a mathematical problem
 - Real algbraic geometry
 - Decidability : Schwartz and Sharir 1982
 - Tarski theorem, Collins decomposition
 - Approximate cell decomposition
- from the 1990's : an algorithmic problem
 - random sampling (1993)
 - asymptotically optimal random sampling (2011)

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Random sampling

Random sampling motion planning methods appeared in the early 1990's

- ► Principle
 - sample random configurations
 - test whether they are collision-free
 - build a roadmap the nodes of which are the free configurations,
 - ▶ and the edges of which are collision-free linear interpolations.

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Probabilistic roadmap (PRM) 1994



Motion planning

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Motion planning

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Image: A matrix and a matrix

Probabilistic roadmap (PRM) 1994



Image: A matrix and a matrix

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Motion planning

Image: A matrix and a matrix

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Probabilistic roadmap (PRM)

- Numerous useless nodes are created
 - this makes the connection of new nodes more time consuming
- Variant : visibility-based PRM
 - only *interesting* nodes are kept.

Visibility-based probabilistic roadmap (Visi-PRM) 1999



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Rapidly exploring Random Tree (RRT) 2000



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Motion planning

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Motion planning

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Random sampling

- Pros :
 - no explicit computation of the configuration space
 - easy to implement,
 - robust.
- ► Cons :
 - no completeness, only probrabilistic completeness
 - difficult to find narrow passages.
- required operators :
 - collision checking
 - for configurations (static)
 - for linear interpolation (dynamic)

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Asymptotically optimal random sampling

Variants of PRM and RRT exist, and are asymptotically optimal :

- when the number of nodes tends to infinity,
- the solution computed by the algorithm tends to the optimal collision-free path.

PRM*

PRM

 $V \leftarrow \emptyset. E \leftarrow \emptyset$ for $i \in \{0, \dots, n\}$ do $x_{rand} \leftarrow \mathsf{SampleFree}_i$ $U \leftarrow G.\text{Near}(x_{rand}, r)$ for all $u \in U$ in order of increasing $||u - x_{rand}||$ do if x_{rand} and u in different connected components then TryConnect (x_{rand}, u) end if end for end for

PRM*

```
V \leftarrow \text{SampleFree}_{i=1,\dots,n}, E \leftarrow \emptyset<br/>for v \in V do<br/>U \leftarrow G.\text{Near}(v, r^*) \setminus v<br/>for all u \in U do<br/>TryConnect (v, u)<br/>end for<br/>end for<br/>v = (\log r(v)/r)^{\frac{1}{2}}
```

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PRM*

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PRM*

 $V \leftarrow \mathsf{SampleFree}_{i=1,\dots,n}, \ E \leftarrow \emptyset$ for $v \in V$ do $U \leftarrow G.\mathsf{Near}(v, r^*) \setminus v$ for all $u \in U$ do TryConnect (v, u)end for end for $r^* = \gamma_{PBM} (\log(n)/n)^{\frac{1}{d}}$

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kPRM*

kPRM

 $V \leftarrow \emptyset. E \leftarrow \emptyset$ for $i \in \{0, \dots, n\}$ do $x_{rand} \leftarrow SampleFree_i$ $U \leftarrow G.$ Nearest (x_{rand}, k) for all $u \in U$ in order of increasing $||u - x_{rand}||$ do if x_{rand} and u in different connected components then TryConnect (x_{rand}, u) end if end for end for

PRM*

```
V \leftarrow \text{SampleFree}_{i=1,\dots,n}, E \leftarrow \emptyset
for v \in V do
U \leftarrow G.\text{Nearest}(v, k^*) \setminus v
for all u \in U do
TryConnect (v, u)
end for
end for
f = k_{\text{Dervelop}}(n), k_{\text{Derve}} \geq o(1 + 1)
```

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kPRM*

kPRM

$$V \leftarrow \emptyset, E \leftarrow \emptyset$$

for $i \in \{0, \dots, n\}$ do
 $x_{rand} \leftarrow \text{SampleFree}_i$
 $U \leftarrow G.\text{Nearest}(x_{rand}, k)$
for all $u \in U$ in order of increa-
sing $||u - x_{rand}||$ do
if x_{rand} and u in different
connected components
then
TryConnect (x_{rand}, u)
end if
end for
end for

kPRM*

 $V \leftarrow \mathsf{SampleFree}_{i=1,\dots,n}, \ E \leftarrow \emptyset$ for $v \in V$ do $U \leftarrow G.\mathsf{Nearest}(v, k^*) \setminus v$ for all $u \in U$ do TryConnect (v, u)end for end for $k^* = k_{PRM} \log(n), \ k_{PRM} > e(1 + \frac{1}{d})$

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Note that :

- PRM*, kPRM* are not iterative anymore,
- making them iterative is not trivial.

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Rapidly Exploring Random trees

There exists also asymptotically optimal variants of RRT

► RRG, RRT*

but they are specific to a given problem $(\mathbf{q}_{init}, \mathbf{q}_{goal})$.

Collision tests

- static : for configurations
 - problem : given
 - two rigid objects made of triangles
 - the relative position of one with respect to the other one

determine whether they are colliding.

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- binary tree of bounding volumes such that
 - each node has two children,
 - leaves are triangles.



Bounding volume hierarchies

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- Algorithm
 - test root nodes of each tree,
 - if two bounding volumes collide, test one with the children of the other one.



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Open source software platform

Several open-source platforms for motion planning are available

- OMPL (Rice University)
 - no kinematic chain,
 - no collision checking.
- Openrave (CMU)
- Movelt (ROS)
 - Integration in ROS of
 - fcl (collision checking), KDL (kinematic chain)
- Humanoid Path Planner
 - numerical constraints (quasi-static equilibrium)
 - advanced manipulation planning

Humanoid Path Planner

https://humanoid-path-planner.github.io/hpp-doc

Motion planning

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