

# Motion planning

Florent Lamiraux

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Introduction
Definitions
Random Sampling
Collision testing
Software

## Motion planning

Florent Lamiraux

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## Motion planning

Introduction

Definitions

Random Sampling

Collision testing

Software

### Context

industrial robot



aerial vehicle



autonomous vehicle



Autonomous mobile systems

- moving in an environment cluttered by obstacles
- possibly subject to kinematic or dynamic constraints

Motion planning: automatically compute a feasible collision-free path between two given configurations.

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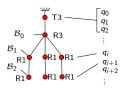
#### Autonomous mobile systems

- moving in an environment cluttered by obstacles
- possibly subject to kinematic or dynamic constraints

Motion planning: automatically compute a feasible collision-free path between two given configurations.

### Robot

Set of rigib bodies  $\mathcal{B}_0, \dots \mathcal{B}_m$ , linked to one another by *joints*.

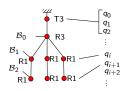


Joint: mobility of a body in the reference frame of its parent, parameterized by one or several numbers.



## Robot configuration

The configuration  $\mathbf{q}$  of a robot is represented by the concatenation of the parameters of each joint.

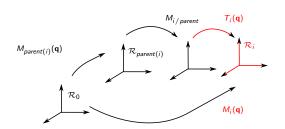




### Forward kinematics

Computation of the position of each joint in world frame.

$$M_i(\mathbf{q}) = M_{parent(i)}(\mathbf{q}) \ M_{i/parent} \ T_i(\mathbf{q})$$





- $lackbox{ Workspace}: \mathcal{W}=\mathbb{R}^2 \text{ or } \mathbb{R}^3: \text{space in which the robot moves}$
- lackbox Workspace obstacle : compact subset of  $\mathcal{W}$ , denoted by  $\mathcal{O}$ .
- ▶ Configuration space : C.
- ▶ Position in configuration **q** of a point  $M \in \mathcal{B}_i : \mathbf{x}_i(M, \mathbf{q})$ .
- ► Configuration space obstacle :

$$C_{obst} = \{ \mathbf{q} \in C, \exists i \in \{1, \dots, m\}, \exists M \in \mathcal{B}_i, \mathbf{x}_i(M, \mathbf{q}) \in \mathcal{O} \text{ ou }$$
  
 $\exists i, j \in \{1, \dots, m\}, \exists M_i \in \mathcal{B}_i, \exists M_j \in \mathcal{B}_j,$   
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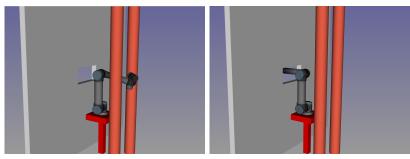
### Motion

- ► Motion :
  - continuous mapping from [0,1] into  $\mathcal{C}$ .
- ► Collision free motion :
  - $\,\blacktriangleright\,$  continuous mapping from [0,1] into  $\mathcal{C}_{\textit{free}}.$

### Motion

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## Motion planning problem



initial configuration

goal configuration

$$\mathcal{C} = [-2\pi, 2\pi]^6$$

## History

- before the 1990's : mainly a mathematical problem
  - Real algbraic geometry
  - Decidability : Schwartz and Sharir 1982
    - ► Tarski theorem, Collins decomposition
  - Approximate cell decomposition
- ▶ from the 1990's : an algorithmic problem
  - random sampling (1993)
  - asymptotically optimal random sampling (2011)

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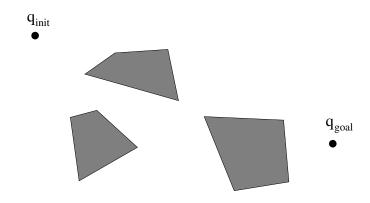
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- ► Principle
  - sample random configurations
  - test whether they are collision-free
  - build a roadmap the nodes of which are the free configurations,
  - ▶ and the edges of which are collision-free linear interpolations.

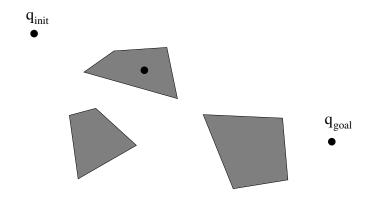
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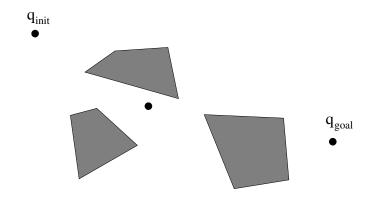
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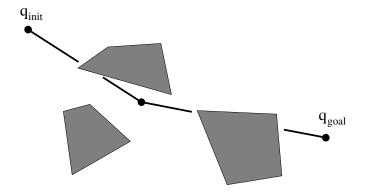
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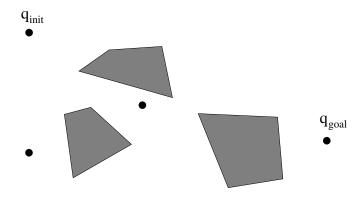
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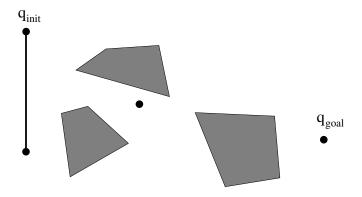


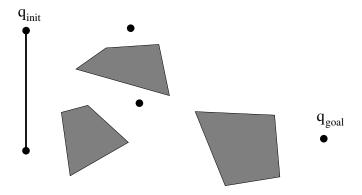


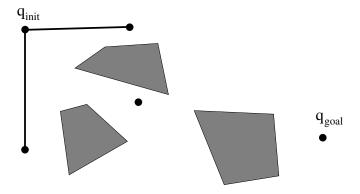


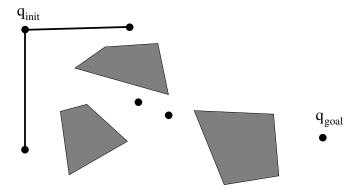


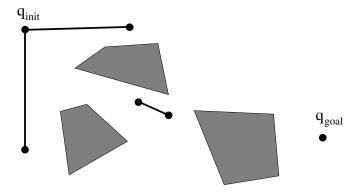


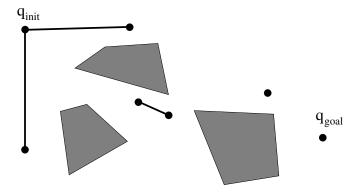


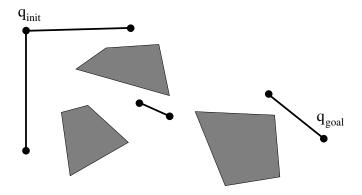


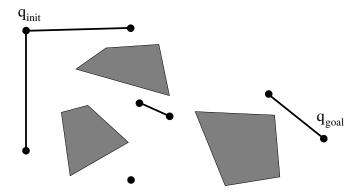


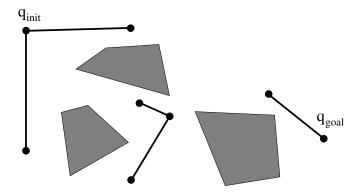


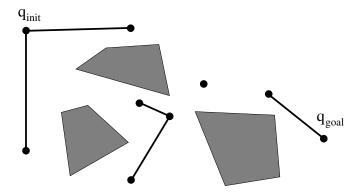


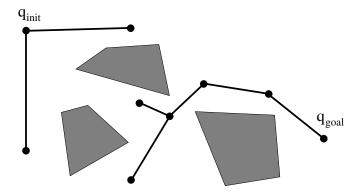


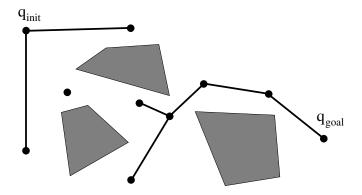


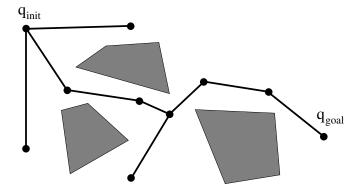




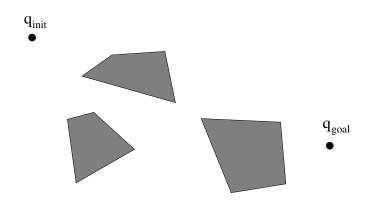


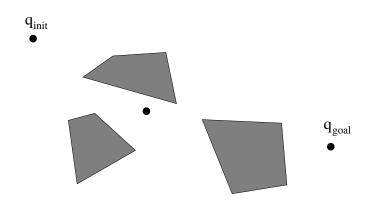


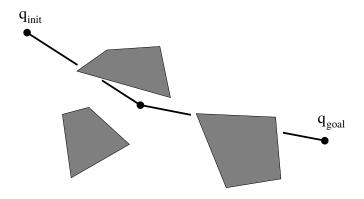


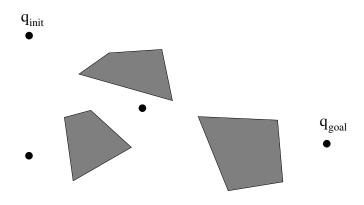


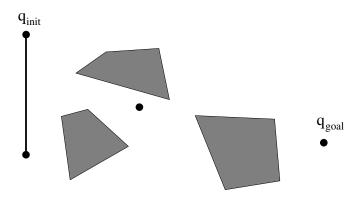
- Numerous useless nodes are created
  - this makes the connection of new nodes more time consuming
- Variant : visibility-based PRM
  - only interesting nodes are kept.

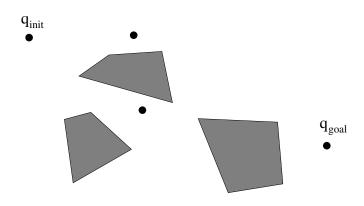


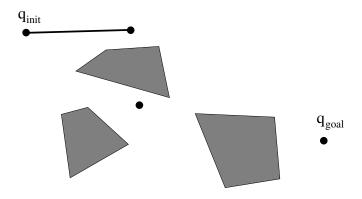


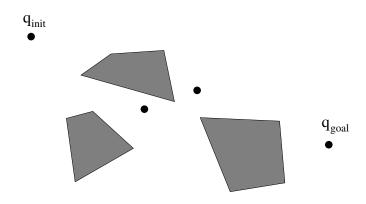


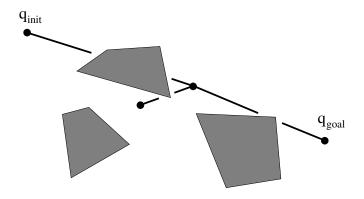


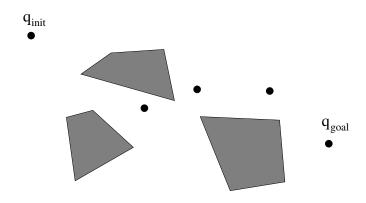


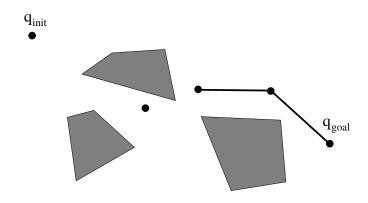


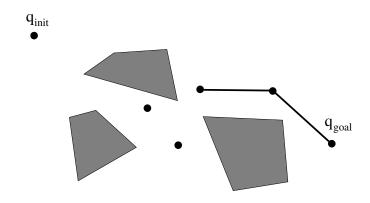


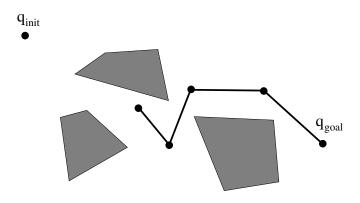


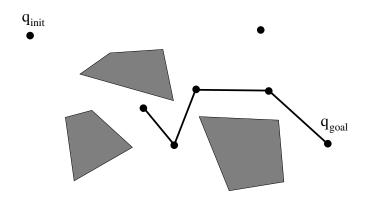


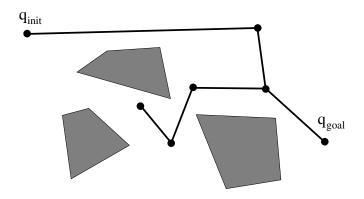


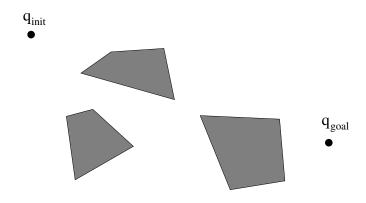


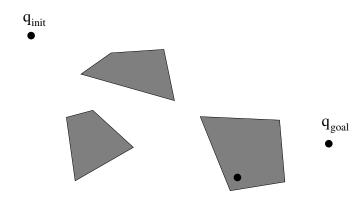


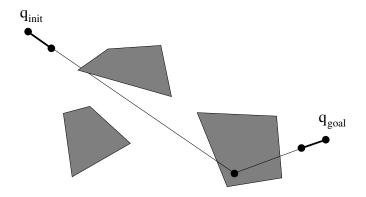


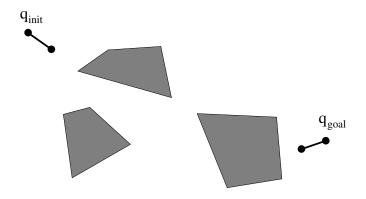


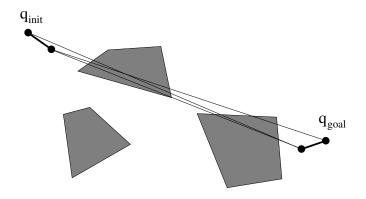


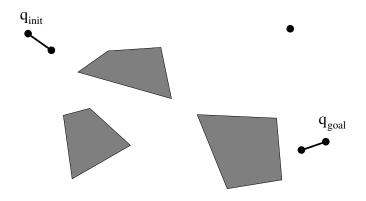


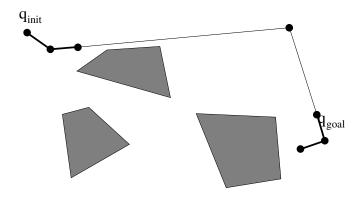


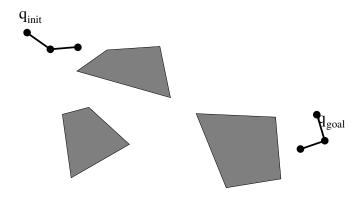


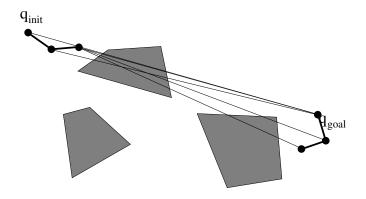


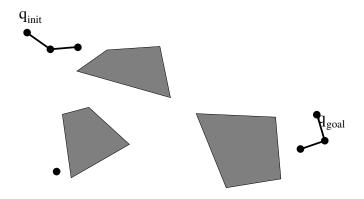


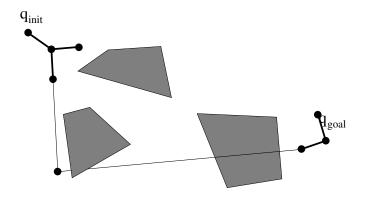


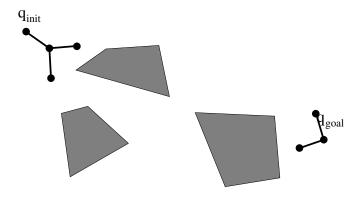


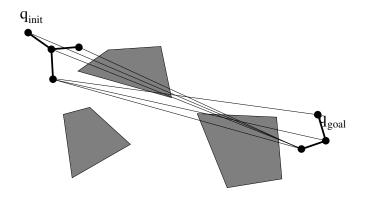


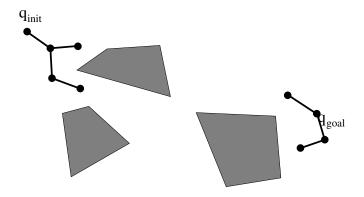


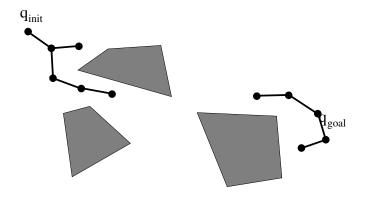


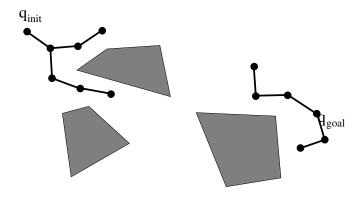


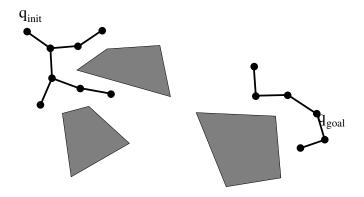


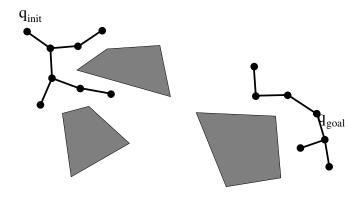


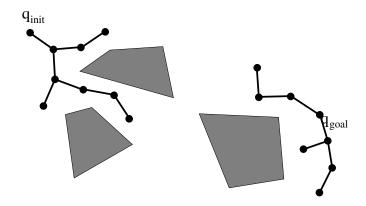


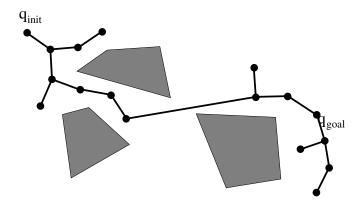












## Random sampling

- ► Pros :
  - no explicit computation of the configuration space
  - easy to implement,
  - robust.
- ► Cons:
  - no completeness, only probrabilistic completeness
  - difficult to find narrow passages.
- required operators :
  - collision checking
    - for configurations (static)
    - ▶ for linear interpolation (dynamic)

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## Asymptotically optimal random sampling

Variants of PRM and RRT exist, and are asymptotically optimal:

- when the number of nodes tends to infinity,
- the solution computed by the algorithm tends to the optimal collision-free path.

#### PRM\*

```
PRM
   V \leftarrow \emptyset. E \leftarrow \emptyset
   for i \in \{0, \dots, n\} do
      x_{rand} \leftarrow \mathsf{SampleFree}_i
      U \leftarrow G.Near(x_{rand}, r)
      for all u \in U in order of increa-
      sing ||u - x_{rand}|| do
         if x_{rand} and u in different
         connected
                               components
         then
             TryConnect (x_{rand}, u)
         end if
      end for
   end for
```

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                                                    PRM*
      x_{rand} \leftarrow \mathsf{SampleFree}_i
                                                        V \leftarrow \mathsf{SampleFree}_{i=1 \dots n}, \ E \leftarrow \emptyset
      U \leftarrow G.Near(x_{rand}, r)
                                                       for v \in V do
      for all u \in U in order of increa-
                                                           U \leftarrow G. \text{Near}(v, r^*) \setminus v
      sing ||u - x_{rand}|| do
                                                           for all u \in U do
          if x_{rand} and u in different
                                                               TryConnect (v, u)
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                                 components
                                                           end for
          then
                                                       end for
              TryConnect (x_{rand}, u)
                                                    r^* = \gamma_{PRM} (\log(n)/n)^{\frac{1}{d}}
          end if
      end for
   end for
```

#### kPRM\*

```
kPRM
   V \leftarrow \emptyset. E \leftarrow \emptyset
   for i \in \{0, \dots, n\} do
      x_{rand} \leftarrow \mathsf{SampleFree}_i
      U \leftarrow G. \text{Nearest}(x_{rand}, k)
      for all u \in U in order of increa-
      sing ||u - x_{rand}|| do
          if x_{rand} and u in different
         connected
                               components
          then
             TryConnect (x_{rand}, u)
          end if
      end for
   end for
```

```
kPRM* V \leftarrow \mathsf{SampleFree}_{i=1,\cdots n}, \ E \leftarrow \emptyset for v \in V do U \leftarrow G.\mathsf{Nearest}(v, k^*) \setminus v for all u \in U do \mathsf{TryConnect}(v, u) end for \mathsf{end} \ \mathsf{for} \mathsf{end} \ \mathsf{for} k^* = k_{PRM} \log(n), \ k_{PRM} > e(1 + \frac{1}{d})
```

#### kPRM\*

```
kPRM
   V \leftarrow \emptyset. F \leftarrow \emptyset
   for i \in \{0, \dots, n\} do
      x_{rand} \leftarrow \mathsf{SampleFree}_i
      U \leftarrow G. \text{Nearest}(x_{rand}, k)
      for all u \in U in order of increa-
      sing ||u - x_{rand}|| do
          if x_{rand} and u in different
          connected
                               components
         then
             TryConnect (x_{rand}, u)
          end if
      end for
   end for
```

```
\begin{aligned} & \mathsf{kPRM*} \\ & V \leftarrow \mathsf{SampleFree}_{i=1,\cdots n}, \ E \leftarrow \emptyset \\ & \mathsf{for} \ v \in V \ \mathsf{do} \\ & U \leftarrow G.\mathsf{Nearest}(v, {\color{red}k^*}) \setminus v \\ & \mathsf{for} \ \mathsf{all} \ u \in U \ \mathsf{do} \\ & \mathsf{TryConnect} \ (v, \ u) \\ & \mathsf{end} \ \mathsf{for} \\ & \mathsf{end} \ \mathsf{for} \\ & {\color{red}k^*} = k_{PRM} \log(n), \ k_{PRM} > e(1+\frac{1}{d}) \end{aligned}
```

### PRM\*, kPRM\*

#### Note that:

- ▶ PRM\*, kPRM\* are not iterative anymore,
- making them iterative is not trivial.

## Rapidly Exploring Random trees

There exists also asymptotically optimal variants of RRT

► RRG, RRT\*

but they are specific to a given problem  $(\mathbf{q}_{init}, \mathbf{q}_{goal})$ .

#### Collision tests

- static : for configurations
  - problem : given
    - two rigid objects made of triangles
    - ▶ the relative position of one with respect to the other one determine whether they are colliding.

- binary tree of bounding volumes such that
  - each node has two children,
  - leaves are triangles.



- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.





- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.





- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.



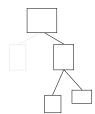


- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.



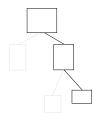


- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.



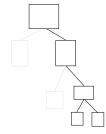


- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.





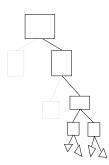
- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.



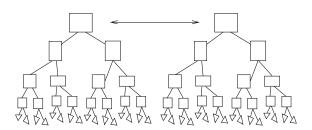


- binary tree of bounding volumes such that
  - ▶ each node has two children,
  - leaves are triangles.

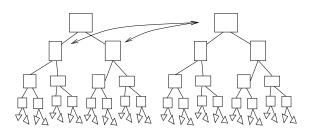




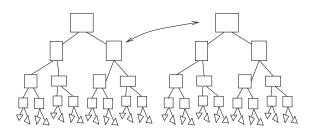
- Algorithm
  - test root nodes of each tree,
  - if two bounding volumes collide, test one with the children of the other one.



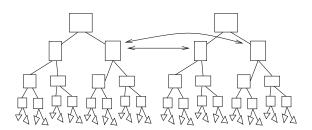
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- Algorithm
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- Algorithm
  - test root nodes of each tree,
  - if two bounding volumes collide, test one with the children of the other one.



### Open source software platform

Several open-source platforms for motion planning are available

- OMPL (Rice University)
  - no kinematic chain,
  - no collision checking.
- Openrave (CMU)
- Movelt (ROS)
  - Integration in ROS of
  - fcl (collision checking), KDL (kinematic chain)
- Humanoid Path Planner
  - numerical constraints (quasi-static equilibrium)
  - advanced manipulation planning



### Humanoid Path Planner

https://humanoid-path-planner.github.io/hpp-doc