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Robot Control

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Robot Control

BRUNO SICILIANO

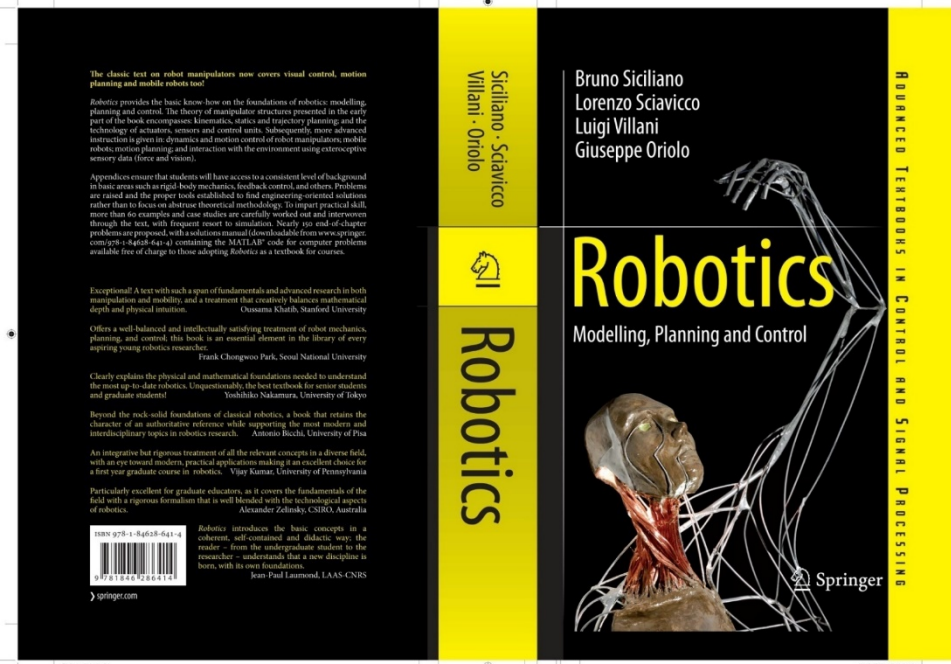


www.prisma.unina.it

- Motion control
- Indirect force control
- Direct force control
- Interaction control using vision and force
- Experiments

B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo, *Robotics: Modelling, Planning and Control*, Springer, London, 2009, DOI 10.1007/978-1-4471-0449-0

- Chapter 8 — Motion Control
- Chapter 9 — Force Control
- Chapter 10 — Visual Control



MOOC Robotics Foundations – Robot Control
 Coming up soon ... <https://youtu.be/JwfRk-U3aPw>

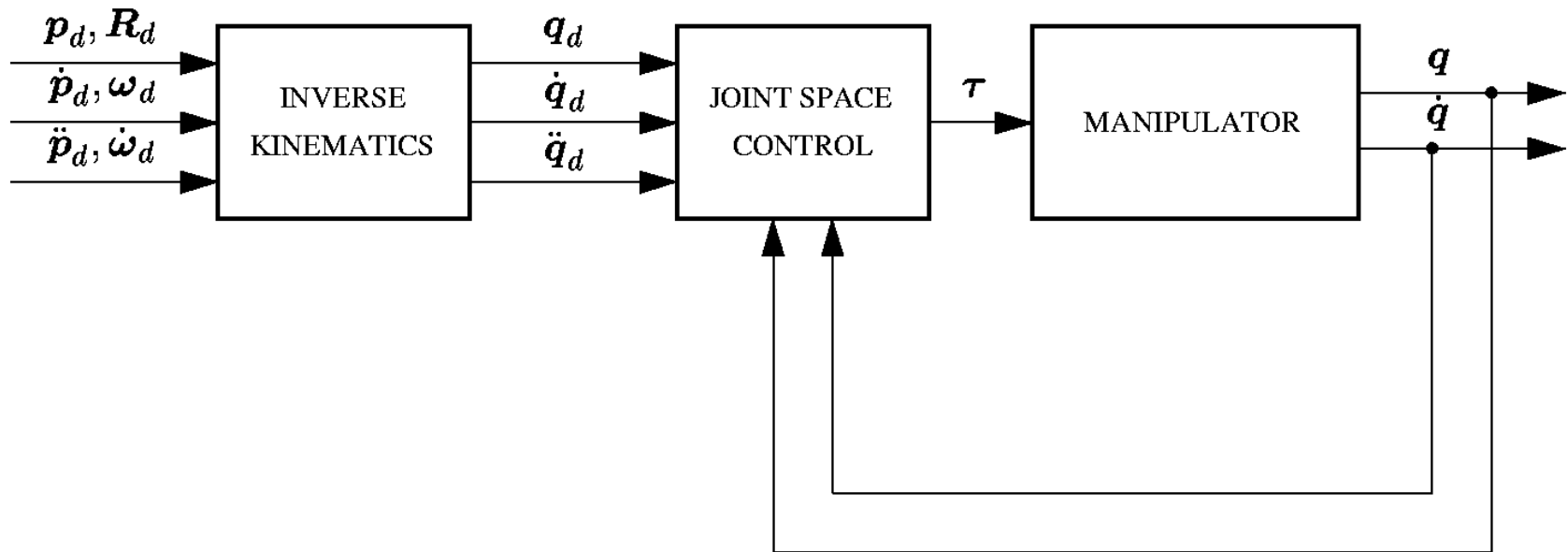
B. Siciliano, O. Khatib, [Springer Handbook of Robotics 2nd Edition](#), Springer, Heidelberg, 2016, DOI [10.1007/978-3-319-32552-1](#)

- Chapter 8 — Motion Control
- Chapter 9 — Force Control
- Chapter 34 — Visual Servoing

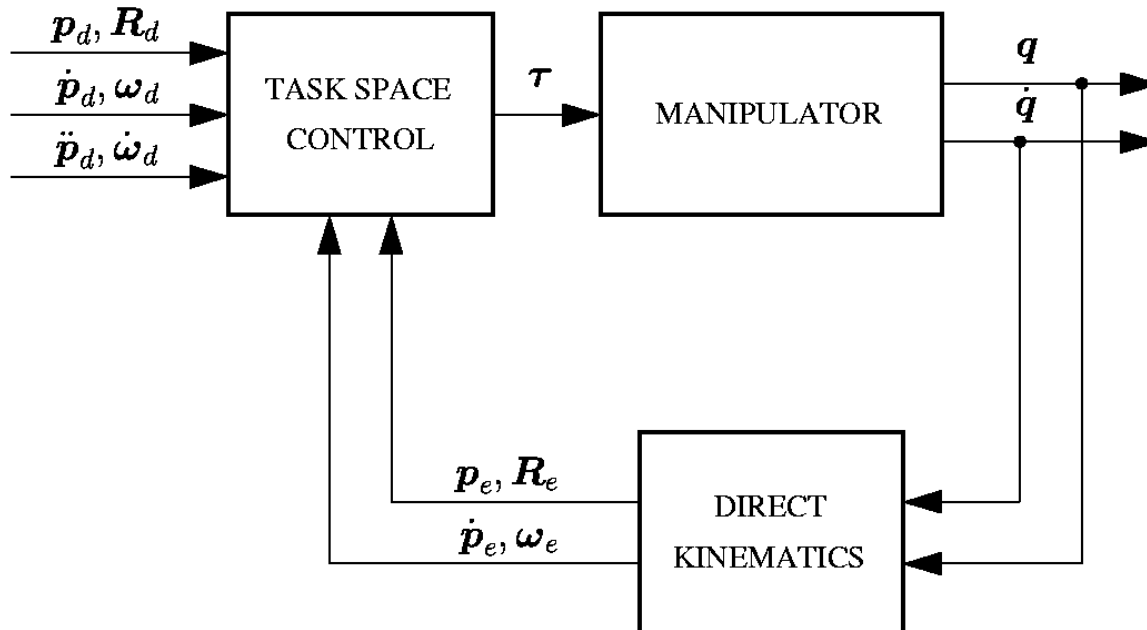


Joint space control

- Task references transformed into joint references
- Redundancy resolution at kinematic level



- Task space control
 - Control directly in task (operational) space
 - Redundancy resolution at dynamic level



- Tracking control
 - Dynamic model-based compensation
 - Euler angles
 - Angle/axis
 - Quaternion
 - Computational issues
 - Redundancy resolution
- Regulation
 - Static model-based compensation
 - Orientation errors

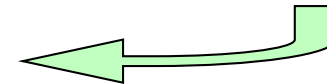


- Inverse dynamics

$$\tau = B(q)\alpha + C(q, \dot{q})\dot{q} + F\dot{q} + g(q)$$

$$\alpha = J^{-1}(q) \left(\begin{bmatrix} a_p \\ a_o \end{bmatrix} - \dot{J}(q, \dot{q})\dot{q} \right)$$

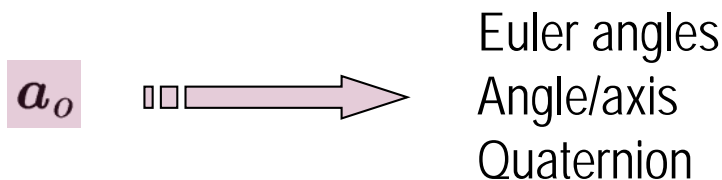
$$\dot{v}_e = J(q)\ddot{q} + \dot{J}(q, \dot{q})\dot{q}$$



- Position control $\Delta p_{de} = p_d - p_e$

$$a_p = \ddot{p}_d + K_{Dp}\Delta\dot{p}_{de} + K_{Pp}\Delta p_{de} \implies \Delta\ddot{p}_{de} + K_{Dp}\Delta\dot{p}_{de} + K_{Pp}\Delta p_{de} = \mathbf{0}$$

- Orientation control



- Orientation error: $\Delta\varphi_{de} = \varphi_d - \varphi_e$

- Resolved angular acceleration

$$\mathbf{a}_o = \mathbf{T}(\varphi_e)(\ddot{\varphi}_d + \mathbf{K}_{Do}\Delta\dot{\varphi}_{de} + \mathbf{K}_{Po}\Delta\varphi_{de}) + \dot{\mathbf{T}}(\varphi_e, \dot{\varphi}_e)\dot{\varphi}_e$$

representation singularities (!)

$$\boldsymbol{\omega}_e = \mathbf{T}(\varphi_e)\dot{\varphi}_e$$

- Error dynamics

$$\dot{\boldsymbol{\omega}}_e = \mathbf{T}(\varphi_e)\ddot{\varphi}_e + \dot{\mathbf{T}}(\varphi_e, \dot{\varphi}_e)\dot{\varphi}_e$$

$$\Delta\ddot{\varphi}_{de} + \mathbf{K}_{Do}\Delta\dot{\varphi}_{de} + \mathbf{K}_{Po}\Delta\varphi_{de} = \mathbf{0}$$

$$\dot{\varphi}_d = \mathbf{T}^{-1}(\varphi_d)\boldsymbol{\omega}_d$$

$$\ddot{\varphi}_d = \mathbf{T}^{-1}(\varphi_d) \left(\dot{\boldsymbol{\omega}}_d - \dot{\mathbf{T}}(\varphi_d, \dot{\varphi}_d)\dot{\varphi}_d \right)$$



■ Orientation error: ${}^e \mathbf{R}_d = \mathbf{R}_e^T \mathbf{R}_d \Rightarrow \varphi_{de}$

■ Resolved angular acceleration

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_d + \mathbf{T}_e(\varphi_{de})(\mathbf{K}_{Do}\dot{\varphi}_{de} + \mathbf{K}_{Po}\varphi_{de}) - \dot{\mathbf{T}}_e(\varphi_{de}, \dot{\varphi}_{de})\dot{\varphi}_{de}$$

$$\dot{\boldsymbol{\omega}}_e = \dot{\boldsymbol{\omega}}_d - \mathbf{T}_e(\varphi_{de})\ddot{\varphi}_{de} - \dot{\mathbf{T}}_e(\varphi_{de}, \dot{\varphi}_{de})\dot{\varphi}_{de}$$

$$\mathbf{T}_e(\varphi_{de}) = \mathbf{R}_e \mathbf{T}(\varphi_{de})$$

choose φ_{de} so that $\mathbf{T}(\mathbf{0})$ is nonsingular (!)

■ Error dynamics

$$\ddot{\varphi}_{de} + \mathbf{K}_{Do}\dot{\varphi}_{de} + \mathbf{K}_{Po}\varphi_{de} = \mathbf{0}$$



- Orientation error: ${}^e \mathbf{R}_d = \mathbf{R}_e^T \mathbf{R}_d \implies {}^e \mathbf{o}_{de} = f(\vartheta_{de}) {}^e \mathbf{r}_{de}$

<i>Representation</i>	$f(\vartheta)$
Classical angle/axis	$\sin(\vartheta)$
Quaternion	$\sin(\vartheta/2)$
Rodrigues parameters	$\tan(\vartheta/2)$
Simple rotation	ϑ

angle axis

- Angle/axis error: ${}^e \mathbf{o}'_{de} = \sin(\vartheta_{de}) {}^e \mathbf{r}_{de}$

- Resolved angular acceleration

$$\mathbf{a}_o = \mathbf{L}^{-1} \left(\mathbf{L}^T \dot{\boldsymbol{\omega}}_d + \dot{\mathbf{L}}^T \boldsymbol{\omega}_d - \dot{\mathbf{L}} \boldsymbol{\omega}_e + \mathbf{K}_{Do} \dot{\mathbf{o}}'_{de} + \mathbf{K}_{Po} \mathbf{o}'_{de} \right)$$

$$\mathbf{o}'_{de} = \mathbf{R}_e {}^e \mathbf{o}'_{de} = \frac{1}{2} (\mathbf{S}(\mathbf{n}_e) \mathbf{n}_d + \mathbf{S}(\mathbf{s}_e) \mathbf{s}_d + \mathbf{S}(\mathbf{a}_e) \mathbf{a}_d)$$

$$\mathbf{L} = -\frac{1}{2} (\mathbf{S}(\mathbf{n}_d) \mathbf{S}(\mathbf{n}_e) + \mathbf{S}(\mathbf{s}_d) \mathbf{S}(\mathbf{s}_e) + \mathbf{S}(\mathbf{a}_d) \mathbf{S}(\mathbf{a}_e))$$

$$\mathbf{n}_e^T \mathbf{n}_d > 0, \mathbf{s}_e^T \mathbf{s}_d > 0, \mathbf{a}_e^T \mathbf{a}_d > 0$$

- Error dynamics

$$\ddot{\mathbf{o}}'_{de} + \mathbf{K}_{Do} \dot{\mathbf{o}}'_{de} + \mathbf{K}_{Po} \mathbf{o}'_{de} = \mathbf{0}$$

- Simpler choice: $\mathbf{R}_e \simeq \mathbf{R}_d \implies \mathbf{L} \simeq \mathbf{I}$

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_d + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{de} + \mathbf{K}_{Po} \mathbf{o}'_{de}$$

- Error dynamics

$$\Delta \dot{\boldsymbol{\omega}}_{de} + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{de} + \mathbf{K}_{Po} \mathbf{o}'_{de} = \mathbf{0}$$

- Stability via Lyapunov argument $\mathcal{Q} = \{\eta, \boldsymbol{\epsilon}\}$

$${}^e \mathbf{o}'_{de} = 2\eta_{de} {}^e \boldsymbol{\epsilon}_{de} \quad \mathbf{K}_{Po} = k_{Po} \mathbf{I} \quad \mathbf{K}_{Do} = k_{Do} \mathbf{I}$$

$$\mathcal{V} = 2k_{Po} {}^e \boldsymbol{\epsilon}_{de}^T {}^e \boldsymbol{\epsilon}_{de} + \frac{1}{2} \Delta \boldsymbol{\omega}_{de}^T \Delta \boldsymbol{\omega}_{de}$$



- Orientation error: $\mathbf{o}_{de}'' = \sin \frac{\vartheta_{de}}{2} {}^e \mathbf{r}_{de} = {}^e \boldsymbol{\epsilon}_{de}$

- Resolved angular acceleration

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_d + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{de} + \mathbf{K}_{Po} \mathbf{R}_e {}^e \boldsymbol{\epsilon}_{de}$$

- Error dynamics

$$\Delta \dot{\boldsymbol{\omega}}_{de} + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{de} + \mathbf{K}_{Po} \mathbf{R}_e {}^e \boldsymbol{\epsilon}_{de} = \mathbf{0}$$

- Stability via Lyapunov argument

$$\mathcal{V} = k_{Po} \left((\eta_{de} - 1)^2 + {}^e \boldsymbol{\epsilon}_{de}^T {}^e \boldsymbol{\epsilon}_{de} \right) + \frac{1}{2} \Delta \boldsymbol{\omega}_{de}^T \Delta \boldsymbol{\omega}_{de}$$



- Number of floating-point operations and function calls

	<i>Resolved acceleration</i>		<i>Trajectory generation</i>	
	<i>Flops</i>	<i>Funcs</i>	<i>Flops</i>	<i>Funcs</i>
<i>Orientation error</i>				
Classical Euler angles	68	8	52	8
Alternative Euler angles	136	8	0	0
Angle/axis	55	0	0	0
Quaternion	60	1	21	1



- Null-space motion

$$\boldsymbol{\tau} = \mathbf{B}(\mathbf{q})\boldsymbol{\alpha} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})$$

$$\boldsymbol{\alpha} = \mathbf{J}^\dagger(\mathbf{q}) \left(\mathbf{a} - \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}} \right) + \boldsymbol{\alpha}_n$$

$$\mathbf{J}^\dagger = \mathbf{B}^{-1} \mathbf{J}^T \left(\mathbf{J} \mathbf{B}^{-1} \mathbf{J}^T \right)^{-1}$$

dynamically consistent
pseudo-inverse

$$\boldsymbol{\alpha}_n = (\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}) \left(\dot{\boldsymbol{\beta}} - \dot{\mathbf{J}}^\dagger \mathbf{J}(\boldsymbol{\beta} - \dot{\mathbf{q}}) + \mathbf{B}^{-1}(\mathbf{K}_n \mathbf{e}_n + \mathbf{C} \mathbf{e}_n) \right)$$

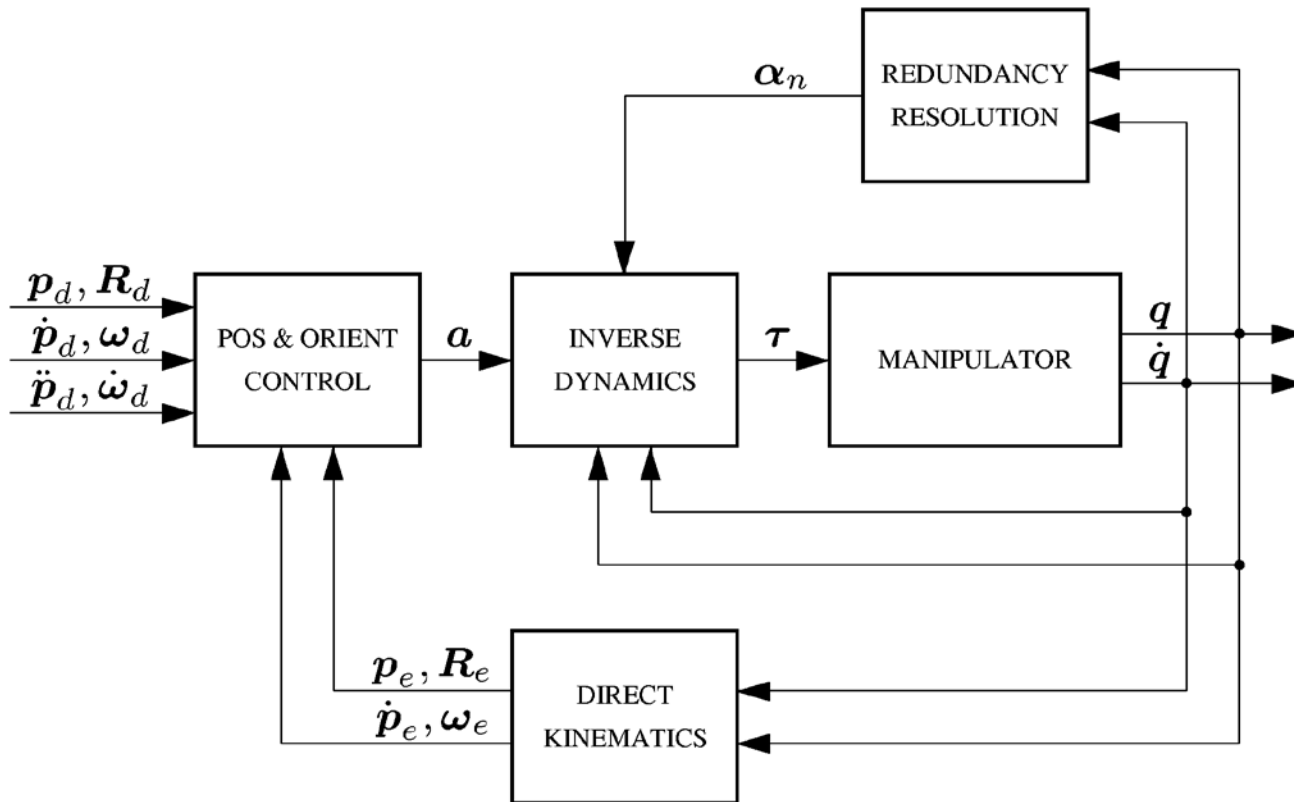
$$\mathbf{e}_n = \left(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q}) \mathbf{J}(\mathbf{q}) \right) (\boldsymbol{\beta} - \dot{\mathbf{q}}) \quad \boldsymbol{\beta} = k_\beta \mathbf{B}^{-1} \left(\frac{\partial w(\mathbf{q})}{\partial \mathbf{q}} \right)$$

- Stability via Lyapunov argument

$$\mathcal{V} = \frac{1}{2} \mathbf{e}_n^T \mathbf{B}(\mathbf{q}) \mathbf{e}_n$$



- Inverse dynamics control with redundancy resolution





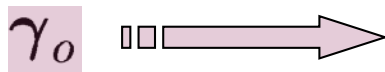
- PD control with gravity compensation

$$\boldsymbol{\tau} = \mathbf{J}^T(\mathbf{q}) \begin{bmatrix} \boldsymbol{\gamma}_p \\ \boldsymbol{\gamma}_o \end{bmatrix} - \mathbf{K}_D \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})$$

- Position control

$$\boldsymbol{\gamma}_p = \mathbf{K}_{Pp} \Delta \mathbf{p}_{de}$$

- Orientation control



Euler angles
Angle/axis
Quaternion



- Euler angles

$$\gamma_o = \mathbf{T}^{-\text{T}}(\varphi_e) \mathbf{K}_{Po} \Delta \varphi_{de}$$

- Alternative Euler angles

$$\gamma_o = \mathbf{T}_e^{-\text{T}}(\varphi_{de}) \mathbf{K}_{Po} \varphi_{de}$$

- Angle/axis

$$\gamma_o = \mathbf{K}_{Po} \mathbf{o}'_{de}$$

- Quaternion

$$\gamma_o = \mathbf{K}_{Po} \mathbf{R}_e^e \boldsymbol{\epsilon}_{de}$$

- For all ... stability via Lyapunov arguments $\mathcal{V} = \frac{1}{2} \dot{\mathbf{q}}^{\text{T}} \mathbf{B}(\mathbf{q}) \dot{\mathbf{q}} + \mathcal{U}_p + \mathcal{U}_o$



- Motion control vs. force control
 - Object manipulation or surface operation requires control of interaction between robot manipulator and environment
 - Use of purely motion control strategy is candidate to fail (task planning accuracy)
 - Control of contact force (compliant behaviour)
 - Use of force/torque sensor (interfaced with robot control unit)
- Indirect vs. direct force control
 - Indirect force control: force control via motion control (w/out explicit closure of force feedback loop)
 - Direct force control: force controlled to desired value (w/ closure of force feedback loop)

- Compliance control
 - Active compliance
 - Experiments
- Impedance control
 - Active impedance
 - Inner motion control
 - Three-DOF impedance control
 - Experiments
 - Six-DOF impedance control
 - Experiments



- Active vs. passive compliance

$$\gamma = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{T}(\varphi_e) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{K}_{Pp} & \mathbf{O} \\ \mathbf{O} & \mathbf{K}_{Po} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{p}_{de} \\ \Delta \varphi_{de} \end{bmatrix}$$

$$\Delta \mathbf{p}_{de} = \mathbf{K}_{Pp}^{-1} \mathbf{f}$$

$$\mathbf{f} = \mathbf{K}_f (\mathbf{p}_e - \mathbf{p}_o)$$

$$\Delta \varphi_{de} = \mathbf{K}_{Po}^{-1} \mathbf{T}^T(\varphi_e) \boldsymbol{\mu}$$

- At steady state (position/force)

$$\mathbf{p}_{e,\infty} = \left(\mathbf{I} + \mathbf{K}_{Pp}^{-1} \mathbf{K}_f \right)^{-1} \left(\mathbf{p}_d + \mathbf{K}_{Pp}^{-1} \mathbf{K}_f \mathbf{p}_o \right)$$

$$\mathbf{f}_\infty = \left(\mathbf{I} + \mathbf{K}_f \mathbf{K}_{Pp}^{-1} \right)^{-1} \mathbf{K}_f (\mathbf{p}_d - \mathbf{p}_o)$$

- Set-up
 - COMAU Smart 3-S robot
 - Open control architecture
- PD control with gravity compensation
 - Large proportional gains
 - Small proportional gains



- Programmable mass-damping-stiffness at the end-effector

$$\boldsymbol{\tau} = \mathbf{B}(\mathbf{q})\boldsymbol{\alpha} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{J}^T(\mathbf{q})\mathbf{h}$$

$$\boldsymbol{\alpha} = \mathbf{J}^{-1}(\mathbf{q}) \left(\begin{bmatrix} \mathbf{a}_p \\ \mathbf{a}_o \end{bmatrix} - \dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \right) \quad \text{force/torque sensor}$$

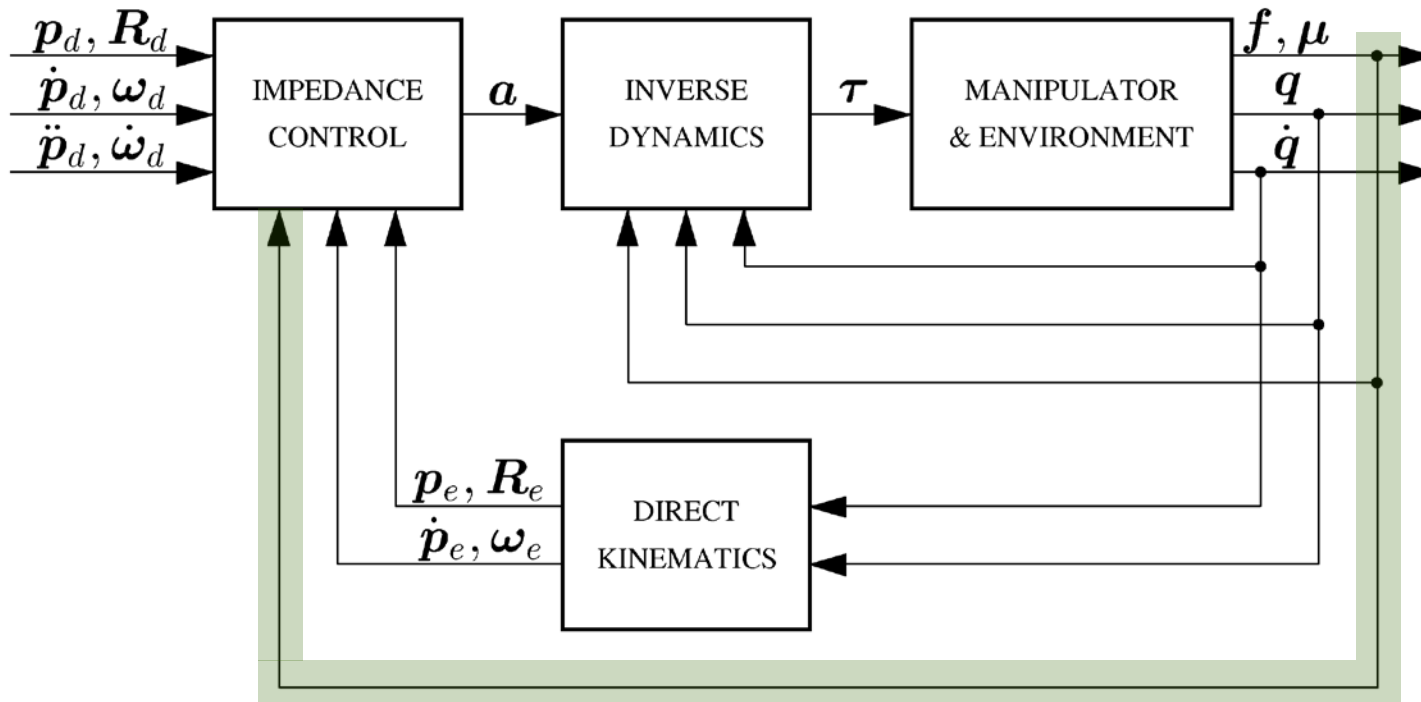
$$\mathbf{a}_p = \ddot{\mathbf{p}}_d + \mathbf{K}_{Mp}^{-1}(\mathbf{K}_{Dp}\Delta\dot{\mathbf{p}}_{de} + \mathbf{K}_{Pp}\Delta\mathbf{p}_{de} - \mathbf{f})$$

$$\mathbf{a}_o = \mathbf{T}(\boldsymbol{\varphi}_e)(\ddot{\boldsymbol{\varphi}}_d + \mathbf{K}_{Mo}^{-1}(\mathbf{K}_{Do}\Delta\dot{\boldsymbol{\varphi}}_{de} + \mathbf{K}_{Po}\Delta\boldsymbol{\varphi}_{de} - \mathbf{T}^T(\boldsymbol{\varphi}_e)\boldsymbol{\mu})) + \dot{\mathbf{T}}(\boldsymbol{\varphi}_e, \dot{\boldsymbol{\varphi}}_e)\dot{\boldsymbol{\varphi}}_e$$



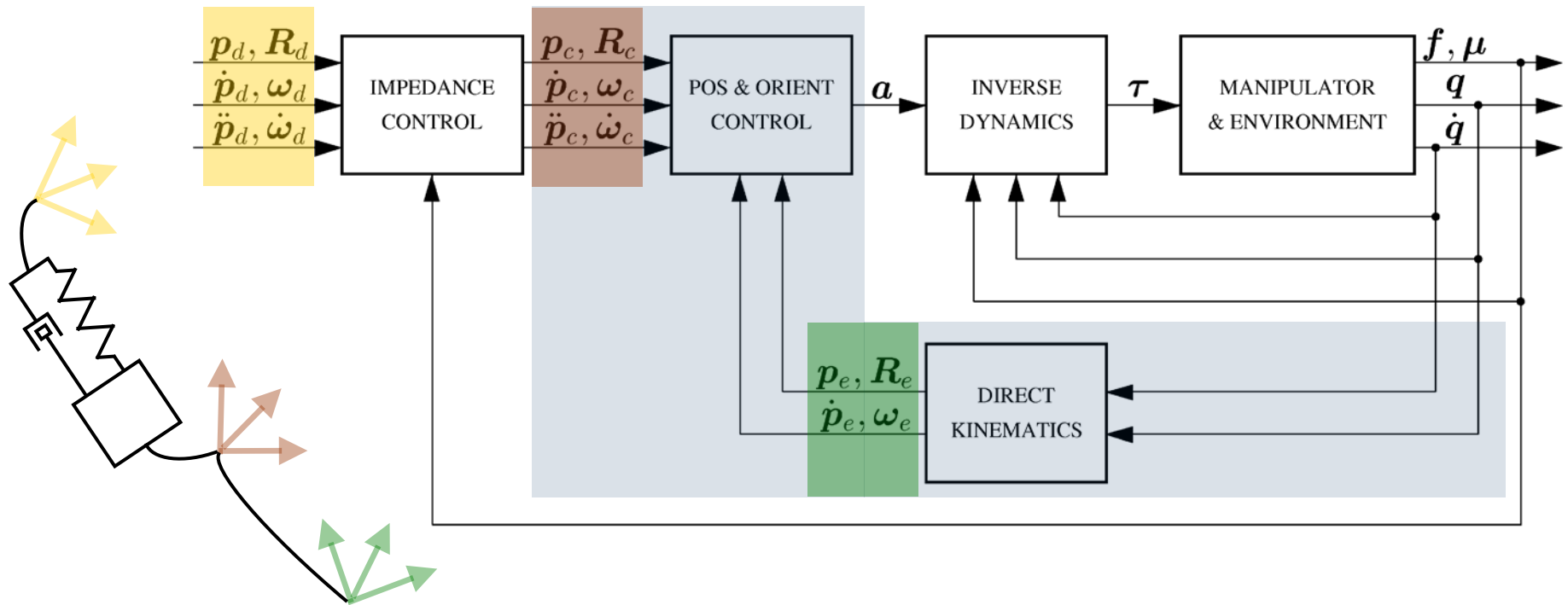
$$\begin{aligned} \mathbf{K}_{Mp}\Delta\ddot{\mathbf{p}}_{de} + \mathbf{K}_{Dp}\Delta\dot{\mathbf{p}}_{de} + \mathbf{K}_{Pp}\Delta\mathbf{p}_{de} &= \mathbf{f} \\ \mathbf{K}_{Mo}\Delta\ddot{\boldsymbol{\varphi}}_{de} + \mathbf{K}_{Do}\Delta\dot{\boldsymbol{\varphi}}_{de} + \mathbf{K}_{Po}\Delta\boldsymbol{\varphi}_{de} &= \mathbf{T}^T(\boldsymbol{\varphi}_e)\boldsymbol{\mu} \end{aligned}$$

- Impedance control
 - Force/torque measurements





- Compliant frame between desired and end-effector frame
 - Enhanced disturbance rejection



- Translational impedance

$$\mathbf{M}_p \Delta \ddot{\mathbf{p}}_{dc} + \mathbf{D}_p \Delta \dot{\mathbf{p}}_{dc} + \mathbf{K}_p \Delta \mathbf{p}_{dc} = \mathbf{f}$$

$$\Delta \mathbf{p}_{dc} = \mathbf{p}_d - \mathbf{p}_c$$

- Linear acceleration (inner motion loop)

$$\mathbf{a}_p = \ddot{\mathbf{p}}_c + \mathbf{K}_{Dp} \Delta \dot{\mathbf{p}}_{ce} + \mathbf{K}_{Pp} \Delta \mathbf{p}_{ce}$$

$$\Delta \mathbf{p}_{ce} = \mathbf{p}_c - \mathbf{p}_e$$

- ATI force/torque sensor
- 3-DOF impedance control
 - Effects of mass, damping and stiffness
 - Contact with unknown surface





- Rotational impedance

- Euler angles

$$\mathbf{M}_o \Delta \ddot{\varphi}_{dc} + \mathbf{D}_o \Delta \dot{\varphi}_{dc} + \mathbf{K}_o \Delta \varphi_{dc} = \mathbf{T}^T(\varphi_c) \boldsymbol{\mu}$$

$$\Delta \varphi_{dc} = \varphi_d - \varphi_c$$

- Infinitesimal orientation displacement

$$\boldsymbol{\mu}_E = \mathbf{T}^{-T}(\varphi_c) \mathbf{K}_o \mathbf{T}^{-1}(\varphi_c) \Delta \boldsymbol{\omega}_{dc} dt \quad \text{task geometric inconsistency}$$

- Angular acceleration (inner motion loop)

$$\mathbf{a}_o = \mathbf{T}(\varphi_e) (\ddot{\varphi}_c + \mathbf{K}_{Do} \Delta \dot{\varphi}_{ce} + \mathbf{K}_{Po} \Delta \varphi_{ce}) + \dot{\mathbf{T}}(\varphi_e, \dot{\varphi}_e) \dot{\varphi}_e$$

$$\Delta \varphi_{ce} = \varphi_c - \varphi_e$$



- Rotational impedance

- Alternative Euler angles

$$\mathbf{M}_o \ddot{\boldsymbol{\varphi}}_{dc} + \mathbf{D}_o \dot{\boldsymbol{\varphi}}_{dc} + \mathbf{K}_o \boldsymbol{\varphi}_{dc} = \mathbf{T}^T(\boldsymbol{\varphi}_{dc})^c \boldsymbol{\mu}$$

$${}^c \mathbf{R}_d = \mathbf{R}_c^T \mathbf{R}_d \implies \boldsymbol{\varphi}_{dc}$$

- Infinitesimal orientation displacement

$$\begin{aligned} {}^c \boldsymbol{\mu}_E &\simeq \mathbf{T}^{-T}(\mathbf{0}) \mathbf{K}_o \mathbf{T}^{-1}(\mathbf{0}) \Delta^c \boldsymbol{\omega}_{dc} dt \\ &= \mathbf{K}_o \Delta^c \boldsymbol{\omega}_{dc} dt \end{aligned}$$

task geometric consistency
(XYZ Euler angles + diagonal stiffness)

- Angular acceleration (inner motion loop)

$$\begin{aligned} \mathbf{a}_o &= \dot{\boldsymbol{\omega}}_d - \dot{\mathbf{T}}_e(\boldsymbol{\varphi}_{de}, \dot{\boldsymbol{\varphi}}_{de}) \dot{\boldsymbol{\varphi}}_{de} \\ &\quad - \mathbf{T}_e(\boldsymbol{\varphi}_{de}) (\ddot{\boldsymbol{\varphi}}_{dc} + \mathbf{K}_{Do} (\dot{\boldsymbol{\varphi}}_{dc} - \dot{\boldsymbol{\varphi}}_{de}) + \mathbf{K}_{Po} (\boldsymbol{\varphi}_{dc} - \boldsymbol{\varphi}_{de})) \end{aligned}$$



- Rotational impedance

- Angle/axis

$$\mathbf{M}_o \Delta^c \dot{\boldsymbol{\omega}}_{dc} + \mathbf{D}_o \Delta^c \boldsymbol{\omega}_{dc} + \mathbf{K}'_o {}^c \mathbf{o}_{dc} = {}^c \boldsymbol{\mu}$$

$${}^c \mathbf{o}_{dc} = f(\vartheta_{dc}) {}^c \mathbf{r}_{dc}$$

$$\mathbf{K}'_o = 2\psi \boldsymbol{\Omega}^T({}^c \mathbf{r}_{dc}, \vartheta_{dc}) \mathbf{K}_o$$

$${}^c \dot{\mathbf{o}}_{dc} = \boldsymbol{\Omega}({}^c \mathbf{r}_{dc}, \vartheta_{dc}) \Delta^c \boldsymbol{\omega}_{dc}$$

- Infinitesimal orientation displacement

$${}^c \boldsymbol{\mu}_E \simeq 2\psi (f'(0))^2 \mathbf{K}_o \Delta^c \boldsymbol{\omega}_{dc} dt \quad \psi = 1/2(f'(0))^2$$

$$= \mathbf{K}_o \Delta^c \boldsymbol{\omega}_{dc} dt$$

task geometric consistency

- Angular acceleration (inner motion loop)

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_c + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{ce} + \mathbf{K}_{Po} \mathbf{o}'_{ce}$$

$$\mathbf{o}'_{ce} = \frac{1}{2} (\mathbf{S}(\mathbf{n}_e) \mathbf{n}_c + \mathbf{S}(\mathbf{s}_e) \mathbf{s}_c + \mathbf{S}(\mathbf{a}_e) \mathbf{a}_c)$$



- Rotational impedance

- Quaternion

$$\mathbf{M}_o \Delta^c \dot{\boldsymbol{\omega}}_{dc} + \mathbf{D}_o \Delta^c \boldsymbol{\omega}_{dc} + \mathbf{K}'_o {}^c \boldsymbol{\epsilon}_{dc} = {}^c \boldsymbol{\mu}$$

$${}^c \mathbf{R}_d = \mathbf{R}_c^T \mathbf{R}_d \implies {}^c \boldsymbol{\epsilon}_{dc}$$

$$\mathbf{K}'_o = 2 \mathbf{E}^T(\eta_{dc}, {}^c \boldsymbol{\epsilon}_{dc}) \mathbf{K}_o$$

$$\mathbf{E}(\eta, \boldsymbol{\epsilon}) = \eta \mathbf{I} - \mathbf{S}(\boldsymbol{\epsilon})$$

- Infinitesimal orientation displacement

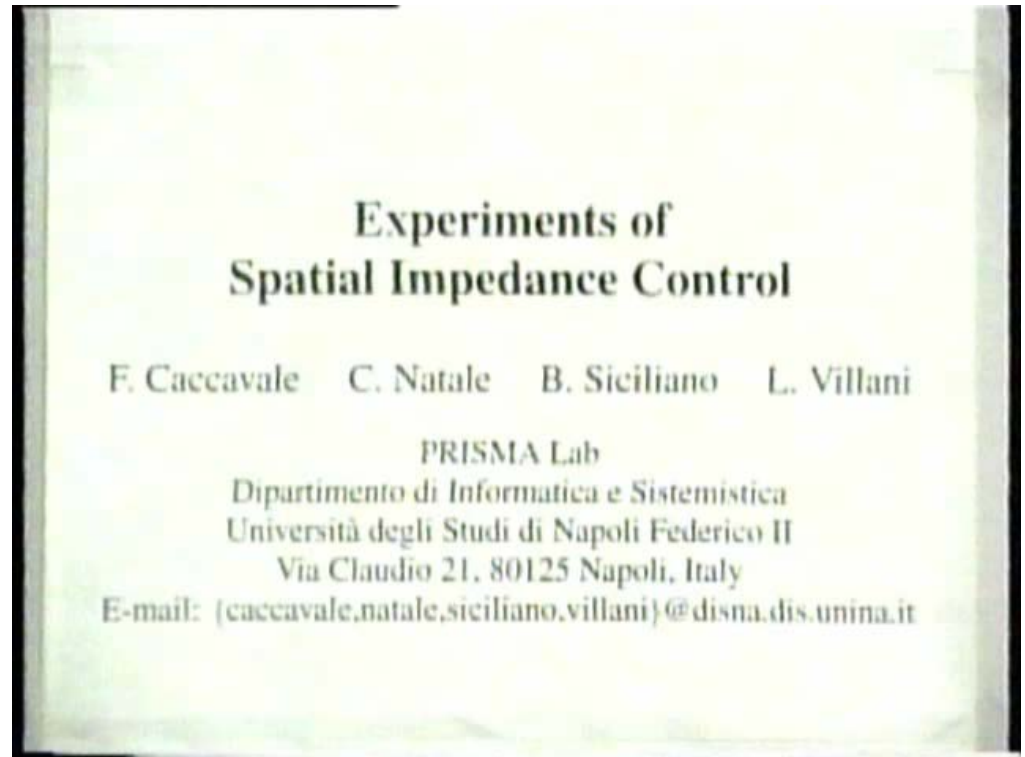
$$\begin{aligned} {}^c \boldsymbol{\mu}_E &\simeq 2\psi (f'(0))^2 \mathbf{K}_o \Delta^c \boldsymbol{\omega}_{dc} dt \\ &= \mathbf{K}_o \Delta^c \boldsymbol{\omega}_{dc} dt \end{aligned}$$

task geometric consistency

- Angular acceleration (inner motion loop)

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_c + \mathbf{K}_{Do} \Delta \boldsymbol{\omega}_{ce} + \mathbf{K}_{Po} \mathbf{R}_e {}^e \boldsymbol{\epsilon}_{ce}$$

- 6-DOF impedance control
 - Accommodation of both force and moment
 - Operational space approach (Euler angles)
 - Geometrically consistent approach





- Regulation of force and moment to desired values

$$\gamma_p = \mathbf{K}_{Pp} \Delta \mathbf{p}_{ce} + \mathbf{f}_d$$

$$\gamma_o = \mathbf{T}^{-T}(\varphi_e) \mathbf{K}_{Po} \Delta \varphi_{ce} + \boldsymbol{\mu}_d$$

- PI control

$$\mathbf{p}_c = \mathbf{K}_{Pp}^{-1} \left(\mathbf{K}_{Fp} \Delta \mathbf{f} + \mathbf{K}_{Ip} \int_0^t \Delta \mathbf{f} d\varsigma \right) \quad \Delta \mathbf{f} = \mathbf{f}_d - \mathbf{f}$$

$$\varphi_c = \mathbf{K}_{Po}^{-1} \left(\mathbf{K}_{Fo} \Delta \boldsymbol{\mu} + \mathbf{K}_{Io} \int_0^t \Delta \boldsymbol{\mu} d\varsigma \right) \quad \Delta \boldsymbol{\mu} = \boldsymbol{\mu}_d - \boldsymbol{\mu}$$

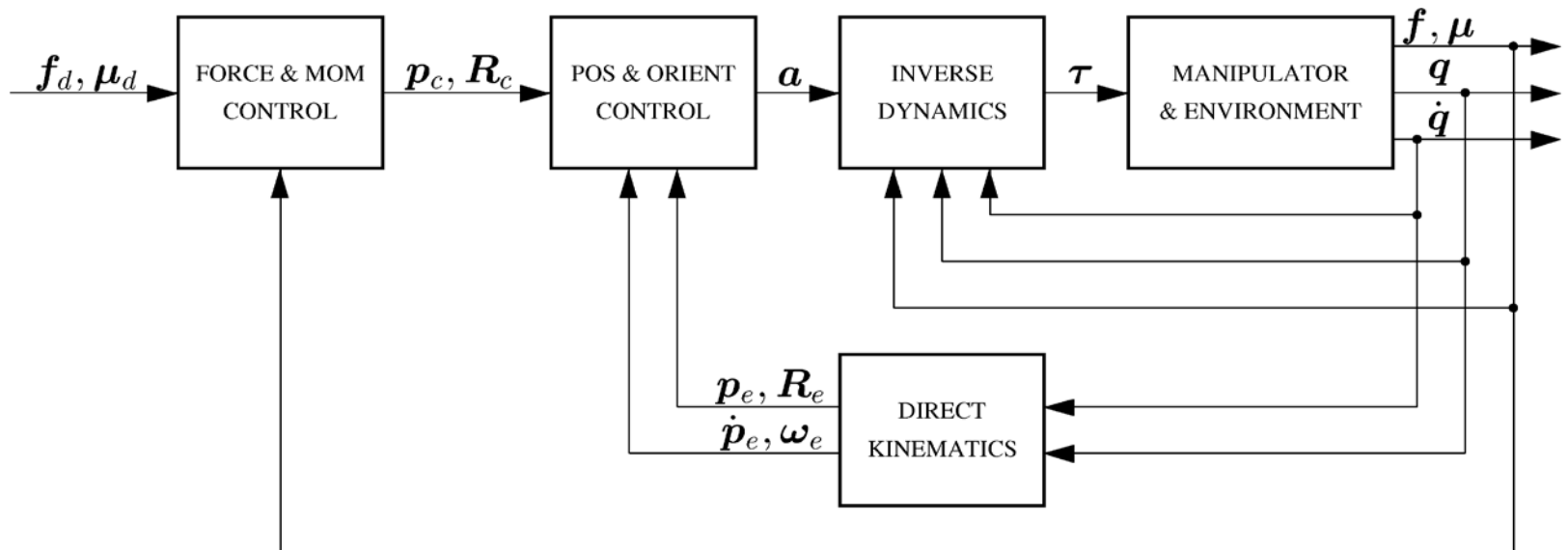
- At steady state

$$\mathbf{f}_\infty = \mathbf{f}_d \quad \boldsymbol{\mu}_\infty = \boldsymbol{\mu}_d$$

- Force and moment control with inner motion control loop
 - Linear and angular accelerations

$$\mathbf{a}_p = -\mathbf{K}_{Dp}\dot{\mathbf{p}}_e + \mathbf{K}_{Pp}\Delta\mathbf{p}_{ce}$$

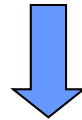
$$\mathbf{a}_o = \mathbf{T}(\varphi_e) (-\mathbf{K}_{Do}\dot{\varphi}_e + \mathbf{K}_{Po}\Delta\varphi_{ce}) + \dot{\mathbf{T}}(\varphi_e, \dot{\varphi}_e)\dot{\varphi}_e$$



- Force control
 - Regulation to zero force
 - Inner position vs. velocity control loop



- Force and motion control
 - Regulation of force but loss of motion control
 - Recover motion control along unconstrained directions while ensuring force control along constrained directions



Parallel control strategy



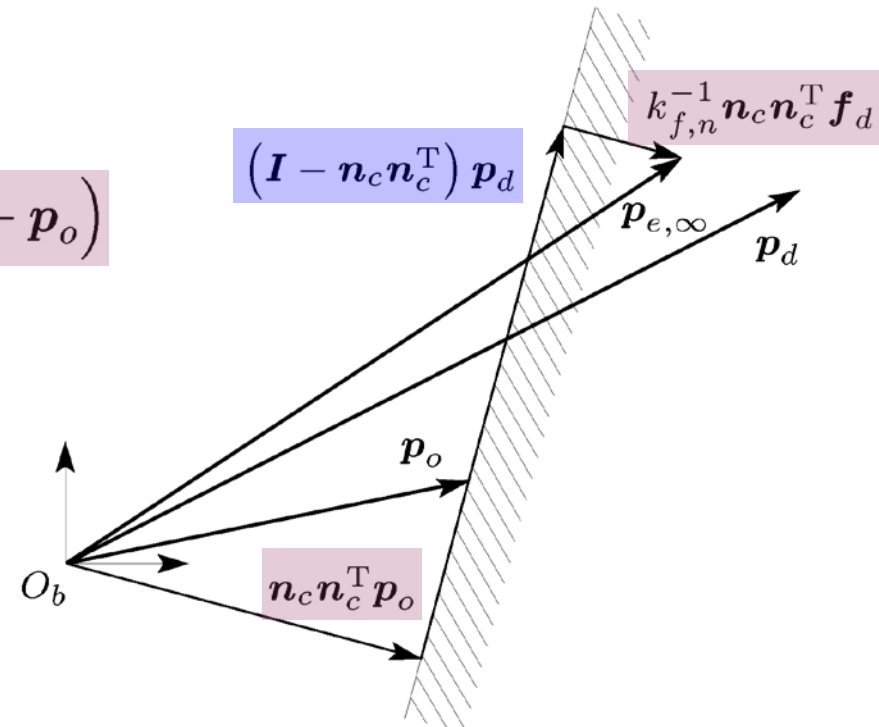
- PD motion control with gravity compensation + force control

$$\boldsymbol{\tau} = \mathbf{J}_p^T(\mathbf{q})\mathbf{K}_{Pp}(\mathbf{p}_r - \mathbf{p}_e) + \mathbf{f}_d - \mathbf{K}_D\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) \quad \mathbf{p}_r = \mathbf{p}_c + \mathbf{p}_d$$

- At steady state

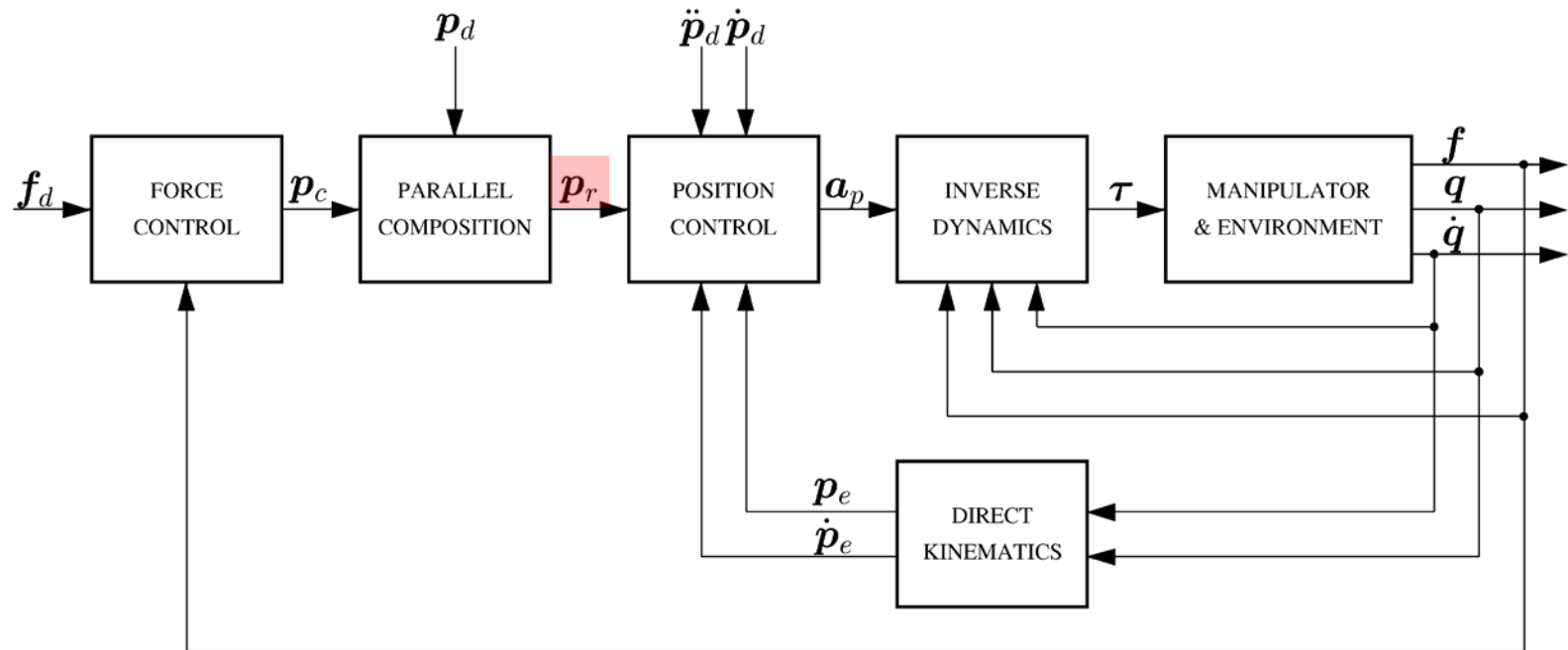
$$\mathbf{p}_{e,\infty} = (\mathbf{I} - \mathbf{n}_c\mathbf{n}_c^T)\mathbf{p}_d + \mathbf{n}_c\mathbf{n}_c^T(k_{f,n}^{-1}\mathbf{f}_d + \mathbf{p}_o)$$

$$\mathbf{f}_\infty = k_{f,n}\mathbf{n}_c\mathbf{n}_c^T(\mathbf{p}_{e,\infty} - \mathbf{p}_o) = \mathbf{f}_d$$



- Parallel force/position control
 - Linear acceleration

$$\mathbf{a}_p = \ddot{\mathbf{p}}_d + \mathbf{K}_{Dp}(\dot{\mathbf{p}}_d - \dot{\mathbf{p}}_e) + \mathbf{K}_{Pp}(\mathbf{p}_r - \mathbf{p}_e)$$



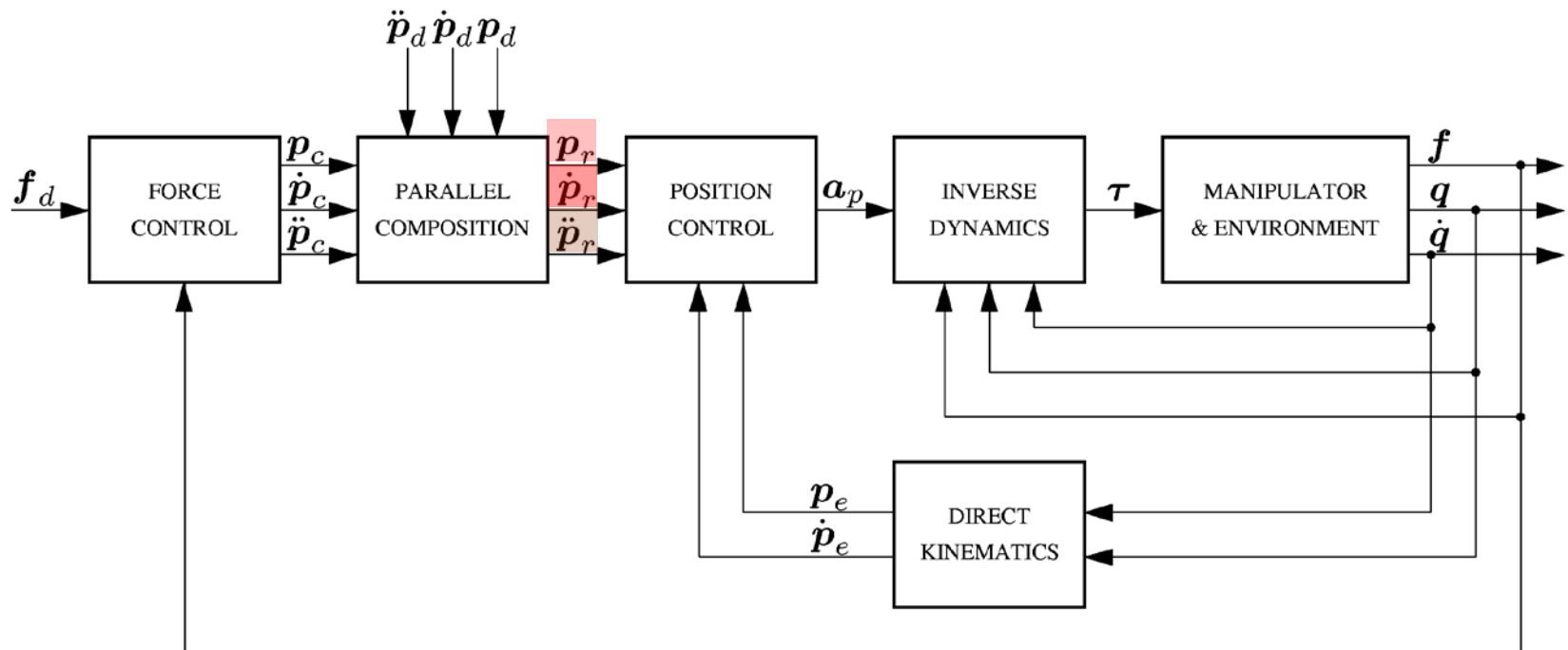


- Force/position control with full parallel composition
 - Linear acceleration

$$\mathbf{a}_p = \ddot{\mathbf{p}}_r + \mathbf{K}_{Dp}(\dot{\mathbf{p}}_r - \dot{\mathbf{p}}_e) + \mathbf{K}_{Pp}(\mathbf{p}_r - \mathbf{p}_e)$$

$$\dot{\mathbf{p}}_r = \dot{\mathbf{p}}_c + \dot{\mathbf{p}}_d$$

$$\ddot{\mathbf{p}}_r = \ddot{\mathbf{p}}_c + \ddot{\mathbf{p}}_d$$



- Parallel force/position control
 - Regulation to zero force with position tracking
 - PD+ position control with PI force control





- Moment/orientation control with full parallel composition

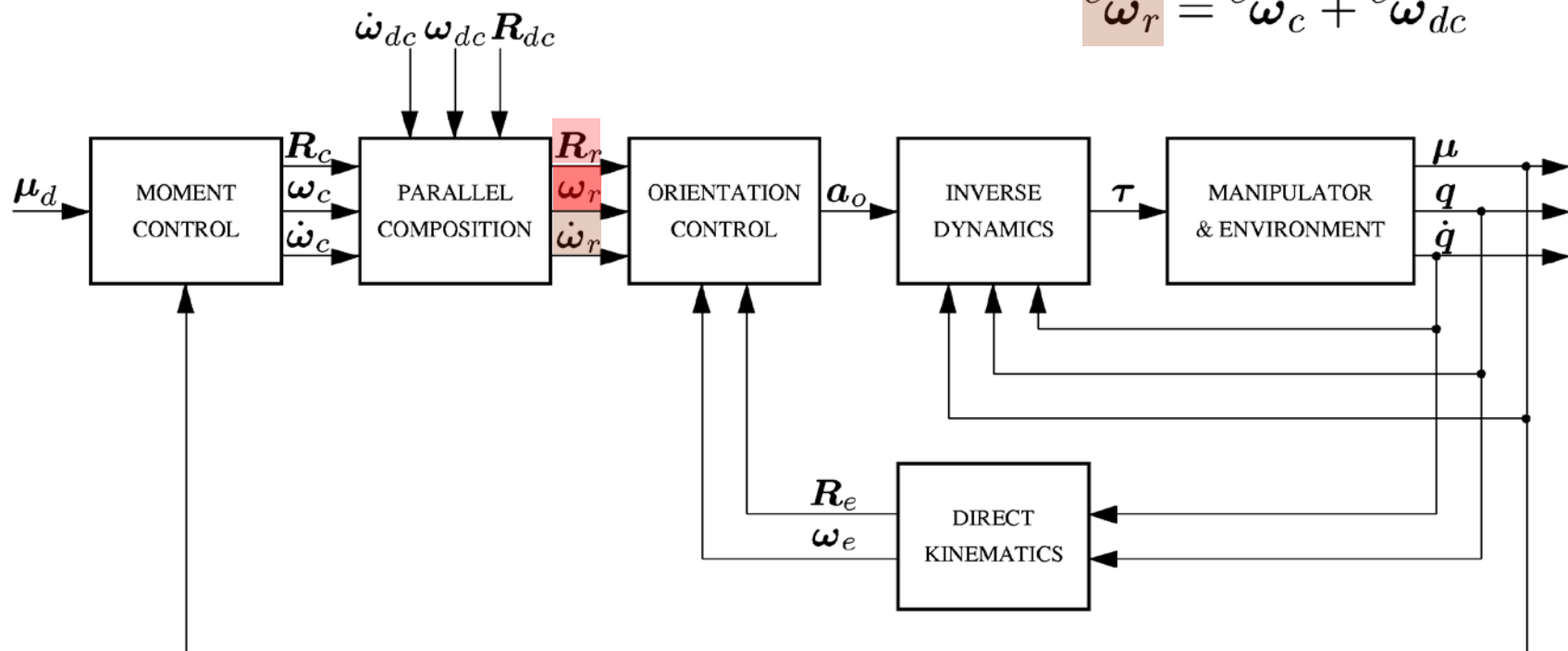
- Linear acceleration

$$\mathbf{a}_o = \dot{\boldsymbol{\omega}}_r + \mathbf{K}_{Do}(\boldsymbol{\omega}_r - \boldsymbol{\omega}_e) + \mathbf{K}_{Po}\mathbf{R}_e^e \boldsymbol{\epsilon}_{re}$$

$$\mathbf{Q}_r = \mathbf{Q}_c * \mathbf{Q}_{dc}$$

$${}^c\boldsymbol{\omega}_r = {}^c\boldsymbol{\omega}_c + {}^c\boldsymbol{\omega}_{dc}$$

$${}^c\dot{\boldsymbol{\omega}}_r = {}^c\dot{\boldsymbol{\omega}}_c + {}^c\dot{\boldsymbol{\omega}}_{dc}$$



- Tracking of time-varying force
 - Full parallel composition

$$\mathbf{K}_{Ap}\ddot{\mathbf{p}}_c + \mathbf{K}_{Vp}\dot{\mathbf{p}}_c = \boldsymbol{\phi}$$

$$\boldsymbol{\phi} = k_{f,n}^{-1} \mathbf{f}_c$$

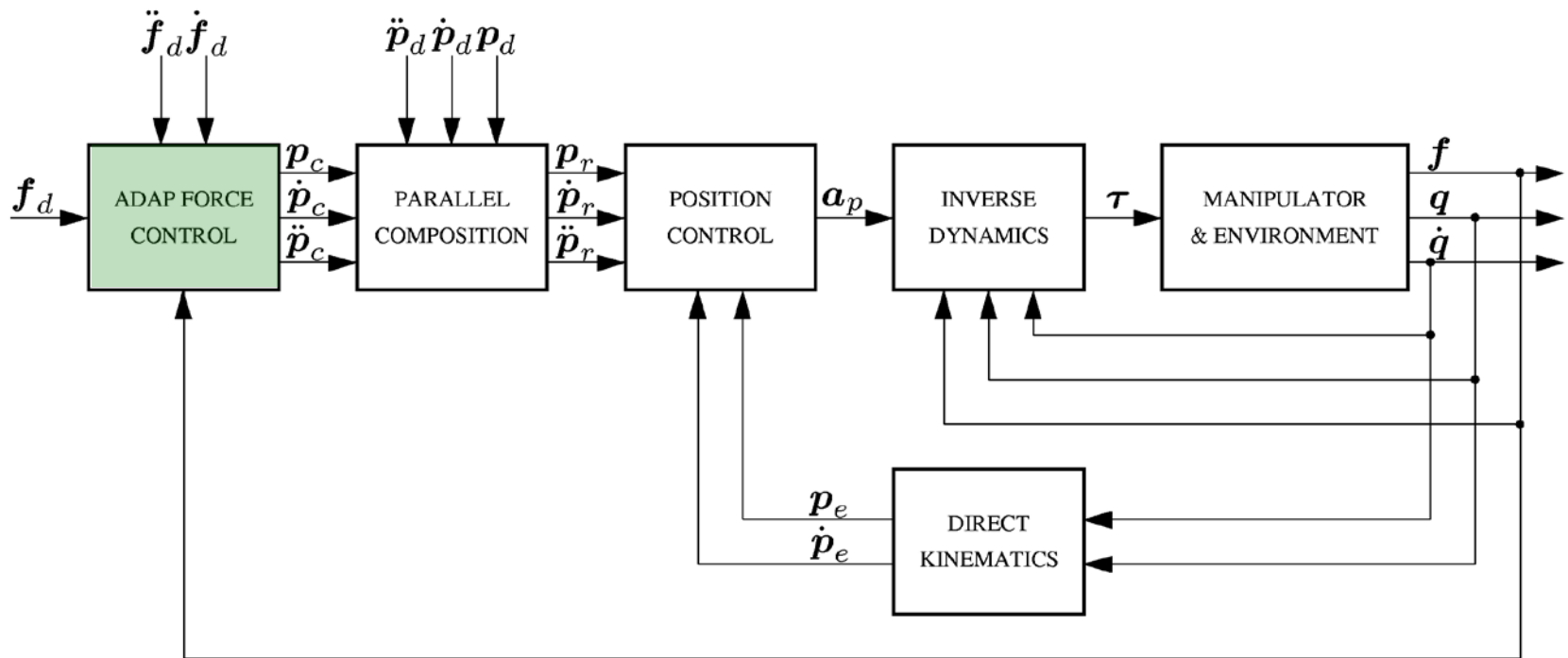
$$\mathbf{f}_c = \mathbf{K}_{Ap}\ddot{\mathbf{f}}_d + \mathbf{K}_{Vp}\dot{\mathbf{f}}_d + \Delta \mathbf{f}$$

- Tracking if $k_{f,n}$ exactly known

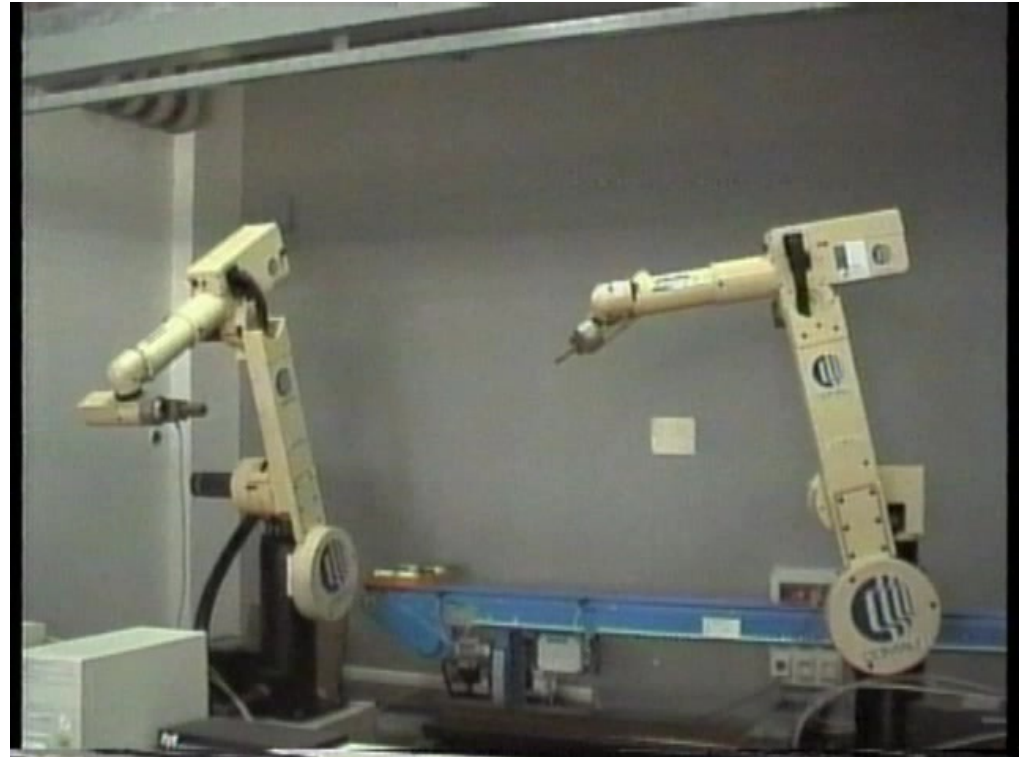
- Contact stiffness adaptation ($\varepsilon = k_{f,n}^{-1}$)

$$\phi = \hat{\varepsilon} f_c + \hat{\varepsilon} \dot{\psi}$$

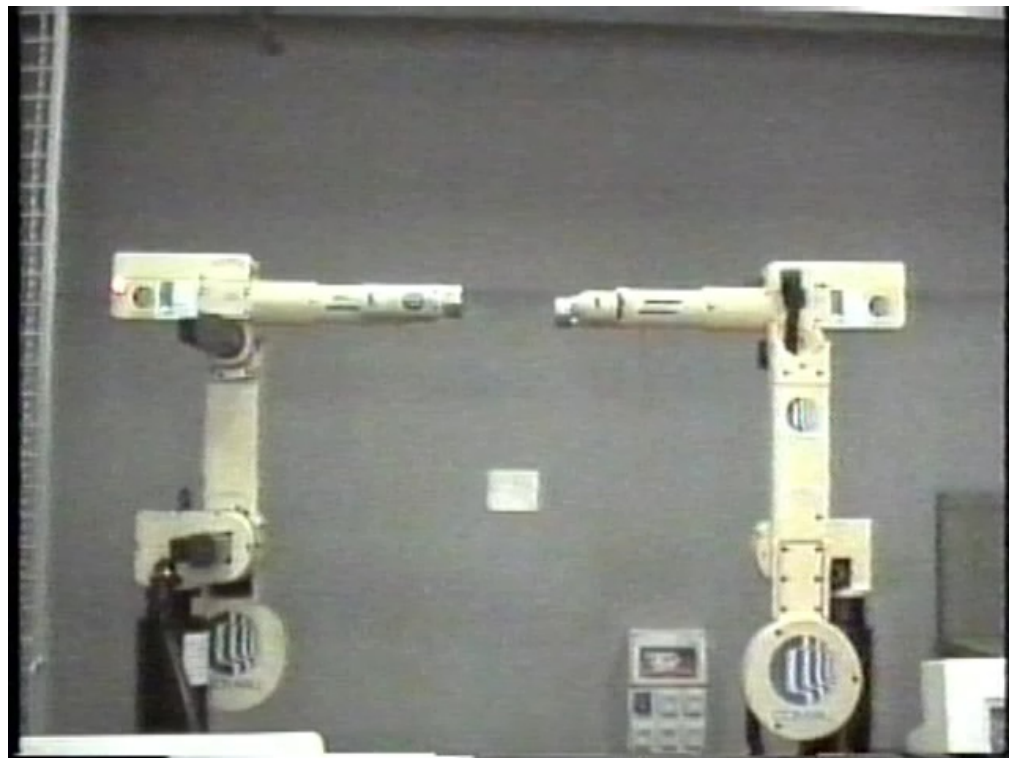
$$\dot{\psi} + \lambda \psi = f_c \quad \dot{\hat{\varepsilon}} = \gamma \psi^T \Delta f$$



- Extension to dual-robot system (loose cooperation)
 - Typical peg-in-hole assembly task
 - Robot holding the hole controlled as 6-DOF impedance
 - Robot holding the peg programmed in PDL-2
 - Accommodation of misalignment and overshoot



- Tight cooperation
 - Two arms tightly grasping a rigid object
 - Control of the object position
 - Control of the internal forces





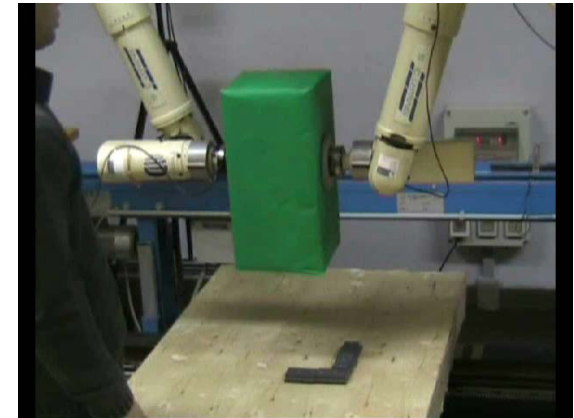
- Dual-arm impedance control



absolute & relative impedance



absolute impedance



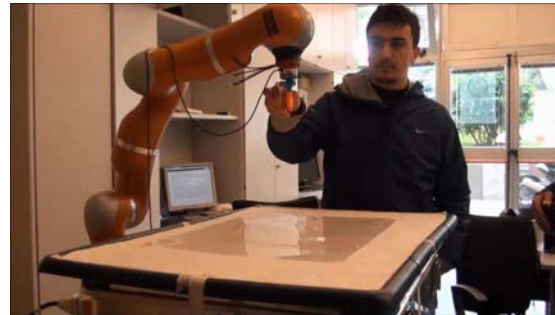
human-object interaction



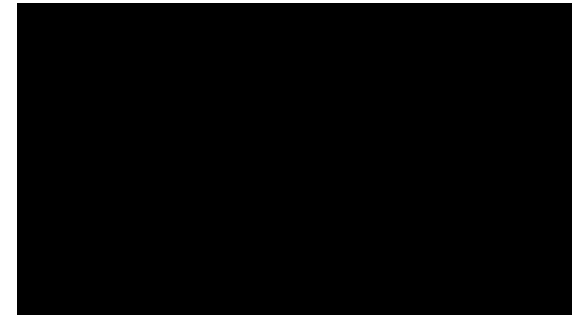
- Human-robot interaction



null-space impedance control

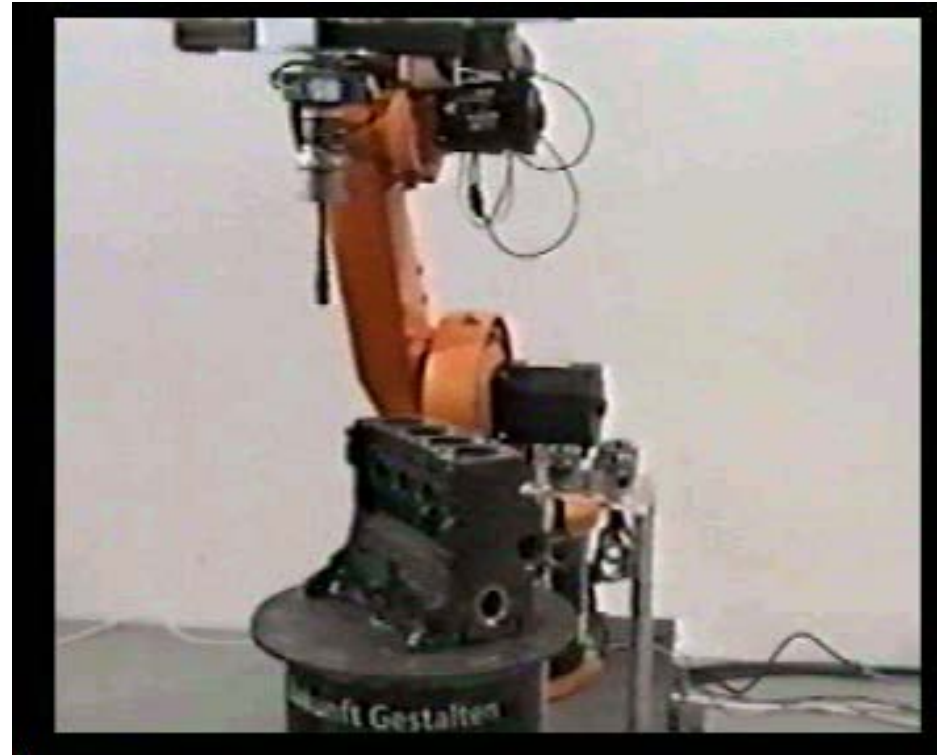


variable impedance control



safe efficient pHRI

- Set-up @ DLR, Germany
 - KUKA robot with force sensor and camera embedded in the gripper
- Integration of vision and force
 - Visual feedback in gross motion
 - Force feedback in fine motion



■ Problem

- Control interaction of a robot manipulator with a rigid object of known geometry but unknown position and orientation

$$\varphi({}^o\mathbf{p}) = 0$$

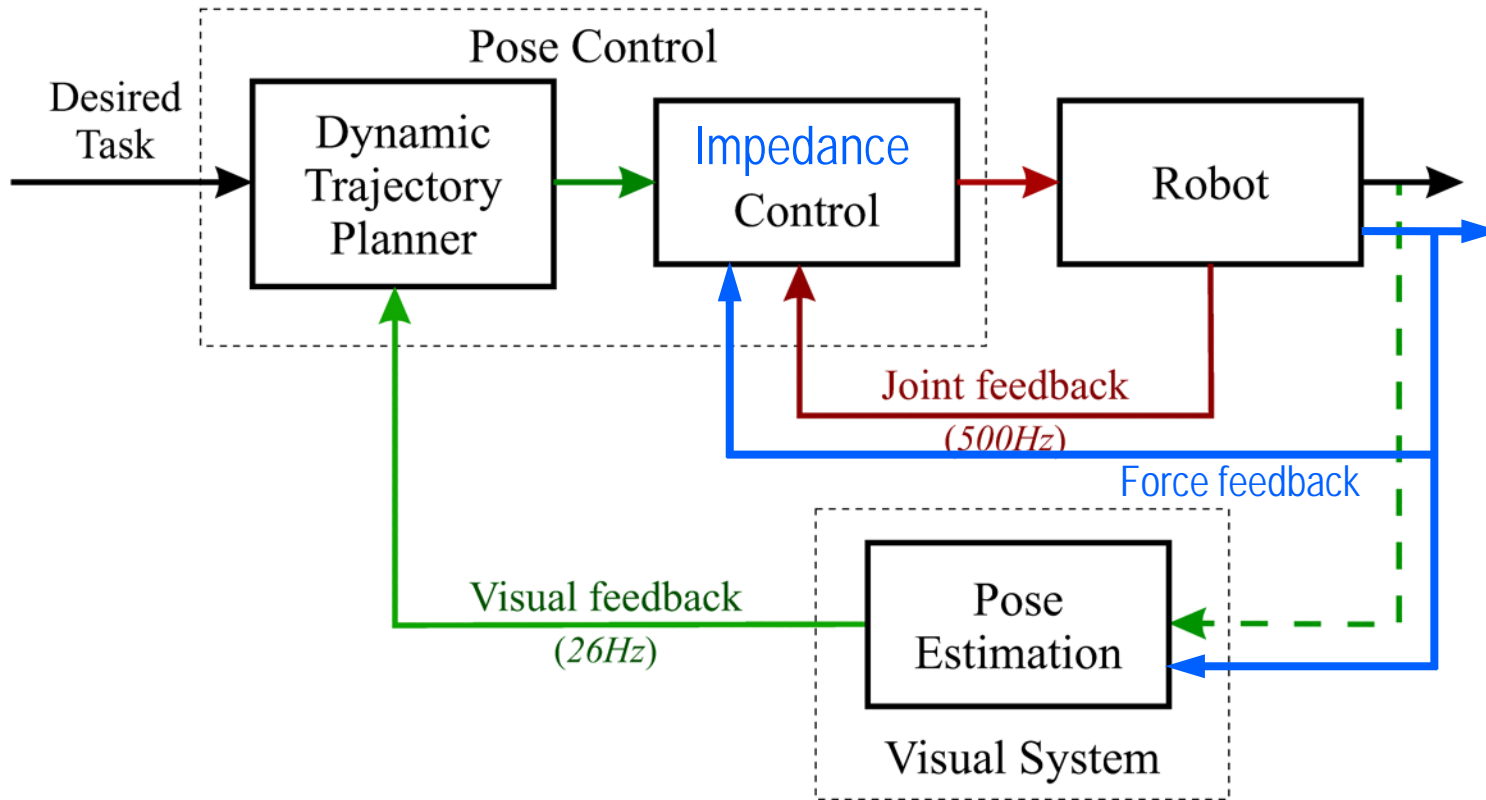
$$\{\mathbf{o}_o, \mathbf{R}_o\} \rightarrow \mathbf{x}_o$$

■ Solution

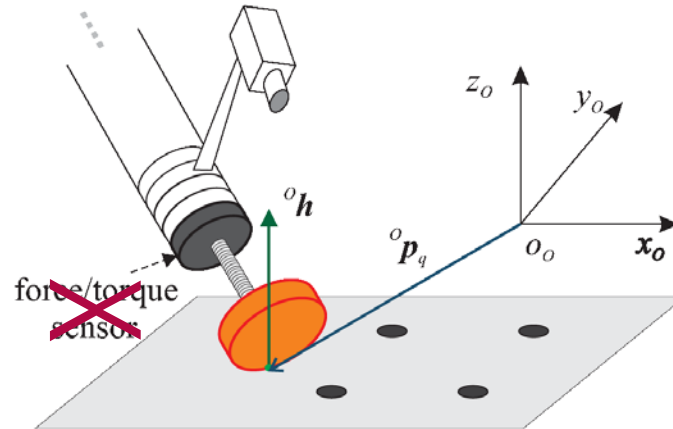
- When robot is far from object
 - Position-based visual servoing is adopted
 - The relative pose of the robot with respect to the object is estimated recursively using only vision
- When robot is in contact with object
 - Any kind of interaction control strategy can be adopted (impedance control, parallel force/position control)
 - The relative pose of the robot with respect to the object is estimated recursively using vision, force and joint position measurements



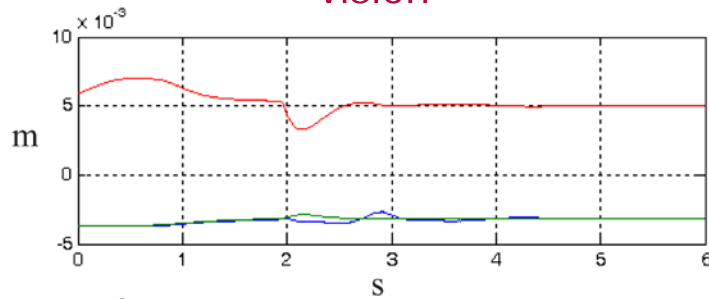
- Position-based visual impedance



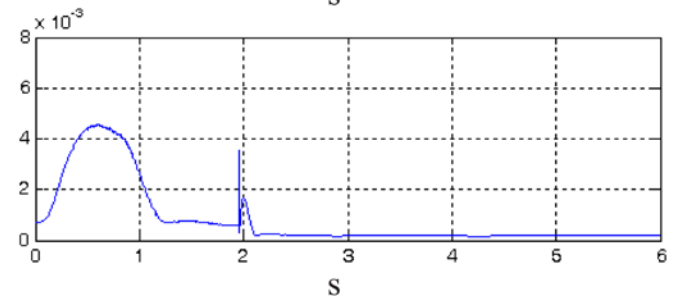
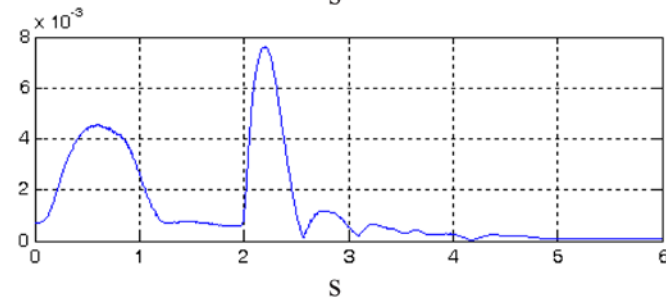
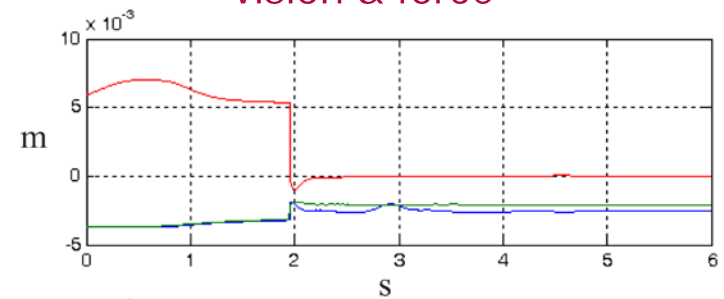
- Pose estimation errors



vision



vision & force



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