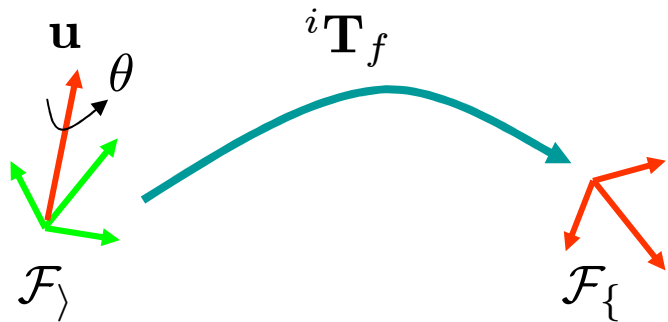


$(\mathbf{u}, \theta)$  representation for orientation



$$T = \begin{pmatrix} s_x & n_x & a_x & P_x \\ s_y & n_y & a_y & P_y \\ s_z & n_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} A & P \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{A} = \mathbf{R} = \begin{pmatrix} s_x & n_x & a_x \\ s_y & n_y & a_y \\ s_z & n_z & a_z \end{pmatrix}$$

From  $(\mathbf{u}, \theta)$  we can fill the  $\mathbf{A}$  matrix

$$A(\underline{u}, \theta) = \begin{pmatrix} u_x^2 \cdot (1 - c\theta) + c\theta & u_x u_y \cdot (1 - c\theta) - u_z \cdot s\theta & u_x u_z \cdot (1 - c\theta) + u_y \cdot s\theta \\ u_x u_y \cdot (1 - c\theta) + u_z \cdot s\theta & u_y^2 \cdot (1 - c\theta) + c\theta & u_y u_z \cdot (1 - c\theta) - u_x \cdot s\theta \\ u_x u_z \cdot (1 - c\theta) - u_y \cdot s\theta & u_y u_z \cdot (1 - c\theta) + u_x \cdot s\theta & u_z^2 \cdot (1 - c\theta) + c\theta \end{pmatrix}$$

From the  $\mathbf{A} = \mathbf{R}$  matrix we can compute  $(\mathbf{u}, \theta)$

$$\begin{cases} [\mathbf{u} \cdot \sin \theta]_{\times} = \frac{1}{2} \cdot (\mathbf{R} - \mathbf{R}^T) \\ \mathbf{u} = \frac{\mathbf{u} \cdot \sin \theta}{\|\mathbf{u} \cdot \sin \theta\|} \\ \theta = \arccos \left( \frac{\text{trace}(\mathbf{R}) - 1}{2} \right) \end{cases}$$

$$\text{trace}(\mathbf{R}) = 1 + 2 \cos \theta$$