

La relation Ressource - Consommateur

III - Rosenzweig-MacArthur

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Septembre 2015

Interaction Ressource-Consommateur

$$\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y} \mu(x, y) \cdot y \\ \dot{y} &= \mu(x, y) \cdot y - \nu(y) \cdot y\end{aligned}$$

Modèle canonique

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y}\mu(x, y) \cdot y \\ \dot{y} &= \mu(x, y) \cdot y - \nu(y) \cdot y\end{aligned}$$

- $f(x) = r \cdot x(1 - \frac{x}{K})$

Rosenzweig-MacArthur

Rosenzweig 1941

Mac Arthur 1930-72

$$\dot{x} = f(x) - \frac{1}{Y}\mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$



Graphical Representation and Stability Conditions of Predator-Prey
Interactions

M. L. Rosenzweig; R. H. MacArthur

American Naturalist, Vol. 97, No. 895 (Jul. - Aug., 1963), 209-223.

Stable URL:

<http://links.jstor.org/sici?&sici=0003-0147%28196307%2F08%2997%3A895%3C209%3AGRASCO%3E2.0.CO%3B2-L>

Rosenzweig-MacArthur

$$\dot{x} = f(x) - \frac{1}{Y}\mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

- $f(x) = r \cdot x(1 - \frac{x}{K})$

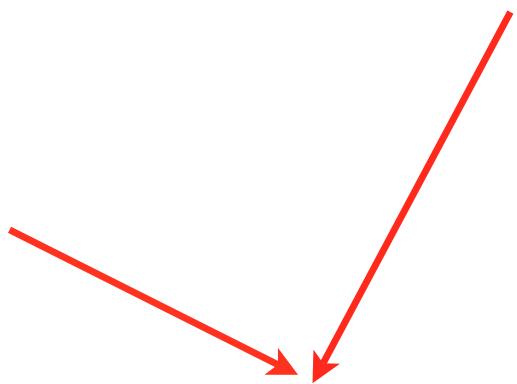
- $\mu(x, y) = \frac{\mu \cdot x}{e + x}$

- $\nu(y) = m$

Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y} \mu(x, y) \cdot y \\ \dot{y} &= \mu(x, y) \cdot y - \nu(y) \cdot y\end{aligned}$$

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$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{y} &= \left(\frac{\mu \cdot x}{e + x} - d\right) \cdot y\end{aligned}$$

Rosenzweig-MacArthur

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- $\nu(y) = m$

$d = \text{"Disparition"}$

$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{y} &= \left(\frac{\mu \cdot x}{e + x} - d \right) \cdot y\end{aligned}$$

Rosenzweig-MacArthur

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m = "Mortalité"

Etude mathématique du modèle de Rosenzweig-MacArthur

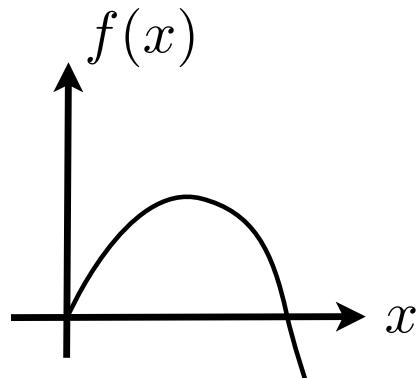
$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{y} &= \left(\frac{\mu \cdot x}{e + x} - m\right) \cdot y\end{aligned}\qquad\qquad\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y} \mu(x) \cdot y \\ \dot{y} &= (\mu(x) - m) \cdot y\end{aligned}$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y$$

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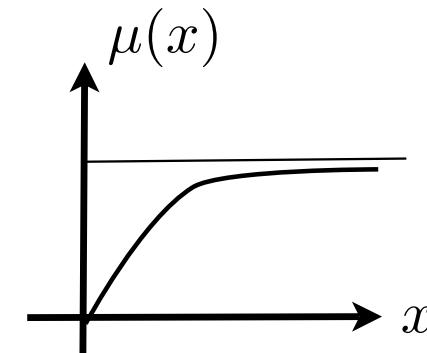
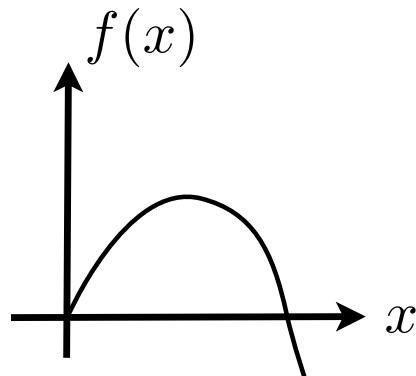


Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y$$

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$$\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y} \mu(x) \cdot y \\ \dot{y} &= (\mu(x) - m) \cdot y\end{aligned}$$



Etude mathématique du modèle de Rosenzweig-MacArthur

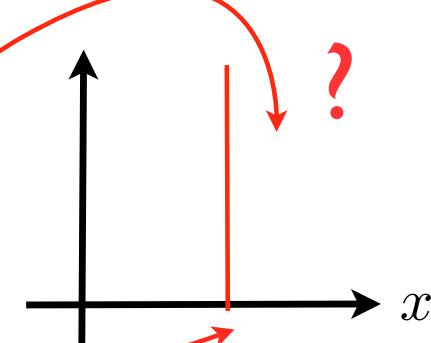
$$\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y} \mu(x) \cdot y & \dot{x} &= f(x) - \mu(x) \cdot \tilde{y} \\ \dot{y} &= (\mu(x) - m) \cdot y & \tilde{y} = \frac{1}{Y} y &\xrightarrow{\quad\quad\quad} & \dot{\tilde{y}} &= (\mu(x) - m) \cdot \tilde{y}\end{aligned}$$

$$\dot{x} = f(x) - \mu(x) \cdot y$$

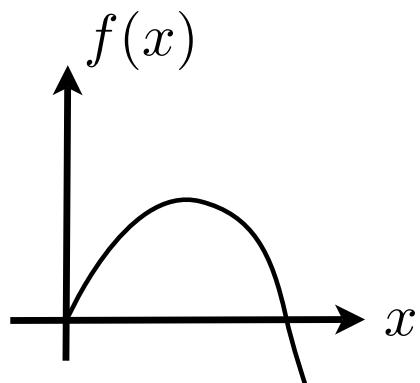
$$\dot{y} = (\mu(x) - m) \cdot y$$

$$\mu(x) = m$$

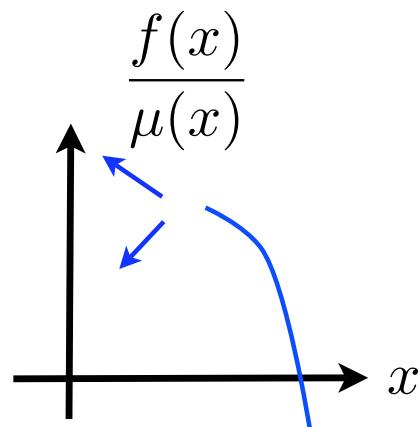
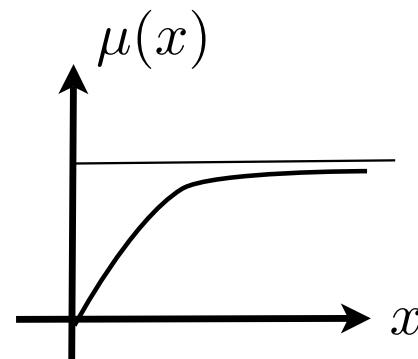
$$y = \frac{f(x)}{\mu(x)}$$



Etude mathématique du modèle de Rosenzweig-MacArthur



$$y = \frac{f(x)}{\mu(x)}$$

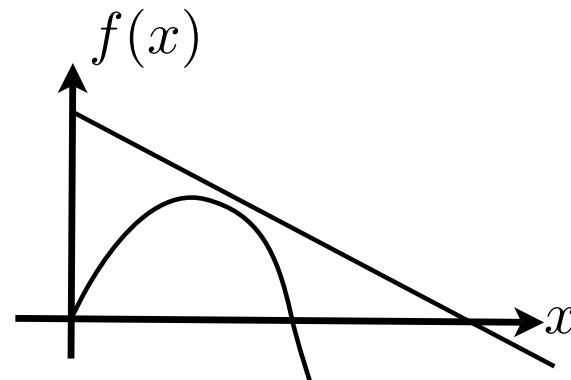
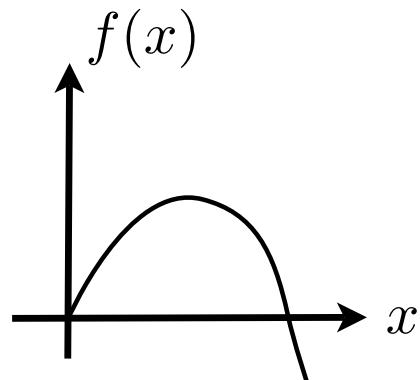


Etude mathématique du modèle de Rosenzweig-MacArthur

Les solutions sont bornées

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$



$$\Rightarrow f(x) < A - m \cdot x$$

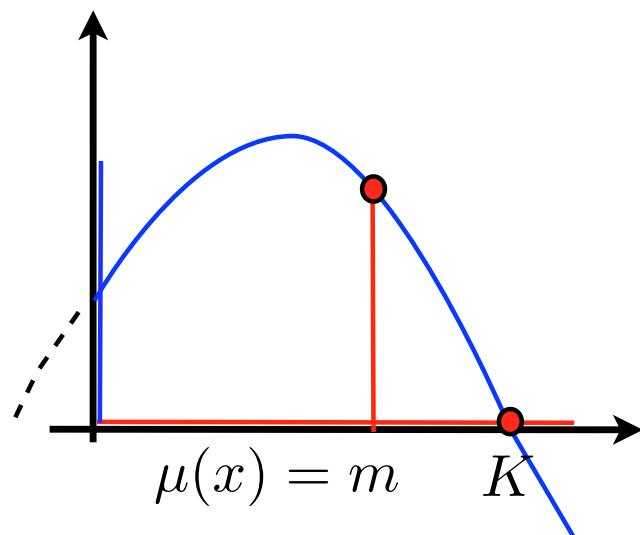
$$\overbrace{x + y}^{\bullet} < A - m(x + y)$$

$$\Rightarrow (x(t) + y(t)) < A/m + e^{-mt}(x_o + y_o - A/m)$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{y} &= \left(\frac{\mu \cdot x}{e + x} - d\right) \cdot y\end{aligned}\quad y = \frac{f(x)}{\mu(x)}$$

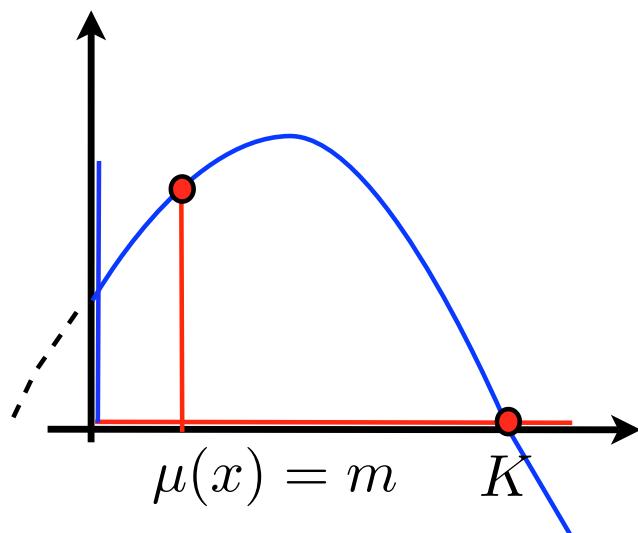
$$\frac{f(x)}{\mu(x)} = \frac{Y \cdot r}{\mu} \cdot (e + x) \cdot \left(1 - \frac{x}{K}\right)$$



Etude mathématique du modèle de Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{y} &= \left(\frac{\mu \cdot x}{e + x} - d\right) \cdot y\end{aligned}\quad y = \frac{f(x)}{\mu(x)}$$

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Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$

Stabilité des équilibres

$$\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

$$\left(\begin{array}{c|c} f'_x & f'_y \\ \hline g'_x & g'_y \end{array} \right)$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = f(x) - \mu(x) \cdot y$$

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Stabilité des équilibres

$$\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

$$\left(\begin{array}{c|c} f'_x & f'_y \\ \hline g'_x & g'_y \end{array} \right) = \left(\begin{array}{c|c} f'(x) - \mu'(x) \cdot y & -\mu(x) \\ \hline \mu'(x) \cdot y & \mu(x) - m \end{array} \right)$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$

Stabilité des équilibres

$$(x, y) = (K, 0) \implies \begin{pmatrix} f'(x) - \mu'(x) \cdot y & | & -\mu(x) \\ \hline \mu'(x) \cdot y & | & \mu(x) - m \end{pmatrix} =$$

$$= \begin{pmatrix} f'(K) & | & -\mu(K) \\ \hline 0 & | & \mu(K) - m \end{pmatrix}$$

Etude mathématique du modèle de Rosenzweig-MacArthur

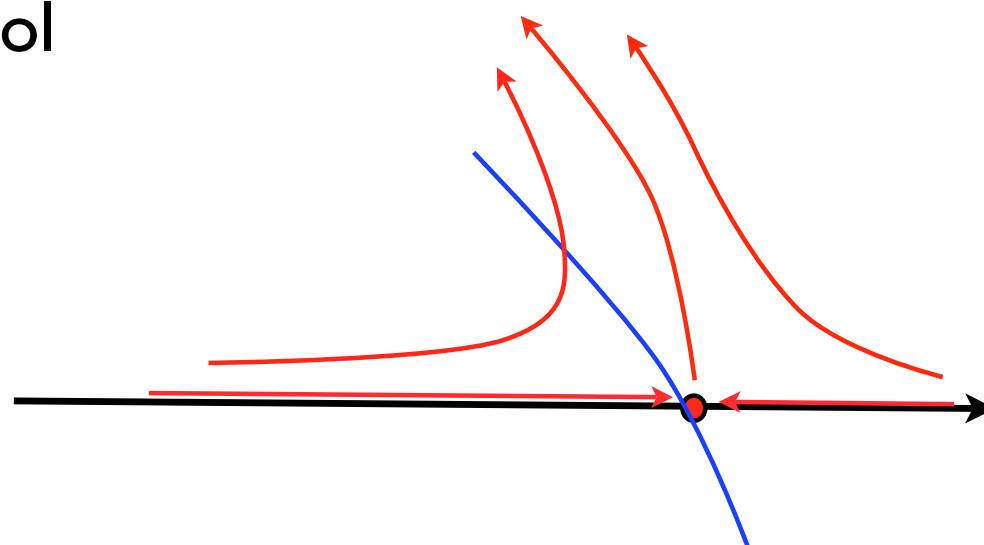
$$\dot{x} = f(x) - \mu(x) \cdot y$$

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Stabilité des équilibres

$$(x, y) = (K, 0) \implies \begin{pmatrix} f'(K) & | & -\mu(K) \\ 0 & | & \mu(K) - m \end{pmatrix} \implies \begin{cases} \lambda_1 = f(K) < 0 \\ \lambda_2 = \mu(K) - m > 0 \end{cases}$$

Col



Etude mathématique du modèle de Rosenzweig-MacArthur

Stabilité des équilibres

$$\dot{x} = f(x) - \mu(x) \cdot y$$

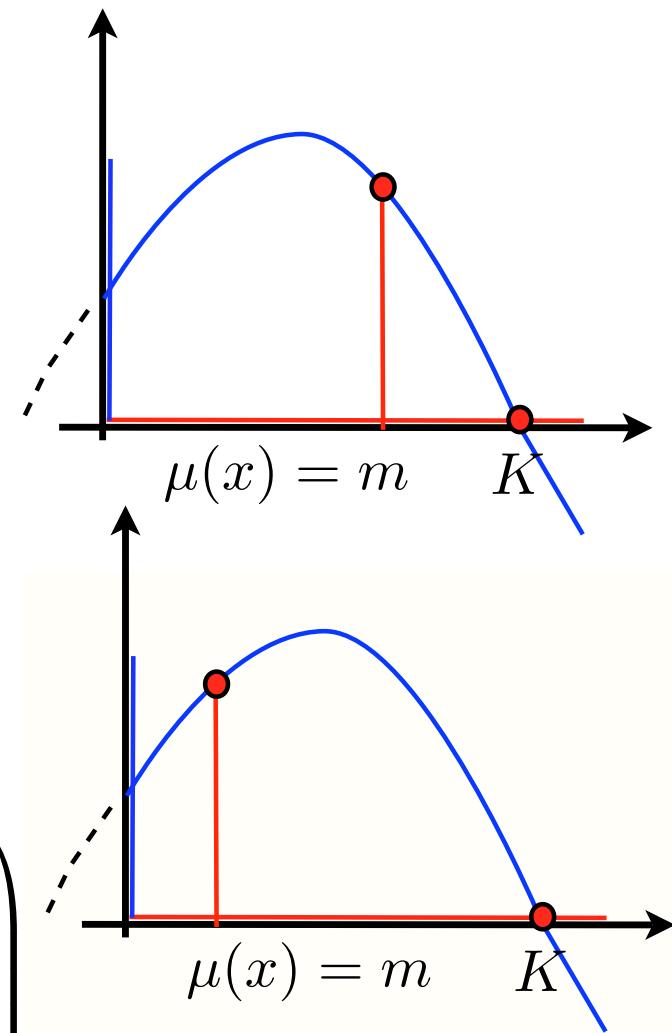
$$\dot{y} = (\mu(x) - m) \cdot y$$

$$x_e \iff \mu(x_e) = m$$

$$y_e = \frac{f(x_e)}{m}$$

$$\left(\begin{array}{c|c} f'(x) - \mu'(x) \cdot y & -\mu(x) \\ \hline \mu'(x) \cdot y & \mu(x) - m \end{array} \right) =$$

$$= \left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$



Etude mathématique du modèle de Rosenzweig-MacArthur

$$\left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$

$$\lambda^2 - \lambda \left(f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Etude mathématique du modèle de Rosenzweig-MacArthur

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Etude mathématique du modèle de Rosenzweig-MacArthur

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Etude mathématique du modèle de Rosenzweig-MacArthur

$$\left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$

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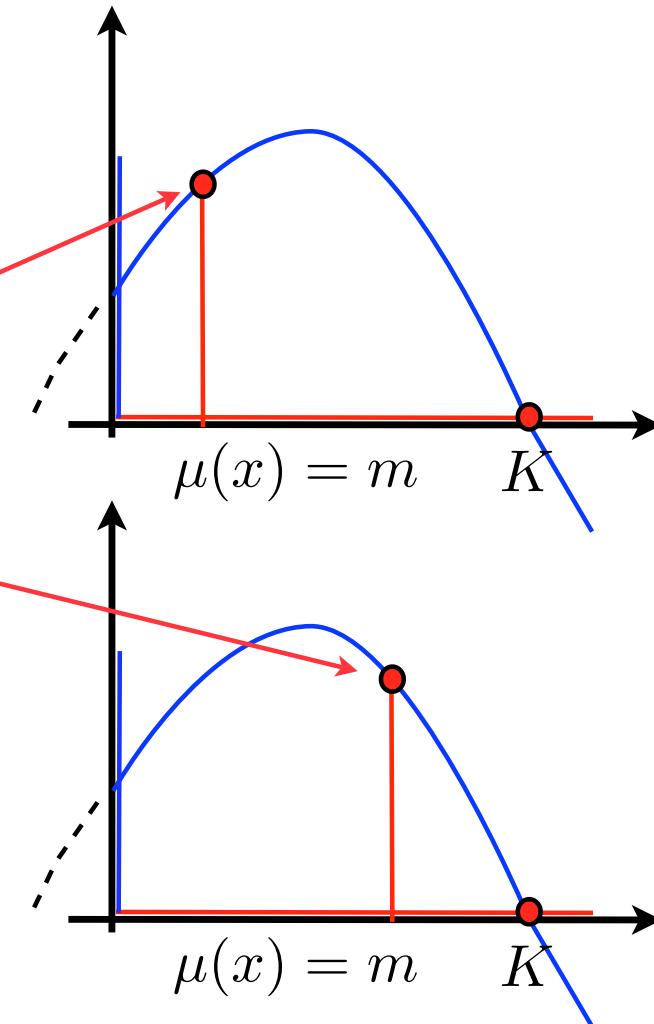
$$\lambda^2 - \lambda \left(\mu(x_e) \frac{\mu(x_e)f'(x_e) - \mu'(x_e) \cdot f(x_e)}{\mu^2(x_e)} \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

$$\lambda^2 - \lambda \left(\mu(x_e) \left(\frac{f}{\mu} \right)'(x_e) \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\lambda^2 - \lambda \left(\mu(x_e) \left(\frac{f}{\mu} \right)'(x_e) \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

$\left(\frac{f}{\mu} \right)'(x_e) > 0 \implies$ instable
 $\left(\frac{f}{\mu} \right)'(x_e) < 0 \implies$ stable



Etude mathématique du modèle de Rosenzweig-MacArthur

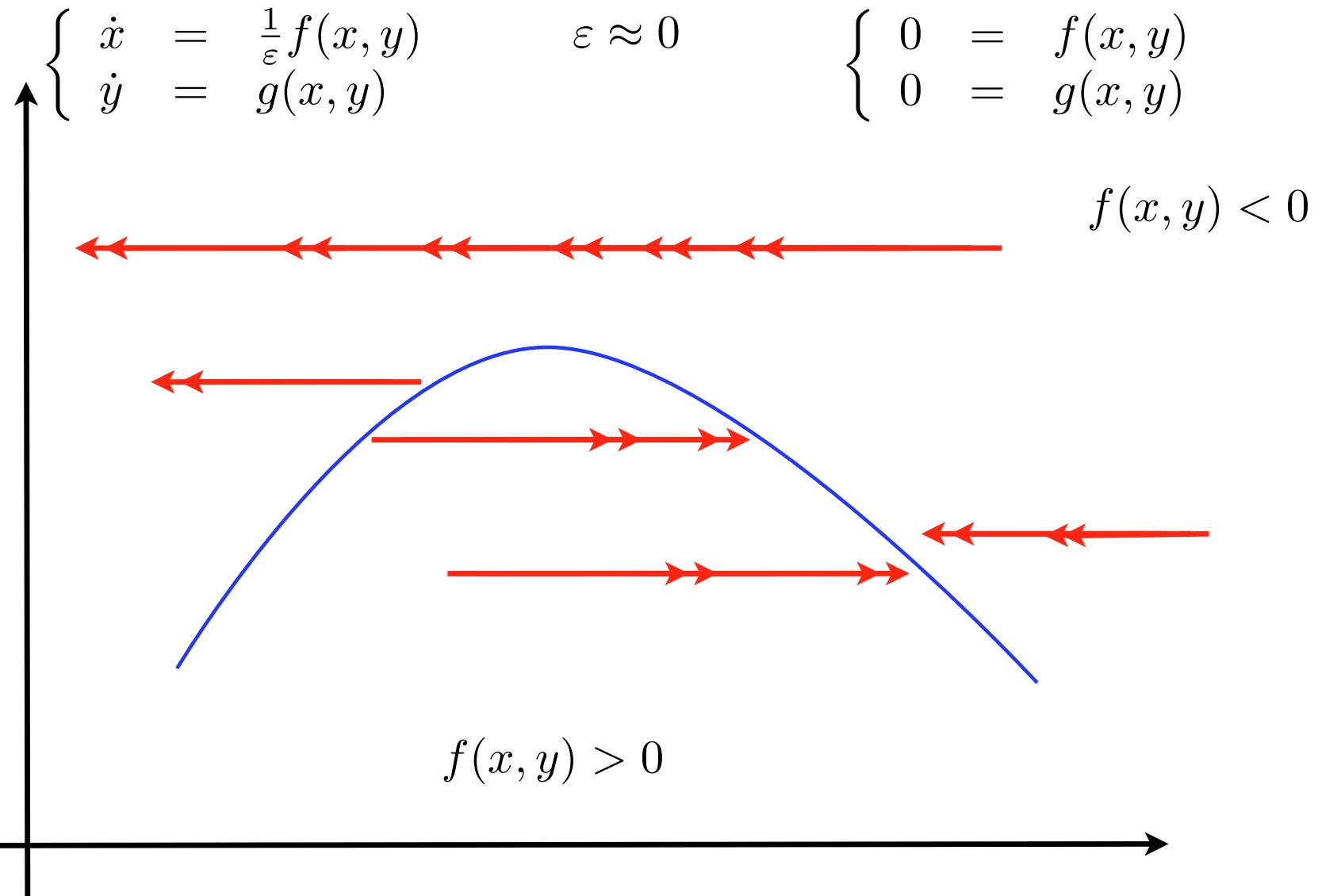
$$\lambda^2 - \lambda \left(\mu(x_e) \left(\frac{f}{\mu} \right)'(x_e) \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Remarque importante

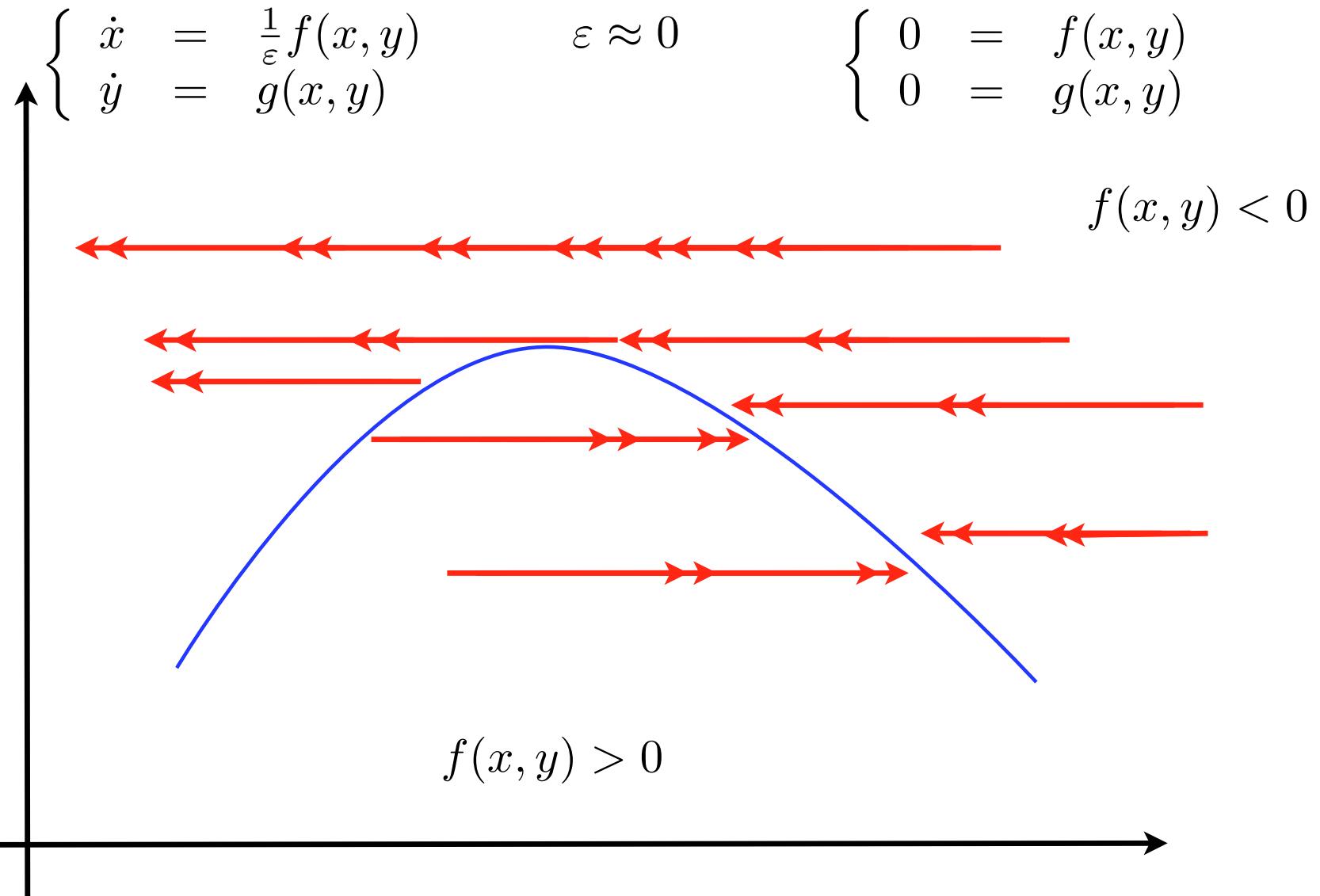
Arguments reposant sur des propriétés qualitatives des fonction f , μ etc...

- Calculs plus élégants
- Valable pour des “classes” de fonctions

L-R = Lent-Rapide

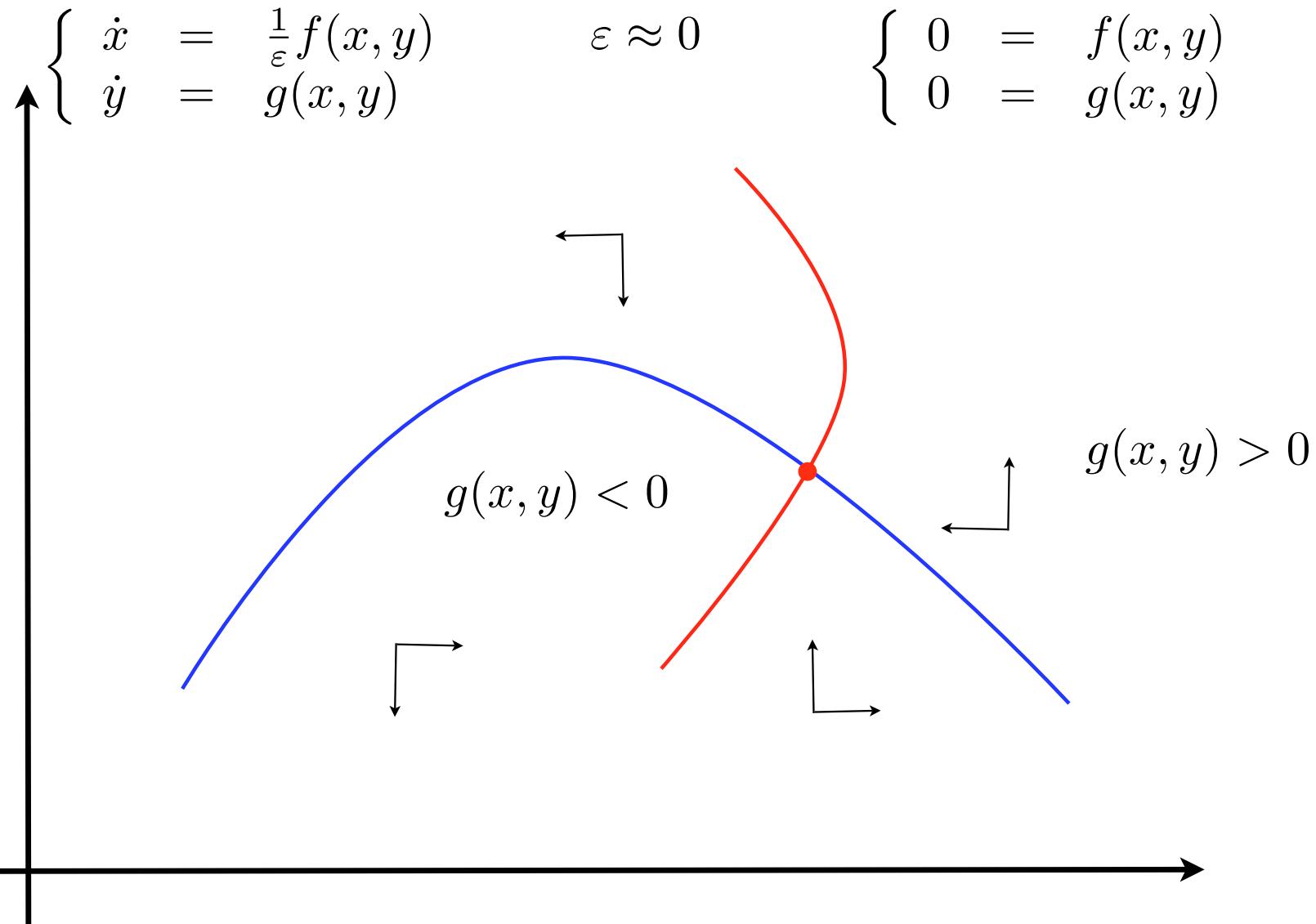


L-R = Lent-Rapide



Intermède mathématique : Systèmes L-R

L-R = Lent-Rapide

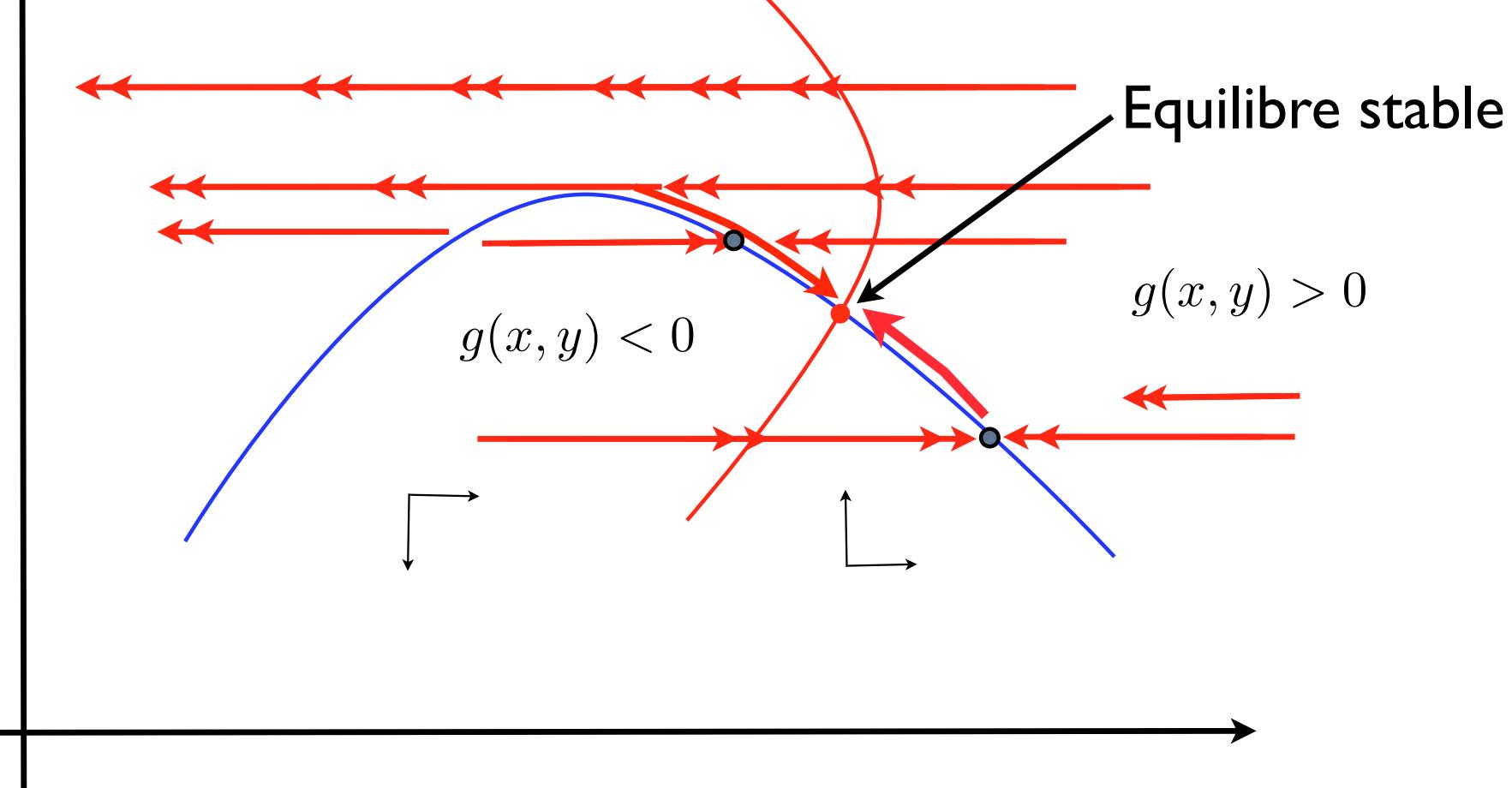


L-R = Lent-Rapide

$$\begin{cases} \dot{x} = \frac{1}{\varepsilon} f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

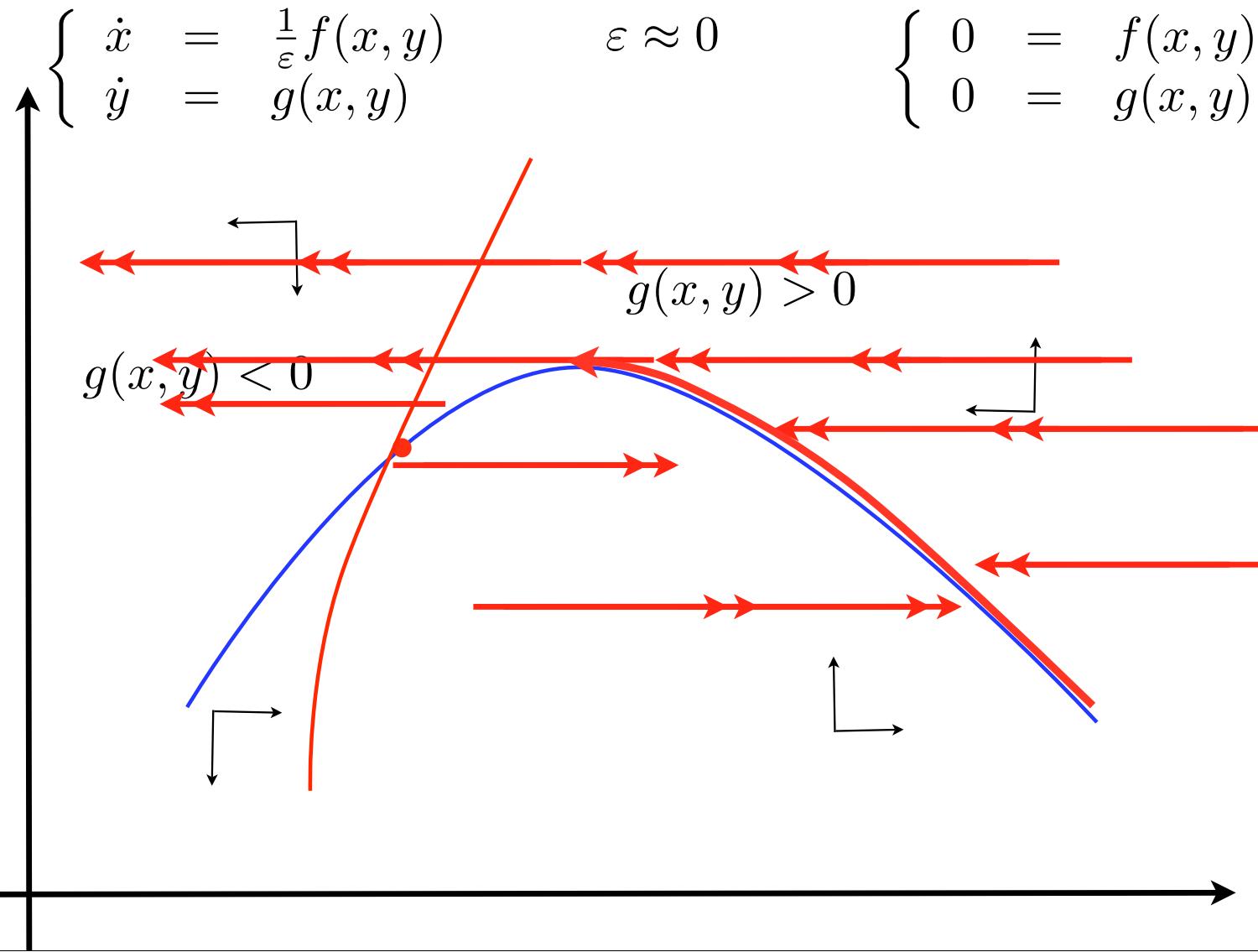
$$\varepsilon \approx 0$$

$$\begin{cases} 0 = f(x, y) \\ 0 = g(x, y) \end{cases}$$



Intermède mathématique : Systèmes L-R

L-R = Lent-Rapide



Etude mathématique du modèle de Rosenzweig-MacArthur

Retour en arrière

Interaction

$$x(t + dt) = x(t) + dt \cdot f(x(t)) - dt \cdot c(x(t), y(t)) \cdot y(t)$$

$$y(t + dt) = y(t) + dt \cdot \mu(x(t), y(t)) \cdot y(t) - dt \cdot \nu(y(t)) \cdot y(t)$$

Une hypothèse forte :

$$\mu(x(t), y(t)) = Y \cdot c(x(t), y(t))$$

Proportionnalité entre ce qui est consommé et
l'accroissement de la population de consommateurs

On remplace $c(x, y)$ par $\frac{1}{Y} \mu(x, y)$

Etude mathématique du modèle de Rosenzweig-MacArthur

On remplace $c(x, y)$ par $\frac{1}{Y}\mu(x, y)$

$$x(t + dt) = x(t) + dt \cdot \left(f(x(t)) - \frac{1}{Y}\mu(x(t), y(t)) \cdot y(t) \right)$$

$$y(t + dt) = y(t) + dt \cdot (\mu(x(t), y(t)) \cdot y(t) - \nu(y(t)) \cdot y(t))$$

$$\frac{dx(t)}{dt} = f(x(t)) - \frac{1}{Y}\mu(x(t), y(t)) \cdot y(t)$$

$$\frac{dy(t)}{dt} = \mu(x(t), y(t)) \cdot y(t) - \nu(y(t)) \cdot y(t)$$

$$\dot{x} = f(x) - \frac{1}{Y}\mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

Etude mathématique du modèle de Rosenzweig-MacArthur

On remplace $\mu(x, y)$ par $Yc(x, y)$

$$\dot{x} = f(x) - c(x, y) \cdot y \quad \dot{x} = f(x) - c(x) \cdot y$$

$$\dot{y} = (Y \cdot c(x, y) - \nu(y)) \cdot y \quad \dot{y} = (Y \cdot c(x) - m)y$$

$c(x)$ noté $\mu(x)$

$$\dot{x} = f(x) - \mu(x) \cdot y \quad m < \varepsilon \cdot \mu(x)$$

$$\dot{y} = (\varepsilon \cdot \mu(x) - m)y \quad m = \varepsilon \cdot \delta$$

$$\begin{array}{ll} \dot{x} = f(x) - \mu(x) \cdot y & \dot{x} = \frac{1}{\varepsilon} (f(x) - \mu(x) \cdot y) \\ \iff & \\ \dot{y} = \varepsilon \cdot (\mu(x) - \delta) \cdot y & \dot{y} = (\mu(x) - \delta) \cdot y \end{array}$$

Etude mathématique du modèle de Rosenzweig-MacArthur

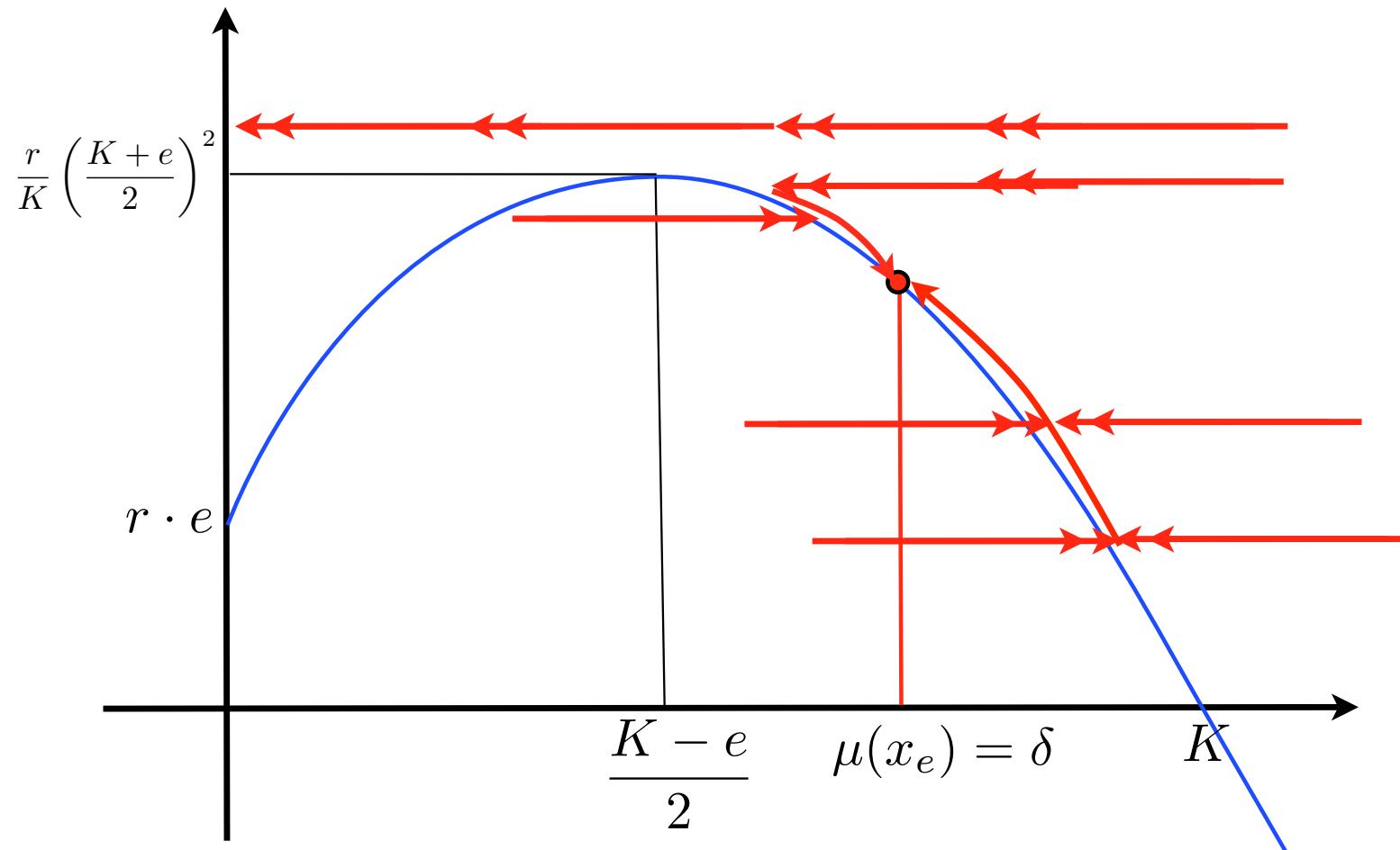
Simulations de RMA

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = \frac{1}{\varepsilon}(f(x) - \mu(x) \cdot y)$$

$$\dot{y} = (\mu(x) - \delta) \cdot y$$

- $f(x) = r \cdot x \cdot (1 - x/K)$
- $\mu(x) = \frac{\mu \cdot x}{e + x}$

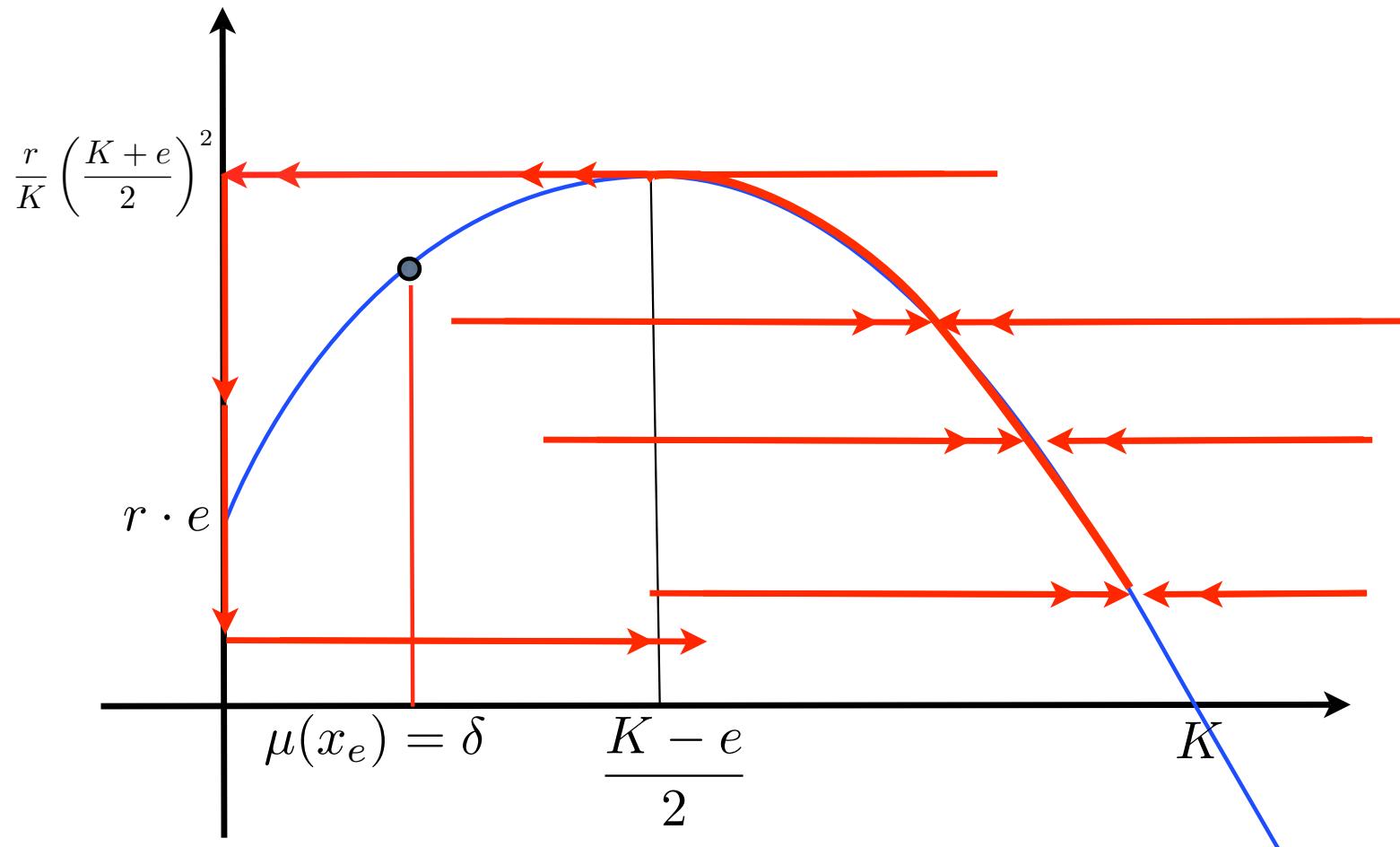


Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = \frac{1}{\varepsilon}(f(x) - \mu(x) \cdot y)$$

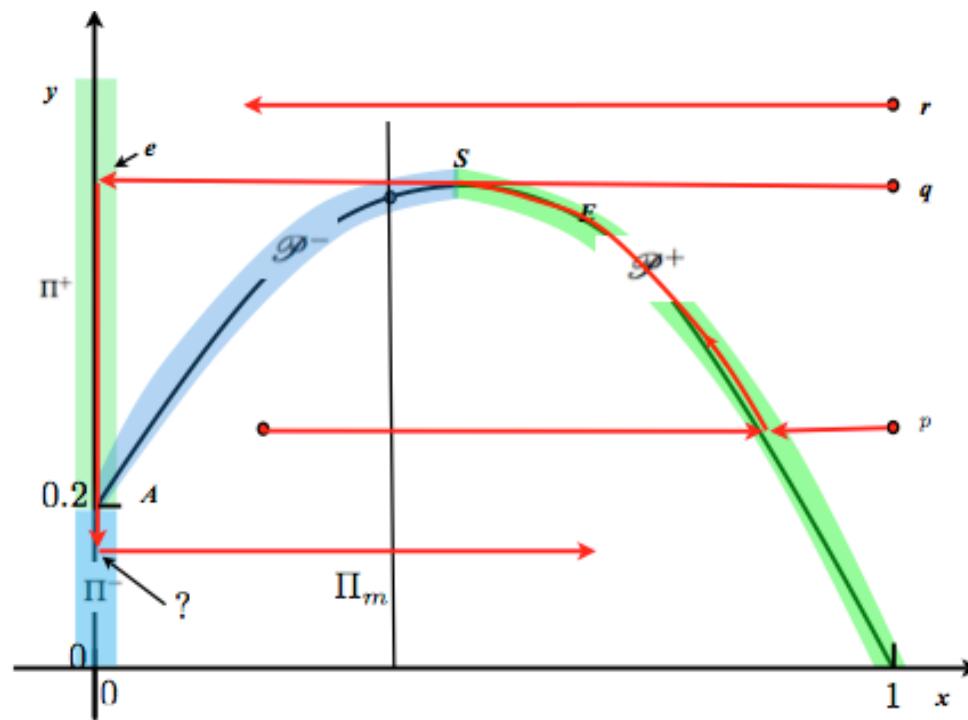
$$\dot{y} = (\mu(x) - \delta) \cdot y$$

- $f(x) = r \cdot x \cdot (1 - x/K)$
- $\mu(x) = \frac{\mu \cdot x}{e + x}$

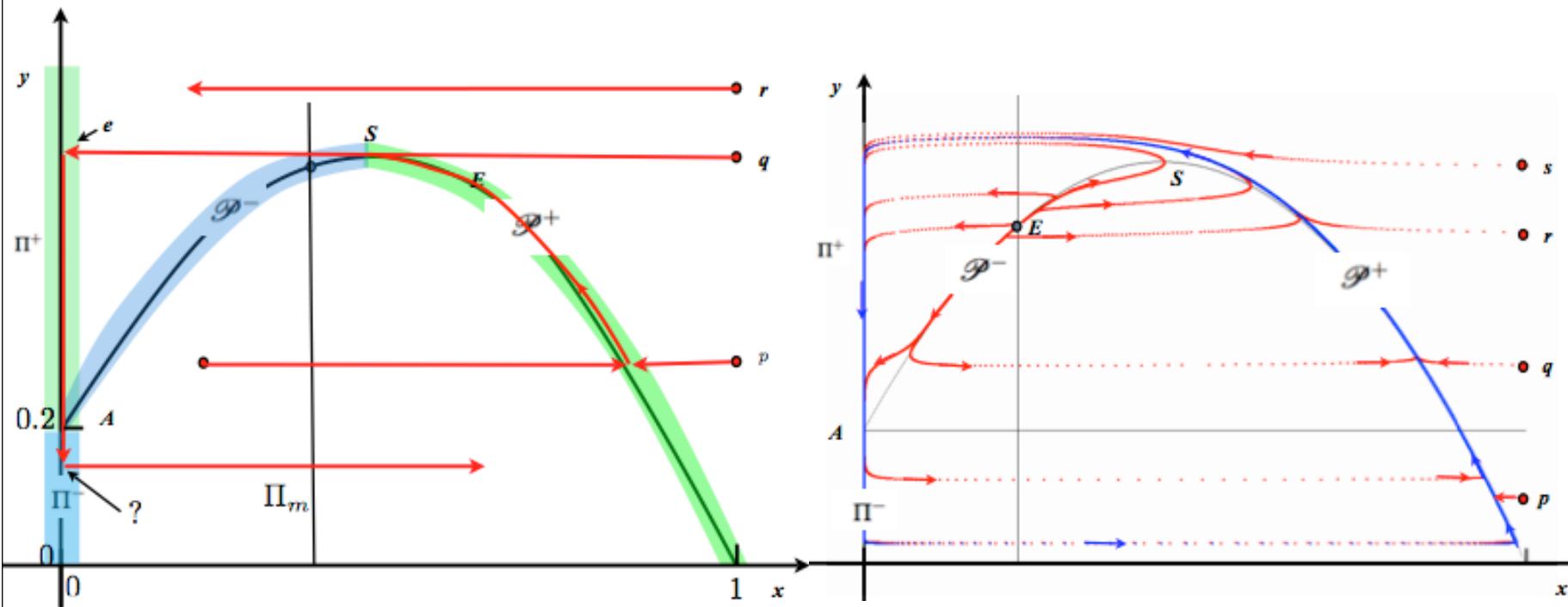


Simulations du modèle de Rosenzweig-MacArthur

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{x}{0.1+x}y \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - \delta \right)y \end{cases}$$



Simulations du modèle de Rosenzweig-MacArthur



Simulations du modèle de Rosenzweig-MacArthur

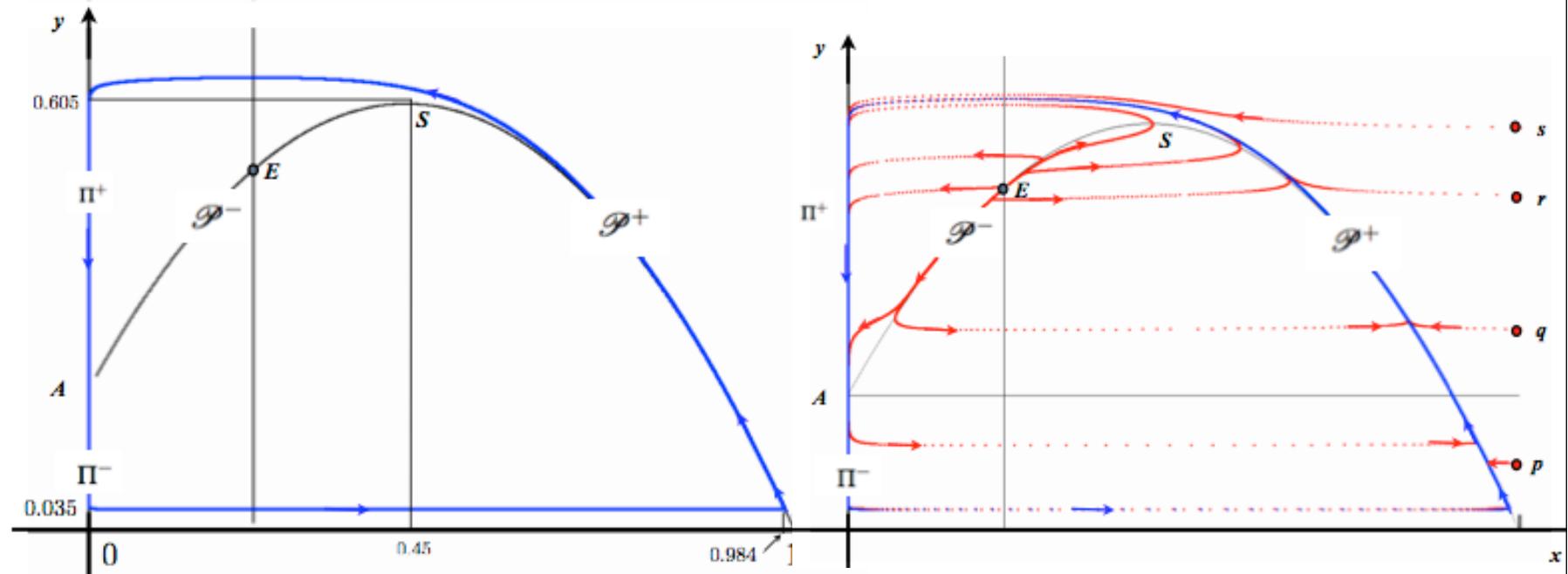


Figure 3: Limit cycle of (3) : $\varepsilon = 0.05$, $m : 0.7$

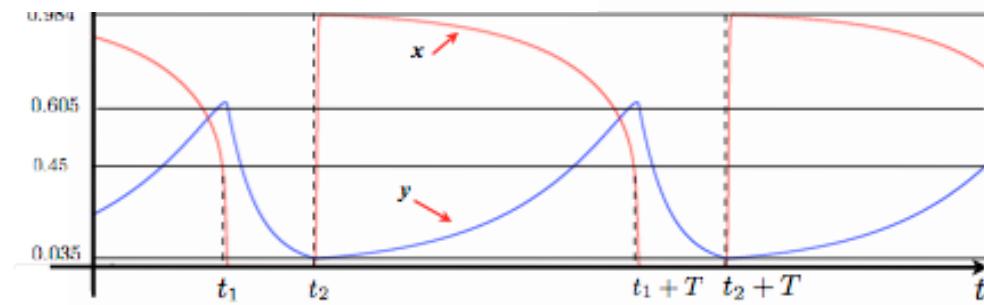
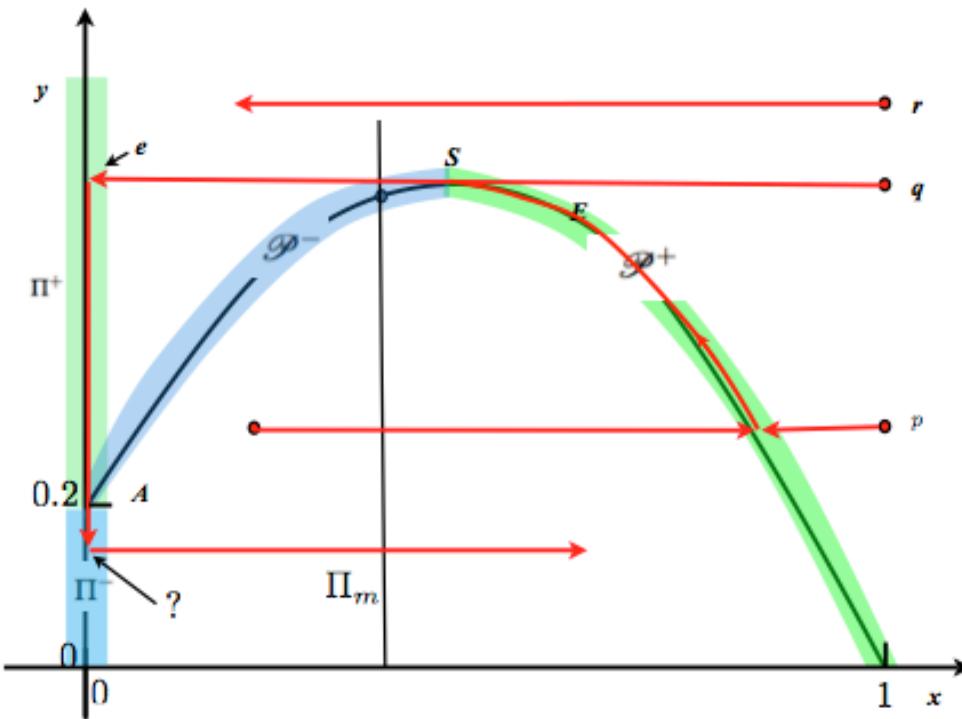


Figure 4: Limit cycle of (3); “jumps” of $x(t)$: $\varepsilon = 0.05$, $m : 0.7$

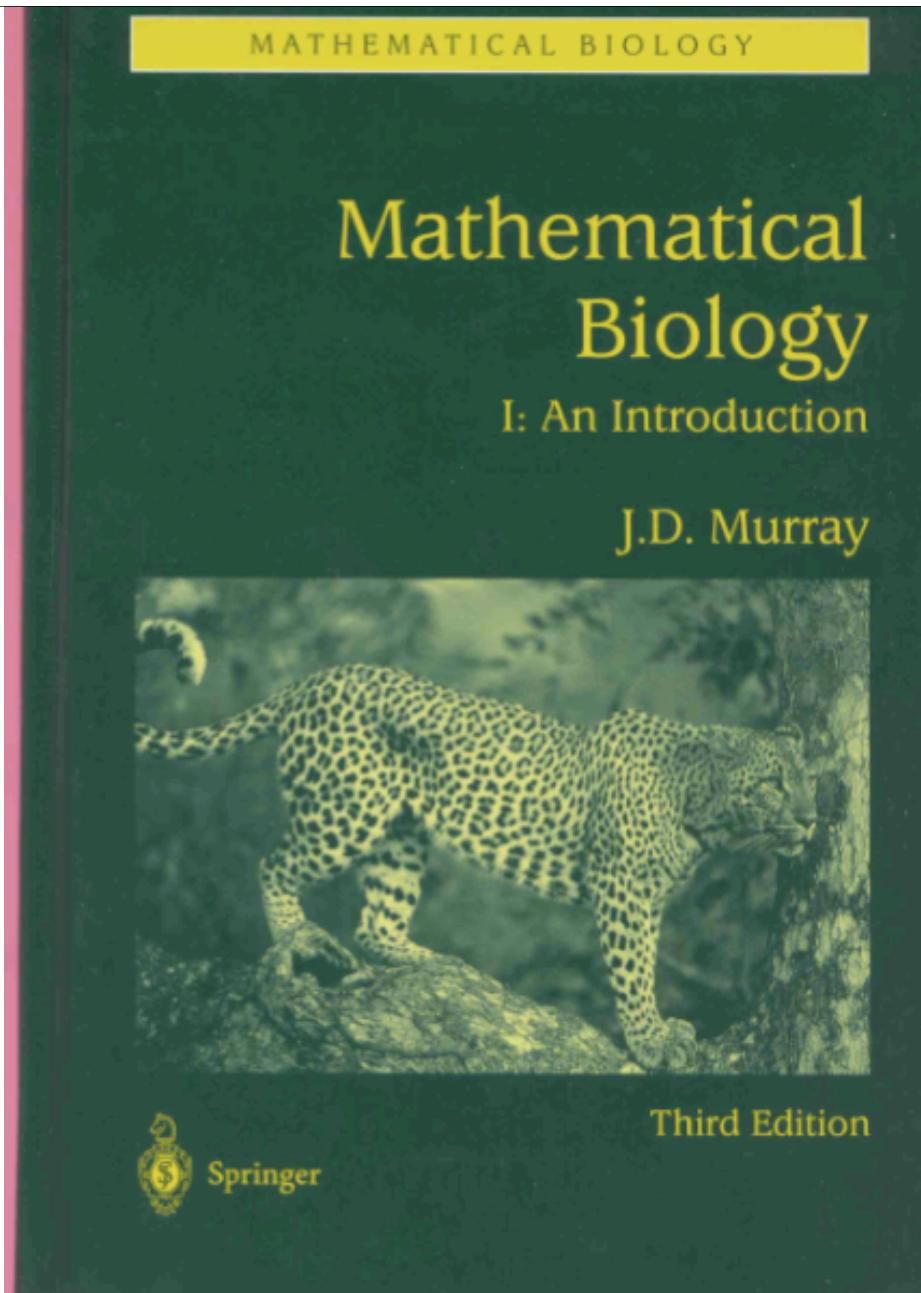
Simulations du modèle de Rosenzweig-MacArthur



y_0	0.4	0.5	0.6	0.7
$\exp\left(\frac{\xi_{min}}{\epsilon}\right)$	$1.66 \cdot 10^{-9}$	$1.79 \cdot 10^{-16}$	$2.14 \cdot 10^{-24}$	$5.34 \cdot 10^{-33}$
x_{min}	$1.62 \cdot 10^{-9}$	$1.69 \cdot 10^{-16}$	$1.90 \cdot 10^{-24}$	$4.31 \cdot 10^{-33}$

Table 2: Values of the parameters : $\epsilon = 0.05$; $m = 0.7$; $x_0 = 0.05$; $\xi_0 = \epsilon \log(x_0)$

Le problème Atto - Fox



Le problème Atto - Fox

On the Spatial Spread of Rabies among Foxes



J. D. Murray; E. A. Stanley; D. L. Brown

Proceedings of the Royal Society of London. Series B, Biological Sciences, Vol. 229, No. 1255 (Nov. 22, 1986), 111-150.

Infected foxes

$$\partial S/\partial T = (a-b)(1-N/K)S - \beta RS,$$

$$\partial I/\partial T = \beta SR - \sigma I - [b + (a-b)N/K]I,$$

$$\partial R/\partial T = \sigma I - \alpha R - [b + (a-b)N/K]R + D\partial^2 R/\partial X^2,$$

Mobility modeled by the diffusion operator

Le problème Atto - Fox

4,6/ km²

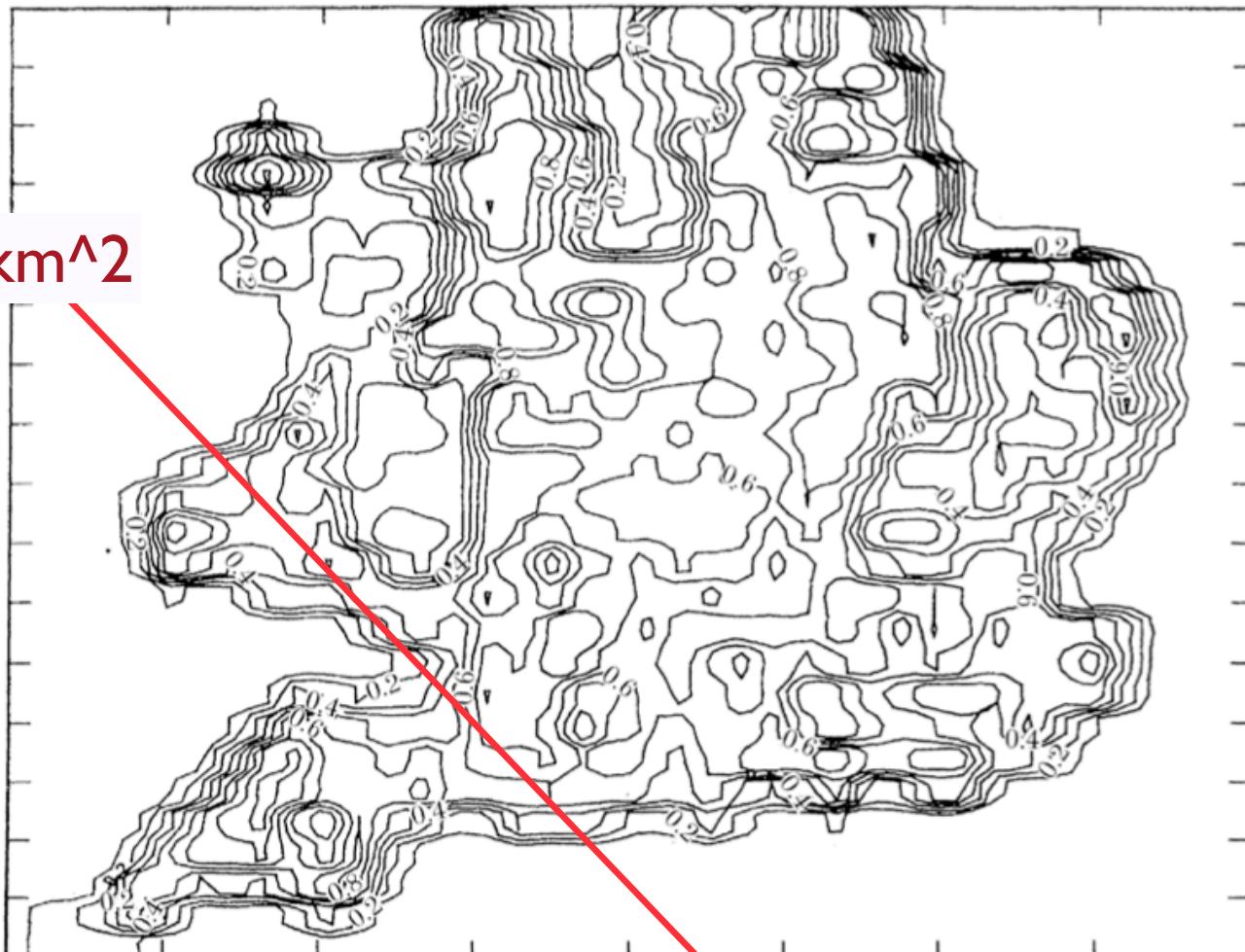


FIGURE 10. Contour plot of fox densities in the southern half of England that were used in our numerical simulations. Values are scaled to lie between 0 and 1, with 1 corresponding to 2.4 adult foxes km^{-2} in springtime, or to an average of 4.6 foxes km^{-2} throughout the year. These values are based on Macdonald's (1980) estimates, who emphasizes that the density map is probably not very accurate but is based on educated estimates.

Le problème Atto - Fox

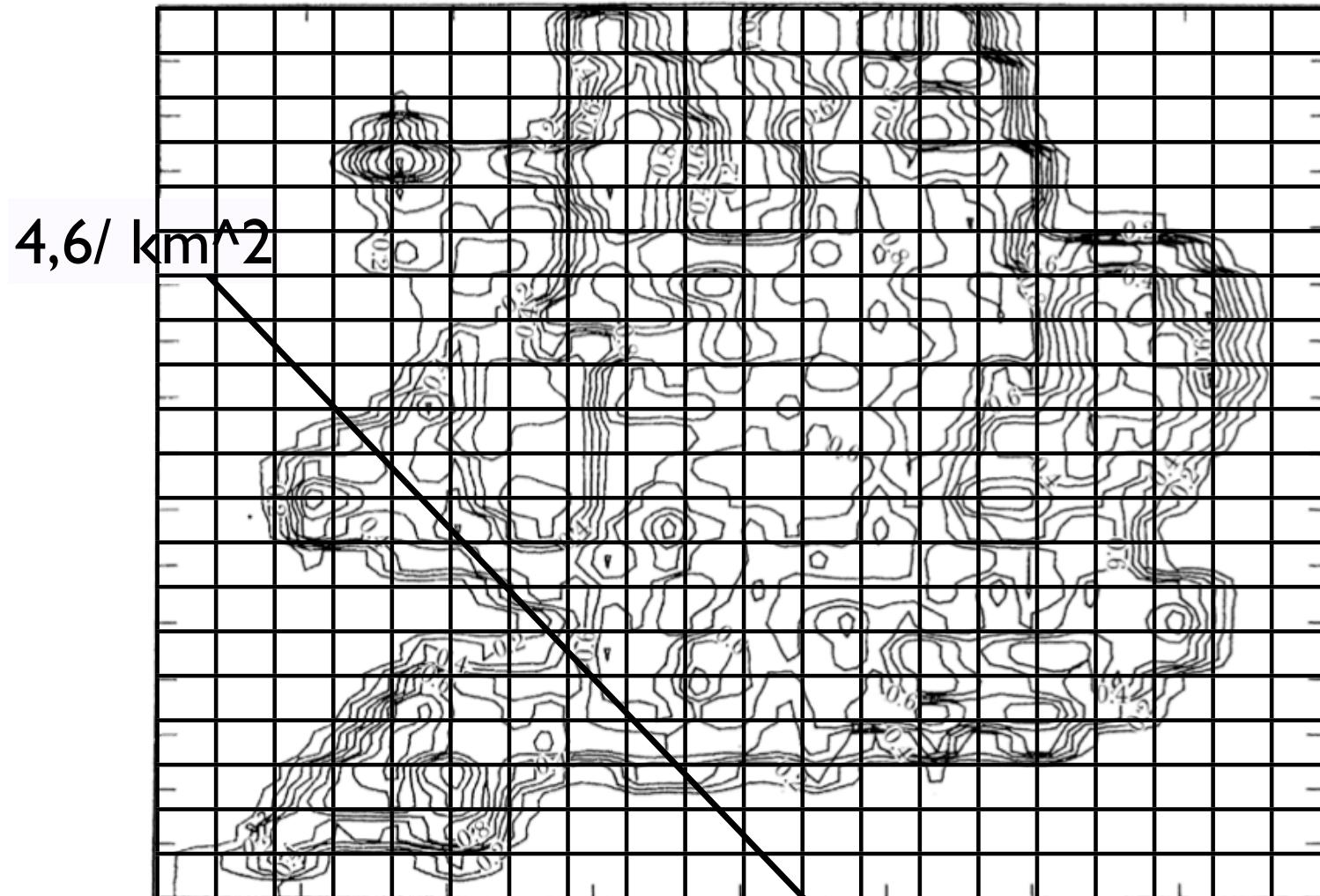


FIGURE 10. Contour plot of fox densities in the southern half of England that were used in our numerical simulations. Values are scaled to lie between 0 and 1, with 1 corresponding to 2.4 adult foxes km^{-2} in springtime, or to an average of 4.6 foxes km^{-2} throughout the year. These values are based on Macdonald's (1980) estimates, who emphasizes that the density map is probably not very accurate but is based on educated estimates.

Le problème Atto - Fox

Math Biosciences (1991) 107, 255-287.

DEPENDENCE OF EPIDEMIC AND POPULATION VELOCITIES ON BASIC PARAMETERS

Denis Mollison

As to the second wave, close inspection shows that the explanation lies, not so much in the determinism of the model, as in its modelling of the population as continuous rather than discrete and its associated inability to let population variables reach the value zero. Thus the density of infected at the place of origin of the epidemic never becomes zero, it only declines to a minimum of around one atto-fox (10^{-18} of a fox, Hughes 1960) per square kilometre. The model then allows this atto-fox to start the second wave as soon as the susceptible population has regrown sufficiently.

Le problème Atto - Fox

Math Biosciences (1991) 107, 255-287.

INFLUENCE OF EPIDEMIC AND POPULATION VELOCITIES ON BASIC PARAMETERS

Denis Mollison

e, close inspection shows that the explanation lies, not so much in the model, as in its modelling of the population as continuous and its associated inability to let population variables reach the density of infected at the place of origin of the epidemic never declines to a minimum of around one atto-fox (10^{-18} of a fox, per kilometre. The model then allows this atto-fox to start the infection as the susceptible population has regrown sufficiently.

$$\begin{cases} \frac{dx}{dt} = xg(x) - h(x,y)y \\ \frac{dy}{dt} = \varepsilon h(x,y)y - \delta y \end{cases} \quad \begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon}(xg(x) - h(x,y)y) \\ \frac{dy}{dt} = (h(x,y) - m)y \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{xy}{0.1+x} \right) \\ \frac{dy}{dt} = (\frac{x}{0.1+x} - m)y \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{xy}{0.1+x} \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$

Le modèle (3) ci-dessus possède pour des valeurs convenables des paramètres, comme par exemple :

$$m = 0.7 ; \varepsilon = 0.05$$

un cycle limite globalement asymptotiquement stable (G.A.S.). Donc, d'un point de vue mathématique, il est persistant. Toutefois, si nous faisons une simulation de ces équations à partir des conditions initiales :

$$x(0) = 0.1 ; y(0) = 0.5$$

nous observerons que la valeur de $x(t)$ va diminuer jusqu'à des quantités de l'ordre de 10^{-26} !! (Nous reviendrons longuement au § suivant sur les raisons de ce phénomène). Pour une telle valeur $x(t)$ ne représente plus rien. En effet supposons que une unité de x représente 10^8 individus (par exemple des renards) la valeur 10^{-26} représenterait un 10^{-18} ème de renard, un "atto-renard"¹.

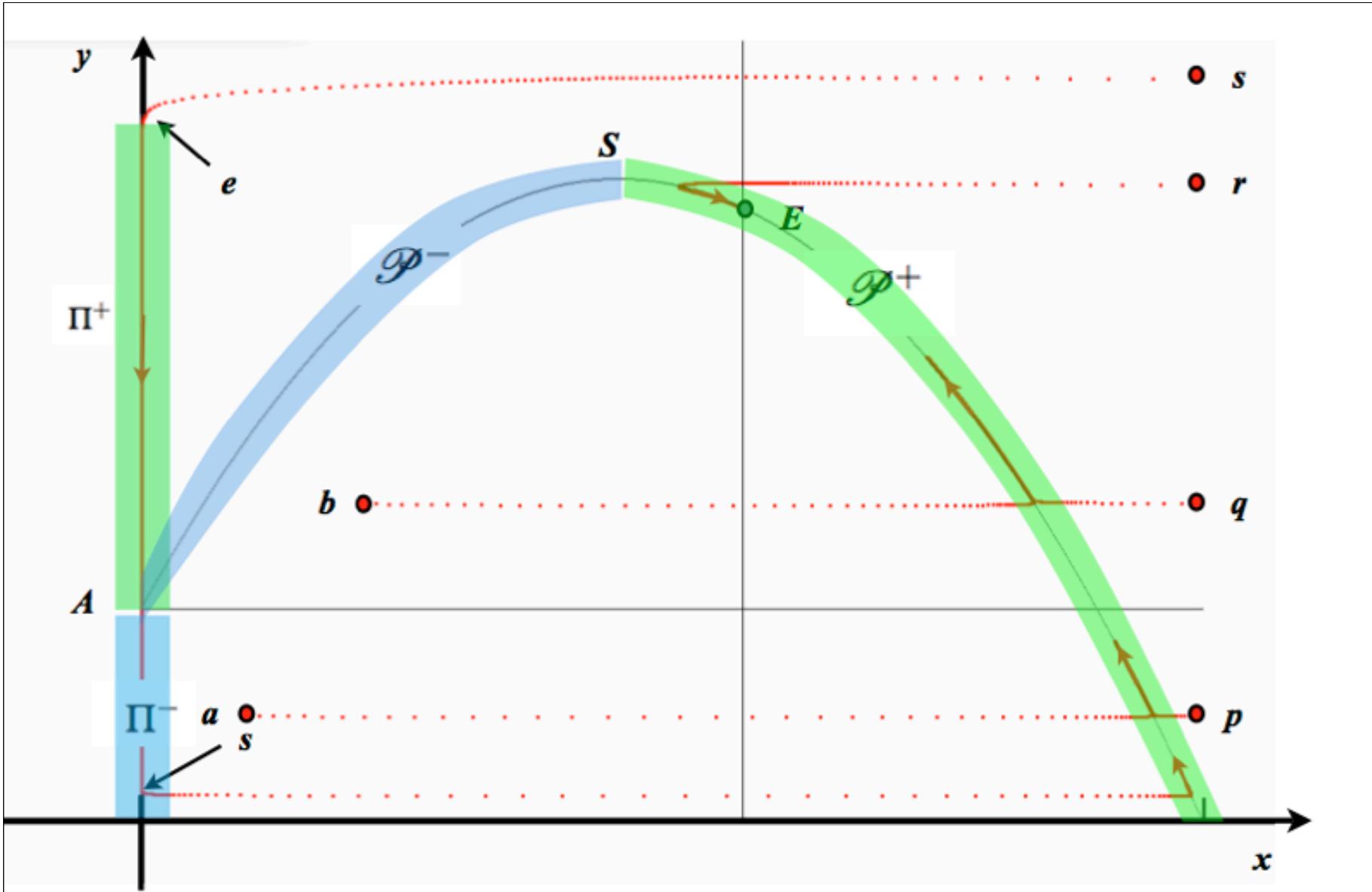


FIG. 2 – Quelques trajectoires avec $\varepsilon = 0.05$ et $m = 0.85$.

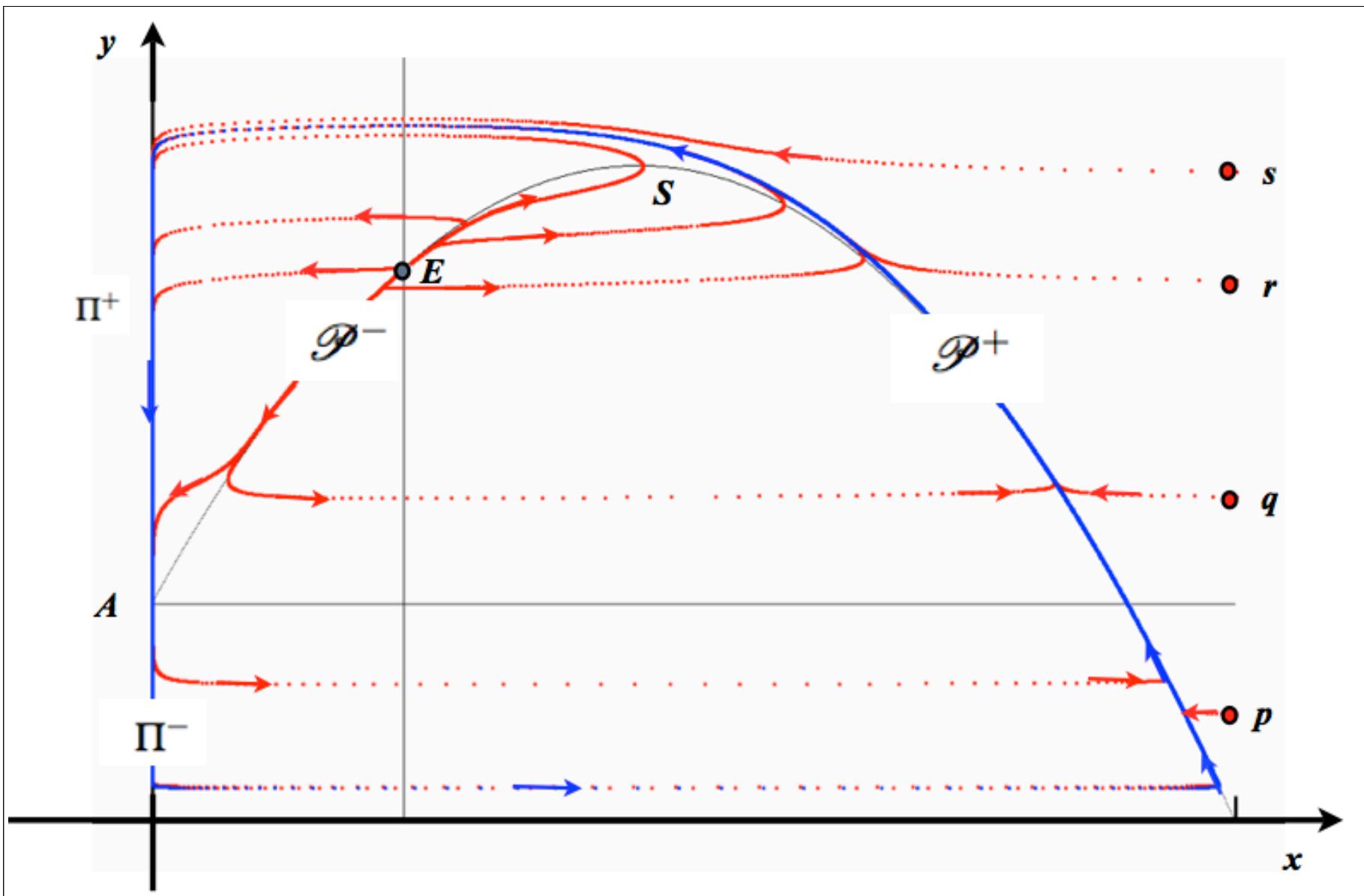


FIG. 3 – Quelques trajectoires avec $\varepsilon = 0.05$ et $m = 0.7$.

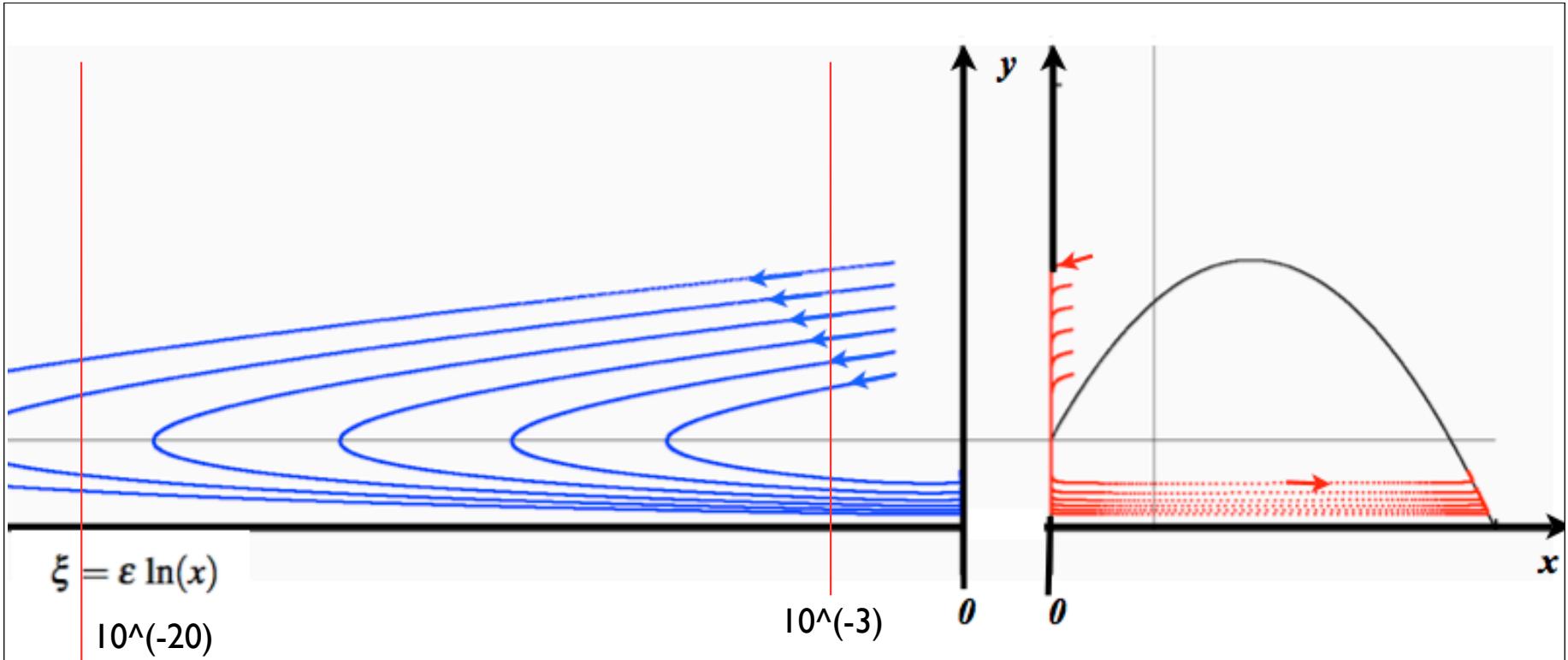


FIG. 4 – Quelques trajectoires en variable ξ avec $\varepsilon = 0.05$ et $m = 0.7$.

$$\begin{cases} \frac{d\xi}{dt} = 2 - 10y \\ \frac{dy}{dt} = -my \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{xy}{0.1+x} \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$

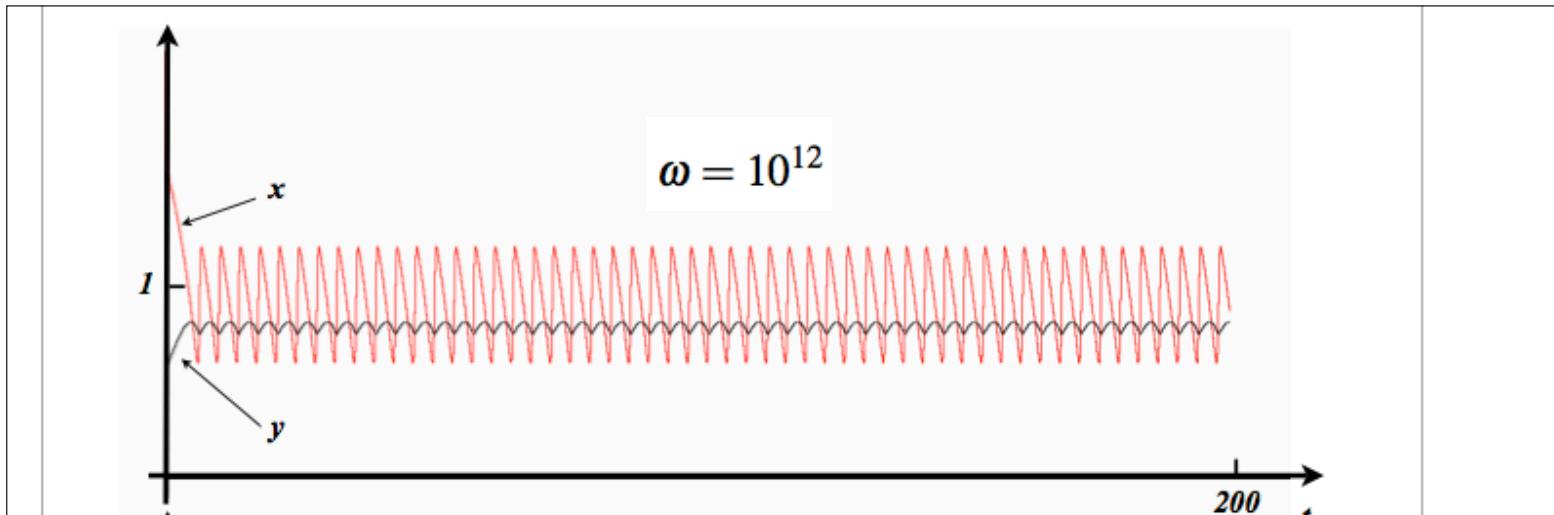
Le même avec du bruit

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} [f(x) - \mu(x)y] \\ \frac{dy}{dt} = (\mu(x) - m)y \end{cases} \quad \begin{aligned} - f(x) &= \frac{1}{2}x(2-x) \\ - \mu(x) &= \frac{x}{0.4+x} \\ - \varepsilon &= 0.02 \end{aligned}$$

$$x(t+dt) - x(t) \approx dt \frac{1}{\varepsilon} [f(x(t)) - \mu(x(t))y(t)] - \sqrt{dt \frac{4}{\omega \varepsilon} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

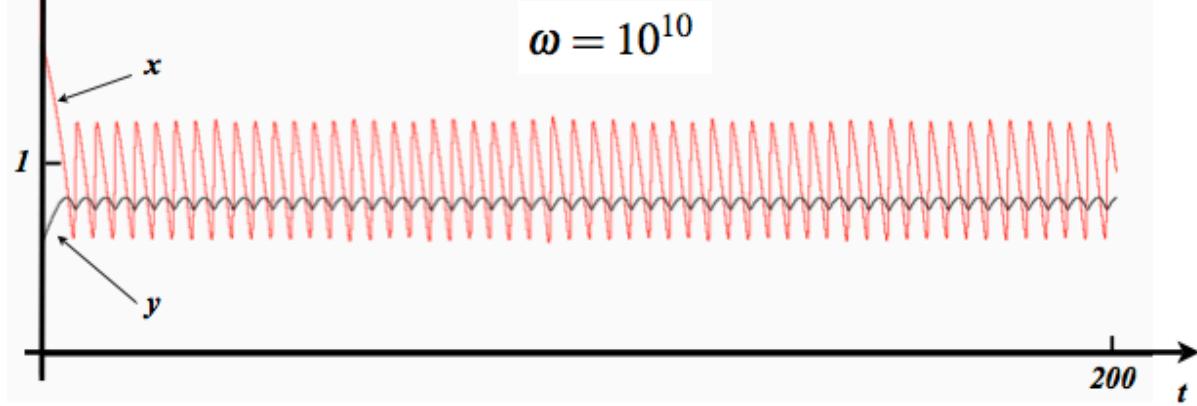
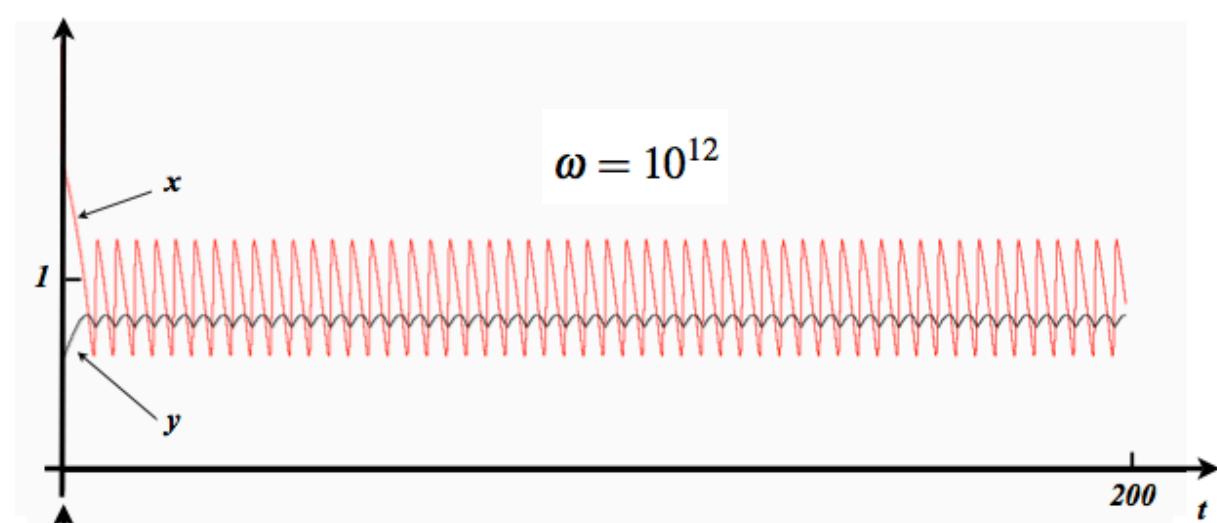
$$y(t+dt) - y(t) \approx dt [\mu(x(t) - m)y(t) + \sqrt{dt \frac{\varepsilon}{\omega} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t]$$

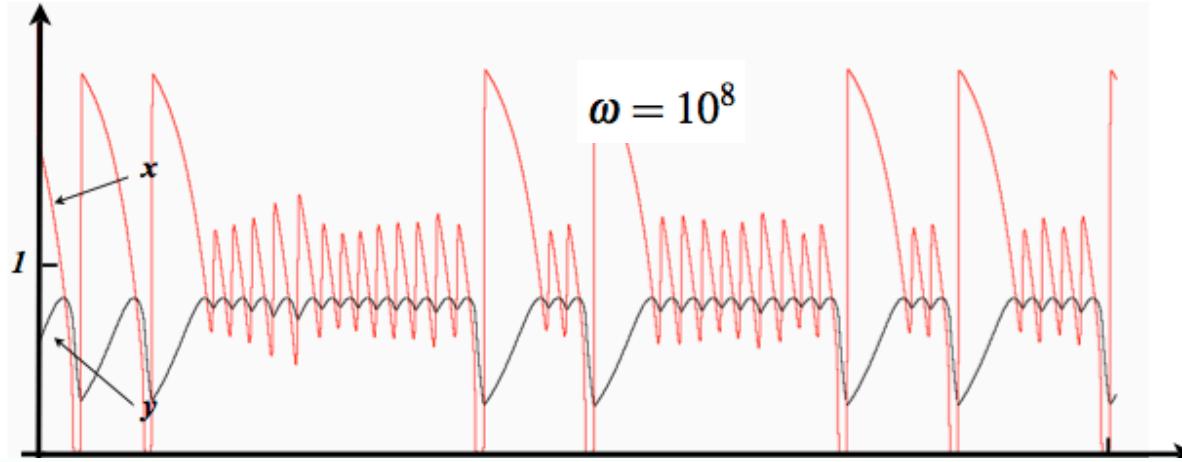
1 unit of x = ω individuals



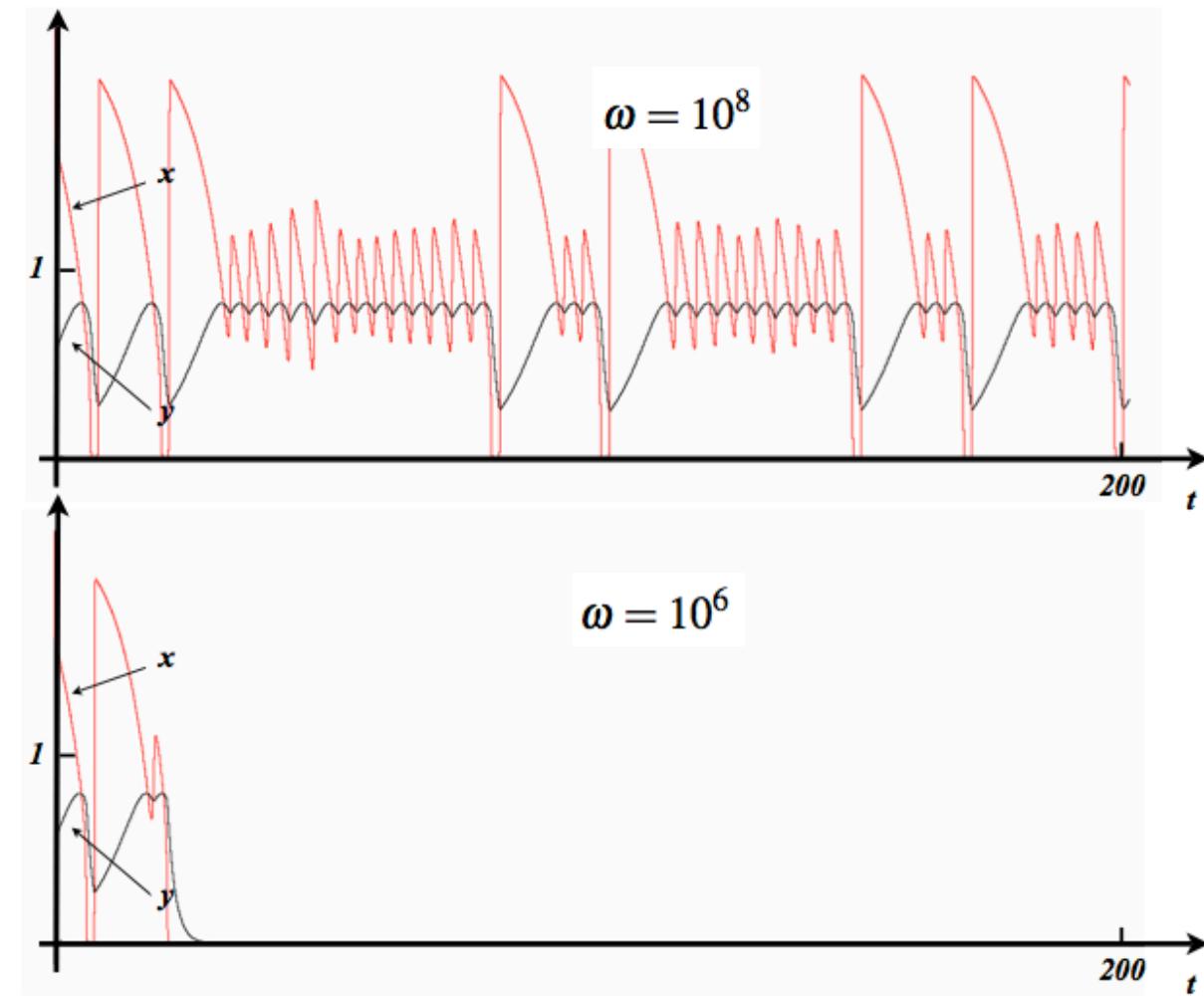
Not a surprise

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} [f(x) - \mu(x)y] \\ \frac{dy}{dt} = (\mu(x) - m)y \end{cases} + \text{very (very) small noise}$$





This is a (big) surprise since the number of individuals is still very large



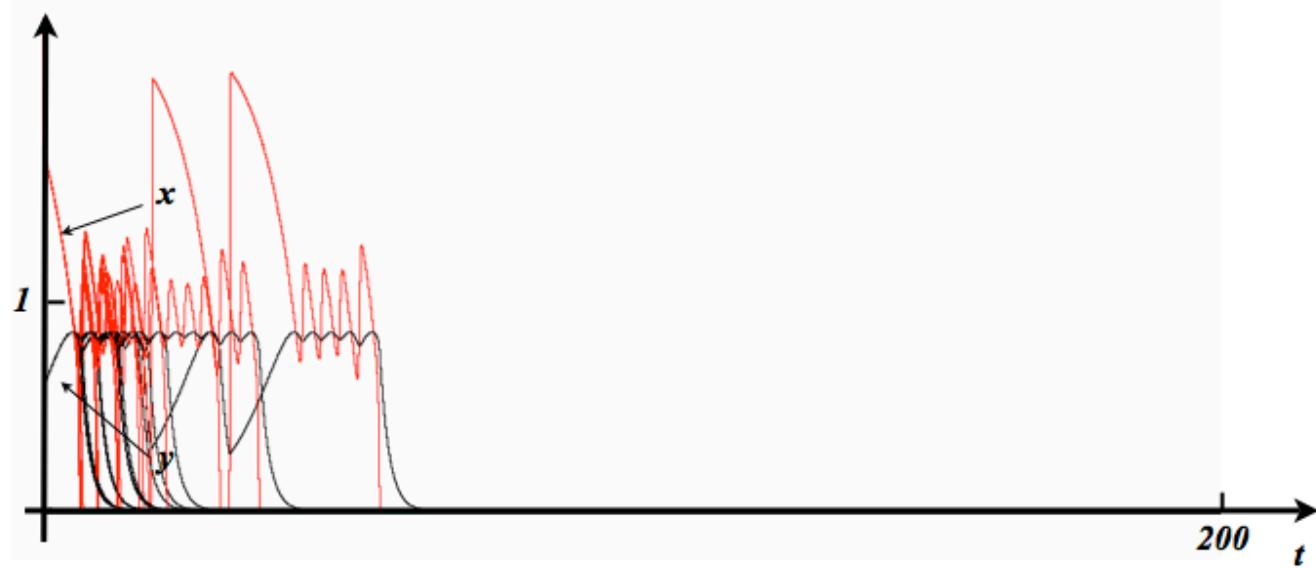


FIG. 3 – Twenty runs with $\omega = 10^6$

$$x(t+dt) - x(t) \approx dt \frac{1}{\varepsilon} [f(x(t)) - \mu(x(t))y(t)] - \sqrt{dt \frac{4}{\omega\varepsilon} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

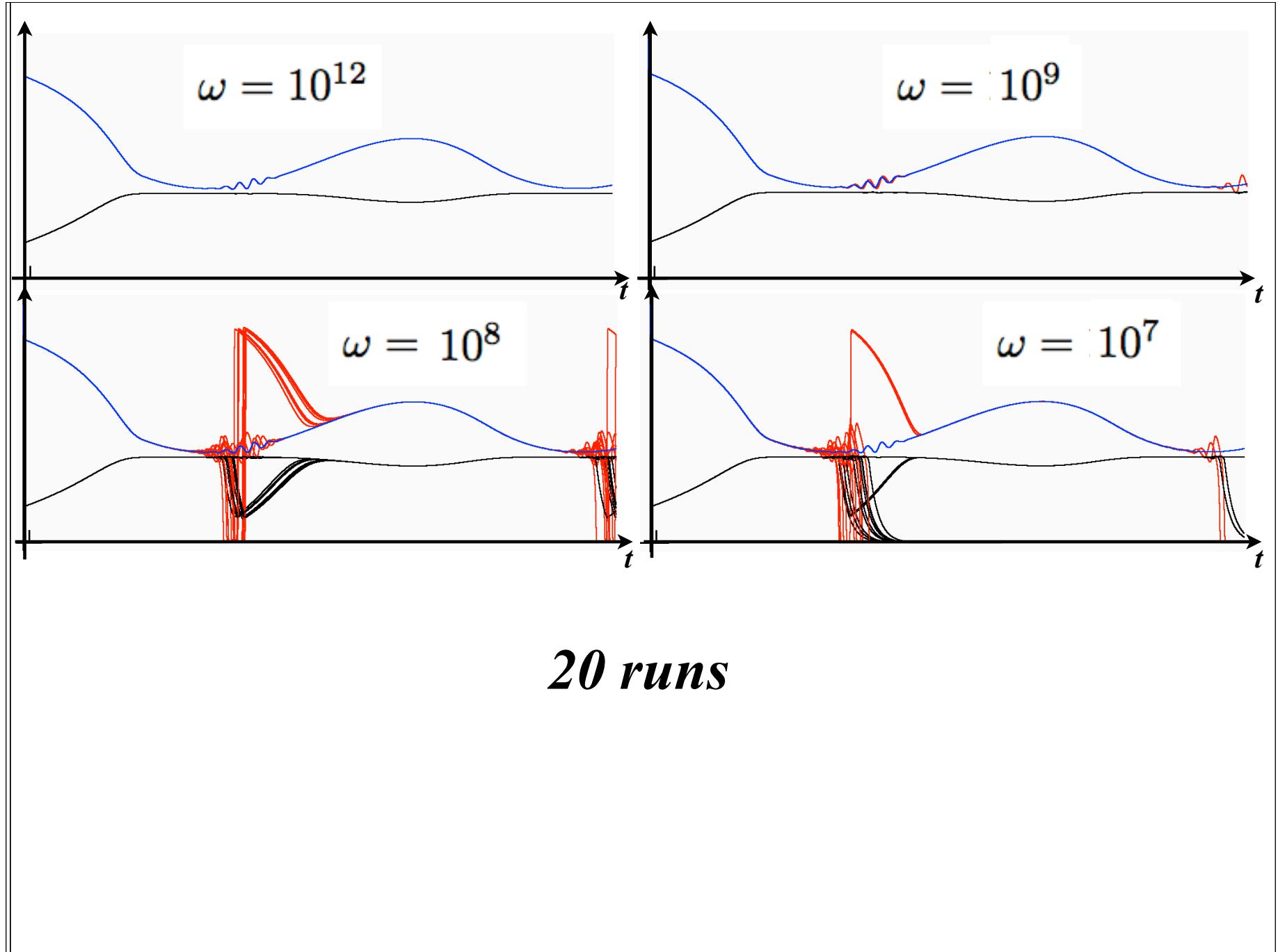
$$y(t+dt) - y(t) \approx dt [\mu(x(t)) - m]y(t) + \sqrt{dt \frac{\varepsilon}{\omega} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$



$$m(t) = a + b \cos(r t)$$

Simulations : In blue = deterministic part only

Simulations : In red = full model





Effect of population size in a predator–prey model

F. Campillo*, C. Lobry

Modemic Team-Project INRA/INRIA, SupAgro, 2 Place Viala, 34060 Montpellier Cedex 2, France

I must confess that, at least for me, these results are hard to believe. I agree that, quite often, deterministic models are interpreted with unacceptably low populations (e.g., a few dozens individuals). But 10^8 is a huge number, far exceeding any real population, except in microbiology. The first question that comes to mind is: aren't these results numerical artifacts? Reading through the paper, I couldn't find a discussion about this.



Effect of population size in a predator-prey model

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This paper is quite interesting. It couples a stochastic model with a differential equation model to describe a predator-prey interacting dynamic system. The major result is that a continuous representation of the population abundance appears to be valid only above 10^8 individuals. If this is true, then almost all continuous models of population dynamics should be thrown away.

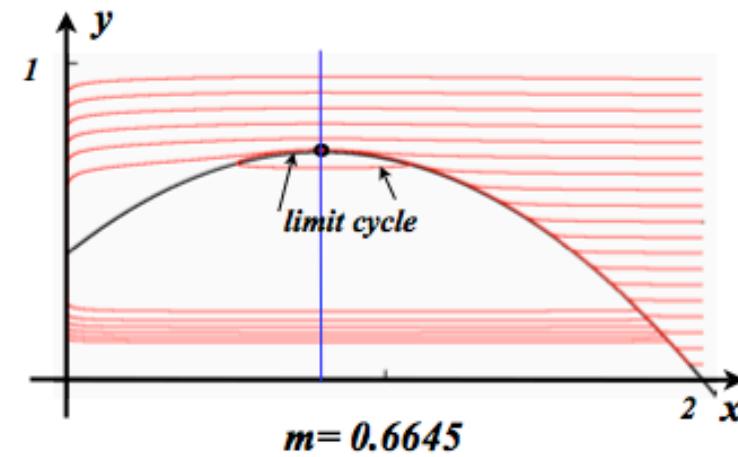
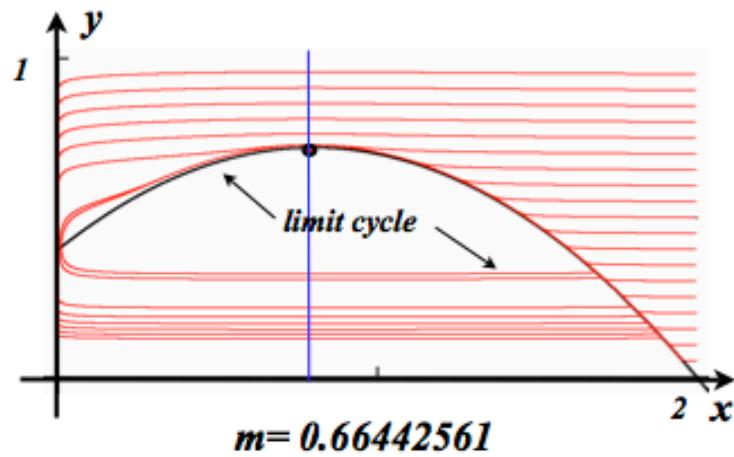
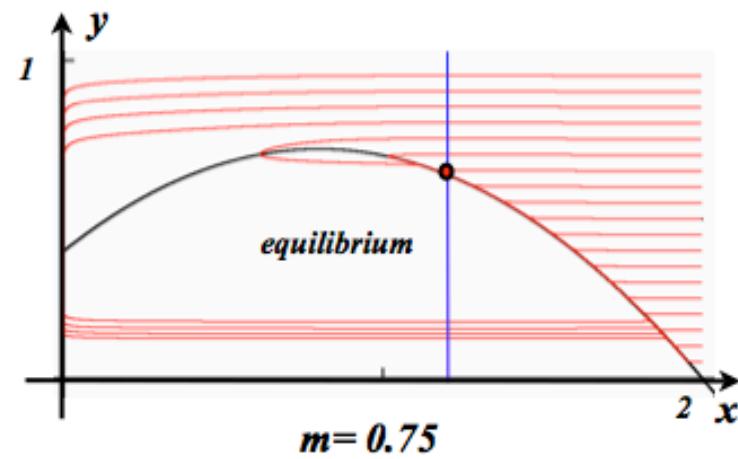
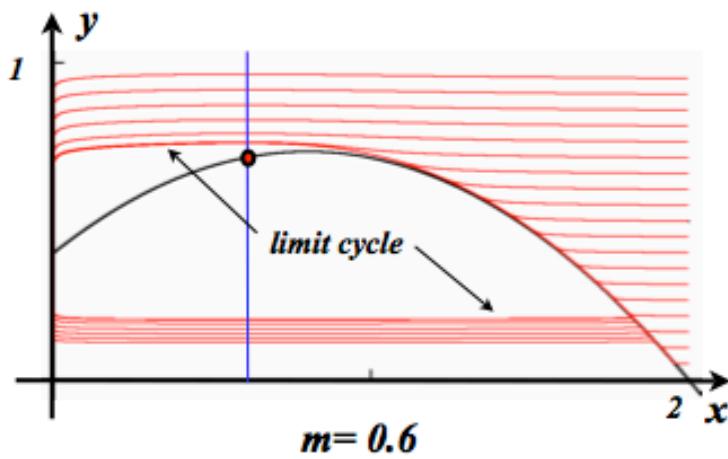


FIG. 6 – Phase portrait of system (7) for different values of m

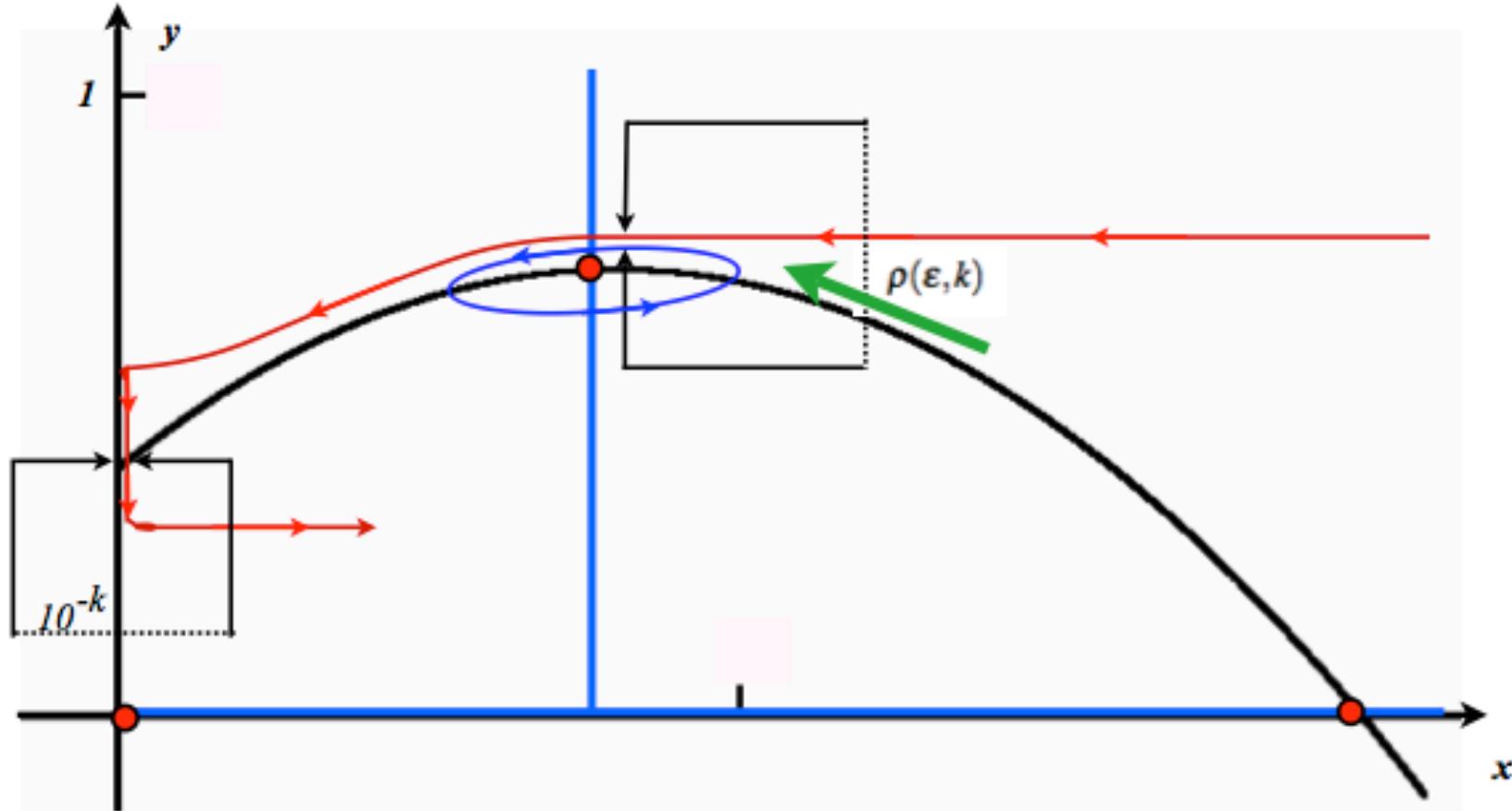
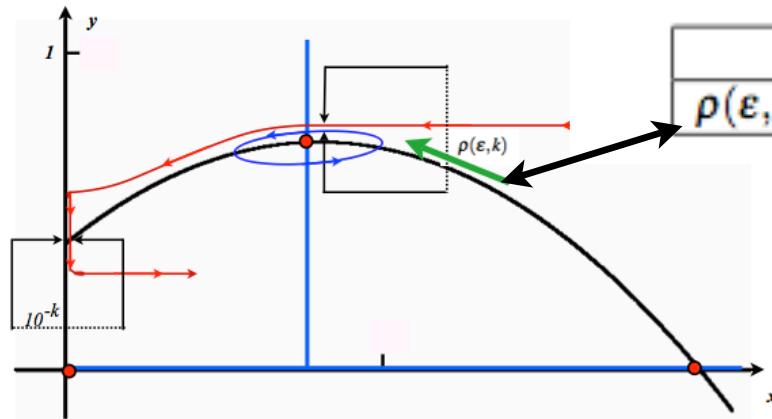


FIG. 9 – The “safety funnel”

ω	10^9	10^8	10^7	10^6
$\rho(\varepsilon, k)$	0.001208	0.000114	0.000054	0.000052

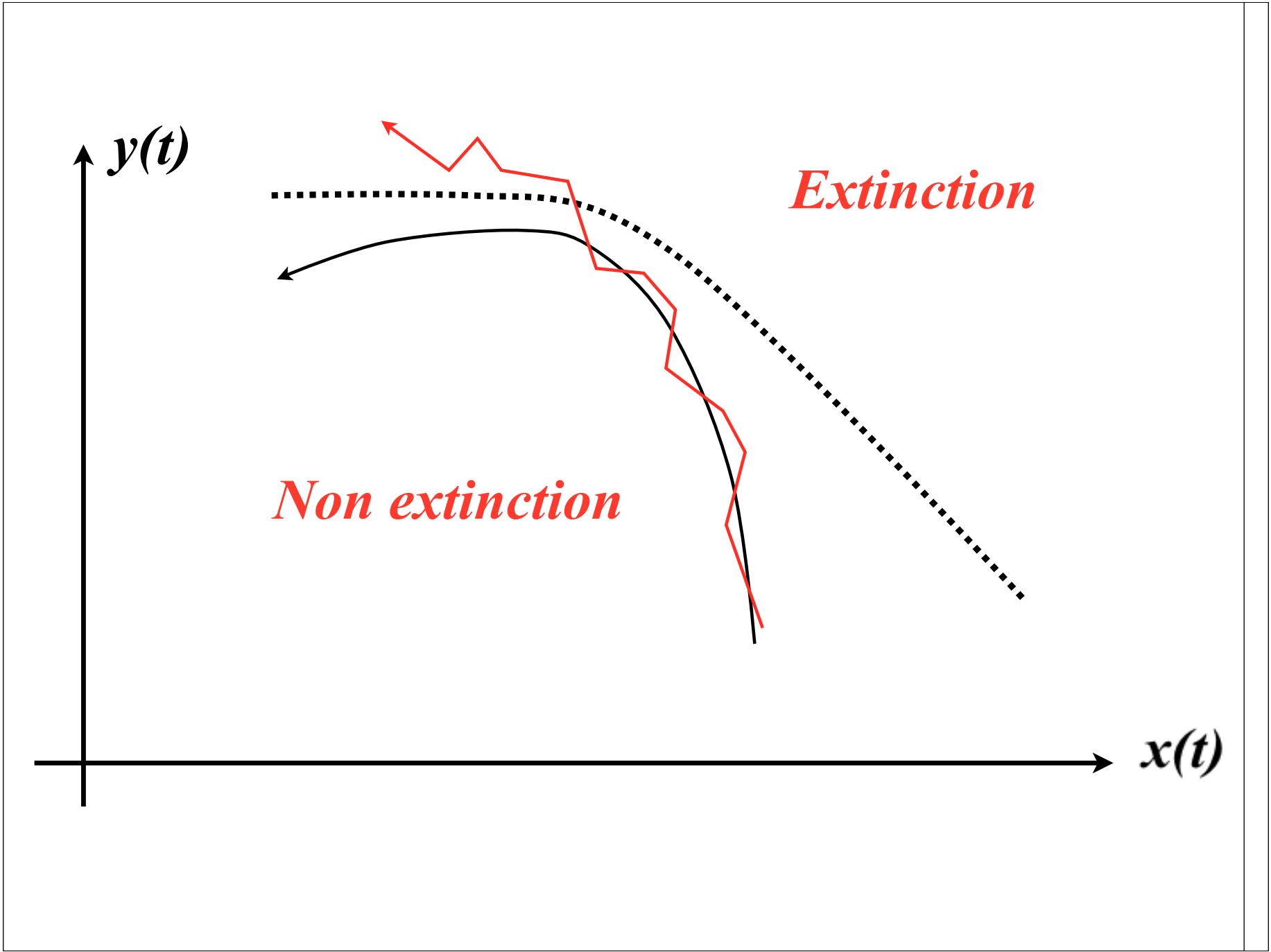


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$\rho(\varepsilon, k)$	0.001208	0.000114	0.000054	0.000052

FIG. 9 – The “safety funnel”

$$x(t+dt) - x(t) \approx dt \frac{1}{\varepsilon} [f(x(t)) - \mu(x(t))y(t)] - \sqrt{dt \frac{4}{\omega\varepsilon} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

$$y(t+dt) - y(t) \approx dt [\mu(x(t) - m)y(t) + \sqrt{dt \frac{\varepsilon}{\omega} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t]$$



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Graphical Representation and Stability Conditions of Predator-Prey Interactions

M. L. Rosenzweig; R. H. MacArthur

American Naturalist, Vol. 97, No. 895 (Jul. - Aug., 1963), 209-223.

Stable URL:

<http://links.jstor.org/sici?&sici=0003-0147%28196307%2F08%2997%3A895%3C209%3AGRASCO%3E2.0.CO%3B2-L>

Now we connect all the points at which the prey population just maintained itself, and call the resulting line the prey isocline $\left(\frac{d \text{ Prey}}{dt} = 0 \text{ for all points on the line} \right)$. Let us also connect the points at which the predator population just maintained itself and call this the predator isocline. If the lines intersect, neither population will be changing at the point of intersection. Such points, if any exist, are called the equilibrium points of the interaction.

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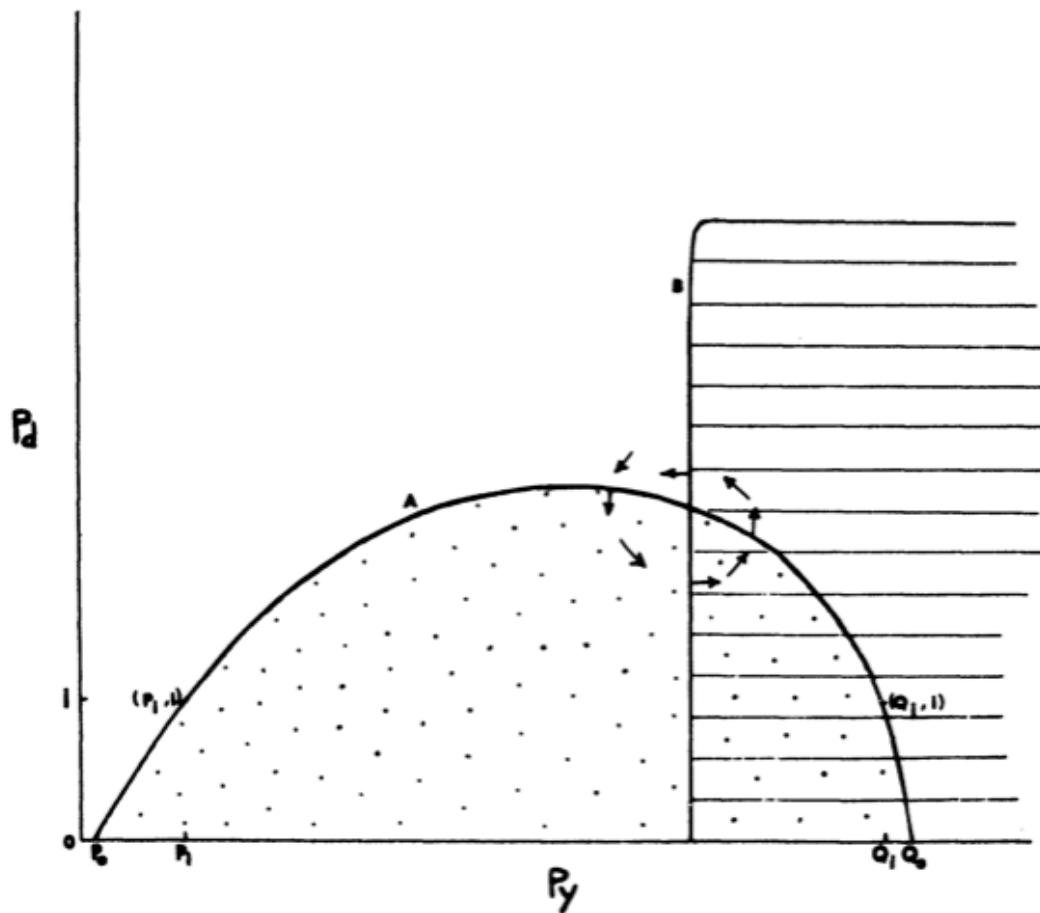


FIGURE 1. The instantaneous model of the interaction of a food-limited predator and its prey. P_y = prey density; P_d = predator density. Line A is the prey isocline, that is, the set of all points for which $\frac{dP_y}{dt} = 0$; line B is the predator isocline. P_y increases in the dotted area only; P_d increases only in the shaded area. The vectors are the instantaneous (general) direction of change of the community at eight qualitatively-different points in the graph.

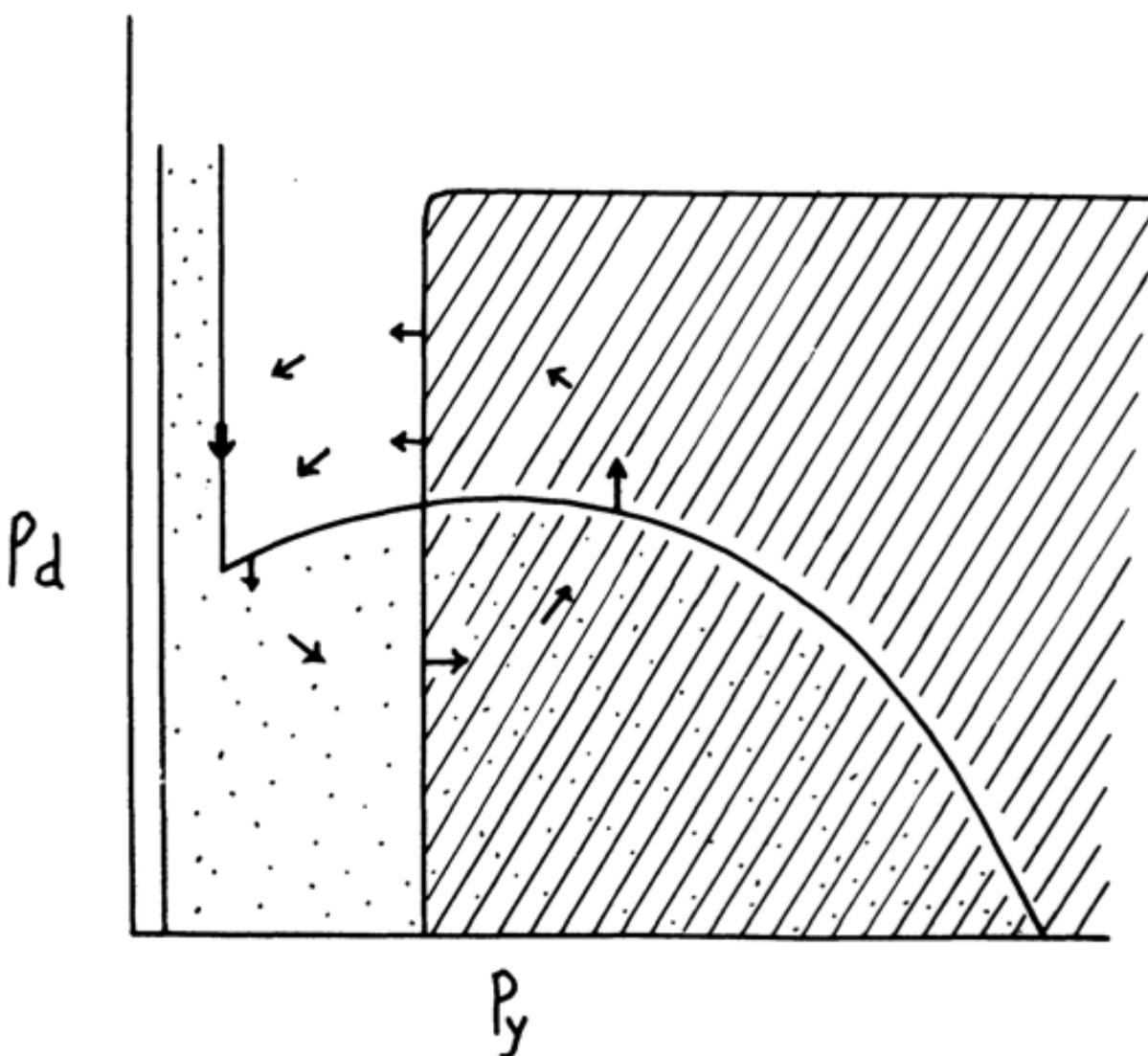


FIGURE 5. The interaction of a predator and its prey where the predator is unable to eat the prey at every prey density. Isoclines, coordinates, and markings as in figure 1. In this case, growing oscillations do not necessarily grow until one or both species becomes extinct, for a limiting oscillation may be reached.

Bibliographie

The predator isocline, $\frac{dP_d}{dt} = 0$, is simpler to deduce. The predator is depending on the prey for its ability to increase. When the prey fall below a certain level, the predators decrease; when they are greater than this level, the predators increase. Since we are dealing with an instantaneous model, the predators will be increasing if and only if they are eating prey at the required rate (Andrewartha and Browning, 1961). Attainment of this rate depends only on the prey density. It is true that the greater the number of predators, the faster the density is reduced; still the instantaneous rate of change of the predator population depends only on the instantaneous rate of kill, which depends on the instantaneous density of prey. This re-

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Manque : *lapillus*,
Balanus

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- Volterra

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- Kolmogorov

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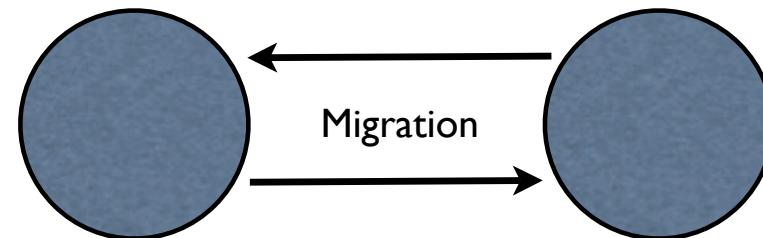
Migrations in the Rosenzweig-MacArthur model and the “atto-fox” problem

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ARIMA



$$\left\{ \begin{array}{lcl} \frac{dx_1}{dt} & = & \frac{1}{\varepsilon} (x_1 g(x_1) - h(x_1)y_1) + \nu_x(x_2, y_2) - \nu_x(x_1, y_1) \\ \frac{dy_1}{dt} & = & (h(x_1) - m)y_1 + \nu_y(x_2, y_2) - \nu_y(x_1, y_1) \\ \frac{dx_2}{dt} & = & \frac{1}{\varepsilon} (x_2 g(x_2) - h(x_2)y_2) + \nu_x(x_1, y_1) - \nu_x(x_2, y_2) \\ \frac{dy_2}{dt} & = & (h(x_2) - m)y_2 + \nu_y(x_1, y_1) - \nu_y(x_2, y_2) \end{array} \right. \quad (10)$$